Research Article

Static and Dynamic Characteristics of Composite Conoidal Shell Roofs

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A thorough review of the existing literature reflects that forced vibration studies of laminated composite conoidal shells with complicated boundary conditions are missing. Hence, the present paper aims to fill the lacuna. A finite element code utilizing eight-noded doubly curved elements together with modified Sanders’ first approximation theory for thin shells is used to study the forced vibration behavior of moderately thin laminated composite conoidal shells subjected to three different uniformly distributed time-dependent forces. Newmark’s direct time integration method is used to solve the dynamic problem. Results obtained using the present code are compared with the values available in the literature, and a good agreement of the results confirms the accuracy of the proposed code. The transient responses of the laminated shell are studied meticulously for parametric variations like boundary conditions and stacking orders of cross and angle-ply laminates and are compared with bending responses of the shell to conclude on the necessity of the dynamic study.

1. Introduction

Laminated composites gained popularity to fabricate plates and shells since second half of the last century. High strength/stiffness to weight ratio, low cost of fabrication, and better durability of the laminated composites popularized them in the weight-sensitive engineering applications. Moreover, the stiffness parameters of this material can be altered by varying the fiber orientations and lamina stacking sequences which made them a lucrative option to the engineers. Naturally, a good number of research reports started publishing on laminated plates and shells. A number of researchers like Reddy [1], Reddy and Chandrashekhara [2], Ribeiro [3], and Nanda and Bandyopadhyay [4, 5] reported bending and dynamic responses of laminated shell configurations.

A conoidal shell is doubly curved, easy to cast, added stiff surface which is used in the civil engineering industry to cover large column free open spaces as one sees in airport terminal buildings, shopping malls, and in car parking lots. Moreover, these shell structures allow entry of daylight and natural air which makes them popular roofing units to medicinal plants and food processing units. In a number of situations like drop hammer condition in the industry, snow loading in low temperature areas, the hit by wind borne debris develops dynamic forces on these shells. To ensure an uninterrupted service life of the shell roofs, their relative static and dynamic performances are needed to be understood in detail.

A number of researchers worked on isotropic and laminated composite conoidal shells. Static bending responses of isotropic conoidal shells were reported by Ghosh and Bandyopadhyay [6, 7]. The authors used finite element method and reported deflections and stress resultants of the conoidal shells. Chakravorty et al. [8] studied fundamental frequencies of composite conoidal shells with cut-outs. They also reported forced vibration responses of laminated conoidal shells with point supported boundaries. Later, Nayak and Bandyopadhyay [9] studied forced vibration responses of stiffened isotropic conoidal shells. Apart from the transient characteristics, the authors [10] also carried out free vibration analysis of composite conoidal shells. Das and Chakravorty [11, 12] worked on bending and free vibration
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of laminated conoidal shells with and without stiffeners, respectively. Bending characteristics of delaminated conoidal shells were reported by Kumari and Chakravorty [13, 14], while Pradyumna and Bandyopadhyay [15, 16], studied vibration and dynamic instability behaviors of laminated conoidal shells using the higher order shear deformation theory.

It is found from the literature survey that although a number of authors worked on dynamic analysis of laminated conoidal shells, none reported forced vibration responses of these shells with complicated boundary conditions. Hence, the present paper aims to study the deflections and stress resultants of laminated conoidal shells with complicated boundary conditions subjected to three different uniformly distributed time-dependent forces. Moreover, the static deflections and fundamental frequencies of these shells are also reported to understand the overall behavior of the conoidal shells.

