Realistic active haptic guided exploration with Cartesian control for force–position tracking in finite time

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Abstract: Perception and interaction with virtual surfaces, through kinaesthetic sensation and visual stimuli, is the basic issue of a haptic interface. When the virtual or real object is in a remote location, and guidance is required to perceive kinaesthetic feedback, a haptic guidance scheme is required. In this document, with purpose of haptic-guided exploration, a new scheme for simultaneous control of force and cartesian position is proposed without using inverse kinematics, and without using the dynamic model of PHANToM, though a strict stability analysis includes the dynamic model of PHANToM. We rely on our previously proposed results to propose a new haptic cartesian controller to reduce the burden of computing cartesian forces in PHANToM. Furthermore, a time base generator for finite-time tracking is also proposed to achieve very fast tracking and high precision, which translated into high fidelity kinaesthetic feedback.

Key words: Haptic guidance, kinaesthetic feedback, haptic device, time base generator, Cartesian control, force–position tracking.

INTRODUCTION

Haptic interface

The kinaesthetic perception is possible by means of an electromechanical device (haptic device) in closed loop with the virtual object (Burdea 1996). In particular, the PHANToM haptic interface (Burdea 1996; Salisbury and Srinivasan 1997) has been successfully used for this purpose, though its application-programming interface GHOST has limited capabilities, since only simple undeformable primitives can be programmed. Therefore, only simple spring-based contact force models can be implemented, though recent GHOST version allows simple dynamic properties for the virtual environments. On the other hand, PHANToM uses simple proportional integral derivative (PID)-based Cartesian stiffness control together with high sampling rate to become a powerful haptic interface. However, it is not evident how to modify the PID control structure to introduce a more complex performance, such as stable interaction with deformable objects. And this is precisely the objective of this article since haptic teleoperation or haptic guidance stands as a very promising applications for haptic interfaces.

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Haptic training

Haptic guidance can be used for training (Feygin *et al.* 2002). We identify four classes of haptic guidance configurations (Fig. 1):

1. *Haptic guidance:* The master sends its position and contact force as the only desired references to the remote haptic interface.
2. *Haptic guidance control:* Same as configuration 1, but the master also controls the remote haptic interface. The difference between 1 and 2 is that in configuration 2, the master station directly controls the remote station, while configuration 1 implements an independent control loop in the remote station.
3. *Haptic guided exploration:* Same as definition 1, but the master performs a recognition task.
4. *Haptic guided exploration control:* Same as definition 3, but the master controls the position and contact force of the remote haptic interface.

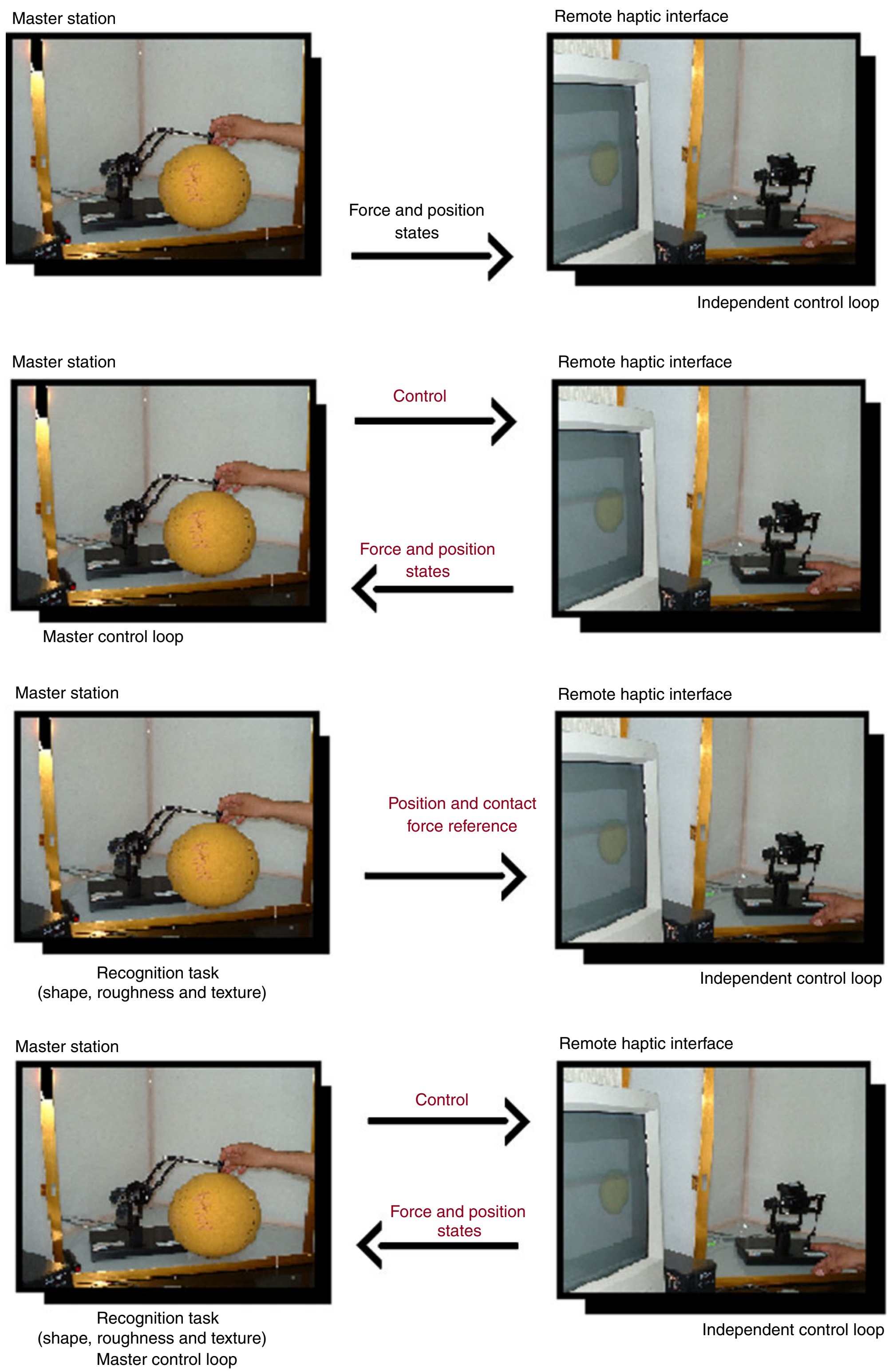

Figure 1 Four classes of haptic guidance: (1) haptic guidance; (2) haptic guidance control; (3) haptic guided exploration; and (4) haptic guided exploration control.

