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Research Article

Autocorrelation Analysis in Time and Frequency Domains for Passive Structural Diagnostics

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In this paper, modal frequency estimation by using autocorrelation functions in both the time and frequency domains for structural diagnostics is discussed. With popular structural health monitoring methods for periodic inspections such as with the "hammering test," hearing is very useful for distinguishing differences between structural conditions. Hearing detects pitch and tone, and it is known that the auditory process is related to wave periodicity calculated from autocorrelation functions. Consequently, on the basis of the hammering test, modal frequencies can be estimated by autocorrelation, the same as hearing. In this paper, modal frequencies were estimated by using autocorrelation for constant structural health monitoring under a nonstationary noise condition. First, fundamental modal frequencies were estimated by using the autocorrelation of the time domain which was inspired by pitch detection of hearing. Second, higher modal frequency compositions were also analyzed by using autocorrelation in the frequency domain as with tones discrimination. From the results by conducting scale-model experiments under unknown nonstationary noise conditions, periods of fundamental modal frequency were derived by using periods histogram of autocorrelation functions. In addition, higher modal frequency estimation under nonstationary noises was also discussed.

1. Introduction

Structural health monitoring is important for our safety, such as in environments where we are surrounded by many buildings and structures. In a previous study, cumulative spectral analysis with decay portions revealed structural spectral characteristics and their decay under a nonstationary noise condition [1]. However, decay portions do not appear frequently in passively observed signals. For structural health monitoring, analysis of damping and frequency changes is a popular way to monitor the health of a structure [2, 3], because if a structure degrades, a structure's spectral characteristics change due to various effects caused by the degrading. In the same way for classic structural diagnostics, the hammering test, which uses hearing, is a popular method for periodically checking construction conditions.

Auditory perception is a useful function for distinguishing acoustical changes as typified by pitch and tones. If acoustical sounds are replaced with structural vibrations, an analogy for the pitch would be the fundamental modal frequency in structural characteristics. The same as with pitch, tone can be replaced with the higher frequency composition in structural vibration. Fundamental modal frequencies contain abundant information related to the composition of frequency resonances. In the auditory model, pitch perception is explained by using autocorrelation functions of the time domain [4, 5]. Consequently, it is supposed that fundamental period estimation with autocorrelation functions can be used for structural diagnostics. After that, higher modal frequencies are also considered by using an autocorrelation function of the frequency domain.

For spectral characteristic analysis for structural diagnostics, the effects of external noise spectra have to be removed in order to extract the characteristics of a building itself. For this reason, external noise information is important to estimate a structure's spectral characteristics [6, 7]. Hirata proposed a short time-interval period (SIP) distribution for structural health monitoring that can estimate a structure's spectral characteristics under an unknown nonstationary noise condition [8, 9]. The SIP distribution is a method for

estimating the shape of a structure's frequency response by collecting a lot of the dominant frequency peaks of short intervals. For this reason, SIP distribution can remove a sound source's effect when analyzing spectral characteristics from unknown observed signals even in poor noise conditions. From the report, spectral peak selection is a useful way of removing sound source spectral characteristics even in calculating autocorrelation functions.

In this paper, period estimation corresponding to fundamental modal frequencies is calculated by using autocorrelation functions with power spectral peak selection, and the autocorrelation functions of the frequency domain also show the relationships between multiple higher modal frequency compositions.

In Section 2, a scale model of impulse responses and their conditions is described. In Section 3, method of fundamental modal frequency estimation for structural health monitoring is described. In Section 4, fundamental modal frequency estimation for simulated nonstationary noise is described. In Section 5, higher modal frequency estimation for simulated nonstationary noise is described.

2. Wooden Scale Model and Impulse Response Measurement for Simulation Experiments

A three-story wooden framework model was prepared in order to measure impulse responses for a structural diagnostic simulation. In this experiment, structural degrading was simulated by using a small number of pieces of lumber. Figure 1 illustrates the wooden scale model and the experimental condition. The wooden model was imitative of a three-story building (18 cm (W) 21 cm (D) 38 cm (H)). The scale model was propped up by four pillars, and each pillar was bound by four pieces of lumber 1 cm square each. Impulse responses were recorded at an Fs of 6 kHz by using a piezoelectric accelerometer (PV-90B, RION) with an impulsive hammer. The upper illustration of Figure 1 shows two prepared scale models for conditions 1 and 4. Structural degrading of the models was simulated by removing a piece of lumber and by linear interpolation of measured impulse responses for conditions 2 and 3. The bottom panel of Figure 1 shows the impulse responses of each condition.

