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Research Article

A New Efficient Method for Solving Two-Dimensional Nonlinear System of Burger's Differential Equations

Shams A. Ahmed,[1] **Mohamed Elbadri**,[1] **and Mohamed Z. Mohamed**[2]

[1]*Department of Mathematic, Faculty of Sciences and Arts, Jouf University, Tubarjal, Saudi Arabia*
[2]*Department of Mathematic, Deanship of Preparatory Programs, University of Hail, Hail, Saudi Arabia*

Correspondence should be addressed to Shams A. Ahmed; shamsalden20@hotmail.com

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In this work, the Sumudu decomposition method (SDM) is utilized to obtain the approximate solution of two-dimensional nonlinear system of Burger's differential equations. This method is considered to be an effective tool in solving many problems. Our results have shown that the SDM offers a much better approximation for solving several numbers of systems of two-dimensional nonlinear Burger's differential equations. To clarify the facility and accuracy of the strategy, two examples are provided.

1. Introduction

Burger's equation is one of the foremost necessary partial differential equations in fluid mechanics. This equation demonstrates the coupling between diffusion and convection processes. Burger's equation describes the structure of shock waves, traffic flow, and acoustic transmission. Additionally, like this, it also appears in varied areas of applied mathematics and physics, such as modelling of gas dynamics [1–5]. Recently, many numerical and analytical methods have been used to study the two-dimensional Burger's equation such as the differential transformation method [6], homotopy perturbation method [7], homotopy analysis method [8], variational iteration method [9], Adomian decomposition method [10–12], cubic B-spline differential quadrature method [13], finite difference method [14], finite element [15], and local discontinuous Galerkin finite element method [16] and also mathematicians have used transform methods coupled with analytical methods [17–30] to solve PDEs. The Sumudu decomposition method (SDM) is one of these methods, and it has been successfully used to solve intricate problems in engineering mathematics and applied science [31–35]. The SDM was first introduced by Kumar [36], to solve nonlinear partial differential equations that show in all aspects of applied science and engineering. This method is an elegant combination of the Sumudu transform method and the Adomian decomposition method. The SDM method generates the solution in a series form whose components are determined by a recursive relationship.

In the current study, we consider the system of two-dimensional nonlinear Burger's equations [9]:

$$\begin{aligned} \theta_t + \theta\theta_\rho + \alpha\theta_\sigma &= \frac{1}{\mathbf{R}}\left(\theta_{\rho\rho} + \theta_{\sigma\sigma}\right), \\ \alpha_t + \theta\alpha_\rho + \alpha\alpha_\sigma &= \frac{1}{\mathbf{R}}\left(\alpha_{\rho\rho} + \alpha_{\sigma\sigma}\right), \end{aligned} \tag{1}$$

with the initial conditions:

$$\begin{aligned} \theta(\rho, \sigma, 0) &= w(\rho, \sigma), \rho, \sigma \in E, \\ \alpha(\rho, \sigma, 0) &= h(\rho, \sigma), \rho, \sigma \in E, \end{aligned} \tag{2}$$

and the boundary conditions:

$$\begin{aligned}\theta(\rho,\sigma,t) &= w_1(\rho,\sigma,t), \rho,\sigma \in \partial E,\\ \alpha(\rho,\sigma,t) &= h_1(\rho,\sigma,t), \rho,\sigma \in \partial E,\end{aligned} \tag{3}$$

where $E = \{(\rho,\sigma) \mid a \le \rho \le b, a \le \sigma \le b\}$ and ∂E is its boundary, $\theta(\rho,\sigma,t)$ and $\alpha(\rho,\sigma,t)$ are the velocity components to be determined, w, h, w_1, and h_1 are the known functions, and R is the Reynolds number.

The major objective of this work is to get analytical and numerical solutions of the system of two-dimensional nonlinear Burger's equations (1) by using SDM. This work is organized as follows: the analysis of the method is given in Section 2. The application of SDM to two examples is given in Section 3. Concluding remarks are given in the last section.