The difference between 3 and 4 is that in configuration 4, the master station directly controls the remote station, while configuration 3 implements an independent control loop in the remote station. Finally, the difference between guidance and guided exploration is that a *guided exploration* configuration involves perception of shape, texture and roughness, in contrast to a *guidance configuration* wherein this object attributes does not play a significant role. For

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instance, in some guided exploration tasks, back and forth and lateral motions might be important, while in guidance explorations, these movements are not important to complete the task. In this work, we are interested in *haptic guidance exploration* (configuration 3), which is useful for rapid training.

Cartesian sliding PID control for force–position tracking of electromechanical device

Cartesian control deals with the problem of designing a controller in terms of desired Cartesian or operational coordinates. This is an important problem since the task is given in operational coordinates, and, in this way, inverse kinematics is not computed. However, a Cartesian controller that uses Cartesian robot dynamics may become a more difficult problem to solve online than a joint controller that uses only inverse kinematics, because Cartesian robot dynamics is even more cost computing than computing the inverse kinematics. For constrained motion, the inverse kinematics are involved not only in mapping Cartesian task into joint tasks but also in checking the consistency of the constrained holonomic equation. Thus, the importance of Cartesian control is more important for constrained motion than for free motion.

The problem

The haptic remote guided exploration of a deformable object is performed when an expert (master station) is training or guiding an inexpert (remote station). This interaction involves two haptic interfaces and two human operators, and therefore compliant interaction arises. Haptic interfaces have been poorly explored, and Feygin *et al.* (2002) have offered an extraordinary review on this subject. The solution of this problem requires contact force based on the dynamics of the whole system. Domínguez-Ramírez and Parra-Vega (2003d) have offered a solution where the high-end kinaesthetic coupling that arises by using the constrained Lagrangian method (Domínguez-Ramírez and Parra-Vega, 2003a) yields a more realistic contact force as a function of the dynamical properties of the whole system. In Domínguez-Ramírez and Parra-Vega (2003d), a haptic guidance scheme for guided exploration is implemented to yield an active haptic exploration for remote training purposes, with simultaneous control of force and position (Parra-Vega *et al.* 2003) on the remote station. Second order Lagrangian dynamics are assigned to the virtual objects and to the PHANToM haptic device. Then, constrained Lagrangian algorithm is implemented to compute the reaction force based on the dynamic properties of the virtual world and the PHANToM haptic device. The components of this contact force is used to reproduce object attributes such as shape, texture and roughness to allow a more realistic contact force compliant to the real sensations of remote exploration. However, to implement control strategy as in Parra-Vega *et al.* (2003), it is necessary for the inverse kinematics to undergo the computationally expensive computations, thus avoiding an excellent opportunity to reproduce object attributes (shape, texture and roughness) as in Domínguez-Ramírez and Parra-Vega (2003c).

Our proposal

A PID-like controller, based on sliding Cartesian PID force and position controller, is proposed for the fast tracking of force and position of robot manipulators (in our case, the PHANToM haptic device in the remote station) without computing inverse kinematics. The control input is smooth, and well-posed terminal attractors are induced in the closed loop to give rise to practical zero tracking error in any given time independent of initial conditions. This Cartesian controller for force–position tracking with a time base generator (TBG) guarantees finite time tracking. The main characteristics of this proposal scheme are as follows:

- The inverse kinematics is not required.
- The haptic device dynamics are not required.
- Exact Jacobean is not required.
- Very fast tracking of force and position trajectories is guaranteed.
- Smooth control activity arises.
- Conservative tuning of feedback gains is required.
- Guarantees finite time tracking.

Simulations on the full non-linear model of a representative haptic device are presented to visualise the closed loop performance of the proposed control scheme.

Organization

The following section discusses briefly the task, and the ‘Non-linear dynamics’ section presents the virtual constrained model methodology for haptic guided exploration of real and virtual objects, the dynamics constraint of PHANToM haptic device in constrained motion, the real and virtual object properties and the constrained Lagrangian as a function of the whole system. The ‘Computation of texture, roughness and shape haptic exploration’ section describes the computation of texture, roughness and shape of haptic exploration of a full non-linear dynamic constrained Lagrangian with these properties. The ‘Haptic Cartesian exploration’ section describes the control design of the Cartesian control in free and constrained motions, and the TBG that allows the finite time tracking. The ‘Simulation’ section discusses the simulation results, and the last section presents the conclusions.

THE TASK

There are two robotic systems involved in the haptic guidance. In the remote station, the remote robot can be a passive linkage robotic arm exploring the real object in contact; this remote arm is equipped with angular position sensors and force sensors to measure angular displacement and real contact forces with the object. At the local station, a haptic display is required to generate the force contact coming from the real contact at the remote station. In this article, we consider that one PHANToM is involved with each side.

Haptic guidance schemes are employed in the remote training tasks. The haptic device defines, in the station teacher, the position references during free motion, and during constrained motion the position and forces that will be reproduced in the remote device in the remote station. To achieve this, we use a non-linear PID control previously proposed in Parra-Vega and Arimoto (2001) for free movement, and a simultaneous control of force and position for constrained movement experiments with the human on the loop we have previously proposed in Domínguez-Ramírez and Parra-Vega (2003d), Parra-Vega *et al.* (2003) and Domínguez-Ramírez and Parra-Vega (2003b). This is a quite demanding task, and to reduce its complexity, a new Cartesian scheme is proposed to avoid any computation of inverse kinematics.

The novelty of our scheme relies on a new Cartesian control for force–position tracking is that inverse kinematics are not longer computed; besides, that a TBG is introduced for finite time tracking. This allows to set up the convergence time at will, voluntarily, thus a fidelity index can be introduced.

The control system under study here can be implemented in a wide variety of tasks in teleoperation and telepresence modes, involving a haptic interface.