Figure 2 shows power spectra and autocorrelation functions of each condition's impulse response. In the left panels of this figure, the impulse responses' frequency components above 250 Hz gradually become shallow from conditions 1 to 4 due to the effect of the degrading simulation. From these results, it can be considered that the difficulty of estimating structural spectral characteristics is higher due to the changes of resonant peaks as the simulation progresses from condition to condition. Additionally, the fuzziness of the spectral peaks had negative effects on period estimation with autocorrelation functions. On the other hand, lowest frequency components associated with fundamental modal frequency form sharp peak. From the peaks, fundamental modal frequencies are indicated as 45 Hz in condition 1, 42 Hz in condition 2, 41 Hz in condition 3, and 41 Hz in condition 4, respectively. The right panels of Figure 2 show autocorrelation functions of each impulse response. For the hearing model, autocorrelation functions are used for pitch detection simulation [4, 5]. Pitch can be represented as a fundamental modal frequency for structural vibration. At the right panels of Figure 2, peaks as periods of fundamental frequency should appear at over 22.222 ms in condition 1, 23.81 ms in condition 2, and 24.39 ms in conditions 3 and 4 on the basis of the fundamental modal frequencies; such peaks were not clearly confirmed due to adjacent peaks of 0 ms between 1 ms and 10 ms. This is because autocorrelation functions are weighted by using a triangular window from the center of the origin. With all these factors, emphasizing the estimation of a period related to the fundamental modal frequency on an autocorrelation function is needed to understand structural conditions.

3. Period Estimation of Fundamental Modal Frequency Using Autocorrelation Functions

Correlation functions are strongly linked frequency characteristics [10], and autocorrelation analysis is known as insusceptible method for external noise. Agneni reported that modal parameters can estimate from autocorrelation functions by random noise simulation [11]. Figure 3 shows the method for estimating the periods of a fundamental modal frequency with autocorrelation functions. In this experiment, random noise convolved with condition 1's impulse response was prepared as simulated observed signal. First, observed signals are each extracted at about 167 ms (1000 samples at an Fs of 6 kHz) as framed signal $x(n)$. Here, n is sample number. Then, triangular window $w_t(n)$ is applied to the framed signal $x(n)$, which is expressed by

$$\begin{gathered} w_t(n) = N - n + 1, \quad (0 \le n \le N), \\ y(n) = w_t(n)\, x(n), \end{gathered} \tag{1}$$

where N denotes length of framed signal. Autocorrelation function ACF (n) is calculated by using the inverse discrete Fourier transform (IDFT) of the power spectrum $P(k)$ of the triangular windowed signal:

$$\begin{gathered} Y(k) = \mathrm{DFT}\left[y(n)\right], \\ P(k) = |Y(k)|^2, \\ \mathrm{ACF}(n) = \mathrm{IDFT}\left[P(k)\right]. \end{gathered} \tag{2}$$

A symmetrical triangular window $w_s(n)$ of 30-ms-long (180 samples each) with zero padding (replacing the outside components of the symmetrical triangular window with zero) is obtained by

$$w_s(n) = \begin{cases} 0, & q \le n \\ q - n + 1, & 0 \le n \le q \\ n + q, & -q \le n < 0 \\ 0, & n \le -q, \end{cases} \tag{3}$$

where $\pm q$ is effectual range of the symmetrical window (q is 180 sample in this experiment). Next, a curved power spectrum is calculated from the symmetrical triangular windowed autocorrelation function $\mathrm{ACF}_w(n)$ by using discrete