2. Analysis of the Method

Now, to obtain the approximate solution of equation (1), apply the Sumudu transformation to equation (1) and using the given condition (2) gives

$$\begin{aligned}S[\theta(\rho,\sigma,u)] &= w(\rho,\sigma) - uS\left[\theta\theta_\rho + \alpha\theta_\sigma\right] + uS\left[\frac{1}{R}\left(\nabla^2\theta\right)\right],\\ S[\alpha(\rho,\sigma,u)] &= h(\rho,\sigma) - uS\left[\theta\alpha_\rho + \alpha\alpha_\sigma\right] + uS\left[\frac{1}{R}\left(\nabla^2\alpha\right)\right],\end{aligned} \tag{4}$$

where $\nabla^2 = (\partial^2/\partial\rho^2) + (\partial^2/\partial\sigma^2)$. Apply the inverse operator S^{-1} to both sides of the equation (4), and it gives

$$\begin{aligned}\theta(\rho,\sigma,t) &= w(\rho,\sigma) - S^{-1}\left[uS\left[\theta\theta_\rho + \alpha\theta_\sigma\right]\right] + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\theta\right)\right]\right],\\ \alpha(\rho,\sigma,t) &= h(\rho,\sigma) - S^{-1}\left[uS\left[\theta\alpha_\rho + \alpha\alpha_\sigma\right]\right] + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\alpha\right)\right]\right].\end{aligned} \tag{5}$$

The Adomian decomposition method suggests that the linear terms $\theta(\rho,\sigma,t)$ and $\alpha(\rho,\sigma,t)$ and the nonlinear terms $\theta\theta_\rho, \alpha\theta_\sigma, \theta\alpha_\rho$, and $\alpha\alpha_\sigma$ are decomposed by an infinite series of components:

$$\begin{aligned}\theta(\rho,\sigma,t) &= \sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t),\\ \theta\theta_\rho &= \sum_{n=0}^{\infty}A_n,\\ \alpha\theta_\sigma &= \sum_{n=0}^{\infty}B_n,\\ \alpha(\rho,\sigma,t) &= \sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t),\\ \theta\alpha_\rho &= \sum_{n=0}^{\infty}C_n,\\ \alpha\alpha_\sigma &= \sum_{n=0}^{\infty}C_n.\end{aligned} \tag{6}$$

For some Adomian polynomials, $A_n(\theta)$ are given by

$$A_n(\theta_0,\theta_1,\theta_2,\ldots,\theta_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^n\theta_n\right)\right]_{\lambda=0}, \quad n = 0,1,2,\ldots. \tag{7}$$

Substituting equation (6) into both sides of equation (5) leads to

$$\begin{aligned}\sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t) &= w(\rho,\sigma) - S^{-1}\left[uS\left[\left[\sum_{n=0}^{\infty}A_n\right] + \left[\sum_{n=0}^{\infty}B_n\right]\right]\right]\\ &\quad + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\left(\sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t)\right)\right)\right]\right],\\ \sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t) &= h(\rho,\sigma) - S^{-1}\left[uS\left[\left[\sum_{n=0}^{\infty}C_n\right] + \left[\sum_{n=0}^{\infty}D_n\right]\right]\right]\\ &\quad + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\left(\sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t)\right)\right)\right]\right].\end{aligned} \tag{8}$$

To construct the recursive relation needed for the determination of the components $(\theta_0,\theta_1,\theta_2,\ldots,\theta_n)$ and $(\alpha_0,\alpha_1,\alpha_2,\ldots,\alpha_n)$, it is important to note that the Adomian method suggests that the zeroth components θ_0 and α_0 are usually defined by the functions $w(\rho,\sigma)$ and $h(\rho,\sigma)$.

Accordingly, the formal recursive relation is defined in (Figures 1 and 2).