NON-LINEAR DYNAMICS

Non-linear constrain dynamics of PHANToM

PHANToM, version 1.0A, is a mechanism of articulate links, with n revolute joints described in generalised joint coordinates $(q^T, \dot{q}^T)^T \in \Re^{2n}$. The dynamics presents restriction in its movement, defined by the following algebraic and differential system of equations (DAE system):

$$M(q)\ddot{q} + (B_0 + C(q, \dot{q}))\dot{q} + G(q) = \tau + \frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|} f_r \tag{1}$$

and

$$\varphi(x) = 0, \tag{2}$$

where $M(q) \in \Re^{3\times3}$ denotes a symmetric positive definite inertial matrix; $B_0 \in \Re^{3\times3}$ stand for a diagonal positive definite matrix composed of damping friction for each joint; $C(q, \dot{q}) \in \Re^{3\times3}$ is a Coriolis and centripetal forces matrix; $G(q) \in \Re^3$models the gravity forces; $\tau \in \Re^3$ stands for the torque input; $f_r \in \Re^r$ is (for $r = 1$ is one-point contact, a scalar) constrained Lagrangian representing the magnitude of the contact force; $\mathcal{J}_\varphi^T / \|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|$ stands for the normalized projection of the Jacobean $\mathcal{J}_\varphi \in \Re^3$; and $\mathcal{J}_\varphi = \mathcal{J}_\varphi(q) \equiv [\frac{\partial}{\partial q_1}\varphi(x)\frac{\partial}{\partial q_2}\varphi(x)\frac{\partial}{\partial q_3}\varphi(x)]\mathcal{J}(q)$, which arises normal at the contact point. The following equations hold while the end-effector is moving on the constraint surface $\varphi(x) = 0$:

$$\frac{d}{dt}\varphi(x(q)) = \frac{\partial}{\partial q}\varphi(x(q))\frac{d}{dt}q \equiv \mathcal{J}_\varphi(q)\dot{q} = 0 \tag{3}$$

and

$$\ddot{\varphi}(x(q)) = \mathcal{J}_\varphi \ddot{q} + \dot{\mathcal{J}}_\varphi \dot{q} \equiv 0. \tag{4}$$

These equations must be satisfied for consistency of the solution of the DAE system.

The local station

(1) *Real remote object:* The surface of the object is described by a geometric function $\varphi(x(q)) = 0$, based on the constrained dynamic model. The real object can be modelled in terms of the operational coordinates $x(q)$ since $\varphi(x(q)) = 0$ as a mass–spring–damper system as follows:

$$m\ddot{\xi}(x(q)) + b\dot{\xi}(x(q)) + k\xi(x(q)) = 0, \tag{5}$$

where m is the mass; b is a damper; and k is a spring. This point-wise model is consistent with the formulation of one-point contact of the DAE system.
(2) *Computation of contact force for local PHANToM:* It is assumed that there exists a force sensor that delivers f_r in the remote station.

The remote station

(1) *Virtual remote object:* Similar to 'The local station' subsection, where now the virtual object is assigned a lumped second order linear dynamics with respect to its inertial frame.
(2) *Computation of contact force for local PHANToM:* PHANToM is not equipped with a force sensor. Then, we propose to compute it by solving the DAE system for f_r as follows. Firstly, for stable interaction $\varphi(x(q))$, the haptic display must stay in contact with the virtual object, then the acceleration $\xi(x(q))$ must equal the acceleration $\varphi(x(q))$, that is, $\xi(x(q)) = \varphi(x(q))$, and then (5) becomes

$$m\ddot{\varphi}(x(q)) + b\dot{\varphi}(x(q)) + k\varphi(x(q)) = 0. \tag{6}$$

Using (4), Equation (6) becomes

$$m\left(\mathcal{J}_\varphi \ddot{q} + \dot{\mathcal{J}}_\varphi \dot{q}\right) + b\dot{\varphi}(x(q)) + k\varphi(x(q)) = 0. \tag{7}$$

Solving (6) by using the DAE system, we obtain:

$$\ddot{q} = M(q)^{-1}\left\{ -(B_0 + C(q, \dot{q}))\dot{q} - G(q) + \tau + \frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|} f_r \right\}. \tag{8}$$

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Now, compute the constrained Lagrangian f_r from (7) and (8) as follows:

$$f_r = \frac{\left\| \mathcal{J}_\varphi \mathcal{J}_\varphi^T \right\|}{m \mathcal{J}_\varphi M(q)^{-1} \mathcal{J}_\varphi^T} \left\{ -b\dot{\varphi}(x(q)) - k\varphi(x(q)) - m\dot{\mathcal{J}}_\varphi \dot{q} + m\mathcal{J}_\varphi M(q)^{-1}\left((B_0 + C(q,\dot{q}))\dot{q} + G(q) - \tau\right) \right\}. \tag{9}$$

Notice that the constrained Lagrangian f_r is the function of $\mathcal{J}_\varphi$, and dynamics of the haptic device and the object.

Reproducing object properties in the remote station

Equation (9) represents the reaction force in terms of (i) the PHANToM dynamics; (ii) the dynamics of the virtual object, and (iii) the controller τ. Notice that acceleration is not required. In this way, the controller τ will track (reproduce) the desired trajectories, that is, the real contact force $f_{r-\text{local}}$ of the local station becomes the desired contact force for the haptic device in the remote station (i.e. $f_{rd-\text{remote}} = f_{r-\text{local}}$), and thus $x(q)_{d-\text{remote}} = x(q)_{\text{local}}$ for position. Since the object in the remote station exhibit roughness and texture through the contact force f_{rd}, then if a controller τ guarantees that $f_{r-\text{remote}}$ converges to $f_{rd-\text{remote}}$, then it also guarantees that the real object properties are perceived in the remote station. Now, since there exists a force sensor in the local station with at least 3 degrees of freedom, then $f_{r-\text{local}} = [f_x, f_y, f_z]^T$, and, in the next section, we propose how to parameterise roughness, shape and texture in terms of $[f_x, f_y, f_z]$ and object parameters. Notice that these properties are parameterised by the operational contact forces at each unitary axis i, j and k, respectively, which are available from the force sensor and friction parameters. In this way, since τ generates tracking of $[f_x, f_y, f_z]$, it will also guarantee tracking of roughness, shape and texture.

COMPUTATION OF TEXTURE, ROUGHNESS AND SHAPE HAPTIC EXPLORATION

How to reproduce object properties with only force sensor measurement when sliding over a real remote object? In this section, we discuss an approach that synthesizes texture, roughness and shape from f_x, f_y and f_z measurements.

