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Oscillatory adaptive yaw-plane control of biorobotic autonomous underwater vehicles using pectoral-like fins

doi:10.1080/11762320801999746

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Abstract: This article considers the control of a biorobotic autonomous underwater vehicle (BAUV) in the yaw plane using biologically inspired oscillatory pectoral-like fins of marine animals. The fins are assumed to be oscillating harmonically with a combined linear (sway) and angular (yaw) motion producing unsteady forces, which are used for fish-like control of BAUVs. Manoeuvring of the BAUV in the yaw plane is accomplished by altering the bias (mean) angle of the angular motion of the fin. For the derivation of the adaptive control system, it is assumed that the physical parameters, the hydrodynamic coefficients, and the fin force and moment are not known. A direct adaptive sampled-data control system for the trajectory control of the yaw-angle using only yaw-angle measurement is derived. The parameter adaptation law is based on the normalised gradient scheme. Simulation results for the set point control, sinusoidal trajectory tracking and turning manoeuvres are presented, which show that the control system accomplishes precise trajectory control in spite of the parameter uncertainties.

Key words: Biorobotic autonomous underwater vehicle, pectoral fin control, adaptive control, yaw-plane control, sampled-data control, oscillatory control.

INTRODUCTION

Aquatic animals are marvellous swimmers and present a wide diversity of manoeuvring behaviours and hydrodynamic mechanism for their locomotion (Azuma 1992; Sfakiotakis 1999; Fish 2004; Walker 2004; Westneat *et al.* 2004). These swimming animals derive excellent manoeuvrability based on the principle of high-lift unsteady hydrodynamics using a variety of oscillating fins (dorsal, caudal, pectoral, etc.). Increasing demand for efficient manoeuvring of autonomous underwater vehicles has led researchers to investigate the potential for incorporating control surfaces resembling those of biological systems.

Presently, there exists considerable interest in designing flapping foils for propulsion and control of biorobotic autonomous underwater vehicles (BAUVs) (Triantafyllou and Triantafyllou 1995; Bandyopadhyay *et al.* 1997, 1999; Lauder and Drucker 2004; Bandyopadhyay 2005; Khan and Agrawal 2007). These biologically-inspired control surfaces can provide autonomous underwater vehicles (AUVs) with greater manoeuvrability as well as efficient propulsion. Research has been conducted on fish morphology, locomotion and application of biologically inspired control surfaces to rigid bodies (Yamamoto *et al.* 1995; Lauder and Drucker 2004; Triantafyllou *et al.* 2004; Westneat *et al.* 2004). Using experimental methods, extensive effort has been made to measure the forces and moments produced by oscillating foils (Yamamoto *et al.* 1995; Bandyopadhyay *et al.* 1999; Kato 2000; Triantafyllou *et al.* 2004). It has been observed that pectoral fins undergoing a combination of lead-lag, feathering and flapping motion have the ability to produce large lift, side force and thrust that can be used for the propulsion and control of AUVs. Computational methods have been used to characterise forces and moments produced by flapping foils (Ramamurti *et al.* 1996; Udaykumar *et al.* 2001; Mittal 2004). An analytical representation of the unsteady hydrodynamics of oscillating foils has been obtained using Theodorsen's theory (Harper *et al.* 1998). Extensive research has been carried out for controlling AUVs using traditional control surfaces (Yoerger and Slotine 1985; Healey and Lienard 1993; Fossen 1994). But it appears from the literature that only little theoretical research has been conducted to design control systems using pectoral fins (Kato 2000; Singh *et al.* 2004).

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An oscillating fin propulsion control system using neural network has been developed and tests have been carried out (Yamamoto *et al*. 1995). The guidance and control of a fish robot equipped with mechanical pectoral fins has been considered, and the rule-based fuzzy control system has been tested in laboratory experiments (Kato 2000, 2002). An adaptive control law for the control of undersea vehicles using dorsal fins in which the control force is generated by cambering the fin (Narasimhan and Singh 2006) has been considered. The optimal and inverse control laws have been designed for regulation, and depth and yaw angle trajectory control using mechanical pectoral fins (Singh *et al*. 2004; Narasimhan *et al*. 2006). For the derivation of these control laws, a parameterisation of periodic fin forces using the computational fluid dynamics (CFD) analysis has been obtained. But the pectoral fin control laws of Singh *et al*. (2004) and Narasimhan *et al*. (2006) have been derived on the assumption that the model parameters are completely known. This is rather a stringent requirement because, in a real case, the vehicle parameters and the hydrodynamic coefficients are not precisely known. Especially the precise knowledge of the forces and moments of unsteadily moving foils is extremely difficult. Furthermore, the parameterisation of the fin forces using the Fourier series of Singh *et al*. (2004) and Narasimhan *et al*. (2006) depends on the order of truncation of the Fourier expansion, and as such different input matrices are obtained as the additional harmonic functions are included in the series representation. Our recent article (Naik *et al*. 2007) has considered the design of an adaptive control system for the state feedback control of BAUVs using pectoral fins. From the practical point of view, the synthesis using state feedback is not attractive, because one must use sensors to measure each state variable. We point out that oscillating fins produce unsteady forces, and this gives rise to non-autonomous (time-varying) mathematical model of the BAUV. Apparently, the design of control systems for non-autonomous biorobotic systems with unknown parameters and limited measurement is indeed an interesting but challenging problem.

The contribution of this article lies in the design of an output feedback adaptive control system for the yaw-plane manoeuvring of a biorobotic AUV using biomimetic mechanism resembling pectoral fins of fish. The pair of fins attached to the AUV are assumed to undergo combined oscillatory linear (sway) and angular (yaw) motion, and consequently generate periodic forces and moments. In this article, the bias (mean) angle of the yaw motion of the fin is treated as a control variable. The model of the AUV considered here is similar to that of Singh *et al*. (2004) and Narasimhan *et al*. (2006), in which the fin forces and moments are parameterised using CFD analysis. For the purpose of design, it is assumed that the vehicle's physical parameters, the hydrodynamics coefficients, and the fin forces and moments are not known and that only the yaw angle is measured for feedback. In this article, a sampled-data adaptive control law is derived for the trajectory control of the yaw angle. The adaptation law for tuning the controller parameters is derived using the normalised gradient method. In the closed-loop system, the yaw angle asymptotically tracks time-varying reference trajectories, and all the signals in the closed-loop system remain bounded. Simulation results for the set point and sinusoidal trajectory control as well as for turning manoeuvres are presented. These results show that the adaptive control system accomplishes precise yaw-angle trajectory control in spite of the parameter uncertainties, and the yaw-angle trajectory remains close to the discrete reference trajectory between the sampling periods.