2. Mathematical Formulation

2.1. Governing Differential Equation. Let us consider a laminated composite conoidal shell (Figure 1) of uniform thickness “h” and radii of curvature “R_y” and “R_x”. Each of the thin laminae may be oriented at an angle “θ” with reference to the x-axis (Figure 3). Lagrange’s equation of motion is derived using Hamilton’s principle to express the governing differential equation of motion for an undamped forced vibration condition of the shell undergoing small displacements,

\[
\frac{d}{dt} \left\{ \frac{\partial U_1}{\partial \dot{d}_e} \right\} - \left\{ \frac{\partial U_2}{\partial \dot{d}_e} \right\} + \left\{ \frac{\partial U_3}{\partial \dot{d}_e} \right\} (U_1 + U_3) = 0,
\]

where \( U_1 \): work done by conservative forces in a shell element and is given by

\[
U_1 = \frac{1}{2} \int \{ \dot{e} \}^T \{ \sigma \} \, dv,
\]

\( U_2 \): kinetic energy of the shell element and is given by

\[
U_2 = \frac{1}{2} \int_A \{ \dot{d} \}^T \{ m \} \{ \dot{d} \} \, dA,
\]

\[
\text{where } \{ \dot{d} \} \text{ represents time derivative of shell deflections with respect to time.}
\]

\( U_3 \): work done by external forces and is given by

\[
U_3 = - \int_A \{ \dot{d} \}^T \{ q \} \, dA,
\]

\[
\text{where } \{ q \} \text{ indicates external transverse load intensity.}
\]

2.2. Laminate Constitutive Relationship. The laminate stress resultant vector \( \{ F \} \) is expressed using the following relationship:

\[
\{ F \} = [D] \{ e \},
\]

where \([D]\) is the laminate constitutive relationship matrix and is adopted from Kumari and Chakravorty [13]. While obtaining the shear stress resultants, proper shear correction factors (as used by Das and Chakravorty [11]) are applied. The laminate stress resultant vector \( \{ F \} \) is expressed as

\[
\{ F \} = \int_{-h/2}^{+h/2} \{ \sigma \} \, dz
\]

\[
= \int_{-h/2}^{+h/2} \{ \sigma_x \ \sigma_y \ \sigma_{xy} \ \sigma_x \cdot z \ \sigma_y \cdot z \ \tau_{xy} \cdot z \ \tau_{xz} \ \tau_{yz} \}^T \, dz
\]

\[
= \{ N_x \ N_y \ N_{xy} \ M_x \ M_y \ M_{xy} \ Q_x \ Q_y \}^T
\]

(see Figure 2).

2.3. Finite Element Formulation. An eight-noded curved quadratic isoparametric element (Figure 3) with \( C^0 \) continuity is used in the proposed finite element code. The element displacement field \( \{ d \} \) is described as follows:

\[
\{ d \} = \{ u \ v \ w \ \alpha \ \beta \}^T,
\]

where \( u, v, \) and \( w \) denote displacements of the shell along \( x, y, \) and \( z \) axes and \( \alpha \) and \( \beta \) define rotations of the normal to the shell midsurface about \( y \) and \( x \) axes, respectively. The element degrees of freedom \( \{ d \} \) are expressed in terms of their nodal values \( \{ d_e \} \) by the following relationship:

\[
\{ d \} = \sum_{i=1}^{8} [N_i] \{ d_e \},
\]

(see Figure 3).
where \([N_e]\) denotes the shape functions of the element and adopted is from Das and Chakravorty [11].

Following the modified Sanders’ first approximation theory for thin shells, the mid-surface strain vector of the shell is expressed as

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{pmatrix}^T = \begin{pmatrix}
\varepsilon^0_x \\
\varepsilon^0_y \\
\gamma^0_{xy} \\
\gamma^0_{xz} \\
\gamma^0_{yz}
\end{pmatrix}^T + z \begin{pmatrix} k_x, k_y, k_{xy}, k_{xz}, k_{yz} \end{pmatrix}^T,
\]

(9)

where

\[
\begin{pmatrix}
\varepsilon^0_x \\
\varepsilon^0_y \\
\gamma^0_{xy} \\
\gamma^0_{xz} \\
\gamma^0_{yz}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial u_0}{\partial x} - \frac{w}{R_{yy}} \\
\frac{\partial v_0}{\partial y} + \frac{\partial w}{\partial x} - \frac{2w}{R_{xy}} \\
\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \alpha \\
\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \beta \\
\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w}{\partial y}
\end{pmatrix}
\]

(10)

The strain vector is related to the nodal degrees of freedom by the strain displacement matrix \([B]\) which is the same used here as those reported by Das and Chakravorty [11].