Roughness perception

The sliding friction between two different materials, with contact area defined by A, is equal to the load W divided by the flow stress P_m of the weaker of the two solids in contact. At this region of contact, the solid forms a number of junctions as if they were welded together. Friction F represents the force required to shear these junctions apart. Mathematically, the theory is expressed as

$$A = \frac{W}{P_m}, \tag{10}$$

$$F = As \tag{11}$$

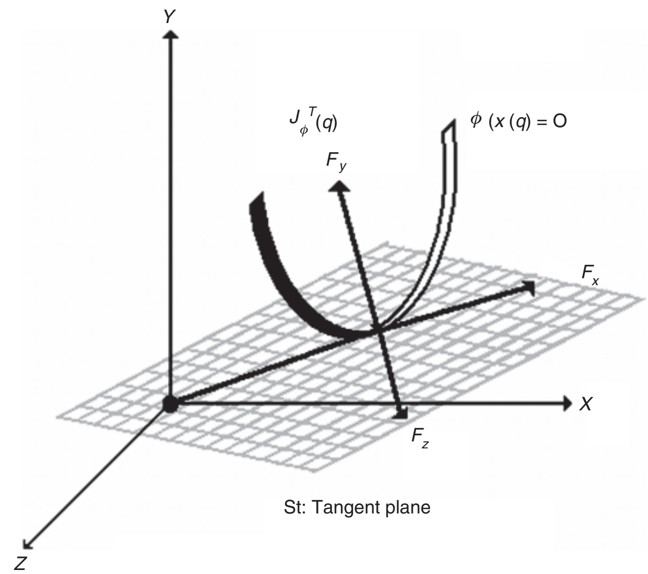

Figure 2 Geometric decomposition in the contact point.

and

$$\mu = \frac{F}{W} = \frac{s}{P_m}, \tag{12}$$

where s is the shear stress. Thus, the coefficient of friction $\mu << 1$ may be represented by the ratio of shear stress to flow stress of the material and becomes its intrinsic property. Roughness arises as a function of the sliding motion over the surface of the object, thus roughness is a function of the tangential friction f_T. Since f_T arises at the tangential plane at the contact point,

$$f_T = \mu\sqrt{\left(f_x^2 + f_z^2\right)}\dot{X}, \tag{13}$$

where $\vec{f}_x, \vec{f}_z \in S_t$ (Fig. 2).

The torque based on the tangential friction force is defined by the following equations:

$$\begin{aligned} \tau_t &= \mathcal{J}^T f_T \\ \tau_t &= \mu\sqrt{\left(f_x^2 + f_z^2\right)}\mathcal{J}^T\dot{X} \end{aligned} \tag{13a}$$

In this way, we can model the roughness by simply assigning values to f_x and f_z, or these variables can be generated online by the master station.

Texture perception

The perception of surface texture is a specific design issue in force feedback interfaces. Manipulation of everyday objects, the perception of surface texture is fundamental to accurately identifying contact points and applying the correct internal contact force. Also, in a virtual environment, haptic texture information can both increase the sense of realism of an object as well as convey information about what the object is and where it is. Phantom haptic device conveys texture by actuating kinaesthetic forces on the users fingers. In this work, we model the texture property as a periodic function

$$\tau_{\text{tex}} = \text{Amp}\,(\sin(2\pi f t) + 1)\,, \tag{14}$$

where Amp stands for half of the maximum value of texture torque; f, frequency in hertz; and t, time in seconds.

Shape perception

Shape is perceived by the normal contact force of an object. Thus, Equation (9) directly provides this perception in absence of roughness and texture.

Full-constrained Lagrangian

Substituting roughness and texture torques into (9), the full-constrained Lagrangian that provides all these effects arises as follows:

$$\begin{aligned} f_r = {} & \frac{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}{m\,\mathcal{J}_\varphi M(q)^{-1}\,\mathcal{J}_\varphi^T} \{-b\dot{\varphi}(x(q)) - k\varphi(x(q)) \\ & - m\,\dot{\mathcal{J}}_\varphi \dot{q} + m\,\mathcal{J}_\varphi M(q)^{-1}((B_0 + C(q,\dot{q}))\dot{q} \\ & + G(q) - \tau - \tau_{tex} + \tau_t)\}\,. \end{aligned} \tag{15}$$

Notice that (15) requires knowledge of PHANToM dynamics, its parameters and the assigned dynamics to the object. Variables $\mathcal{J}_\varphi(q)$ and $\varphi(x(q))$ are provided by the user, and $x(q)$ and $\dot{x}(q)$ by GHOST.

HAPTIC CARTESIAN EXPLORATION

For completeness, the 'Time base generator: time varying feedback gain $\alpha(t)$' and 'A stable switching algorithm' subsections are included, which can be reviewed in detail in Domínguez-Ramírez and Parra-Vega (2003c).

Free motion Cartesian control of the remote station

A Cartesian sliding PID control is proposed for the haptic guidance task in free movement. This control compensates the non-linear dynamics in continuous mechanical plants with tracking capability. Assume that initial condition and desired trajectories are within a space free of singularities. Consider the robot dynamics in closed loop with the controller given by

$$\tau = -K_d\left[\mathcal{J}^{-1}(q)\,\dot{X} - \hat{\mathcal{J}}^{-1}(q)\,\dot{X}_r\right] \tag{16}$$

for K_d as a $n \times n$ diagonal symmetric positive definite matrix. Then, for large enough gain K_d and small enough errors of initial condition, local exponential tracking is assured provided that

$$\begin{aligned} K_i \geq {} & \left\| \frac{d}{dt}\hat{\mathcal{J}}(q)\,\hat{S}_q + \hat{\mathcal{J}}(q)\frac{d}{dt}\hat{S}_q + \frac{d}{dt}\hat{\mathcal{J}}(q)\,\mathcal{J}(q)\,\dot{X}_r \right. \\ & \left. + \hat{\mathcal{J}}(q)\dot{\mathcal{J}}(q)\dot{X}_r \right\|, \end{aligned}$$

where $\hat{\mathcal{J}}^{-1}(q)$ stands as an estimate of $\mathcal{J}^{-1}(q)$ such that rank $(\hat{\mathcal{J}}^{-1}(q)) = n\ \forall\, q \in \Omega$, where $\Omega = \{q/\text{rank}\,(\mathcal{J}(q)) = n\}$. Let $\dot{X}_r$ be as follows:

$$\dot{X}_r = \dot{X}_d - \alpha\Delta X + S(t_0)\exp^{-k(t-t_0)} - K_i\sigma \tag{17}$$

and

$$\dot{\sigma} = \text{sgn}\left\{\Delta\dot{X} + \alpha\Delta X - S(t_0)\exp^{-k(t-t_0)}\right\}, \tag{18}$$

where $\Delta X = X - X_d$, $K_i = K_i^T \in \Re_+^{3\times3}$ and $\text{sgn}(x)$ as the discontinuous entry wise function *signum*, and $k > 0$ and feedback gains α are the diagonal positive definite (3×3) matrix.