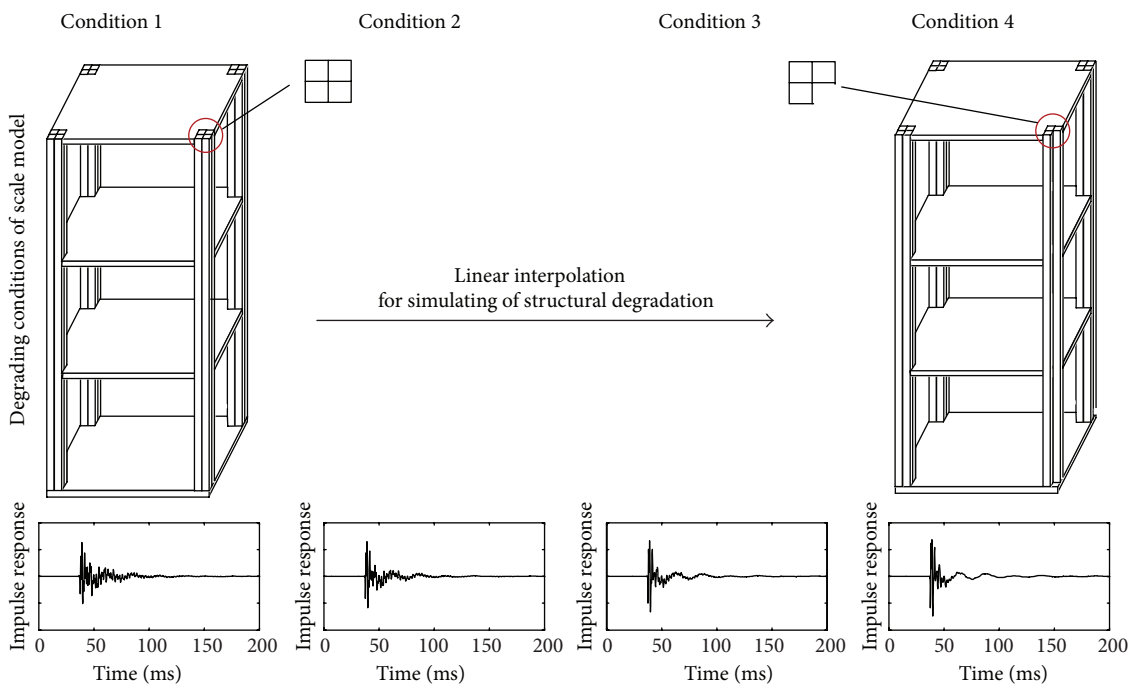

Figure 1: Scale models and corresponding measured and interpolated impulse responses.

Fourier transform (DFT) as follows:

$$\begin{aligned} \mathrm{ACF}_w(n) &= w_s(n)\,\mathrm{ACF}(n), \\ C_p(k) &= \mathrm{DFT}\left[\mathrm{ACF}_w(n)\right]. \end{aligned} \tag{4}$$

After that, a simple power spectrum is prepared by peak selection of the power spectrum for removing sound source effects, the same as that for SIP distribution [8, 9]. Here, peak selection is derived by flexion point seeking using differential approach as follows:

$$D_C(k) = C_P(k) - C_P(k+1), \tag{5}$$

where $D_c(k)$ is differential data. Peak selected linear spectrum $L_{\mathrm{sp}}(k)$ is given by

$$L_{\mathrm{sp}}(k) = \begin{cases} 1, & D_C(k) < 0,\ D_c(k+1) \geq 0, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

In this experiment, 6 line spectra were preserved in descending order. Finally, an autocorrelation function $E_{\mathrm{ACF}}(n)$ for fundamental modal frequency estimation, which comprises about 6 unit line spectra, is calculated by using an IDFT as follows:

$$E_{\mathrm{ACF}}(n) = \mathrm{IDFT}\left[L_{\mathrm{sp}}(n)\right]. \tag{7}$$

Figure 4 shows the process of period estimation with autocorrelation functions. Figure 4(a) shows the waveform of a framed observed signal $x(n)$. Figure 4(b) shows triangular windowed waveforms $y(n)$. Figure 4(c) shows the power spectrum $P(k)$ of a triangular windowed waveform. Figure 4(d) represents the autocorrelation function $\mathrm{ACF}(n)$ that is calculated from the power spectrum. Figure 4(e) shows a symmetric triangular windowed autocorrelation function $\mathrm{ACF}_w(n)$. Figure 4(f) represents the curved power spectrum $C_p(k)$ and unit line spectra $L_{\mathrm{sp}}(k)$ that are calculated by using the DFT of the symmetric triangular windowed autocorrelation. In this figure, the solid line denotes a curved power spectrum, and the posts denote the peak selected unit line spectra. Figure 4(g) shows results of period estimation of fundamental modal frequency $E_{\mathrm{ACF}}(n)$. In Figure 4(g), the peak at 22 ms was maximum point of the figure counted from 1 ms. These results matched condition 1's fundamental modal frequency 45 Hz which was calculated from magnitude spectrum of impulse response. The results indicated that triangular windowed autocorrelation function can estimate fundamental modal frequency. Finally, Figure 4(h) represents the average period estimation results for 30 s long white noise convolved with condition 1's impulse response. In Figure 4(h), the solid line denotes averaged results of period estimation, and the dotted line denotes the period estimation results of condition 1's impulse response for the same method. In the result, maximum peak except 0 ms of impulse response result was coincident with the maximum peak of averaged result. Accordingly, expected peaks appeared to be 22 ms for the method of period estimation that did not appear for the normal standard autocorrelation function of Figure 2(e) when using the triangular weights of the autocorrelation function.