Using the strain displacement and laminate constitutive relations, (2) can be expressed as

\[
U_1 = \frac{1}{2} \iint_A \{d_e\}^T [B]^T [D] [B] \{d_e\} \, dx \, dy.
\]

(11)

Using (8), (3) is given by

\[
U_2 = \frac{1}{2} \iint_A \{\dot{d}_e\}^T [N]^T [m] [N] \{d_e\} \, dx \, dy,
\]

(12)

where

\[
[m] = \begin{pmatrix}
m & 0 & 0 & 0 \\
0 & m & 0 & 0 \\
0 & 0 & m & 0 \\
0 & 0 & 0 & I
\end{pmatrix}; \quad m = \rho h, \quad I = \frac{\rho h^3}{12}.
\]

(13)

Similarly, we get from (4),

\[
U_3 = -\iint_A \{\dot{d}_e\}^T [N]^T \{q\} \, dx \, dy.
\]

(14)

Using (11), (12), and (14), (1) can be expressed as

\[
\begin{pmatrix}
\iint_A \{N\}^T [m] [N] \, dx \, dy \end{pmatrix} \{\ddot{d}\} + \begin{pmatrix}
\iint_A [B]^T [D] [B] \, dx \, dy
\end{pmatrix} \{\dot{d}\} = \begin{pmatrix}
\iint_A [N]^T \{q\} \, dx \, dy
\end{pmatrix}.
\]

(15)

The coefficient of the element acceleration vector \(\{\ddot{d}_e\}\) of (15) represents the element mass matrix \([M_e]\), and that of the element displacement vector \(\{d_e\}\) represents the element stiffness matrix \([K_e]\). The term on the right-hand side of (15) represents the element load vector \(\{Q_e\}\).

Thus, (15) results in

\[
[M_e] \{\ddot{d}_e\} + [K_e] \{d_e\} = \{Q_e\},
\]

(16)

where

\[
[M_e] = \iint_A [N]^T [m] [N] \, dx \, dy,
\]

(17)

\[
[K_e] = \iint_A [B]^T [D] [B] \, dx \, dy,
\]

\[
[Q_e] = \iint_A [N]^T \{q\} \, dx \, dy.
\]

The element mass matrix, stiffness matrix, and load vector are transformed to isoparametric coordinates \(\xi, \eta\) for numerical integration by Gauss quadrature rule. The element matrices are assembled to get the respective global matrices with due consideration for curvature of the conoidal shell surface [17].

Consider

\[
[K] = \sum_{i=1}^n [K_e], \quad [M] = \sum_{i=1}^n [M_e], \quad [Q] = \sum_{i=1}^n [Q_e].
\]

(18)

The dynamic equation of motion in the global form is

\[
[M] \{\ddot{d}\} + [K] \{d\} = \{Q\}.
\]

(19)
2.4. The Static Problem. If the inertia force term of (19) is dropped, and the displacement and load vectors are assumed to be time independent, then the following equation of static equilibrium is obtained:

\[ [K] \{d\} = \{Q\} . \tag{20} \]

The previous equation is solved by the Gauss elimination method [18].

2.5. The Free Vibration Problem. If the load vector of (19) is dropped, the equation of free vibration is obtained as

\[ [K] \{d\} + [M] \{\ddot{d}\} = 0 , \]

or,

\[ [K] - \omega^2 [M] = 0 . \tag{21} \]

The free vibration analysis involves determination of natural frequencies and is solved by subspace iteration algorithm [18].

2.6. The Forced Vibration Problem. The global force vector in (19) is transient in nature and solved using Newmark’s direct time integration method [18].
(hence forth referred as DMF) are calculated as the ratios of maximum dynamic responses to the corresponding static ones. The material and geometric properties of the conoidal shells are furnished with the tables. In case of bending analysis, the magnitude of the static load is considered equal to the peak step load value of load-time history considered here.

4. Results and Discussion

Static deflections and fundamental frequencies obtained using the present formulation are in close agreement [refer to Tables 1 and 2] with the values reported by Reddy [1]. Close agreement of static deflections as reported in Table 1 ensures accurate bending formulation for laminated shells in the current code. The deflection values monotonically increase as the finite element mesh is made finer in steps because a coarse mesh indicates a rigid modelling of the plate stiffness. For the finite element mesh is made finer instead of a coarse code. The deflection values increase gradually and this is why the DMF values are marginally greater than unity.