The organisation of this article is as follows. The AUV model and the problem formulation are presented in the 'AUV model and control problem' section. An adaptive law for yaw-angle control is derived in the section 'adaptive control law', and the section 'simulation results for yaw manoeuvres' presents the simulation results.

AUV MODEL AND CONTROL PROBLEM

Figure 1 shows the schematic diagram of a typical AUV. Two fins resembling the pectoral fins of fish are symmetrically attached to the vehicle. The vehicle moves in the yaw plane ($X_I - Y_I$ plane), where $O_I X_I Y_I$ is an inertial coordinate system. $O_B X_B Y_B$ is body-fixed coordinate system with its origin at the centre of buoyancy. X_B is in the forward direction. Each fin oscillates harmonically and produces unsteady forces. Use of two fins instead of one fin gives larger control force without increasing the size of the fin. Moreover, symmetric fin forces of the two fins allow yawing without rolling.

Fin force and moment

We assume that the combined sway–yaw motion of the fin is described as follows:

$$\delta(t) = \delta_m \sin(2\pi f t),$$
$$\theta_y(t) = \beta + \theta_{ym} \sin(2\pi f t + \nu), \tag{1}$$

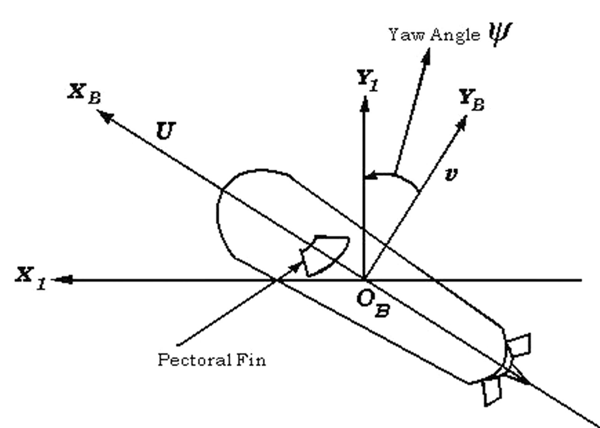

Figure 1 AUV model with pectoral fins.

where δ and θ_y correspond to sway and yaw angle of the fin, δ_{m} and θ_{ym} are the amplitudes of linear and angular oscillations β is the bias (mean) angle, f (Hz) is the frequency of oscillations and ν is the phase difference between the sway and yaw motion. It is pointed out that the forces and moments produced by the oscillating fins are functions of the oscillation parameters (bias angle, phase angle, amplitude and frequency), and these could be used as control inputs for the design of control system. The CFD analysis of Narasimhan *et al.* (2006) and the experimental study of Triantafyllou *et al.* (2004) show that the bias angle would be an appropriate parameter for control purposes. Motivated by this, we have chosen the bias angle as the primary control variable.

On the basis of the CFD analysis, it has been shown in Singh *et al.* (2004) and Narasimhan *et al.* (2006) that the periodic lateral force (f_{y}) and yawing moment (m_{y}) generated by the oscillating fin can be described by the Fourier series given by

$$f_{\mathrm{y}}(t) = \sum_{n=0}^{N} \left[f_n^s(\beta)\sin(n\omega_f t) + f_n^c(\beta)\cos(n\omega_f t) \right],$$
$$m_{\mathrm{y}}(t) = \sum_{n=0}^{N} \left[m_n^s(\beta)\sin(n\omega_f t) + m_n^c(\beta)\cos(n\omega_f t) \right], \tag{2}$$

where f_n^a and f_n^b, $a \in \{s, c\}$ are the Fourier coefficients, and N is an arbitrarily large integer such that the neglected harmonics have insignificant effect. (The control law designed here does not depend on N.) The Fourier coefficients are non-linear functions of the bias angle. Assuming that β is small, fin force and moment can be approximated as ($k = 1, 2, 3, \ldots$).

$$f_k^a(\beta) = f_k^a(0) + \left(\frac{\partial f_k^a(0)}{\partial \beta} \right)\beta,$$
$$m_k^a(\beta) = m_k^a(0) + \left(\frac{\partial m_k^a(0)}{\partial \beta} \right)\beta, \tag{3}$$

where a $\in \{s, c\}$. Defining a vector $\phi(t)$ of sinusoidal signals

$$\phi(t) = [1, \sin\omega_f(t), \cos\omega_f(t), \ldots, \sin N\omega_f(t), \times\cos N\omega_f(t)]^T \in R^{2N+1}, \tag{4}$$

and using Equations (2)–(4), one obtains

$$f_{\mathrm{y}}(t) = \phi^T(f_a + \beta f_b),$$
$$m_{\mathrm{y}}(t) = \phi^T(m_a + \beta m_b), \tag{5}$$

where f_a, f_b, m_a, and m_b are approximate vectors, which can be obtained from (2) and (3).

Yaw dynamics

We assume that vehicle's forward speed U is held constant by some control mechanism. The equations of motion of a neutrally buoyant vehicle is described by Fossen (1999)

$$m(\dot{v} + Ur + X_{\mathrm{G}}\dot{r} - Y_{\mathrm{G}}r^2) = Y_{\dot{r}}\dot{r} + (Y_{\dot{v}}\dot{v} + Y_r Ur) + Y_v Uv + F_{\mathrm{y}}$$
$$I_z\dot{r} + m(X_G\dot{v} + X_G Ur + Y_G vr) = N_{\dot{r}}\dot{r} + (N_{\dot{v}}\dot{v} + N_r Ur) + N_v Uv + M_y$$
$$\dot{\psi} = r \tag{6}$$

where ψ is the heading angle, $r = \dot{\psi}$ is the yaw rate, v is the lateral velocity along the Y_{B}-axis, $(X_{\mathrm{G}}, Y_{\mathrm{G}}) = (X_{\mathrm{G}}, 0)$ is the coordinate of the centre of gravity with respect to O_{B}, m is the mass and I_z is the moment of inertia of the vehicle. $Y_{\dot{v}}$, $N_{\dot{r}}$, Y_v, etc. are the hydrodynamic coefficients, and F_{y} and M_{y} are the net fin force and moment. The global position coordinates X and Y of the vehicle are described by the kinematic equations

$$\dot{X} = U\cos(\psi) - v\sin(\psi),$$
$$\dot{Y} = U\sin(\psi) + v\cos(\psi). \tag{7}$$

For small motion of the vehicle, linearising Equation (6) gives

$$\begin{bmatrix} m - Y_{\dot{v}} & mX_{\mathrm{G}} - Y_{\dot{r}} & 0 \\ mX_{\mathrm{G}} - N_{\dot{v}} & I_z - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v U & Y_r U - mU & 0 \\ N_v U & N_r U - mX_{\mathrm{G}}U & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} F_{\mathrm{y}} \\ M_{\mathrm{y}} \\ 0 \end{bmatrix} \tag{8}$$

