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Research Article

A New Solution of Time-Fractional Coupled KdV Equation by Using Natural Decomposition Method

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In this article, we applied a new technique for solving the time-fractional coupled Korteweg-de Vries (KdV) equation. This method is a combination of the natural transform method with the Adomian decomposition method called the natural decomposition method (NDM). The solutions have been made in a convergent series form. To demonstrate the performances of the technique, two examples are provided.

1. Introduction

In the latest years, the branch fractional calculus [1–5] has played a significant role in applied mathematics; this is evident when it is used in phenomena of physical science and engineering, which are described by fractional differential equations. Fractional partial differential equations (FPDEs) have lately been studied and solved in several ways [6, 7]. Many transforms coupled with other techniques were used to solve differential equations [8–10]. The coupled natural transform [11–14] and Adomian decomposition method [15–17] called the natural decomposition method (NDM) is introduced in [18, 19] to solve differential equations, and it presents the approximate solution in the series form. The natural decomposition method has been used by many researchers to find approximate analytical solutions; it has shown reliable and closely converged results of the solution. In [20], Eltayeb et al. used the NDM to take out an analytical solution of fractional telegraph equation. Khan et al. in [21] obtained the solution of fractional heat and wave equations by NDM. Rawashdeh and Al-Jammal [22] gave the solution of fractional ODEs using the NDM, and in [23], Shah et al. obtained the solution of fractional partial differential equations with proportional delay by using the NDM. Many analytical and numerical methods were used to solve the fractional coupled KdV equation, such as spectral collection method [24], HPM [25], DTM [26], VIM [27], and meshless spectral method [28]. In this paper, we provided the application of the NDM to find the approximate solutions of nonlinear time-fractional coupled KdV equations given by

$$\begin{aligned}\frac{\partial^{\alpha}\phi(x,t)}{\partial t^{\alpha}} &= \eta\frac{\partial^{3}\phi(x,t)}{\partial x^{3}} + \gamma\phi(x,t)\frac{\partial\phi(x,t)}{\partial x} + \mu\psi(x,t)\frac{\partial\psi(x,t)}{\partial x},\\ \frac{\partial^{\beta}\psi(x,t)}{\partial t^{\beta}} &= \lambda\frac{\partial^{3}\psi(x,t)}{\partial x^{3}} - \nu\phi(x,t)\frac{\partial\psi(x,t)}{\partial x},\end{aligned} \tag{1}$$

subject to initial conditions

$$\phi(x,0) = f_1(x), \quad \psi(x,0) = f_2(x), \tag{2}$$

where $\eta, \lambda < 0$; γ,μ, and ν are constant parameters.

The KdV equation arose in many physical phenomena and application in the study of shallow-water waves, and it has been studied by many researchers. This work is organized as follows. In Section 2, we give definitions and properties of the natural transform. In Section 3, the NDM is made.

Section 4 discusses the new technique and compares it with two different techniques by two examples and presents tables and graphs to offer the validation of the NDM. Discussion and conclusion are included.

2. Natural Transform

Definition 1. (see [8–11]). The natural transform of the function $y(t)$ is defined by

$$\mathbb{N}^+[y(t)] = G(s,u) = \int_0^\infty y(ut)e^{-st}dt, \quad u,s>0, \tag{3}$$

where u and s are the transform variables.

Definition 2. The inverse natural transform of $G(s,u)$ is defined by

$$\mathbb{N}^-[G(s,u)] = y(t) = \int_{\epsilon-i\infty}^{\epsilon-i\infty} G(s,u)\,e^{st}ds, \quad \epsilon\in\mathbb{R}. \tag{4}$$

Now, we introduce some properties of the natural transform given as follows.

Property 3.

$$\mathbb{N}^+[1] = \frac{1}{s}. \tag{5}$$

Property 4.

$$\mathbb{N}^+[t^\propto] = \frac{u^\alpha\,\Gamma[\propto+1]}{s^{\alpha+1}}, \quad \propto>-1. \tag{6}$$

Theorem 5. (see [8–10]). *If $n\in\mathbb{N}$, where $n-1\le\alpha<n$ and $G(s,u)$ is the natural transform of the function $y(t)$, then the natural transform of the Caputo fractional derivative $\partial^\propto y(x,t)/\partial t^\propto$ is given by*

$$\mathbb{N}^+\left[\frac{\partial^\propto y(x,t)}{\partial t^\propto}\right] = \frac{s^\alpha}{u^\alpha}G(s,u) - \sum_{k=0}^{n-1}\frac{s^{\propto-(k+1)}}{u^{\propto-k}}\left[\frac{\partial^\propto y(x,t)}{\partial t^\propto}\right]_{t=0}. \tag{7}$$

3. Analysis of Method

We explain the algorithm of the NDM by considering the fractional coupled KdV equations (1) and (2).