The complicated boundary conditions considered here (Figure 5) have equal numbers of support degrees of freedom locked but arranged in a different manner. Naturally, a practicing engineer may be curious to know the relative performances of the conoidal shells in terms of these two-edge conditions. It is interesting to note from Table 3 that for any given lamination the CCSS boundary condition shows less static deflections when they are compared with the values obtained for the SSCC one. The conoidal shell is curved along one of its plan directions and has straight edges along the other plan direction. The arch direction of the shell is relatively stronger than the beam direction as it combines the bending and axial rigidity together to sustain external loads. But unlike the cylindrical shell, the conoidal shell has its curved boundaries at different elevations. This is why the bending stiffness of the shell is enhanced substantially on clamping the lower parabolic arch of the shell, and the static deflections decrease. The fundamental frequencies of the conoidal shell listed in the same table also vary in a similar manner as it was observed in case of the static deflections, although the symmetric angle-ply laminations show marginally higher values for the SSCC shell. The $45^0/-45^0/45^0$ and $45^0/-45^0/-45^0/45^0$ laminates yield 3.71% and 2.18% higher frequency values, respectively, for the SSCC boundary condition. This observation leads to conclude that clamping the lower arch of the conoid is advantageous to achieve higher bending and dynamic stiffnesses.

The dynamic deflections of all the shell options considered here are studied for three different load cases: step load of infinite duration (load case I), step load of finite duration (load case II), and half sinusoidal pulse load (dynamic load case III). The results are furnished in Table 4. The results show that the dynamic deflections follow the same pattern as it was noted in cases of static deflections and fundamental frequencies where the CCSS shell performed better than the SSCC one. Although the magnification factors for all the load cases listed in Table 3 vary in a different manner where in a number of cases the SSCC shell shows lesser DMF values, it is found from the table that the DMF values for dynamic load cases I and II are all greater than 2. The load-time history of load cases I and II is identical up to 1 second, and after that the dynamic load corresponding to load case I continues to act with the same intensity, and for load case II, the load is withdrawn. Interestingly, the magnification factors are same for both the load cases or marginally higher for load case II. In load case III, both the application and withdrawn of pressure are gradual and this is why the DMF values are marginally greater than unity.
Table 3: Nondimensional responses of the conoidal shell.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Lamination (degree)</th>
<th>Static deflections</th>
<th>Fundamental frequency</th>
<th>Dynamic magnification factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Load case I</td>
</tr>
<tr>
<td>CCSS</td>
<td>0/90</td>
<td>0.000327 (15)</td>
<td>55.61 (15)</td>
<td>2.10 (4)</td>
</tr>
<tr>
<td></td>
<td>0/90/0</td>
<td>0.000256 (11)</td>
<td>70.52 (8)</td>
<td>2.37 (14)</td>
</tr>
<tr>
<td></td>
<td>0/90/0/90</td>
<td>0.000252 (10)</td>
<td>65.81 (13)</td>
<td>2.07 (2)</td>
</tr>
<tr>
<td></td>
<td>0/90/90/0</td>
<td>0.000243 (9)</td>
<td>71.37 (7)</td>
<td>2.24 (11)</td>
</tr>
<tr>
<td></td>
<td>45/−45</td>
<td>0.000104 (3)</td>
<td>70.06 (9)</td>
<td>2.18 (8)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45</td>
<td>0.000116 (4)</td>
<td>82.46 (6)</td>
<td>2.08 (3)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/−45</td>
<td>0.000083 (1)</td>
<td>89.15 (1)</td>
<td>2.15 (5)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/45</td>
<td>0.000091 (2)</td>
<td>86.90 (4)</td>
<td>2.16 (7)</td>
</tr>
<tr>
<td>SCC</td>
<td>0/90</td>
<td>0.000394 (16)</td>
<td>53.10 (16)</td>
<td>2.06 (1)</td>
</tr>
<tr>
<td></td>
<td>0/90/0</td>
<td>0.000279 (12)</td>
<td>68.36 (10)</td>
<td>2.38 (16)</td>
</tr>
<tr>
<td></td>
<td>0/90/0/90</td>
<td>0.000306 (14)</td>
<td>62.16 (14)</td>
<td>2.15 (6)</td>
</tr>
<tr>
<td></td>
<td>0/90/90/0</td>
<td>0.000279 (13)</td>
<td>68.21 (11)</td>
<td>2.37 (15)</td>
</tr>
<tr>
<td></td>
<td>45/−45</td>
<td>0.000186 (8)</td>
<td>68.13 (12)</td>
<td>2.25 (12)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45</td>
<td>0.000137 (6)</td>
<td>85.52 (5)</td>
<td>2.29 (13)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/−45</td>
<td>0.000138 (7)</td>
<td>87.25 (3)</td>
<td>2.18 (9)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/45</td>
<td>0.000133 (5)</td>
<td>88.80 (2)</td>
<td>2.22 (10)</td>
</tr>
</tbody>
</table>