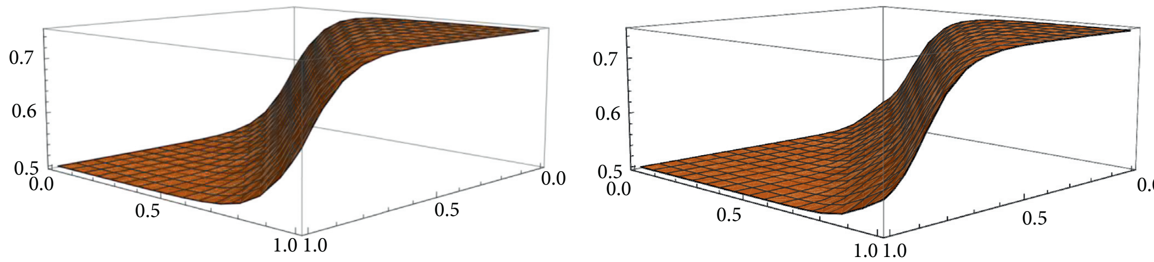
$$\begin{aligned}\theta_0(\rho,\sigma,t) &= w(\rho,\sigma),\\ \theta_{k+1}(\rho,\sigma,t) &= -S^{-1}\left[uS\left[A_k + B_k\right]\right]\\ &\quad + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2(\theta_k)\right)\right]\right], \quad k \ge 0,\\ \alpha_0(\rho,\sigma,t) &= h(\rho,\sigma),\\ \alpha_{k+1}(x,y,t) &= -S^{-1}\left[uS\left[C_k + D_k\right]\right]\\ &\quad + S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2(\alpha_k)\right)\right]\right], \quad k \ge 0.\end{aligned} \tag{9}$$

Having determined these components, substitute it into $\theta(\rho,\sigma,t) = \sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t)$ and $\alpha(\rho,\sigma,t) = \sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t)$ to obtain the solution in a series form.

3. Application

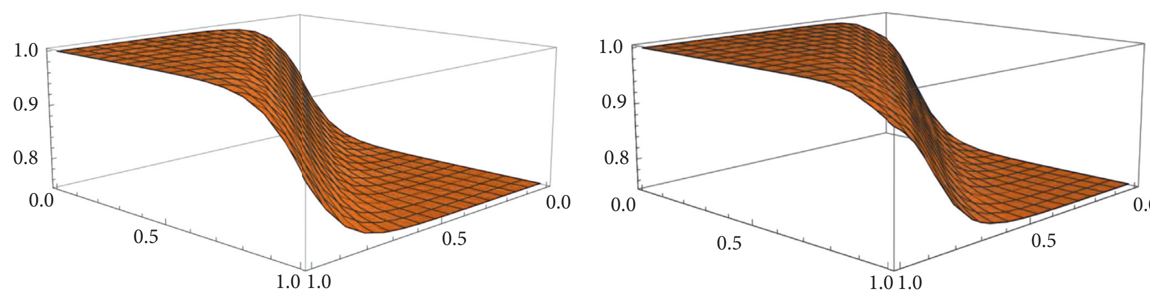
In this part, two examples are provided to illustrate the method.

Example 1. Consider the system of two-dimensional Burger's equation (1), with the following initial conditions [9]:



(a) (b)

Figure 1: Distributions of approximation solutions for $\theta(\rho,\sigma,t)$ at (a) $t=0.01$ and (b) $t=0.5$ with $R=100$, for Example 2.



(a) (b)

Figure 2: Distributions of approximation solutions for $\alpha(\rho,\sigma,t)$ at (a) $t=0.01$ and (b) $t=0.5$ with $R=100$, for Example 2.

$$\begin{aligned}\theta(\rho,\sigma,0)&=\rho+\sigma,\ \rho,\sigma\in E,\\ \alpha(\rho,\sigma,0)&=\rho-\sigma,\ \rho,\sigma\in E.\end{aligned}\tag{10}$$

Solution. Subsequent to the discussion presented above, the system of equation (8) becomes

$$\begin{aligned}\sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t)&=\rho+\sigma-S^{-1}\left[uS\left[\left[\sum_{n=0}^{\infty}A_n\right]+\left[\sum_{n=0}^{\infty}B_n\right]\right]\right]\\&\quad+S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\left(\sum_{n=0}^{\infty}\theta_n(\rho,\sigma,t)\right)\right)\right]\right],\\ \sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t)&=\rho-\sigma-S^{-1}\left[uS\left[\left[\sum_{n=0}^{\infty}C_n\right]+\left[\sum_{n=0}^{\infty}D_n\right]\right]\right]\\&\quad+S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2\left(\sum_{n=0}^{\infty}\alpha_n(\rho,\sigma,t)\right)\right)\right]\right].\end{aligned}\tag{11}$$