Applying natural transform to equation (1), we have

$$\begin{aligned}
&\frac{s^\alpha}{u^\alpha}\Phi(x,s,u) - \frac{s^{\alpha-1}}{u^\alpha}\phi(x,0)\\
&\quad = \mathbb{N}^+\left[\eta\frac{\partial^3\phi(x,t)}{\partial x^3} + \gamma\phi(x,t)\frac{\partial\phi(x,t)}{\partial x}\right.\\
&\qquad \left. + \mu\psi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right],\\
&\frac{s^\alpha}{u^\alpha}\Theta(x,s,u) - \frac{s^{\alpha-1}}{u^\alpha}\psi(x,0)\\
&\quad = \mathbb{N}^+\left[\lambda\frac{\partial^3\psi(x,t)}{\partial x^3} - \nu\phi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right].
\end{aligned} \tag{8}$$

Substituting the initial conditions of equation (2) into equation (8), we get

$$\begin{aligned}
\Phi(x,s,u) &= \frac{f_1(x)}{s} + \frac{u^\alpha}{s^\alpha}\,\mathbb{N}^+\left[\eta\frac{\partial^3\phi(x,t)}{\partial x^3} + \gamma\phi(x,t)\frac{\partial\phi(x,t)}{\partial x}\right.\\
&\qquad \left. + \mu\psi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right],\\
\Theta(x,s,u) &= \frac{f_2(x)}{s} + \frac{u^\alpha}{s^\alpha}\,\mathbb{N}^+\left[\lambda\frac{\partial^3\psi(x,t)}{\partial x^3} - \nu\phi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right].
\end{aligned} \tag{9}$$

Operating the inverse natural transform of equation (9), we obtain

$$\begin{aligned}
\phi(x,t) &= f_1(x) + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\,\mathbb{N}^+\left[\eta\frac{\partial^3\phi(x,t)}{\partial x^3} + \gamma\phi(x,t)\frac{\partial\phi(x,t)}{\partial x}\right.\right.\\
&\qquad \left.\left. + \mu\psi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right]\right],\\
\psi(x,t) &= f_2(x) + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\,\mathbb{N}^+\left[\lambda\frac{\partial^3\psi(x,t)}{\partial x^3}\right.\right.\\
&\qquad \left.\left. - \nu\phi(x,t)\frac{\partial\psi(x,t)}{\partial x}\right]\right].
\end{aligned} \tag{10}$$

The natural decomposition method represents the solution as infinite series

$$\begin{aligned}
\phi(x,t) &= \sum_{n=0}^\infty \phi_n(x,t),\\
\psi(x,t) &= \sum_{n=0}^\infty \psi_n(x,t),
\end{aligned} \tag{11}$$

and the nonlinear terms decomposed as

$$\phi(x,t)\frac{\partial\phi(x,t)}{\partial x} = \sum_{n=0}^{\infty} A_n,$$
$$\psi(x,t)\frac{\partial\psi(x,t)}{\partial x} = \sum_{n=0}^{\infty} B_n, \tag{12}$$
$$\phi(x,t)\frac{\partial\psi(x,t)}{\partial x} = \sum_{n=0}^{\infty} C_n,$$

where A_n, B_n, and C_n are Adomian polynomial, which can be calculated by

$$\frac{1}{n!}\frac{d^n}{d\beta^n}\left[N\sum_{i=0}^{n}\beta^i U_i\right]_{\beta=0}. \tag{13}$$

Substituting equations (11) and (12) into equation (10) yields

$$\sum_{n=0}^{\infty}\phi_n(x,t) = f_1(x) + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[\eta\sum_{n=0}^{\infty}\frac{\partial^3\phi_n(x,t)}{\partial x^3} + \gamma\sum_{n=0}^{\infty}A_n + \mu\sum_{n=0}^{\infty}B_n\right]\right],$$
$$\sum_{n=0}^{\infty}\psi_n(x,t) = f_2(x) + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[\lambda\sum_{n=0}^{\infty}\frac{\partial^3\psi_n(x,t)}{\partial x^3} - \nu\sum_{n=0}^{\infty}C_n\right]\right]. \tag{14}$$

We get the general recursive formula

$$\phi_0(x,t) = f_1(x),$$
$$\phi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[\eta\frac{\partial^3\phi_n(x,t)}{\partial x^3} + \gamma A_n + \mu B_n\right]\right], \quad n\geq 0, \tag{15}$$

$$\psi_0(x,t) = f_2(x),$$
$$\psi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[\lambda\frac{\partial^3\psi_n(x,t)}{\partial x^3} - \nu C_n\right]\right], \quad n\geq 0. \tag{16}$$

Finally, the approximate solutions can be written as

$$\phi(x,t) = \sum_{n=0}^{\infty}\phi_n(x,t),$$
$$\psi(x,t) = \sum_{n=0}^{\infty}\psi_n(x,t). \tag{17}$$

4. Applications

Now, we explain the appropriateness of the technique by the following examples.