Values in parentheses indicate ranks for respective shell actions.

Table 4: Nondimensional dynamic deflections of the conoidal shell.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Lamination (degree)</th>
<th>Dynamic deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Load case I</td>
</tr>
<tr>
<td>CCSS</td>
<td>0/90</td>
<td>0.000687 (15)</td>
</tr>
<tr>
<td></td>
<td>0/90/0</td>
<td>0.000598 (11)</td>
</tr>
<tr>
<td></td>
<td>0/90/0/90</td>
<td>0.000522 (9)</td>
</tr>
<tr>
<td></td>
<td>45/−45</td>
<td>0.000227 (3)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45</td>
<td>0.000241 (4)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/−45</td>
<td>0.000178 (1)</td>
</tr>
<tr>
<td></td>
<td>45/−45/45/45</td>
<td>0.000196 (2)</td>
</tr>
</tbody>
</table>

Values in parentheses indicate ranks for respective shell actions.

It is also interesting to observe that for both the boundary conditions considered here the angle-ply shells yield lesser values of static and dynamic deflections when they are compared with their counterparts among cross-ply shells. It is further noted from Table 3 that among the symmetric and antisymmetric angle-ply laminates, the four-layered anti-symmetric one (45°/−45°/45°/−45°) shows the best overall performance for the CCSS boundary condition. This means that in order to get the least possible static and dynamic deflections of a conoidal shell, an engineer’s choice will be the CCSS boundary condition with the four-layered anti-symmetric angle ply laminate (45°/−45°/45°/−45°). The above conclusion is further reinforced by the fact that in terms of fundamental frequency also this particular laminate
The dynamic deflections for load cases I and II are reported along with the static deflection in Figure 7, and they are observed to oscillate around the static deflection up to 1 second. After the sudden removal of the dynamic load in case of load case II, the shell response magnifies drastically and even results in upward movements. This is presented with the figures. The loading conditions for load cases I and II are in fact identical up to time = 1 second. This is why the response curves up to 1 second for these two load cases coincides. In both of these cases since the load is suddenly applied, the elastic rebound of the structure is quite strong, and sharp closely spaced crests and troughs are noted. For load case III, however, as the load is very gradually applied and withdrawn and load is of lesser magnitude, the deflection response curve almost resembles the shape of the loading curve.

The dynamic deflections for load cases I and II are reported along with the static deflection in Figure 7, and they are observed to oscillate around the static deflection up to 1 second. After the sudden removal of the dynamic load in case of load case II, the shell response magnifies drastically and even results in upward movements. This is
why the DMF values for load cases I and II are almost the same or marginally higher for load case II. This observation also brings out the importance of forced vibration analysis as static simplification of dynamic problems using a suitable factor cannot account for such reversal of dynamic displacements. It is also important to note here that in case of the dynamic loads, its time variation is more significant than the magnitude. The dynamic deflections for load case I continue its same vibrating pattern for 2 seconds. On the other hand for load case III when the load is applied and withdrawn gradually, the shell vibrates with positive and negative peaks of equal magnitude and in a frequency which is nearly equal to its fundamental frequency.