4. Fundamental Modal Frequency Estimation in Nonstationary Noise Simulation

In autocorrelation analyses, white random noise is an easy case to calculate spectral characteristics, because white noise

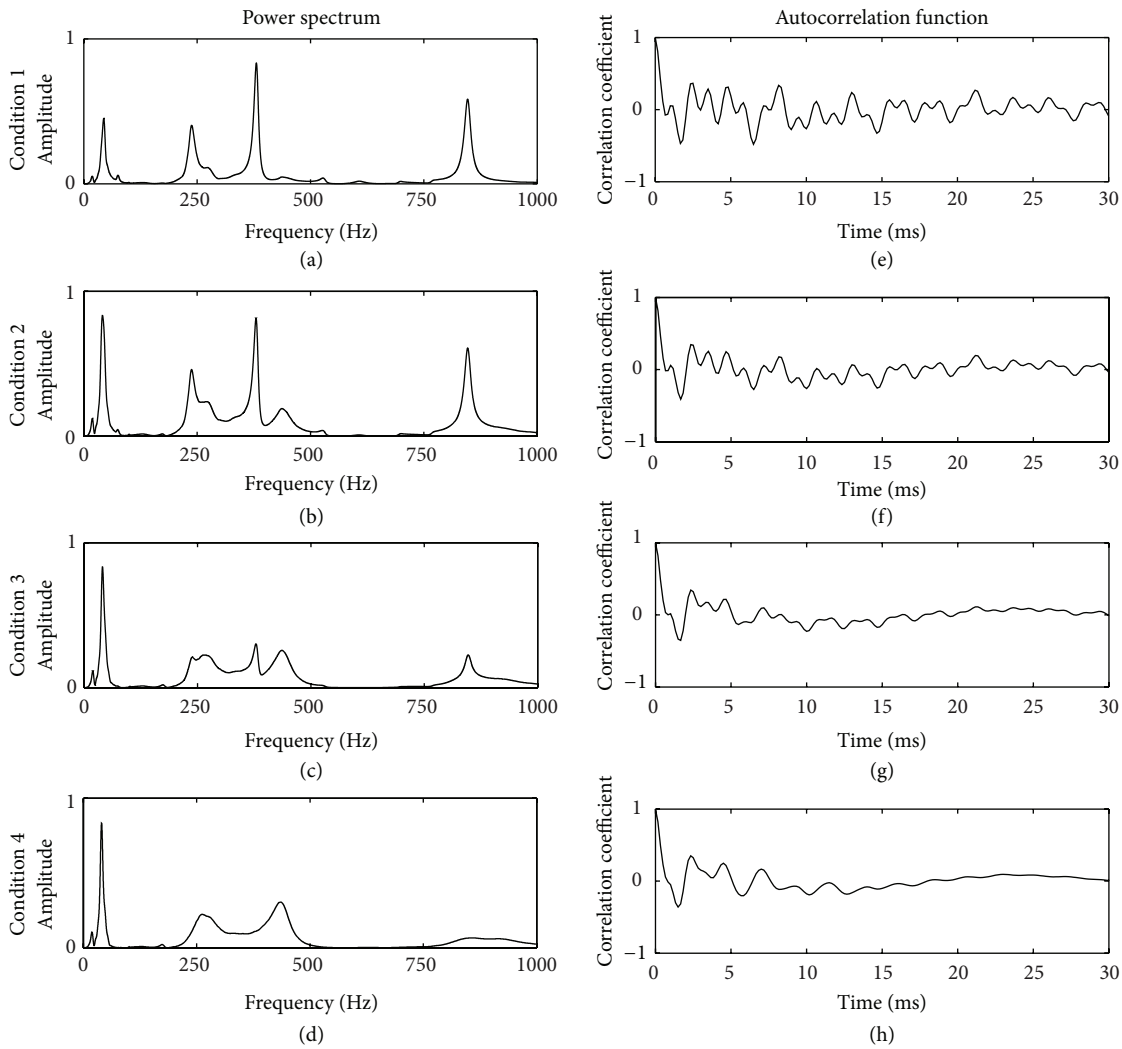

Figure 2: Impulse response power spectra: condition 1 (a), condition 2 (b), condition 3 (c), and condition 4 (d). Autocorrelation functions: condition 1 (e), condition 2 (f), condition 3 (g), and condition 4 (h).

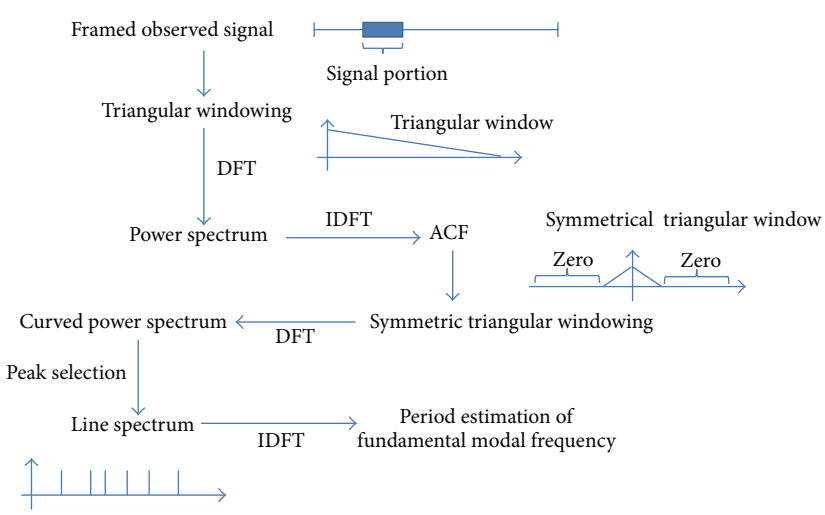

Figure 3: Method of period estimation of fundamental modal frequency.