Example 1. Consider the fractional coupled KdV equation (1) with $\eta = -a$, $\gamma = -6a$, $\mu = 2b$, $\lambda = -r$, and $\nu = 3r$ subject to

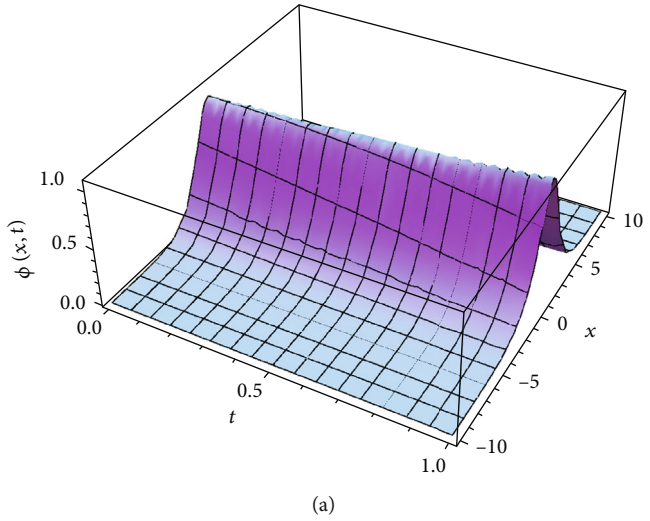
$$\phi(x,0) = \frac{\zeta}{a}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2, \quad \psi(x,0) = \frac{\zeta}{\sqrt{2a}}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2. \tag{18}$$

Solution 1. Applying the method formulated in Section 3 leads to the following:

$$\sum_{n=0}^{\infty}\phi_n(x,t) = \frac{\zeta}{a}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2 + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-a\sum_{n=0}^{\infty}\frac{\partial^3\phi_n(x,t)}{\partial x^3} - 6a\sum_{n=0}^{\infty}A_n + 2b\sum_{n=0}^{\infty}B_n\right]\right],$$
$$\sum_{n=0}^{\infty}\psi_n(x,t) = \frac{\zeta}{\sqrt{2a}}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2 + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-r\sum_{n=0}^{\infty}\frac{\partial^3\psi_n(x,t)}{\partial x^3} - 3r\sum_{n=0}^{\infty}C_n\right]\right]. \tag{19}$$

We define the following recursive formulas:

$$\varphi_0(x,t) = \frac{\zeta}{a}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2,$$
$$\psi_0(x,t) = \frac{\zeta}{\sqrt{2a}}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2,$$
$$\phi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-a\frac{\partial^3\phi_n(x,t)}{\partial x^3} - 6aA_n + 2bB_n\right]\right], \quad n\geq 0,$$
$$\psi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-b\frac{\partial^3\psi_n(x,t)}{\partial x^3} - 3bC_n\right]\right], \quad n\geq 0. \tag{20}$$



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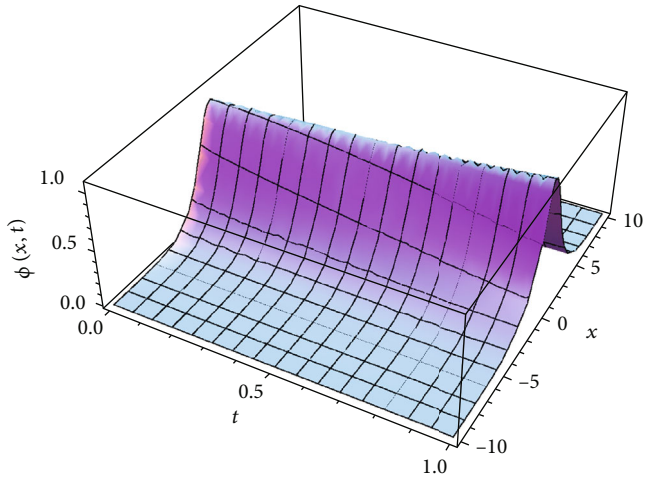