The recursive relation can be constructed from equation (11) given by

$$\begin{aligned}\theta_0(\rho,\sigma,t)&=\rho+\sigma,\\ \theta_{k+1}(\rho,\sigma,t)&=-S^{-1}\left[uS\left[A_k+B_k\right]\right]\\&\quad+S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2(\theta_k)\right)\right]\right],\quad k\geq 0,\\ \alpha_0(\rho,\sigma,t)&=\rho-\sigma,\\ \alpha_{k+1}(x,y,t)&=-S^{-1}\left[uS\left[C_k+D_k\right]\right]\\&\quad+S^{-1}\left[uS\left[\frac{1}{R}\left(\nabla^2(\alpha_k)\right)\right]\right],\quad k\geq 0.\end{aligned}\tag{12}$$

We get the next couple of components, and upon setting $R=1$, we have

$$\begin{aligned}(\theta_0,\alpha_0)&=(\rho+\sigma\ \ ,\rho-\sigma),\\ (\theta_1,\alpha_1)&=(-2\rho t,-2\sigma t),\\ (\theta_2,\alpha_2)&=\left(2\rho t^2+2\sigma t^2,2\rho t^2-2\sigma t^2\right),\\ (\theta_3,\alpha_3)&=\left(-4\rho t^3,-4\sigma t^3\right),\\ (\theta_4,\alpha_4)&=\left(4\rho t^4+4\sigma t^4,4\rho t^4-4\sigma t^4\right),\end{aligned}\tag{13}$$

TABLE 1: The (SDM) results for $\theta(\rho,\sigma,t)$ for first four approximations for $R=1$, with mesh points $\rho=0.1$ and $\sigma=0.1$, for Example 2.

t	Exact $\theta(\rho,\sigma,t)$	SDM $\theta(\rho,\sigma,t)$	$\lvert\theta_{\text{Exact}}-\theta_{\text{SDM}}\rvert$
0.05	0.6249023437698682	0.6249023437698682	0
0.1	0.6248046876589456	0.6248046876589457	$1.1102230E-16$
0.15	0.6247070317864406	0.6247070317864418	$1.2212453E-15$
0.2	0.6246093762715608	0.6246093762715658	$4.9960036E-15$
0.25	0.6245117212335117	0.6245117212335268	$1.5099033E-14$
0.3	0.6244140667914967	0.6244140667915343	$3.7636561E-14$
0.35	0.6243164130647162	0.6243164130647978	$8.1601392E-14$
0.4	0.6242187601723671	0.6242187601725259	$1.5876189E-13$
0.45	0.6241211082336422	0.6241211082339287	$2.8654856E-13$
0.5	0.6240234573677299	0.624023457368215	$4.8505644E-13$

TABLE 2: The (SDM) results for $\alpha(\rho,\sigma,t)$ for first four approximations for $R=1$, with mesh points $\rho=0.1$ and $\sigma=0.1$, for Example 2.

t	Exact $\alpha(\rho,\sigma,t)$	SDM $\alpha(\rho,\sigma,t)$	$\lvert\alpha_{\text{Exact}}-\alpha_{\text{SDM}}\rvert$
0.05	0.875097656230132	0.875097656230132	0
0.1	0.875195312341054	0.875195312341054	$1.1102230E-16$
0.15	0.875292968213559	0.875292968213558	$1.2212453E-15$
0.2	0.875390623728439	0.875390623728434	$4.9960036E-15$
0.25	0.875488278766488	0.875488278766473	$1.5099033E-14$
0.3	0.875585933208503	0.875585933208466	$3.7636561E-14$
0.35	0.875683586935284	0.875683586935202	$8.1601392E-14$
0.4	0.875781239827633	0.875781239827474	$1.5887291E-13$
0.45	0.875878891766358	0.875878891766071	$2.8654856E-13$
0.5	0.87597654263227	0.875976542631785	$4.8505644E-13$