Defining the state vector $x = (v, r, \psi)^T \in R^3$ and using (8) gives the state variable form

$$\dot{x} = Ax + B_v \begin{bmatrix} F_y \\ M_y \end{bmatrix} \tag{9}$$

where A and B_v are appropriate matrices. The net lateral force and moment due to two fins is given by $F_{\mathrm{y}} = 2f_{\mathrm{y}}$ and $M_{\mathrm{y}} = 2(d_{\mathrm{f}} \cdot f_{\mathrm{y}} + m_{\mathrm{y}})$, respectively, where d_{f} is the moment arm due to the fin location. Then substituting the fin force and moment from Equation (5) in Equation (9), gives the state variable representation

$$\dot{x} = Ax + B\Phi(t)f_c + B\Phi(t)f_v\beta,$$
$$y(t) = [0\ 0\ 1]x(t), = Cx(t), \tag{10}$$

where $y = \psi$ is selected as the controlled output variable, B is an appropriate matrix satisfying $B[f_{\mathrm{y}}, m_{\mathrm{y}}]^T = B_v[F_y, M_y]^T$, $f_c = (f_a^T, m_a^T)^T \in R^{4N+2}$, $f_v = (f_b^T, m_b^T)^T \in R^{4N+2}$ and

$$\Phi(t) = \begin{bmatrix} \phi^T(t) & 0 \\ 0 & \phi^T(t) \end{bmatrix}. \tag{11}$$

For the purpose of design, we assume that the system matrices A and B, and the parameter vectors f_c and f_v are not known. We assume that only the yaw angle is available for feedback. Let $y_m(t)$ be a given yaw-angle reference trajectory. We are interested in designing an adaptive control law such that in the closed-loop system, all the signals are bounded, and the yaw angle ψ asymptotically tracks $y_m(t)$.

ADAPTIVE CONTROL LAW

This section presents the derivation of an adaptive control law using output feedback. The system (10) is time-varying but periodic. The design of control system for a time-varying unknown system is not simple. Moreover, to obtain a meaningful use of the parameterisation of the fin force and moment using the CFD analysis, we proceed to design a sampled-data adaptive control system.

We assume that the bias angle changes at a regular interval $T = m_o T_o$, where m_o is an integer and $T_o = 1/f$ is the fundamental period. That is, the bias angle switches after the completion of m_o cycles of the oscillation of the fins and is kept constant between the switching instants. Discretising the state (10) yields a time-invariant system given by

$$\begin{aligned} x[(k+1)T] &= A_d x(kT) + B_d \beta_k + d_u \\ y(kT) &= C x(kT) \end{aligned} \tag{12}$$

where A_d, B_d and d_u are constant vectors, β_k (a constant) is the bias angle over $t \in [kT, (k+1)T)$, and $k = 0, 1, 2, \ldots$ We assume that the matrices A_d, B_d and d_u are unknown to the designer. Here we treat d_u as a constant disturbance input vector.

In the sequel, z denotes the z-transform variable or an advance operator (i.e. $zq(kT) = q[(k+1)T]$). Solving Equation (12), the output $y(z)$ can be written as follows:

$$\begin{aligned} y(z) &= C(zI - A_d)^{-1} B_d \beta_k(z) + C(zI - A_d)^{-1} d_u(z) \\ &\triangleq k_p \frac{n_p(z)}{d(z)} \beta_k(z) + \frac{n_d(z)}{d(z)} d_u(z) \end{aligned} \tag{13}$$

where $n_p(z)$ and $d(z)$ are monic polynomials of degree $m = 2$ and $n = 3$, respectively, and $n_d(z)$ is a polynomial. For the derivation of the control law, the following assumptions are needed:
Assumptions :

(A.1) $n_p(z)$ is a stable polynomial.
(A.2) The degree n of $d(z)$ is known.
(A.3) The sign of k_p and the upper bound k_p^o of $|k_p|$ is known.
(A.4) The relative degree $n^* = n - m > 0$ is known.
(A.5) The disturbance $d_u(t)$ is bounded.

For the vehicle model, $n_p(z)$ is a stable polynomial (i.e. both of its roots are strictly within the unit disk in the complex plane), but the denominator polynomial $d(z)$ is unstable. We point out that the stability of the polynomial $n_p(z)$ depends on the choice of fin location on the vehicle, and it is found that for small d_f (distance of point of attachment of fin to centre of buoyancy), the system is minimum phase. The relative degree of the system is 1; therefore, we choose a reference model of the form

$$y_m(kT) = W_m(z) r(kT), \quad k = 0, 1, 2, \ldots, \tag{14}$$

where $r(kT)$ is a discrete-time command input and

$$W_m(z) = \frac{1}{z} \tag{15}$$

is the delay operator (i.e. $y_m[(k+1)T] = r[kT]$).

First we consider the existence of the control law assuming that the system parameters are exactly known. Then this control law is modified for the case where the parameters are not known. The design of the control law follows the steps described in Tao (2003). Consider a control law

$$\begin{aligned} u^*(kT) = \beta^*(kT) &= \theta_1^{*T} \omega_1(kT) + \theta_{2a}^{*T} \omega_{2a}(kT) \\ &\quad + \theta_{20}^* y(kT) + \theta_3^* r(kT) + \theta_4^* \end{aligned} \tag{16}$$

where $\omega_1(kT) = a_\lambda(z)[u](t)$, $u(kT) = \beta(kT)$, and $\omega_{2a}(kT) = a_\lambda(z)[y](t)$ with $a_\lambda(z) = [z^{-n+1}, \ldots, z^{-1}]^T$, $n = 3$ and θ_1^*, $\theta_{2a}^* \in R^2$, and θ_3^*, θ_4^*, $\theta_{20}^* \in R$ are to be chosen properly. In the control law (16), the gain vectors are such that the transfer function from r to y is equal to $W_m(z)$, and θ_4^* asymptotically cancels the contribution of the disturbance d_u in the output. The signals $w_1(kT)$ and $w_{2a}(kT)$ in (16) are obtained as the states of the two filters

$$\begin{aligned} w_1[(k+1)T] &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} w_1(kT) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(kT) \\ &\triangleq A_0 w_1(kT) + b_0 u(kT), \end{aligned} \tag{17}$$