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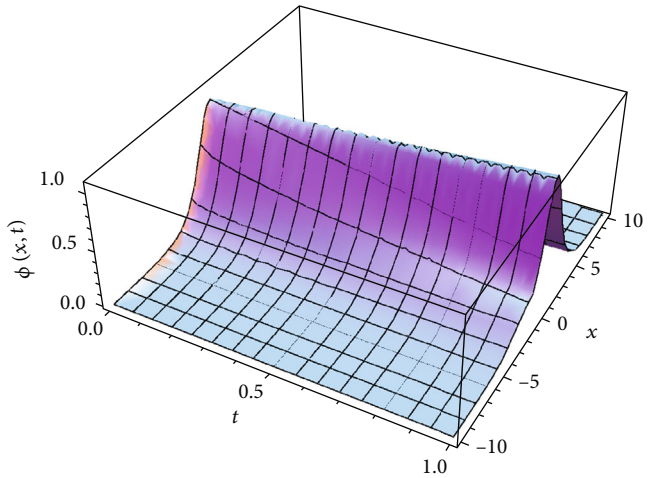
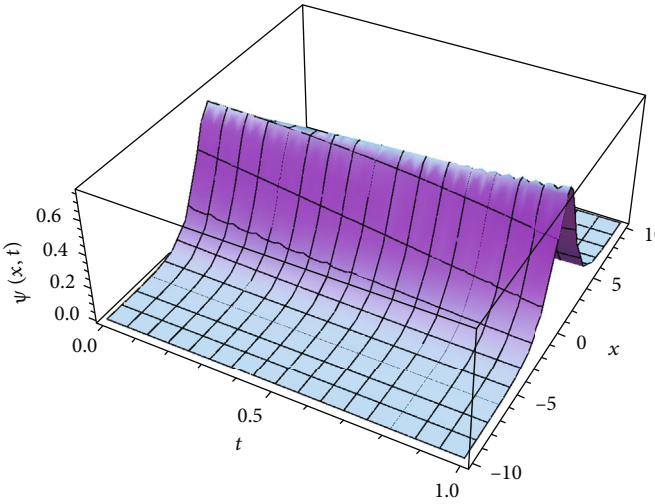


(c)

Figure 1: The NDM solutions $\phi(x,t)$ for Example 1 when $a=1$, $b=3$, $\zeta=1$, and $r=1$: (a) $\alpha=1$, $\beta=1$; (b) $\alpha=0.75$, $\beta=0.75$; (c) $\alpha=0.5$, $\beta=0.5$.



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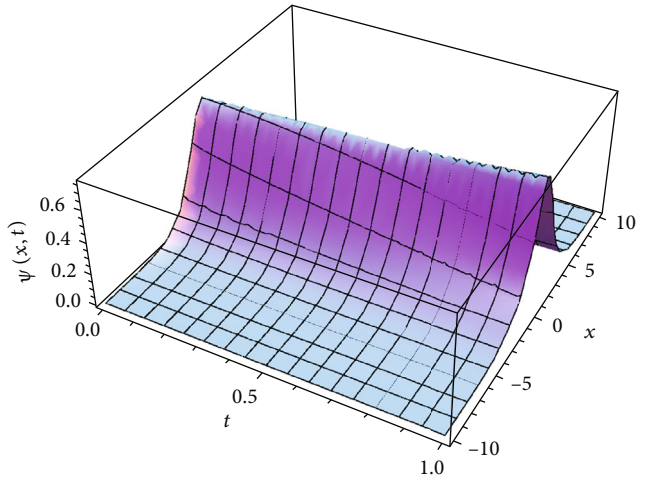

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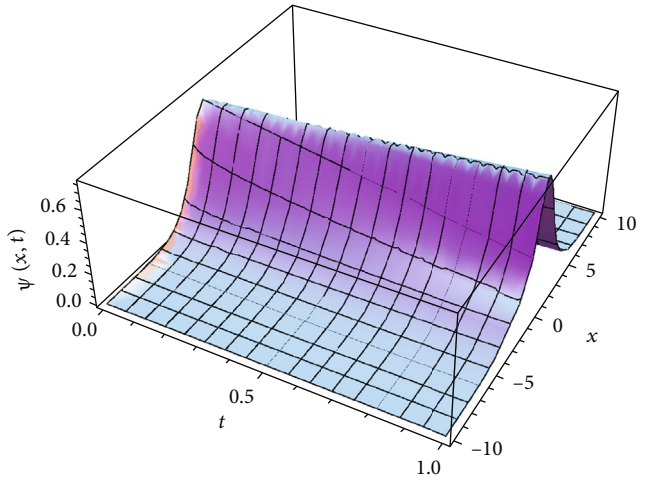


(c)

Figure 2: The NDM solutions $\psi(x,t)$ for Example 1 when $a=1$, $b=3$, $\zeta=1$, and $r=1$: (a) $\alpha=1$, $\beta=1$; (b) $\alpha=0.75$, $\beta=0.75$; (c) $\alpha=0.5$, $\beta=0.5$.