Constrained motion Cartesian control of the remote station

Consider the following state feedback continuous control law:

$$\begin{aligned} \tau = {} & -K_d S_r + \frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}[-f_d + \eta\Delta F] \\ & + \gamma_2 \frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}\left[\text{sgn}\left(\mu S_{qf}\right) + \eta\int_{t_0}^{t}\text{sgn}\left(S_{qf}\right)\right], \end{aligned} \tag{19}$$

where the description of variables and gains are defined as follows. The nominal reference is given as

$$\dot{q}_r = \dot{q}_{rp} + \beta\frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}\left\{\Delta F - S_{df} + \gamma\int_{t_0}^{t}\text{sgn}\left(S_{qf}\right)\right\}, \tag{20}$$

where $\dot{q}_{rp}$ is the nominal reference of position; $\Delta F = \int_{t_0}^{t}(f_r - f_d)\varsigma d\varsigma$, $\beta > 0$, $\gamma > 0$; and $\text{sgn}\left(S_{qf}\right)$ is the function sign of vector S_{qf}. With the purpose to eliminate the calculation of the inverse kinematics, we have

$$\dot{q}_r = Q\mathcal{J}^{-1}(q)\,\dot{X}_r, \tag{21}$$

the new nominal reference Cartesian position $\dot{X}_r$, given as

$$\dot{X}_r = \dot{X}_d - \alpha\Delta X + S_{dp} - \gamma_p\int_{t_0}^{t}\text{sgn}\left(S_{qp}\right), \tag{22}$$

the new nominal reference with force and position, is

$$\begin{aligned} \dot{q}_r = {} & Q\mathcal{J}^{-1}(q)\left\{\dot{X}_d - \alpha\Delta X + S_{dp} - \gamma_p\int_{t_0}^{t}\text{sgn}\left(S_{qp}\right)\right\} \\ & + \beta\frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}\left\{\Delta F - S_{df} + \gamma_f\int_{t_0}^{t}\text{sgn}\left(S_{qf}\right)\right\}. \end{aligned} \tag{23}$$

Finally, S_r is given as

$$\begin{aligned} S_r = {} & Q\mathcal{J}^{-1}(q)\left\{\dot{X} - \dot{X}_r\right\} \\ & - \beta\frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi \mathcal{J}_\varphi^T\|}\left\{\Delta F - S_{df} + \gamma_f\int_{t_0}^{t}\text{sgn}\left(S_{qf}\right)\right\} \end{aligned} \tag{24}$$

$$S_r = Q\mathcal{J}^{-1}(q)\,S_{vp} - \beta\frac{\mathcal{J}_\varphi^T}{\|\mathcal{J}_\varphi\mathcal{J}_\varphi^T\|}S_{vf}, \tag{25}$$

where the extend orthogonalised manifolds of Cartesian force S_{vf} and position S_{vp} are defined as

$$S_{vp} = S_{qp} + \gamma_p\int_{t_0}^{t}\text{sgn}\left(S_{qp}\right) \tag{26}$$

$$S_{vf} = S_{qf} + \gamma_f\int_{t_0}^{t}\text{sgn}\left(S_{qf}\right) \tag{27}$$

and, finally, $Q(q) = I - \mathcal{J}_\varphi^T(q)(\mathcal{J}_\varphi(q)\mathcal{J}_\varphi^T(q))^{-1}$ stands for the orthogonal projection of the normal of a matrix $\mathcal{J}_\varphi \in \Re^{1\times 3}$, and on $\mathcal{J}_\varphi^T$.

Time base generator: Time-varying feedback gain $\alpha(t)$

An ill-posed *TBG* has been exposed in the context of non-linear mappings (see for instance, Morasso *et al*. 1997, and references therein). For completeness, we present the basics of a well-posed TBG (Parra-Vega 2001). Consider the following first order unforced time-varying linear differential equation:

$$\dot{z} + \alpha(t)\,z = 0 \rightarrow \dot{z} = \alpha(t)\,z, \tag{28}$$

where

$$\alpha(t) = \alpha_0\frac{\dot{\xi}}{(1-\xi)+\delta} \tag{29}$$

with $\alpha_0 = 1+\varepsilon$, $0 < \varepsilon << 1$, and $0 < \delta << 1$.

The *TBG* $\xi = \xi(t) \in C^2$ must be provided by the user so that ξ goes smoothly from 0 to 1 in finite time $t = t_b > 0$, and $\dot{\xi} = \dot{\xi}(t)$ is a bell-shaped derivative of ξ such that $\dot{\xi}(t_0) = \dot{\xi}(t_b) \equiv 0$. In this conditions, the solution of (28) is

$$z(t) = z(t_0)\,[(1-\xi)+\delta]^{1+\varepsilon} \tag{30}$$

and the gain $\alpha(t)$ is now well posed in contrast to Morasso *et al.* (1997), with $\alpha(t_b) > 0$. Note that t_b is independent of any initial conditions, and hence

$$\xi(t_b) = 1 \rightarrow z(t_b) = z(t_0)\,\delta^{1+\varepsilon} > 0 \tag{31}$$

can be made arbitrarily small in arbitrary finite time t_b. Also note that the transient of $z(t)$ is shaped by $\xi(t)$ over time. However, the extension into a non-linear equation does not apply directly; the main difficulty stems from the fact that TBG gain may grow unboundedly if it is introduced into a linear differential equation with a non-linear

Table 1 **Estimated dynamic haptic parameters**

Parameter	Value
m_a (kg)	17.5×10^{-3}
m_c (kg)	10.4×10^{-3}
m_{be} (kg)	0.2214
m_{df} (kg)	0.1106
l_1 (mm)	139.7
l_2 (mm)	139.7
l_3 (mm)	32.5
l_4 (mm)	36.8
l_5 (mm)	52.7
g (m/s^2)	9.81