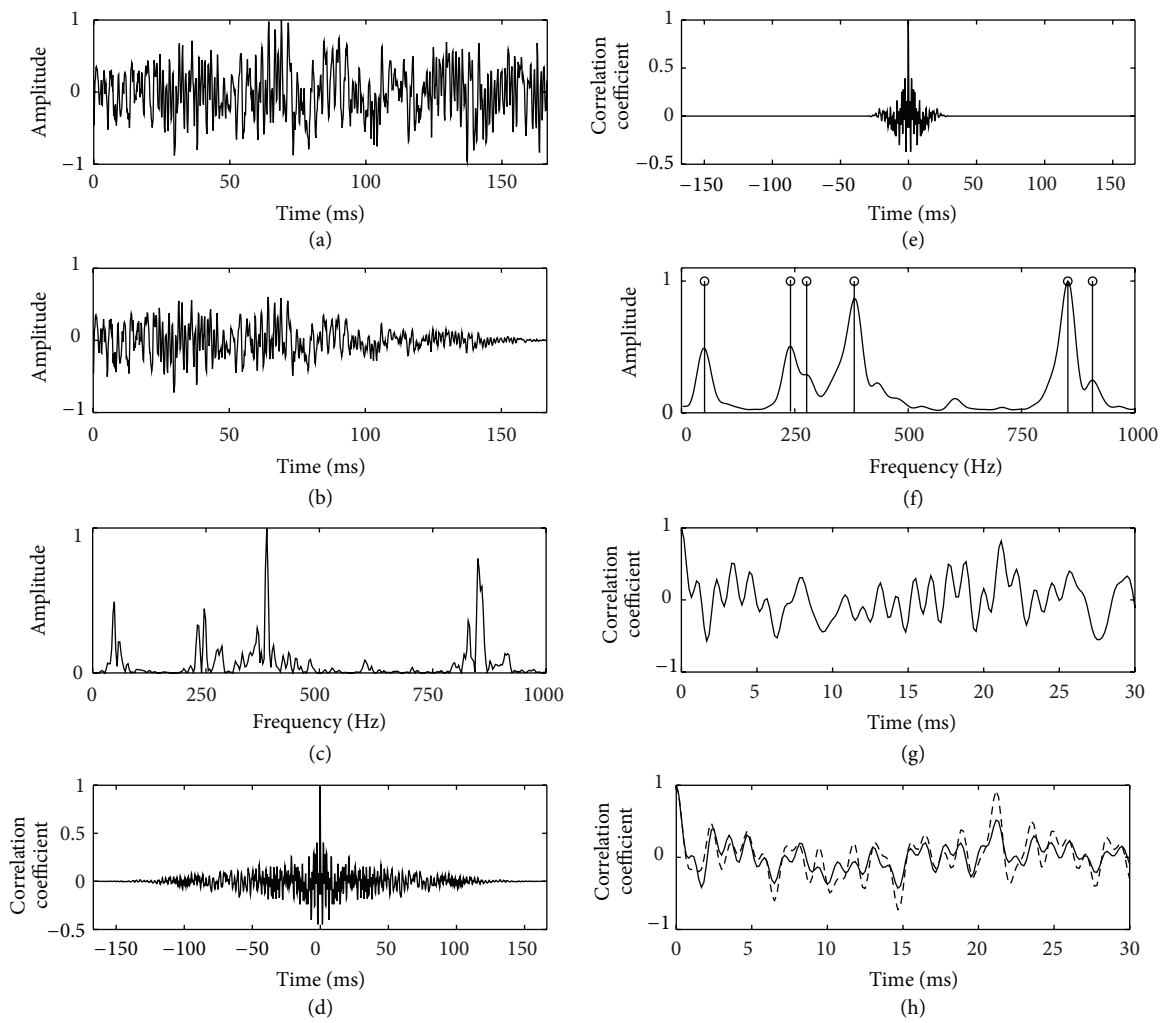

Figure 4: Process of period estimation of fundamental modal frequency. Waveform of framed observed signal (a), triangular windowed framed signal waveform (b), power spectrum of triangular windowed waveform (c), autocorrelation function calculated from power spectrum (d), symmetric triangular windowed autocorrelation function (e), curved power spectrum and unit line spectra (f), period estimation of fundamental modal frequency (g), and average of period estimation results for 30 s long white noise convolved with condition 1's impulse response (solid line: simulated observed signal; dotted line: original impulse response) (h).

is an uncorrelated signal. On the other hand, nonstationary noise is a difficult case for autocorrelation analyses that due to nonstationary modulated waveform makes irregularly multiple periods in the results.