TABLE 3: The (SDM) results for $\theta(\rho,\sigma,t)$, for first four approximations, and for $R=1$, with mesh points $\rho=0.3$ and $\sigma=0.1$, for Example 2.

t	Exact $\theta(\rho,\sigma,t)$	SDM $\theta(\rho,\sigma,t)$	$\lvert\theta_{\text{Exact}}-\theta_{\text{SDM}}\rvert$
0.05	0.6233399413556532	0.6233399413556536	$4.4408921E-16$
0.1	0.6232423033622646	0.6232423033622709	$6.3282712E-15$
0.15	0.6231446675140766	0.6231446675141092	$3.2529535E-14$
0.2	0.6230470339301989	0.6230470339303031	$1.0424994E-13$
0.25	0.6229494027297301	0.6229494027299877	$2.5757174E-13$
0.3	0.6228517740317571	0.6228517740322975	$5.4045657E-13$
0.35	0.6227541479553547	0.6227541479563675	$1.0127454E-12$
0.4	0.6226565246195848	0.6226565144551626	$1.7474910E-12$
0.45	0.622558904143496	0.6225588896714487	$2.8308467E-12$
0.5	0.6224612866461229	0.622461286650486	$4.3630655E-12$

and so on. Consequently, the solution in a series form is given by

$$(\theta,\alpha)=\begin{pmatrix}\rho\left(1+2t^2+4t^4+\cdots\right)-2\rho t\left(1+2t^2+\cdots\right)+\sigma\left(1+2t^2+4t^4+\cdots\right),\\ \rho\left(1+2t^2+4t^4+\cdots\right)-2\sigma t\left(1+2t^2+\cdots\right)-\sigma\left(1+2t^2+4t^4+\cdots\right)\end{pmatrix}, \tag{14}$$

and in a closed form it is

$$\left(\theta(\rho,\sigma,t),\alpha(\rho,\sigma,t)\right)=\left(\frac{\rho+\sigma-2\rho t}{1-2t^2},\frac{\rho-\sigma-2\sigma t}{1-2t^2}\right), \tag{15}$$

which is the exact solution of two-dimensional Burger's equations [9].

Example 2. Consider another system of Burger's equations (1), with the following initial conditions [9]:

TABLE 4: The (SDM) results for $\alpha(\rho,\sigma,t)$, for first four approximations, and for $R=1$, with mesh points $\rho=0.3$ and $\sigma=0.1$, for Example 2.

t	Exact $\alpha(\rho,\sigma,t)$	SDM $\alpha(\rho,\sigma,t)$	$\lvert\alpha_{\text{Exact}}-\alpha_{\text{SDM}}\rvert$
0.05	0.876660058644347	0.876660058644346	$4.4408921E-16$
0.1	0.876757696637735	0.876757696637729	$6.3282712E-15$
0.15	0.876855332485923	0.876855332485891	$3.2529535E-14$
0.2	0.876952966069801	0.876952966069697	$1.0424994E-13$
0.25	0.87705059727027	0.877050597270012	$2.5757174E-13$
0.3	0.877148225968243	0.877148225967703	$5.4045657E-13$
0.35	0.877245852044645	0.877245852043633	$1.0127454E-12$
0.4	0.877343475380415	0.877343485544838	$1.7474910E-12$
0.45	0.877441095856504	0.877441095853673	$2.8309577E-12$
0.5	0.877538713353877	0.877538713349514	$4.3630655E-12$

$$\theta(\rho,\sigma,0)=\frac{3}{4}-\frac{1}{4\left(1+e^{(R(\sigma-\rho)/8)}\right)},$$
$$\alpha(\rho,\sigma,0)=\frac{3}{4}+\frac{1}{4\left(1+e^{(R(\sigma-\rho)/8)}\right)},\tag{16}$$

with the exact solutions:

$$\theta(\rho,\sigma,t)=\frac{3}{4}-\frac{1}{4\left(1+e^{(R(4\sigma-4\rho-t)/32)}\right)},$$
$$\alpha(\rho,\sigma,t)=\frac{3}{4}+\frac{1}{4\left(1+e^{(R(4\sigma-4\rho-t)/32)}\right)}.\tag{17}$$