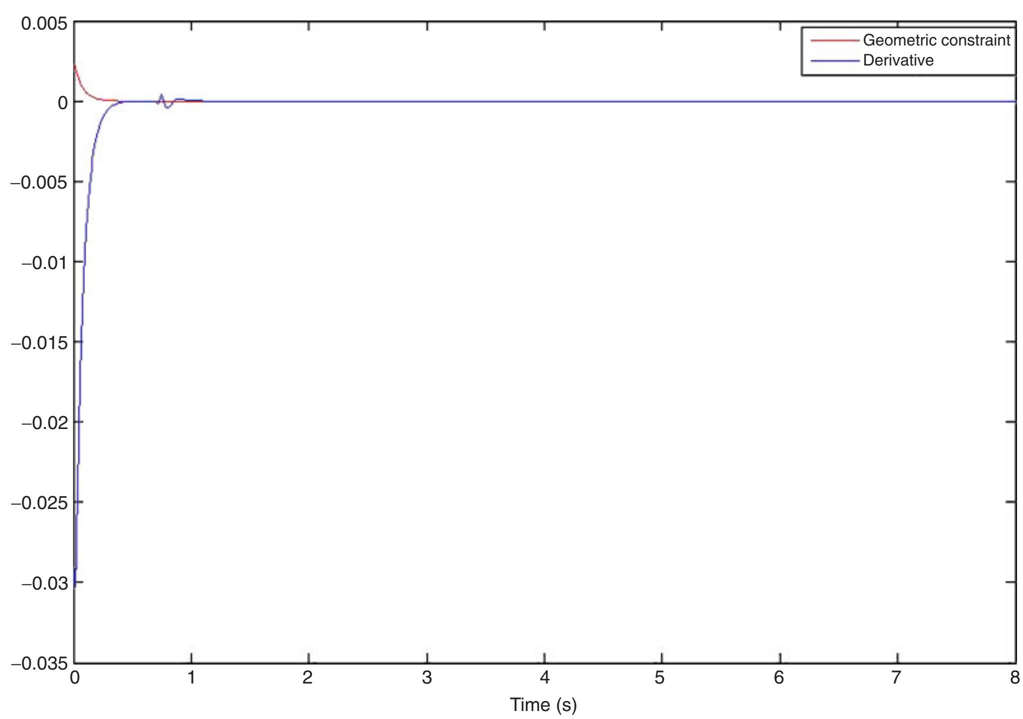

Figure 3 Geometric constraint and its derivative.

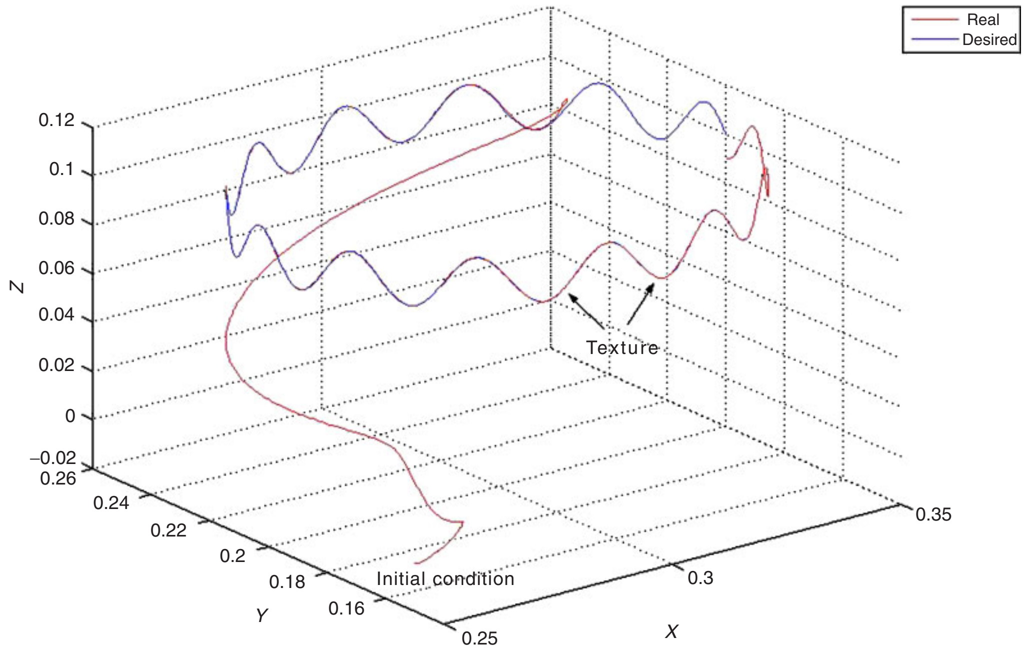

Figure 4 Cartesian workspace, real and desired.

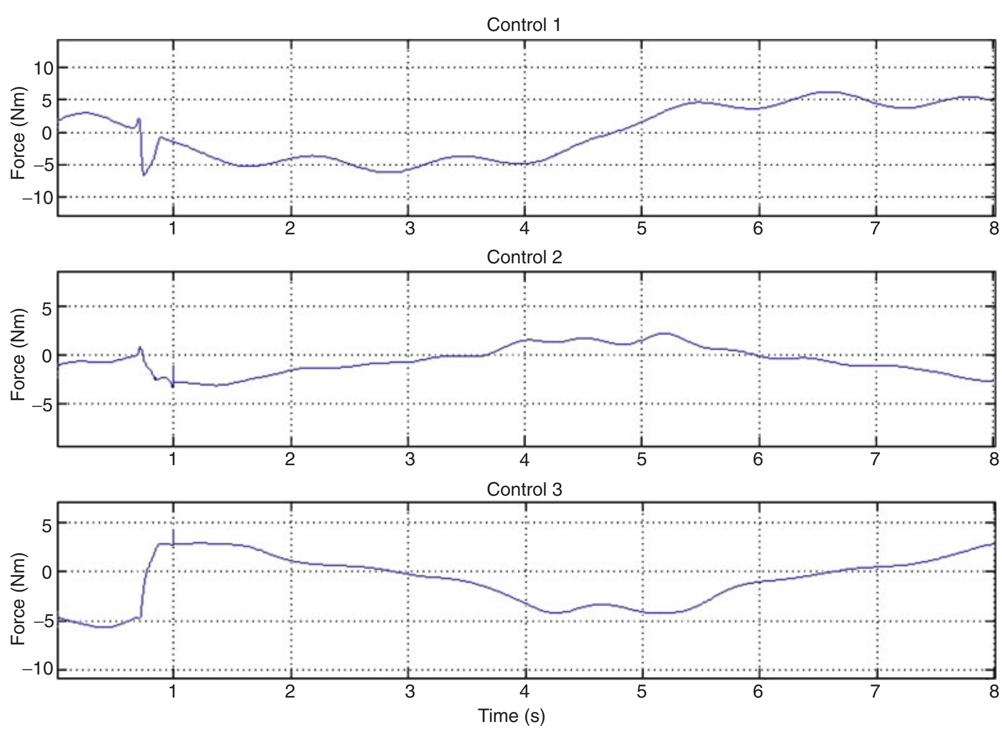

Figure 5 Joint control.

non-vanishing input, which eventually may lead to unstable dynamics. Thus, if our controller yields an equation similar to (28) (i.e. $\dot{z} + \alpha(t) z = 0$), for z, the position-tracking errors of the robot, then finite time convergence arises.