In this experiment, fundamental modal frequency estimation under unknown nonstationary condition is carried out. 30 s of nonstationary noise was prepared. The noise was Cauchy distribution noise with Gaussian noise for nonstationary noise simulation. The Cauchy distribution noise was made by two Gaussian noises for which $C(n)$ is defined as

$$C(n) = \frac{G_A(n)}{G_B(n)}, \tag{8}$$

where $G_A(n)$ and $G_B(n)$ each mean Gaussian noise.

Figure 5 represents nonstationary noise waveforms. In this figure, stand-out amplitude changes are generated at short intervals that generally cause autocorrelation functions to jam. For a nonstationary noise experiment, each condition impulse response was convolved to the nonstationary noise to produce simulated observed signals.

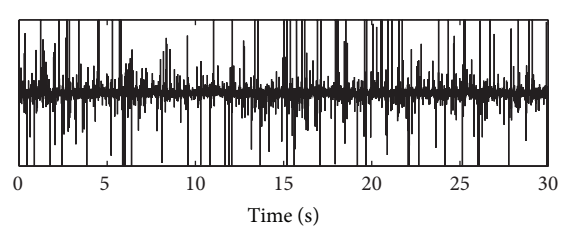

Figure 5: Nonstationary noise waveform.

Figure 6 shows the fundamental modal frequency estimation of simulated observed signals. In this figure, left panels show the averaged period estimation results calculated from the non-stationary simulated observed signals, and