Solution. Using the previous aforesaid discussion, we get

$$\theta_0(\rho,\sigma,t)=\frac{3}{4}-\frac{1}{4\left(1+e^{(R(-\rho+\sigma)/8)}\right)},$$
$$\alpha_0(\rho,\sigma,t)=\frac{3}{4}+\frac{1}{4\left(1+e^{(R(-\rho+\sigma)/8)}\right)},$$
$$\theta_1(\rho,\sigma,t)=-\frac{e^{(1/8)R(-\rho+\sigma)}Rt}{64\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^3}-\frac{e^{(1/4)R(-\rho+\sigma)}Rt}{64\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^3}+\frac{e^{(1/8)R(-\rho+\sigma)}Rt}{128\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^2},$$
$$\alpha_1(\rho,\sigma,t)=\frac{e^{(1/8)R(-\rho+\sigma)}Rt}{64\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^3}+\frac{e^{(1/4)R(-\rho+\sigma)}Rt}{64\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^3}-\frac{e^{(1/8)R(-\rho+\sigma)}Rt}{128\left(1+e^{(1/8)R(-\rho+\sigma)}\right)^2},$$
$$\theta_2(\rho,\sigma,t)=-\frac{e^{(R\rho/8)+(R\sigma/8)}\left(-e^{(R\rho/8)}+e^{(R\sigma/8)}\right)R^2t^2}{8192\left(e^{(R\rho/8)}+e^{(R\sigma/8)}\right)^3},\tag{18}$$
$$\alpha_2(\rho,\sigma,t)=\frac{e^{(R\rho/8)+(R\sigma/8)}\left(-e^{(R\rho/8)}+e^{(R\sigma/8)}\right)R^2t^2}{8192\left(e^{(R\rho/8)}+e^{(R\sigma/8)}\right)^3},$$
$$\theta_3(\rho,\sigma,t)=-\frac{e^{(R\rho/8)+(R\sigma/8)}\left(e^{(R\rho/4)}+e^{(R\sigma/4)}-4e^{(R\rho/8)+(R\sigma/8)}\right)R^3t^3}{786432\left(e^{(R\rho/8)+(R\sigma/8)}\right)^4},$$
$$\alpha_3(\rho,\sigma,t)=\frac{e^{(R\rho/8)+(R\sigma/8)}\left(e^{(R\rho/4)}+e^{(R\rho/4)}-4e^{(R\rho/8)+(R\sigma/8)}\right)R^3t^3}{786432\left(e^{(R\rho/8)+(R\sigma/8)}\right)^4}.$$

Therefore, the solution $\theta(\rho,\sigma,t)$ and $\alpha(\rho,\sigma,t)$ in the series form is given by

$$\theta(\rho,\sigma,t)=\theta_0(\rho,\sigma,t)+\theta_1(\rho,\sigma,t)+\theta_2(\rho,\sigma,t)+\theta_3(\rho,\sigma,t),$$
$$\alpha(\rho,\sigma,t)=\alpha_0(\rho,\sigma,t)+\alpha_1(\rho,\sigma,t)+\alpha_2(\rho,\sigma,t)+\alpha_3(\rho,\sigma,t).\tag{19}$$

Numerical outcomes shown in Tables 1–4 illustrate that the accuracy of SDM agrees good with the exact solutions of the system of two-dimensional Burger's equation, and absolute errors are very small for the present choice of ρ, σ, R, and t.

4. Conclusion

In this paper, SDM had been successfully applied to find the solutions of the system of two-dimensional nonlinear Burger's equations. The numerical studies showed that SDM offers accurate results for two-dimensional nonlinear Burger's equations in comparison with another analytical methods. This fact is shown in the second example. Therefore, this method may be a favourable method to solve other nonlinear partial differential equations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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