Table 1: Numerical outcomes of the exact and approximate solutions and the results obtained in [27].

x	t	ϕ_{Exact}	$\phi_{\text{approximate}}$	[24]	Error of [24]	Error approximate
-10	0.1	0.000164305	0.000164334	0.000164384	2.99039×10^{-8}	2.95039×10^{-8}
-10	0.2	0.00014867	0.000148901	0.000148991	2.33335×10^{-7}	2.30335×10^{-7}
-5	0.1	0.0240923	0.0240963	0.02409673	3.96592×10^{-6}	3.93592×10^{-6}
-5	0.2	0.0218248	0.0218556	0.02185586	0.0000338049	0.0000308049
5	0.1	0.0293476	0.0293435	0.02934364	3.97592×10^{-6}	0.000202966
5	0.2	0.0323838	0.0323501	0.0323459	0.0000378049	0.000515586
10	0.1	0.000200679	0.000200648	0.000200649	2.96039×10^{-8}	2.11254×10^{-8}
10	0.2	0.000221782	0.000221527	0.000221544	2.37335×10^{-7}	2.28173×10^{-7}
x	t	ψ_{Exact}	$\psi_{\text{approximate}}$	[24]	Error of [24]	Error approximate
-10	0.1	0.000116181	0.000116202	0.000116232	2.18624×10^{-8}	2.08624×10^{-8}
-10	0.2	0.000105126	0.000105289	0.000105259	1.64872×10^{-7}	1.62872×10^{-7}
-5	0.1	0.0170358	0.0170386	0.0170387	2.88312×10^{-6}	2.78312×10^{-6}
-5	0.2	0.0154325	0.0154542	0.0154552	0.0000287824	0.0000217824
5	0.1	0.0207519	0.020749	0.0207509	2.98312×10^{-6}	2.9094×10^{-6}
5	0.2	0.0228988	0.022875	0.0228740	0.0000247824	0.0000238036
10	0.1	0.000141901	0.000141879	0.000141880	2.09624×10^{-8}	2.19313×10^{-8}
10	0.2	0.000156823	0.000156643	0.000156650	1.72872×10^{-7}	1.79991×10^{-7}

This gives

$$\phi_0(x,t) = \frac{\zeta}{a}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2,$$

$$\phi_1(x,t) = \left(\frac{1}{2}\zeta\left(\frac{\zeta}{a}\right)^{3/2}\left(7-2b+\cosh\left(\sqrt{\frac{\zeta}{a}}x\right)\right)\cdot \operatorname{sech}^4\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\tanh\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)\frac{t^\alpha}{\Gamma[\alpha+1]},$$

$$\phi_2(x,t) = \frac{t^\alpha\zeta^4}{32a^3\Gamma[2\alpha+1]\Gamma[\alpha+\beta+1]}\left(8bt^\beta r\left(2+\cosh\left(\sqrt{\frac{\zeta}{a}}x\right)-\cosh\left(2\sqrt{\frac{\zeta}{a}}x\right)\right)\Gamma[2\alpha+1]+at^\alpha\left(40-32b+(-345+104b)\cosh\left(\sqrt{\frac{\zeta}{a}}x\right)-8(-15+4b)\cosh\left(2\sqrt{\frac{\zeta}{a}}x\right)+\cosh\cdot\left(3\sqrt{\frac{\zeta}{a}}x\right)\right)\Gamma[\alpha+\beta+1]\right)\operatorname{sech}^8\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right),$$

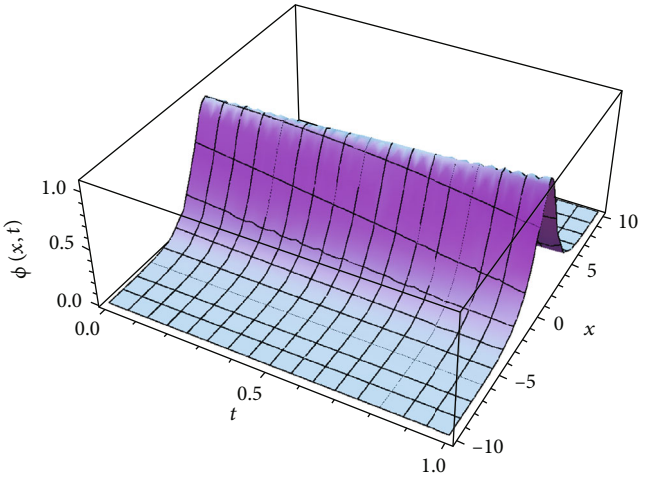
$$\cdots,$$

$$\psi_0(x,t) = \frac{\zeta}{\sqrt{2a}}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)^2,$$

$$\psi_1(x,t) = \left(4\sqrt{2}\sqrt{a}\,r\left(\frac{\zeta}{a}\right)^{5/2}\operatorname{csch}^3\left(\sqrt{\frac{\zeta}{a}}x\right)\cdot \sinh^4\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)\frac{t^\beta}{\Gamma[\beta+1]},$$

$$\psi_2(x,t) = \frac{-1}{8\sqrt{2}a^{7/2}\Gamma[2\beta+1]\Gamma[\alpha+\beta+1]}\cdot\left(r\zeta^4t^\beta\operatorname{sech}^6\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\cdot\left(-rt^\beta\left(9-14\cosh\left(\sqrt{\frac{\zeta}{a}}x\right)+\cosh\left(2\sqrt{\frac{\zeta}{a}}x\right)\right)\Gamma[\alpha+\beta+1]-12at^\alpha\left(7-2b+\cosh\left(\sqrt{\frac{\zeta}{a}}x\right)\right)\cdot\Gamma[2\beta+1]\tanh^2\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}x\right)\right)\right),$$