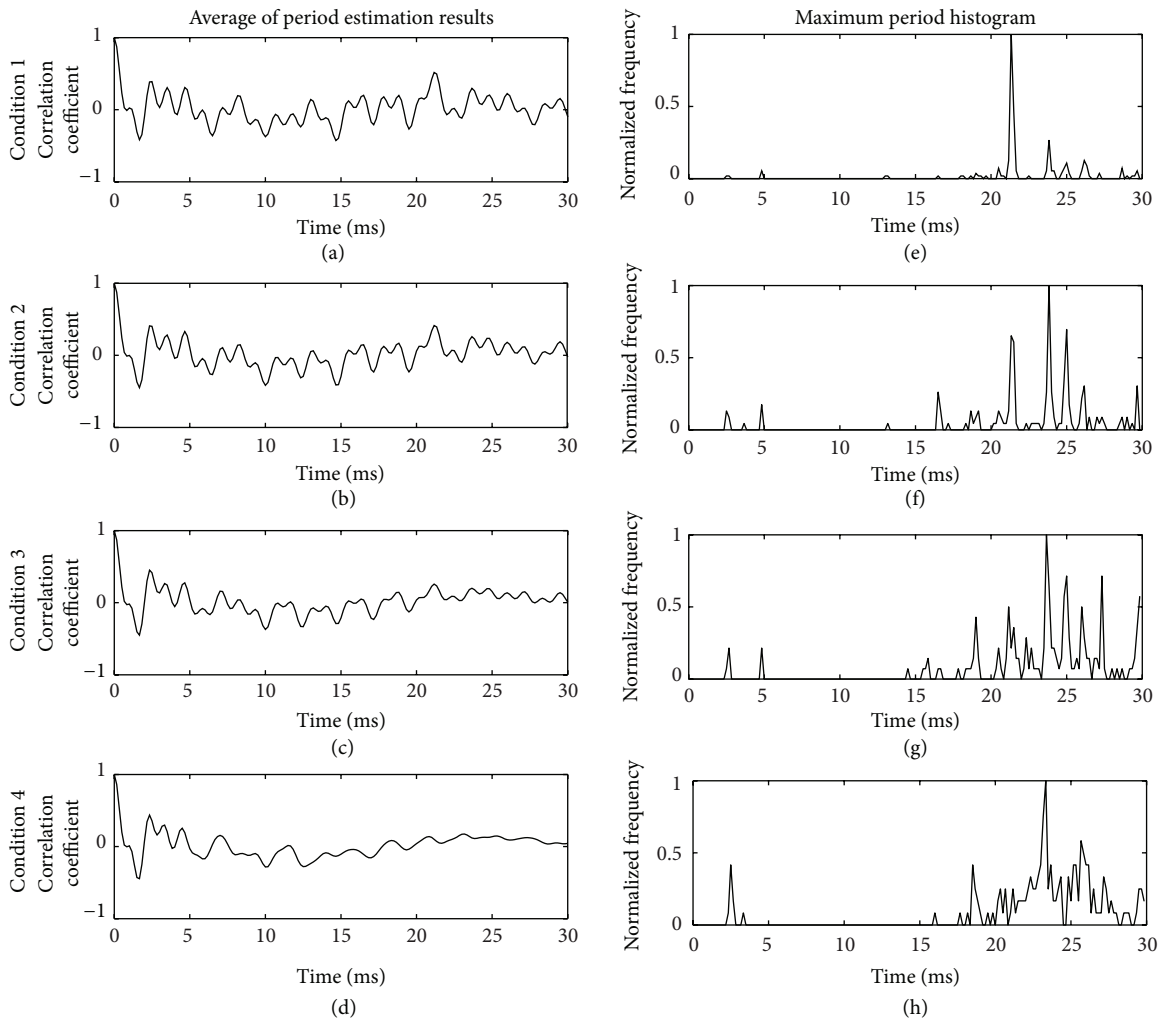

Figure 6: Period estimation results of fundamental modal frequency: condition 1 (a), condition 2 (b), condition 3 (c), and condition 4 (d). Period histogram of maximum peak collection frame by frame: condition 1 (e), condition 2 (f), condition 3 (g), and condition 4 (h).

right panels show histogram of maximum periods which counted maximum period frequency frame by frame between 1 ms and 30 ms. In the histogram, frequency was normalized by maximum frequency in the result. In the estimation results of the left panels, the periods of fundamental modal frequency are not precisely except condition 1. Under the nonstationary condition, the periods around 22 to 24 ms from fundamental frequency are buried way other periods. It denotes that nonstationary noise has strong interference effects on autocorrelation estimation. On the other hand, the histogram of maximum periods on the right panels shows very clear periods of fundamental modal frequency. These results indicated that statistical approach is a good way to periods estimation under nonstationary noise condition such as Cauchy noise. From the results, if observed signals are nonstationary unknown signals, it would be possible to estimate the periods of fundamental modal frequency by the maximum periods histogram even in passive constant monitoring.

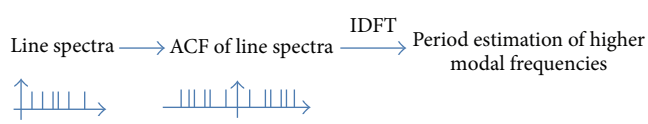

Figure 7: Method of higher modal frequency estimation.

5. Higher Modal Frequency Estimation by Frequency Domain Autocorrelation

The fundamental modal frequency gives important information on structural condition monitoring. In comparison, if the fundamental modal frequency is the pitch in auditory perception used for the hammering test, higher modal frequencies, which are related with frequency component composition, can be considered to be the tone in hearing. Figure 7 represents the method of period estimation of higher modal frequencies with frequency domain's autocorrelation