That is, we now are looking for a controller that guarantees $\dot{z} + \alpha(t) z = 0 \rightarrow S = \Delta\dot{X} + \alpha(t) \Delta X = 0$ for all time. And that is precisely what our previous article (Parra-Vega and Arimoto, 2001) provides, but with a constant α. Thus, in this article, we introduce a time-based generator $\alpha(t)$ to induce terminal attractors (Parra-Vega and Arimoto, 2001).

A stable switching algorithm

Remote exploration involves free and constrained motion, that is, at least two controllers are switching over time (it

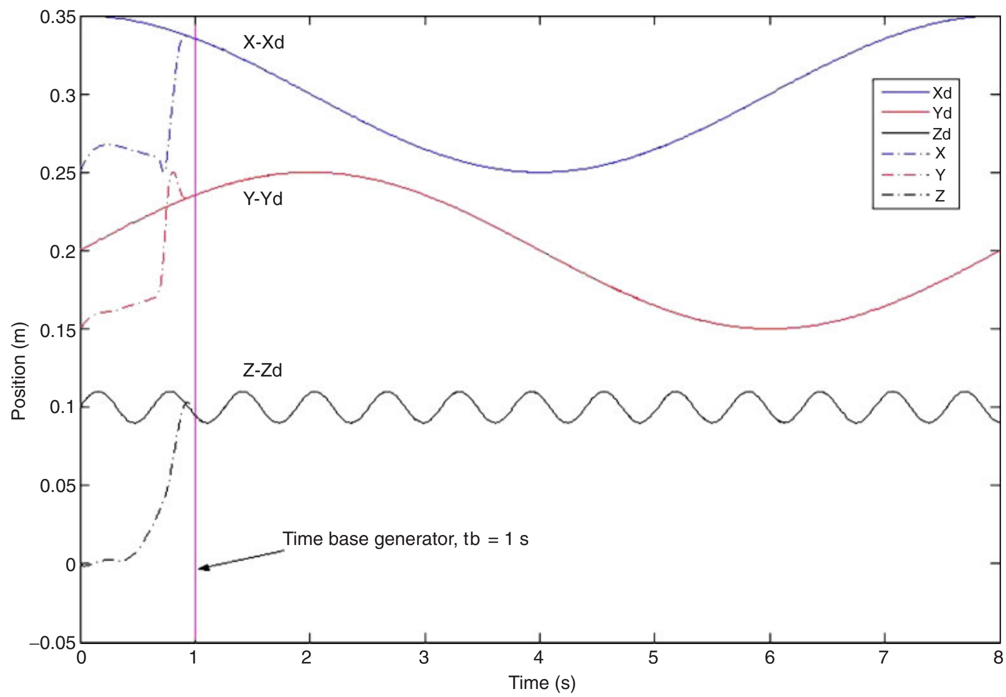

Figure 6 The Cartesian trajectories X, Y and Z, real and desired (tracking in $t_b = 1$ s).

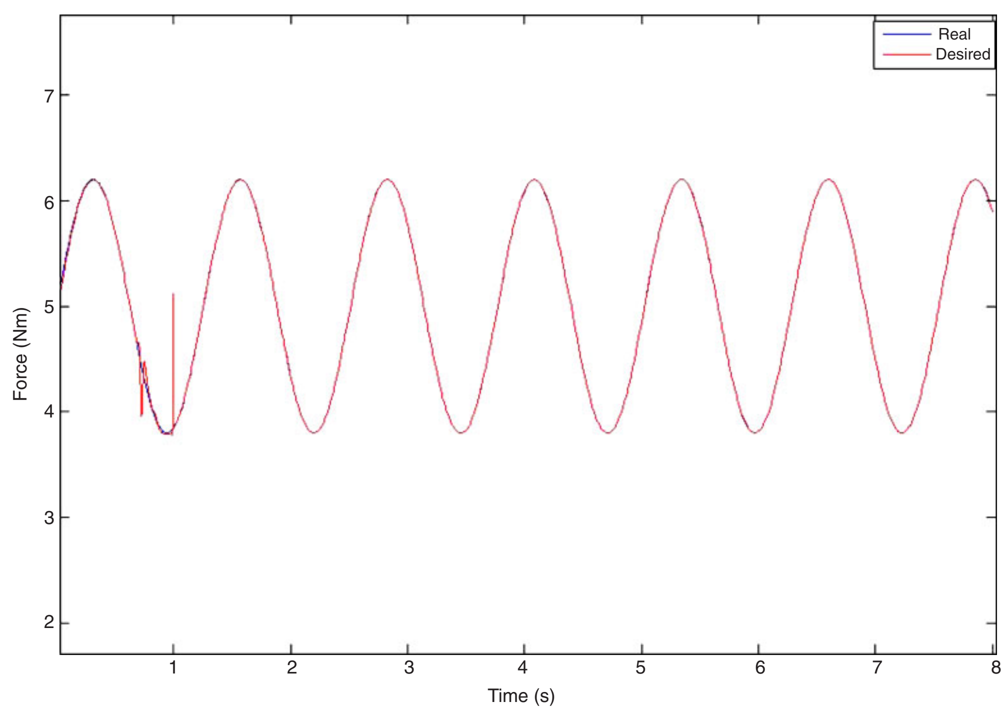

Figure 7 Contact force, real and desired (tracking in $t_b = 1$ s).

was shown that switching of the above controllers is stable). The algorithm is as follows:

Phase a (without interaction): $\varphi(q) > \varepsilon \rightarrow$ free motion control;
Phase b (collision detection: within a band a contact action is defined): $-\varepsilon \leq \varphi(q) \leq \varepsilon \rightarrow$ constrained motion control; and
Phase c (stable interaction with deformation): $\varphi(q) < -\varepsilon \rightarrow$ constrained motion control,

where $\varepsilon = 1 \times 10^{-6}\ m$. It can be seen that the application of the constrained Lagrangian method, in contrast to the penalty-based method, involves low frequencies over the virtual object. This allows a stable interaction, without trembling, for deformable objects.