$$w_{2a}[(k+1)T] = A_0 w_{2a}(kT) + b_0 y(kT). \tag{18}$$

Then the closed-loop system (12), using Equations (17) and (18) is given by

$$\begin{aligned} &\begin{bmatrix} x[(k+1)T] \\ w_1[(k+1)T] \\ w_{2a}[(k+1)T] \end{bmatrix} \\ &= \begin{bmatrix} A_d + B_d \theta_{20}^* C & B_d \theta_1^{*T} & B_d \theta_{2a}^{*T} \\ b_0 \theta_{20}^* C & A_0 + b_0 \theta_1^{*T} & b_0 \theta_{2a}^{*T} \\ b_0 C & 0 & A_0 \end{bmatrix} \\ &\times \begin{bmatrix} x(kT) \\ w_1(kT) \\ w_{2a}(kT) \end{bmatrix} + \begin{bmatrix} B_d \\ b_0 \\ 0 \end{bmatrix} \theta_3^* r(kT) \end{aligned}$$

$$+\begin{bmatrix} B_d \\ b_0 \\ 0 \end{bmatrix}\theta_4^* + \begin{bmatrix} d_u \\ 0 \\ 0 \end{bmatrix}$$
$$\triangleq A_a X_c(kT) + B_a\theta_3^* r(kT) + B_a\theta_4^* + d_a,$$
$$y(kT) = [C \quad 0 \quad 0]X_c(kT) = C_a X_c(kT), \tag{19}$$

where

$$X_c(kT) = \left[x^T(kT), w_1^T(kT), w_{2a}^T(kT)\right]^T \in R^7.$$

Since the system (12) is controllable and observable, under assumption (A.1), there exist $\theta_1^*, \theta_{2a}^*, \theta_{20}^*$ and

$$\theta_3^* = (k_p)^{-1}, \tag{20}$$

satisfying (Tao 2003)

$$\theta_1^{*T} a_\lambda(z)d(z) + \left[\theta_{2a}^{*T}, \theta_{20}^*\right]\left[a_\lambda^T(z), 1\right]^T k_p n_p(z) = d(z) - n_p(z)z \tag{21}$$

such that

$$C_a(zI - A_a)^{-1}B_a\theta_3^* = W_\mathrm{m}(z), \tag{22}$$

and in the closed-loop system the output is given by

$$\begin{aligned} y(z) &= C_a(zI - A_a)^{-1}B_a\theta_3^* r(z) \\ &\quad + C_a(zI - A_a)^{-1}B_a\theta_4^*\left(\frac{z}{z-1}\right) \\ &\quad + C_a(zI - A_a)^{-1}d_a\left(\frac{z}{z-1}\right) \\ &\triangleq W_\mathrm{m} r(z) + \Delta(z), \end{aligned} \tag{23}$$

where $\rho^* = k_p$ and

$$\Delta(z) = [C_a(zI - A_a)^{-1}B_a\theta_4^* + C_a(zI - A_a)^{-1}d_a]\left(\frac{z}{z-1}\right).$$

Since matrix A_a is Schur and d_a is a constant vector, $\Delta(kT)$ asymptotically tends to a constant value given by

$$\begin{aligned} \Delta_\infty &= \lim_{k\to\infty}\Delta(kT) = \lim_{z\to 1}(z-1)\Delta(z) \\ &= C_a(-A_a)^{-1}[B_a\theta_4^* + d_a]. \end{aligned} \tag{24}$$

For cancelling the effect of the disturbance vector d_a on the output, there exists θ_4^* in Equation (24) such that $\Delta_\infty = 0$.

For the system with unknown parameters, the control law is chosen as

$$u(kT) = \theta_1^T(kT)\omega_1(kT) + \theta_2^T(kT)\omega_2(kT) + \theta_3^T(kT)r(kT) + \theta_4, \tag{25}$$

where $\omega_2(kT) = [w_{2a}^T, y]^T$, $\theta_1(kT) \in R^2$, $\theta_2(kT) = [\theta_{2a}^T \quad \theta_{20}]^T \in R^3$, and $\theta_3(kT), \theta_4(kT) \in R$ are time-varying estimates of $\theta_i^*, i = 1, \ldots, 4$. Define

$$\begin{aligned} \omega(kT) &= \left[\omega_1^T(kT), \omega_2^T(kT), y_m((k+1)T), 1\right],^T \\ \theta(kT) &= \left[\theta_1^T(kT), \theta_2^T(kT), \theta_3(kT), \theta_4\right],^T \\ e(kT) &= y(kT) - y_\mathrm{m}(kT), \tilde{\theta}(kT) = \theta(kT) - \theta^*. \end{aligned} \tag{26}$$

Using the control law (25), the closed-loop system takes the form

$$X_a[(k+1)T] = A_a X_c(kT) + B_a(\theta_3^* r(kT) + \theta_4^*) + d_a + B_a\tilde{\theta}^T(kT)w(kT), y = C_a X_a, \tag{27}$$

and now the output is

$$y(kT) = W_\mathrm{m}(z)r(kT) + \Delta(z) + \rho^* W_m(z)\tilde{\theta}^T(kT)w(kT). \tag{28}$$

$\Delta(kT)$ is an exponentially decaying signal by the choice of θ_4^*. Therefore, ignoring the decaying signal in Equation (28) gives the tracking error

$$\begin{aligned} e(kT) &= \rho^* W_\mathrm{m}(z)\tilde{\theta}^T(kT)w(kT) \\ &= -\rho^*(\theta^{*T}\omega((k-1)T)) \\ &\quad -\theta^T((k-1)T)w((k-1)T)). \end{aligned} \tag{29}$$

We are interested in deriving an adaptation law such that the tracking error asymptotically tends to be 0. We define estimation error as follows:

$$\epsilon(kT) = e(kT) + \rho(kT)\xi(kT), \tag{30}$$

where $\rho(kT)$ is an estimate of $\rho^* = k_p$ and

$$\begin{aligned} \xi(kT) &= \theta^T(kT)w((k-1)T) \\ &\quad -\theta^T((k-1)T)w((k-1)T), \end{aligned} \tag{31}$$

substituting Equations (29) and (31) into Equation (30), we obtain the error equation

$$\epsilon(kT) = \rho^*\tilde{\theta}^T(kT)w((k-1)T) + \tilde{\rho}(kT)\xi(kT) \tag{32}$$

where $\tilde{\rho}(kT) = \rho(kT) - \rho^*$. This error equation is linear in the parameter errors $\tilde{\theta}^T(kT)$ and $\tilde{\rho}(kT)$. This equation is important for the derivation of the adaptation law.