The force and moment resultants of the composite shell (Figures 8, 9, 10, and 11) follow the same pattern as it was observed in case of the dynamic deflections. Reversals of all the force and moment resultants are noted in the figures although it is more prominent for the moment resultants compared to than that of the force resultants in case of load case I where the dynamic load continued to act for 2 seconds.

5. Conclusion

The following conclusions can be drawn from this study.

(1) The present code accurately incorporates the finite element method to solve static, free, and forced vibration problems of laminated composite conoidal shells as it is indicated by the solutions of the benchmark problems.

(2) It is concluded that among the boundary conditions considered in the present study, the lower parabolic arch of the conoidal shell should be clamped to achieve higher bending and dynamic stiffnesses.

(3) It is further indicated that among the considered laminations, the 45°/−45°/45°/−45° laminate shows the best overall performance by yielding the least static and dynamic deflections and the highest fundamental frequency.

(4) When the dynamic responses of the composite shell are studied for varying time, it is found that the deflections and stress resultants show reversal in nature. Hence a full dynamics analysis is needed to be carried out to get the magnitudes of the reversed stresses, imposing a factor of safety on the static values cannot take care of this phenomenon.

**Notations**

- $A$: Area of the shell
- $a, b$: Length and width of shell in plan
- $D$: Flexural rigidity matrix of the shell
- $\{u\}$: Global displacement vector
- $\ddot{u}$: Global acceleration vector
- $E_{11}, E_{22}, E_{33}$: Young's moduli
- $1, 2$ and $3$: Local coordinates of a lamina
- $G_{12}, G_{13}, G_{23}$: Shear moduli
- $h_h, h_l$: Higher and lower heights of the conoidal shell, respectively
- $t$: Shell thickness
- $M_x, M_y, M_{xy}$: Moment resultants per unit length
- $\bar{M}_x, \bar{M}_y$: Nondimensional moment resultants [$= (M_x \text{ or } M_y)/qa^2$]
- $\bar{M}_{xy}$: Nondimensional torsion resultant [$= M_{xy}/qab$]
- $N_x, N_y, N_{xy}$: Force resultants per unit length
- $\bar{N}_x, \bar{N}_y$: Nondimensional inplane force resultants [$= (N_x \text{ or } N_y)/qa$]
- $\bar{N}_{xy}$: Nondimensional inplane shear resultant [$= N_{xy}/qa$]
- $n_e$: Number of elements
- $Q_x, Q_y$: Transverse shear resultants per unit length
- $q_0$: Peak value of the transient loads
- $R_{xy}$: Radius of cross curvature of conoidal shell
- $R_{yy}$: Radius of curvature of conoidal shell along $y$-axis
- $R$: Radius of the spherical shell
- $u, v, w$: Translational degree of freedoms along $x$-, $y$-, and $z$-directions, respectively
- $V$: Volume of the shell
- $\bar{w}$: Nondimensional transverse displacement [$= wE_{22}h^3/(qa^4)$]
- $x, y, z$: Global coordinates axes
- $\alpha, \beta$: Rotations of the shell about $y$- and $x$-axes, respectively
- $\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$: Shear strains
- $\varepsilon_x, \varepsilon_y$: Inplane strains along $x$- and $y$-axes, respectively
- $\theta$: Angle of lamination with respect to $x$-axis of the conoidal shell
- $\nu_{ij}$: Poisson’s ratio
\( \xi, \eta: \) Natural coordinates of isoparametric elements

\( \rho: \) Mass density

\( \sigma_x, \sigma_y: \) Inplane stresses along \( x \)- and \( y \)-axes, respectively

\( \tau_{xy}, \tau_{xz}, \tau_{yz}: \) Shear stresses

\( k_x, k_y, k_{xy}: \) Curvatures of conoidal shell due to load

\( \omega: \) Fundamental frequency in radian/sec

\( \bar{\omega}: \) Nondimensional fundamental frequency

\[ \bar{\omega} = \omega^2 \left( \frac{\rho}{E_{22} h^2} \right)^{1/2}. \]

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References