$$\cdots. \tag{21}$$



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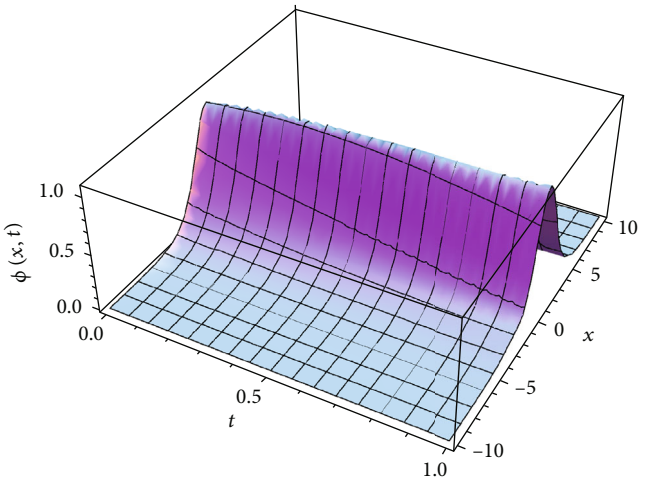

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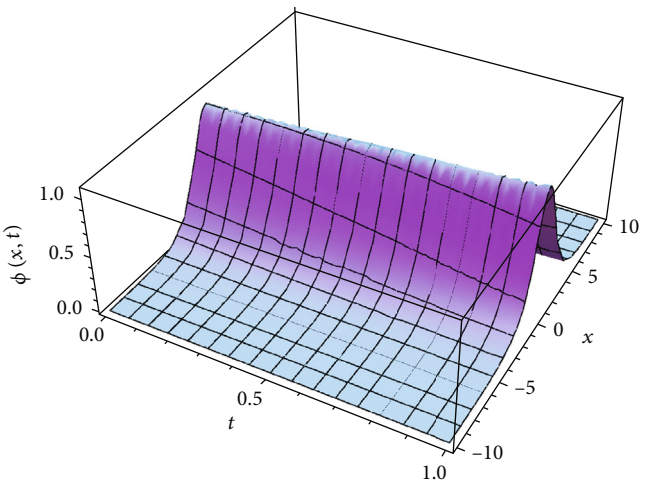
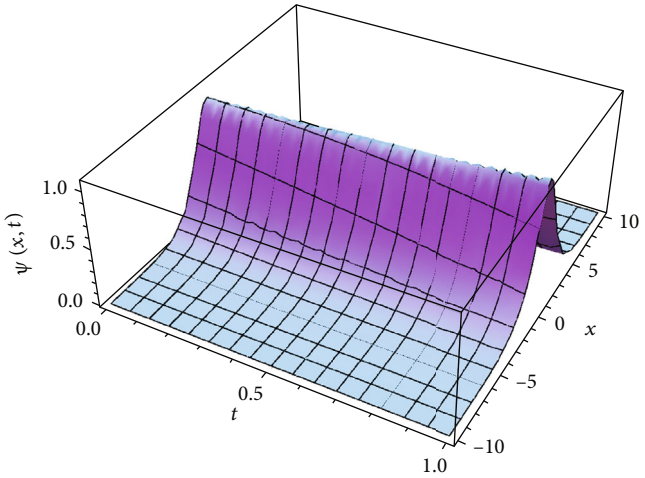


(c)

Figure 3: The NDM solutions $\phi(x, t)$ for Example 2 when $c = 1$: (a) $\alpha = 1, \beta = 1$; (b) $\alpha = 0.75, \beta = 0.5$; (c) $\alpha = 1, \beta = 0.90$.



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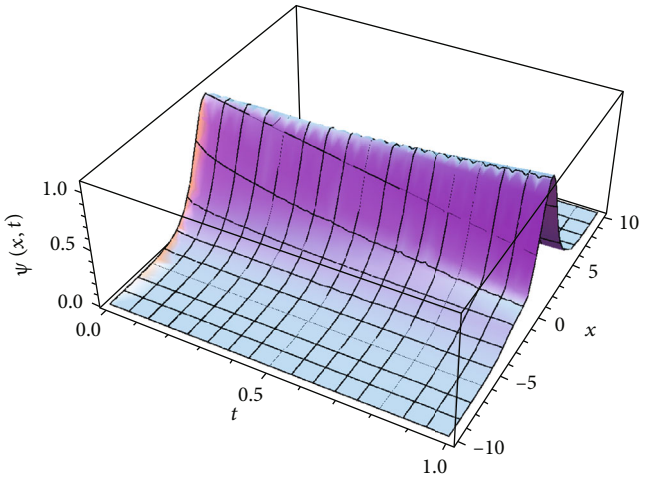