Now following Tao (2003), the normalised gradient based control law is chosen as

$$\begin{aligned} \theta((k+1)T) &= \theta(kT) - \frac{\mathrm{sign}(k_p)\Gamma w((k-1)T)\epsilon(kT)}{m^2(kT)}, \\ \rho((k+1)T) &= \rho(kT) - \frac{\gamma\xi(kT)\epsilon(kT)}{m^2(kT)} \end{aligned} \tag{33}$$

where the symmetric positive definite adaptation gain matrix Γ satisfies $0 < \Gamma = \Gamma^T < \frac{2}{k_p^o} I_{7\times 7}$, $0 < \gamma < 2$, and

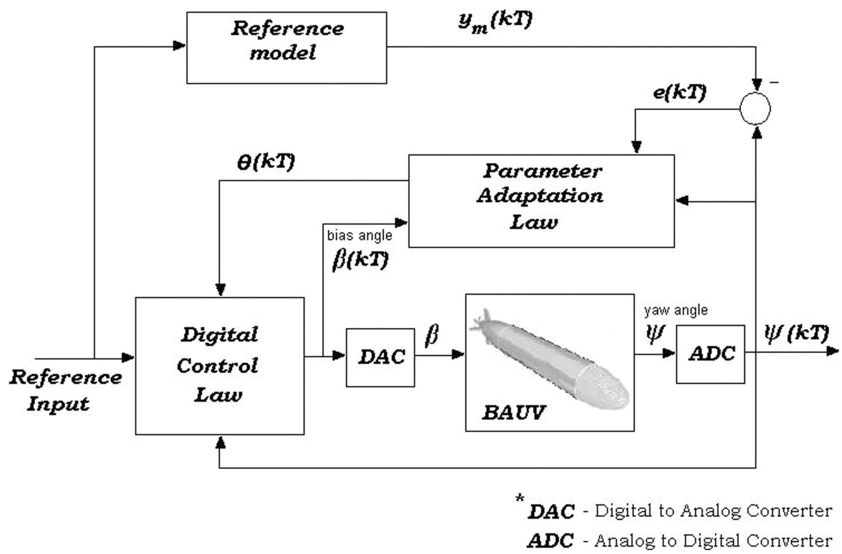

Figure 2 The complete closed-loop system with parameter adaptation.

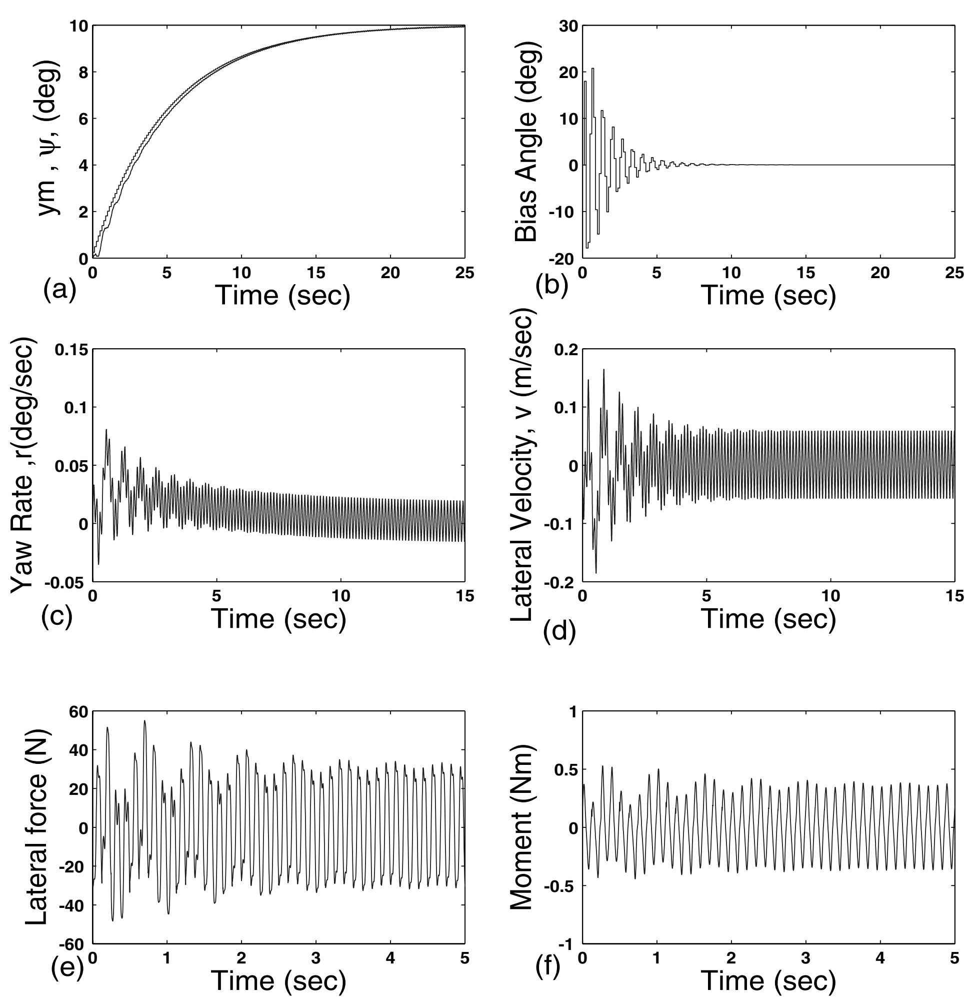

Figure 3 Adaptive set point control: Frequency of flapping = 8 Hz for $\psi^* = 10(^\circ)$ and parameter uncertainty = 50%; (a) Yaw angle, ψ (solid) and reference yaw angle (staircase)(°); (b) Bias angle (°); (c) Yaw rate (°/s); (d) Lateral velocity (m/s); (e) Lateral force (N); and (f) Moment (Nm). Note: For clarity a different time scale is used for each plot.

$$m^2(kT) = 1 + w^T((k-1)T)w((k-1)T) + \xi^2(kT).$$

For the stability analysis, one chooses the Lyapunov function

$$V(\tilde{\theta}, \tilde{\rho}) = |\rho^*|\tilde{\theta}^T(kT)\Gamma^{-1}\tilde{\theta}(kT) + \gamma^{-1}\tilde{\rho}^2(kT) \quad (34)$$

and following Tao (2003) shows that

$$V((k+1)T) - V(kT) \leq -\alpha_1 \frac{\epsilon^2(kT)}{m^2(kT)}, \quad (35)$$

where $\alpha_1 > 0$. This implies that $\theta(kT), \rho(kT), \frac{\epsilon(kT)}{m(kT)} \in L^\infty$ (the set of bounded functions), and

$$\frac{\epsilon(kT)}{m(kT)}, (\theta((k+i_0)T) - \theta(kT)), (\rho((k+i_0)T) - \rho(kT)) \in L^2$$

(the set of square summable functions) for any integer $i_0 > 0$. Furthermore, one can show that $e(kT) \to 0$ and all the signals in the closed-loop system are bounded. This completes the derivation of the adaptive control law for the yaw-plane manoeuvring.

Figure 2 shows the complete closed-loop system with parameter adaptation law. It clarifies how the different modules of the control system for BAUV are connected. The command input signal is applied to the reference model. The digital controller provides the discrete bias angle control input according to Equation (25). The control output from the controller being discrete is converted to analog before applying to the BAUV. Furthermore, the bias angle along with yaw angle and output error is used to tune the parameters of the controller using gradient algorithm. Thus the combined system works harmoniously and controls the BAUV in the yaw plane.