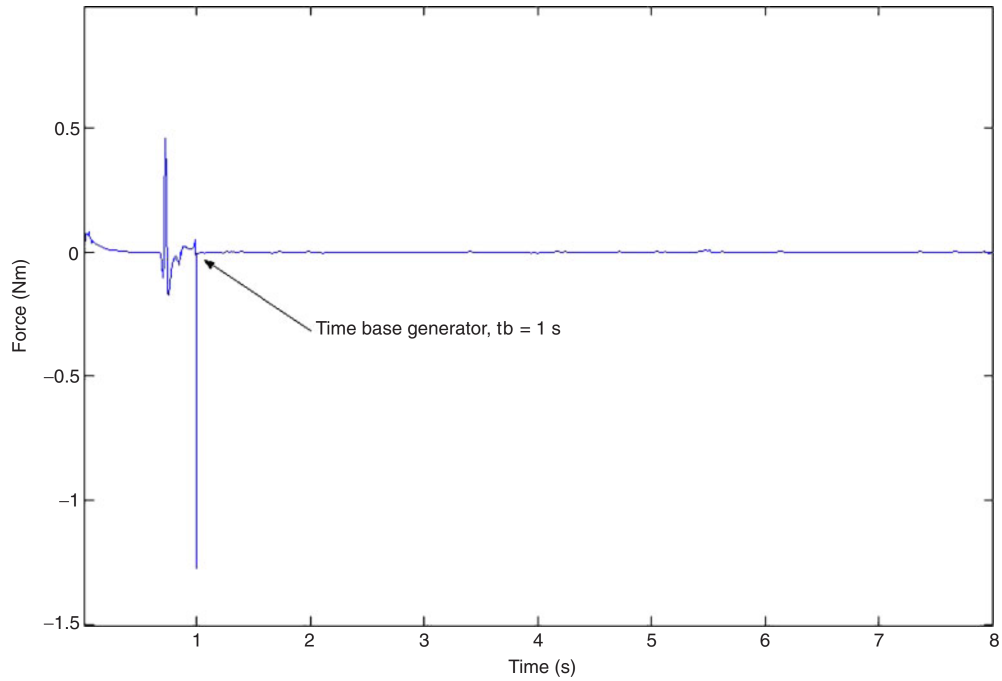

Figure 8 Contact force error (tracking in $t_b = 1$ s).

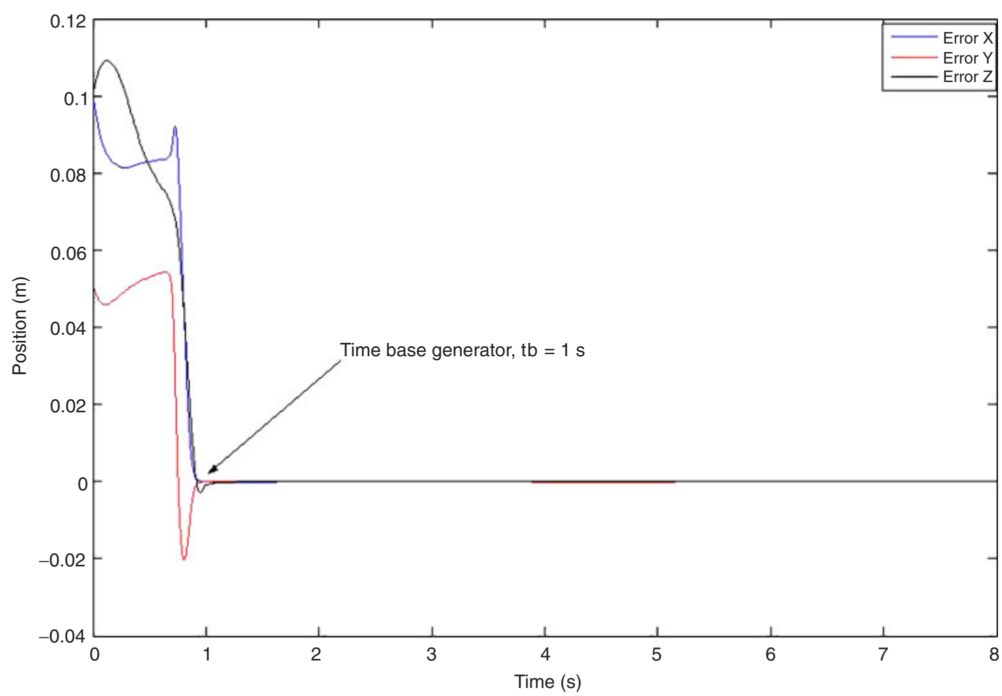

Figure 9 Cartesian error in X, Y and Z (tracking in $t_b = 1$ s).

SIMULATIONS

Digital simulations on the PHANToM haptic device model are presented (Domínguez-Ramírez and Parra-Vega 2003b). The stiff numerical solver of Matlab 7 was implemented. The PHANToM parameters are given in Table 1.

Desired Cartesian trajectories

The desired Cartesian trajectories of position and force are $x = h + r\cos(wt)$, $y = k + r\sin(wt)$, $z = z_1 + z_2\sin(w_2 t)$ and $f_d = f_1 + f_2\sin(w_3 t)$, where $h = 0.3$, $k = 0.2$, $r = 0.05$, $f_1 = 5$, $f_2 = 1.1$, $w = \pi/4$, $w_2 = 10$, $w_3 = 5$ and the tracking time for the timer base generator is $t_b = 1$.

Simulations results

The PHANToM in the remote station tracks the desired Cartesian trajectory without any knowledge of the PHANToM dynamics and without the inverse kinematics. After $t_b = 1$ the simultaneous force–position exponential tracking is established. Figures 3–9 show the PHANToM haptic device in constrained motion.

CONCLUSIONS

The Cartesian controller for force–position tracking with TBG for finite time tracking, without using model, has been proposed for the purposes of haptic guided exploration for remote training. A general framework based on constrained robot dynamics renders a Lagrangian-based contact force controller in a systematic way to produce shape, roughness and texture properties of the remote object under exploration. Even deformation of the object can be perceived. This is particularly appealing for medical applications. Numerical simulation results allow to verify the excellent closed loop performance, which in practice means high fidelity of the kinaesthetic coupling. Development is under way on a real PHANToM, version 1.0A, device, and preliminary experimental results confirm these conclusions.

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