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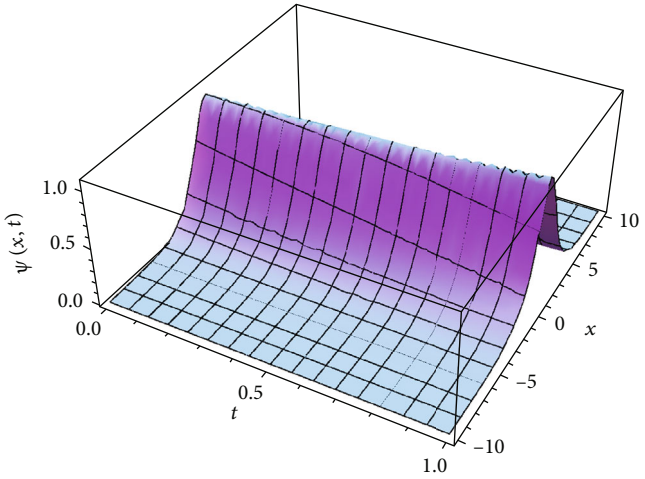

(c)

Figure 4: The NDM solutions $\psi(x, t)$ for Example 2 when $c = 1$: (a) $\alpha = 1, \beta = 1$; (b) $\alpha = 0.75, \beta = 0.5$; (c) $\alpha = 1, \beta = 0.90$.

Table 2: Numerical outcomes of the exact and approximate solutions and the results obtained in [26].

x	t	ϕ_{Exact}	$\phi_{\text{approximate}}$	[23]	Error of [23]	Error approximate
-10	0.1	0.000164305	0.000164334	0.000164334	2.95039×10^{-8}	2.95039×10^{-8}
-10	0.2	0.00014867	0.000148901	0.000148901	2.30335×10^{-7}	2.30335×10^{-7}
-5	0.1	0.0240923	0.0240963	0.0240963	3.93592×10^{-6}	3.93592×10^{-6}
-5	0.2	0.0218248	0.0218556	0.0218556	3.08049×10^{-5}	3.08049×10^{-5}
5	0.1	0.0293576	0.0293435	0.0293435	4.11452×10^{-6}	4.11452×10^{-6}
5	0.2	0.0323838	0.0323501	0.0323501	3.36633×10^{-5}	3.36633×10^{-5}
10	0.1	0.000200679	0.000200648	0.000200648	3.10155×10^{-8}	3.10155×10^{-8}
10	0.2	0.000221782	0.00221527	0.00221527	2.54546×10^{-7}	2.54546×10^{-7}
x	t	ψ_{Exact}	$\psi_{\text{approximate}}$	[23]	Error of [23]	Error approximate
-10	0.1	0.000164305	0.000164334	0.000164334	2.95039×10^{-8}	2.95039×10^{-8}
-10	0.2	0.00014867	0.000148901	0.000148901	2.30335×10^{-7}	2.30335×10^{-7}
-5	0.1	0.0240923	0.0240963	0.0240963	3.93592×10^{-6}	3.93592×10^{-6}
-5	0.2	0.0218248	0.0218556	0.0218556	3.08049×10^{-5}	3.08049×10^{-5}
5	0.1	0.0293576	0.0293435	0.0293435	4.11452×10^{-6}	4.11452×10^{-6}
5	0.2	0.0323838	0.0323501	0.0323501	3.36633×10^{-5}	3.36633×10^{-5}
10	0.1	0.000200679	0.000200648	0.000200648	3.10155×10^{-8}	3.10155×10^{-8}
10	0.2	0.000221782	0.00221527	0.00221527	2.54546×10^{-7}	2.54546×10^{-7}

Therefore, the series solution can be written in the form

$$\phi(x,t) = \sum_{n=0}^{\infty} \phi_n(x,t), \tag{22}$$

$$\psi(x,t) = \sum_{n=0}^{\infty} \psi_n(x,t). \tag{23}$$

For $\alpha = \beta = 1$, $b = 3$, and $a = r$, the exact solution of example (1) is

$$\begin{aligned} \varphi(x,t) &= \frac{\zeta}{a}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}(x-\zeta t)\right)\right)^2, \\ \psi(x,t) &= \frac{\zeta}{\sqrt{2a}}\left(\operatorname{sech}\left(\frac{1}{2}\sqrt{\frac{\zeta}{a}}(x-\zeta t)\right)\right)^2. \end{aligned} \tag{24}$$

The approximate solutions (22) and (23) for the special cases are shown in Figures 1 and 2; the numerical results in Table 1 where $\alpha = \beta = 1$, $a = 1$, $b = 3$, $\zeta = 1$, and $r = 1$ show that the solutions obtained by the NDM are nearly identical with the exact solution. Also, it can be seen from Table 1 that the solutions obtained by the NDM are more accurate than those obtained in [27].