SIMULATION RESULTS FOR YAW MANOEUVRES

In this section, simulation results using the MATLAB/SIMULINK for yaw-angle control are presented. Various time-varying reference trajectories are considered

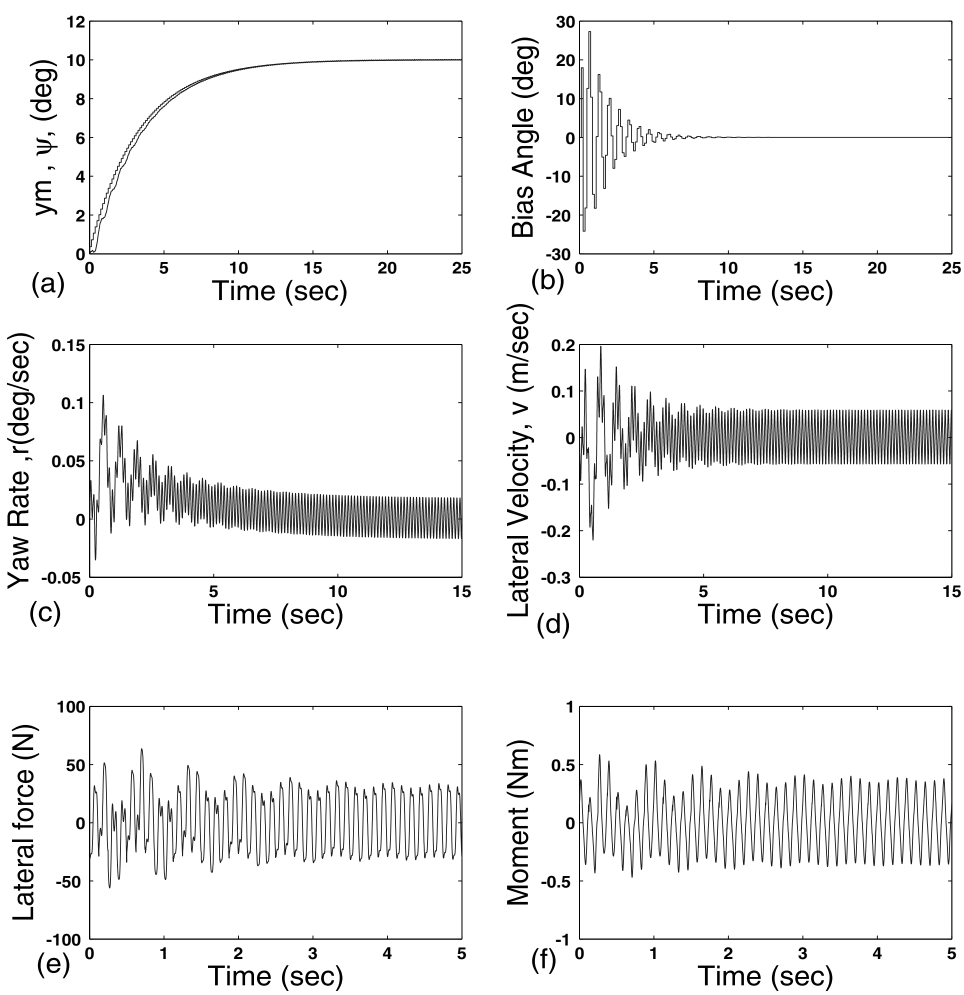

Figure 4 Adaptive set point control: Frequency of flapping = 8 Hz for $\psi^* = 10(^\circ)$, parameter uncertainty = 50% and faster command; (a) Yaw angle, ψ (solid) and reference yaw angle (staircase)(○); (b) Bias angle (○); (c) Yaw rate (○/s); (d) Lateral velocity (m/s); (e) Lateral force (N); and (f) Moment (Nm).

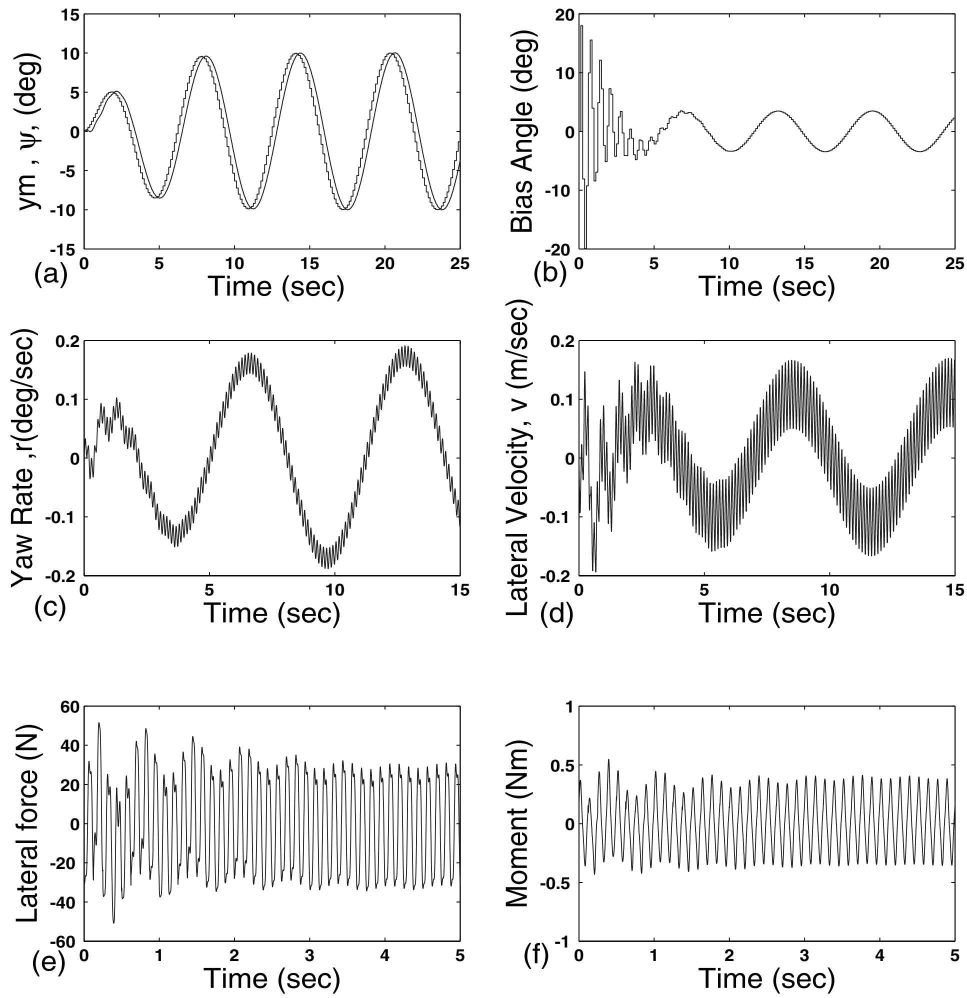

Figure 5 Adaptive sinusoidal trajectory control: Frequency of flapping = 8 Hz for $y_m = 10 \sin 2\pi fkT$ (°) and parameter uncertainty = 50%; (a) Yaw angle, ψ (solid) and reference yaw angle (staircase)(°); (b) Bias angle (°); (c) Yaw rate (°/s); (d) Lateral velocity (m/s); (e) Lateral force (N); and (f) Moment (Nm).