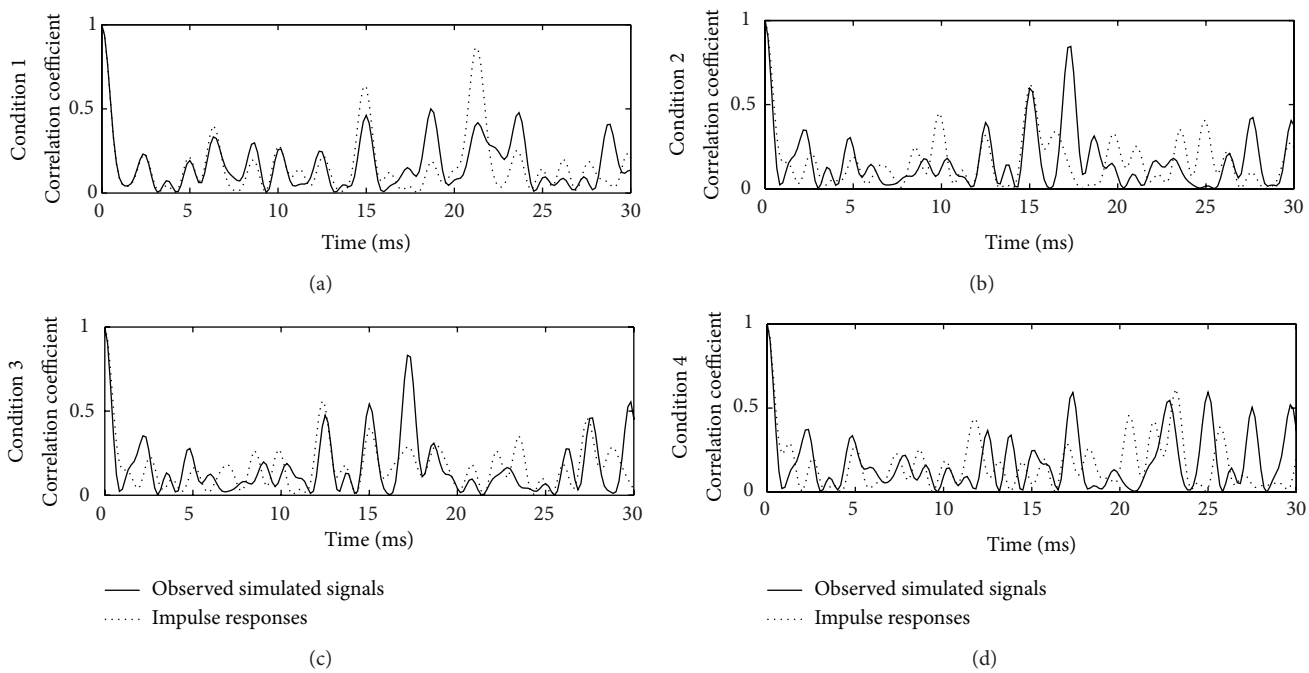

Figure 8: Higher modal frequency estimation results of impulse response and nonstationary noise: condition 1 (a), condition 2 (b), condition 3 (c), and condition 4 (d). (Solid line: simulated observed signal; dotted line: original impulse response.)

functions. In this method, peak selected line spectra L_{sp}, the same as those from the method of fundamental modal frequency estimation, were used to calculate the fine relationship between frequency components by using the frequency domain autocorrelation function. Autocorrelation function of line spectra $\text{ACF}_L(k)$ is given by

$$\text{ACF}_L(k) = \sum_k L_{\text{sp}}(k) L_{\text{sp}}(k-l). \tag{9}$$

After calculating the autocorrelation function in the frequency domain from unit line spectra, period estimation of higher modal frequencies was derived by using the IDFT of the autocorrelation results to change the time domain in order to average the framed results of continuous values:

$$\text{ACF}_F(n) = \text{IDFT}\left[\text{ACF}_L(k)\right]. \tag{10}$$

In this method, period estimation of higher modal frequencies showed up as peaks in the result data. Figure 8 shows the results of period estimation of higher modal frequency compositions. In this figure, the solid line denotes averaged results of period estimation of higher modal frequency, and the dotted line denotes the period estimation results of each impulse response for the same method. In the impulse response results of Figure 8, the peaks could be confirmed clearly through conditions. In comparison, conditions change from 1 to 4 and the remarkable peaks gradually decrease. These results dovetailed with the fact that the power spectra were also not as sharp as the spectra above 250 Hz of Figures 2(c) and 2(d). These results indicate that effects of those vague spectra were also present in the method of period estimation, because the same way as that used for SIP distribution [8, 9] was used even when using peak selected unit line spectra. In comparison, the results of nonstationary simulated observed signals had similar periodic shapes in the panel (a) of condition 1's results. On the other hand, Figures 8(c) and 8(d) also indicate that periods of higher modal frequencies composition is not similar to impulse responses results due to vague spectra conditions such as those in Figures 2(c) and 2(d) from noisy observed signals. Consequently, those results confirmed that higher modal frequency composition does not show clear condition trends under the nonstationary noise. From those results, one factor that has been the spiky nonstationary waveform had negative effects on period estimation with autocorrelation functions. In the autocorrelation function, if continuous projecting waveform is observed at the noise, period estimation is strongly drawn to interval of projecting waveform. However, those projecting nonstationary waveforms contain worthy information to observe structural condition changes [1].

6. Summary

In this paper, on the basis of the hammering test with hearing, structural modal frequency characteristics were estimated by changing the time domain information of periods with autocorrelation functions both for the time and frequency domains. First, periods of fundamental modal frequency were conducted from autocorrelation functions by using triangular window and peak selected power spectrum. Period estimation results of the fundamental frequency indicated that hidden periods appeared due to autocorrelation functions with peak selected unit line spectra. Under the unknown nonstationary noise, maximum period histogram

showed periods of fundamental modal frequency clearly. Second, estimation of higher modal frequencies was calculated from autocorrelation functions in the frequency domain of peak selected line spectra. This indicated that higher modal frequencies can estimate the peaks of the results with line spectra autocorrelations when stractural frequency components are sharp. Consequently, the fundamental modal frequency could be estimated by using simple histogram of constant observed signal results, and higher modal frequency would estimated frequency components are sharp or nonstationary noise is not very spiky. On the other hand, if nonstationary noise has strongly negative effect on autocorrelations due to many stand-out waveforms, stractural modal parameters can estimate successfully from decay portions of observed signals.

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