Example 2. Consider the fractional coupled KdV in the form equation (1) with $\eta = -1$, $\gamma = -6$, $\mu = 3$, $\lambda = -1$, and $\nu = 3$ subject to

$$\phi(x,0) = \frac{4c^2 e^{cx}}{(1+e^{cx})^2}, \quad \psi(x,0) = \frac{4c^2 e^{cx}}{(1+e^{cx})^2}. \tag{25}$$

Solution 2. Applying the method formulated in Section 3 leads to the following:

$$\begin{aligned} \sum_{n=0}^{\infty} \phi_n(x,t) &= \frac{4c^2 e^{cx}}{(1+e^{cx})^2} + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-\sum_{n=0}^{\infty}\frac{\partial^3 \phi_n(x,t)}{\partial x^3} - 6\sum_{n=0}^{\infty} A_n + 3\sum_{n=0}^{\infty} B_n\right]\right], \\ \sum_{n=0}^{\infty} \psi_n(x,t) &= \frac{4c^2 e^{cx}}{(1+e^{cx})^2} + \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-\sum_{n=0}^{\infty}\frac{\partial^3 \psi_n(x,t)}{\partial x^3} - 3\sum_{n=0}^{\infty} C_n\right]\right]. \end{aligned} \tag{26}$$

The component of the solution given by

$$\phi_0(x,t) = \frac{4c^2e^{cx}}{(1+e^{cx})^2},$$
$$\psi_0(x,t) = \frac{4c^2e^{cx}}{(1+e^{cx})^2},$$
$$\phi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-\frac{\partial^3\phi_n(x,t)}{\partial x^3} - 6A_n + 3B_n\right]\right], \quad n \geq 0,$$
$$\psi_{n+1}(x,t) = \mathbb{N}^-\left[\frac{u^\alpha}{s^\alpha}\mathbb{N}^+\left[-\frac{\partial^3\psi_n(x,t)}{\partial x^3} - 3C_n\right]\right], \quad n \geq 0. \quad (27)$$

This gives

$$\phi_0(x,t) = \frac{4c^2e^{cx}}{(1+e^{cx})^2},$$
$$\phi_1(x,t) = \left(\frac{4c^5e^{cx}(-1+e^{cx})}{(1+e^{cx})^3}\right)\frac{t^\alpha}{\Gamma[\alpha+1]},$$
$$\phi_2(x,t) = \frac{1}{(1+e^{cx})^6\Gamma[2\alpha+1]\Gamma[\alpha+\beta+1]} \cdot \left(4c^8e^{cx}t^\alpha\left(-24e^{cx}\left(1-3e^{cx}+e^{2cx}\right)t^\beta\Gamma[2\alpha+1] + \left(1+22e^{cx}-78e^{2cx}+22e^{3cx} + e^{4cx}\right)t^\alpha\Gamma[\alpha+\beta+1]\right)\right),$$
$$\cdots,$$
$$\psi_0(x,t) = \frac{4c^2e^{cx}}{(1+e^{cx})^2},$$
$$\psi_1(x,t) = \left(\frac{4c^5e^{cx}(-1+e^{cx})}{(1+e^{cx})^3}\right)\frac{t^\beta}{\Gamma[\beta+1]},$$
$$\psi_2(x,t) = \frac{1}{(1+e^{cx})^6\Gamma[2\beta+1]\Gamma[\alpha+\beta+1]} \cdot \left(4c^8e^{cx}t^\beta\left(\left(1-14e^{cx}+18e^{2cx}-14e^{3cx}+e^{4cx}\right) \cdot t^\beta\Gamma[\alpha+\beta+1] + 12e^{cx}(1+e^{cx})^2t^\alpha\Gamma[2\beta+1]\right)\right),$$
$$\cdots. \quad (28)$$

Therefore, the series solution can be written in the form

$$\phi(x,t) = \sum_{n=0}^{\infty}\phi_n(x,t), \quad (29)$$
$$\psi(x,t) = \sum_{n=0}^{\infty}\psi_n(x,t). \quad (30)$$

For $\alpha = \beta = 1$, the exact solution is

$$\phi(x,t) = \psi(x,t) = \frac{4c^2e^{c\left(x-c^2t\right)}}{\left(1+e^{c(x-c^2t)}\right)^2}. \quad (31)$$

The approximate solutions (29) and (30) for the special cases are shown in Figures 3 and 4; the numerical results in Table 2 where $\alpha = \beta = c = 1$ show that the solutions obtained by the NDM are nearly identical with the exact solution. Also, it can be seen from Table 2 that the solutions obtained by the NDM are the same with those obtained in [26].

5. Conclusion

In this paper, the approximate solution of the time-fractional coupled KdV equation has been successfully done by using the NDM. We have tested the method on two examples, which revealed that the method is highly efficacious by comparing the approximate solutions with two different methods and exact solutions.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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