for tracking, and the performance of the adaptive controller in the presence of parameter uncertainties is examined. It is pointed out that the derived adaptive control system is applicable to any minimum-phase BAUV model; and of course the choice of fin location can be selected to obtain a minimum-phase model. However, for the purpose of illustration, the fin forces and moments obtained by CFD analysis of Narasimhan *et al.* (2006) and the model parameters of Ridley *et al.* (2003) are used for simulation. But the designed adaptive control system can be used to other vehicle models equipped with oscillating fins having different oscillation parameters (amplitude, phase and frequency). The AUV is assumed to be moving with a constant forward velocity of 0.7 m/s.

The vehicle parameters are $l = 1.391$ m, mass = 18.826 kg, $I_z = 1.77$ kgm^2, $X_G = -0.012$, $Y_G = 0$. The hydrodynamic parameters for a forward velocity of 0.7 m/s derived from Ridley *et al.* (2003) are $Y_{\dot r} = -0.3781$, $Y_{\dot v} = -5.6198$, $Y_r = 1.1694$, $Y_v = -12.0868$, $N_{\dot r} = -0.3781$, $N_{\dot v} = -0.8967$, $N_r = -1.0186$ and $N_v = -4.9587$. It is assumed that $d_f = 0.01$ m and the fin oscillation frequency is $f = 8$ Hz. The vectors f_a, f_b, m_a and m_b are found to be (Narasimhan *et al.*, 2006)

$$\begin{aligned}
f_a &= (0, -40.0893, -43.6632, -0.3885, 0.6215,\\
&\quad 6.2154, -10.17, -0.1554, 0.6992)\\
f_b &= (68.9975, 0.4451, -16.4704, 64.1009, -19.5864,\\
&\quad -0.8903, -2.2257, 2.2257, 4.8966)\\
m_a &= (0.0054, 0.6037, 0.4895, 0, -0.0054, 0,\\
&\quad -0.0925, 0, -0.0054)\\
m_b &= (-0.5297, -0.3739, -0.0935,\\
&\quad -0.2493, 0.1246, 0.0312, -0.0312, 0.0935, 0)
\end{aligned}$$

It is pointed out that these parameters are obtained using the Fourier decomposition of the fin force and moment, and are computed by multiplying the Fourier coefficients by $\frac{1}{2}\rho . W_a . U_\infty{}^2$ and $\frac{1}{2}\rho . W_{a.chord} . U_\infty{}^2$, respectively, where W_a is the surface area of the foil. For simulation, the initial conditions of the vehicle are assumed to be $x(0) = 0$.

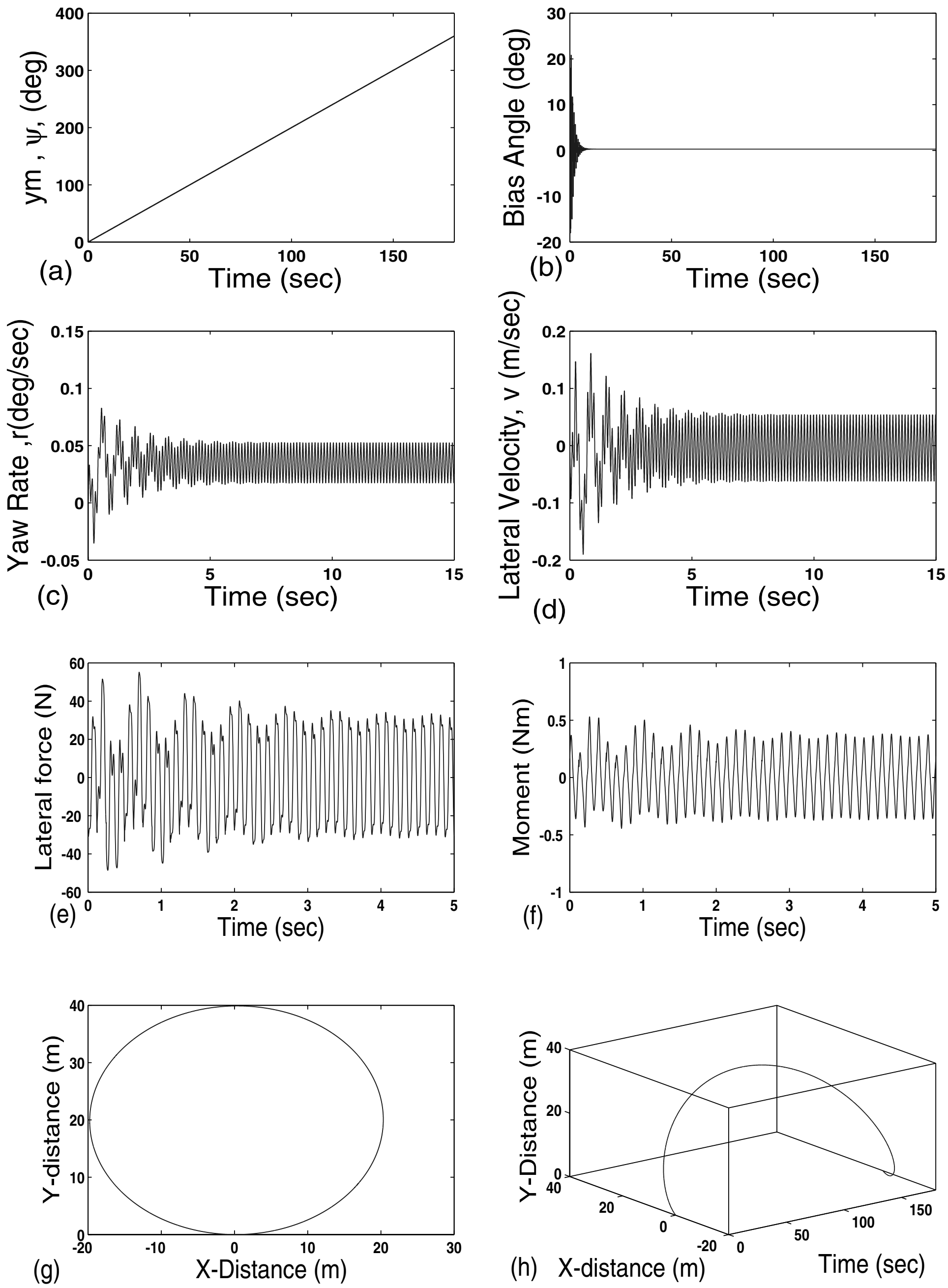

Figure 6 Adaptive turning manoeuvre: Turn rate of 2°/s and parameter uncertainty = 50%; (a) Yaw angle, ψ (solid) and reference yaw angle (staircase)(○); (b) Bias angle (○); (c) Yaw rate (○/s); (d) Lateral velocity (m/s); (e) Lateral force (N); (f) Moment (Nm); (g) Global position of the BAUV (X and Y coordinates); and (h) Three-dimensional plot of X and Y distances with time.

The closed-loop system (10) and (25) with the update law (33) is simulated. The bias angle is changed to a new value every $T = T_o$ seconds, where $T_o = 1/f$ is the fundamental period of f_p and m_p. For the set point control, the terminal value of the yaw angle is taken as $\psi^* = 10°$. Thus one desires to control the BAUV to a heading angle of 10°. For the update law, the adaptation gains are selected as $\Gamma = 0.0001(2/k_p^o)I_{7\times7}$ and $\gamma = 0.002$; where $k_p^o = 0.08 \geq |k_p|$. Using the values of AUV model, it is found that the actual feedback gains satisfying Equations (20) and (21) for model matching are $\theta_1^* = (0.4195 - 0.4323)^T$, $\theta_2^* = (99.0671 - 303.9286\ 309.4949)^T$, $\theta_3^* = -104.6334$ $\theta_4^* = 0.0001$ and $\rho^* = k_p = -0.0096$. Of course, these gains (θ_i^*, ρ^*) cannot be computed because the BAUV model parameters are not known; and therefore the adaptation law (33) is used to obtain the estimates of these for synthesis. The open-loop zeros and poles of the discretised system are (−0.8990, 0.4667) and (1.0000, 1.0864, 0.8715), respectively. Therefore, the transfer function is minimum-phase.

Case A: Adaptive set point control: Parameter uncertainty 50% off-nominal for yaw angle 10°

For smooth control, the reference input $r(kT)$ (in rad) is selected as

$$r(kT) = [1 - \exp(-0.2(k)T)]10\pi/180,$$

where the sampling time is $T = 0.125$ s. Thus the control law is updated at the completion of each cycle of oscillation. To show the robustness of the adaptive control system, off-nominal values of the controller gains (which differ from the actual feedback gains θ_i^*, ρ^*) are used as an initial estimate of the parameters $(\theta(kT), \rho(kT))$ in the adaptation law (33), but the BAUV model with actual parameters is simulated to examine the performance of the adaptive control system. Assuming 50% uncertainty, the initial estimates $\theta(0)$ and $\rho(0)$ are set to $0.5\theta^*$ and $0.5\rho^*$. This way, the control law gains are 50% lower than the exact θ^*. Figure 3 shows the simulated results. It can be seen that the adaptive controller achieves accurate heading angle control to the target set point in about 23 s. The control input (bias angle) magnitude required is around 20°, which can be provided by the pectoral fins. The plots of the lateral force and moment produced by the fins are also provided in the figure. In the steady-state, the lateral fin force and moment exhibit bounded periodic oscillations, as one would have expected.

Case B: Adaptive set point control: Parameter uncertainty 50% off-nominal for Yaw angle 10° but for a faster command

For this case, the reference input $r(kT)$ (in rad) is selected as

$$r(kT) = [1 - \exp(-0.3(k)T)]10\pi/180,$$

which is faster than the previous command. The control parameters of Figure 3 are retained for simulation. Simulated results are shown in Figure 4. It is seen that the response time is only about 10–12 s. But the control input (28°) required to track this faster command trajectory is larger compared with Case A for slow command as expected.

Case C: Adaptive sinusoidal trajectory control: Parameter uncertainty 50 % off-nominal

To examine the time-varying tracking ability of the controller, a sinusoidal reference trajectory is generated using the command input $r(kT) = 10\times\ (\pi/180)\ [1 - \exp(-0.4(k)T)]\ \sin(kT)$ (rad). It is assumed that $\theta(0) = 0.5\theta^*$ and $\rho(0) = 0.5\rho^*$ giving 50% uncertainty. The responses are shown in Figure 5. It is seen that, after the initial transients, the heading angle smoothly tracks the sinusoidal command trajectory with a minor tracking error. The control input (bias angle) magnitude required is 20°.

Case D: Adaptive turning manoeuvre: Parameter uncertainty 50% off-nominal

The turning manoeuvre is an important practical manoeuvre that BAUVs frequently need to perform. For constant turning rate, a smooth trajectory is generated using the command input $r(kT) = 2kT\ (\pi/180)$ rad. As seen in Figure 6 the trajectory tracked by the system is almost a circle due to the small magnitude of the time-varying lateral velocity. (The segments of the intersample yaw trajectory remain close to the circle.) This requires a control input magnitude of 20° and less than 200 s to make a complete circle. It is possible to have a faster turning rate, however, that requires larger control forces.

Simulations for other off-nominal (lower and higher) choices of estimates of $(\theta(0), \rho(0))$ have been performed. (The plots are not shown here in order to save space.) It is found that the control system performs relatively well for the choice of underestimated initial values of the adaptive gains $(\theta(0), \rho(0))$. Of course, the responses also depend on the choice of the command generator and the adaptation gain matrix Γ and γ of the update law.

CONCLUSION

In this article, the design of an adaptive control system for the yaw-plane control of a BAUV using biologically-inspired pectoral-like fins was considered. The periodic fin forces and moments produced by the oscillating fins were parameterised using CFD analysis. The system parameters were assumed to be unknown. The bias angle was treated as the control input. A sampled-data adaptive control law was derived for the control of the yaw-angle using only the yaw-angle feedback. The control system included a normalised gradient adaptation law for tuning the controller gains. In the closed-loop system, it was shown that the yaw angle asymptotically follows the prescribed time-varying yaw-angle trajectories. The designed output feedback adaptive control system was simulated for various types of yaw-plane manoeuvres. The simulations also showed that using pectoral fins, one can perform precise and rapid manoeuvres in the presence of model uncertainties using a single sensor. This is important because in a real situation, one does not have the knowledge of the system parameters. Especially, the precise characterisation of the pectoral fin forces and moments is not easy. Flexibility exists in the choice of the design parameters, which can be selected to obtain desirable response characteristics in the closed-loop system.

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