Slip effects on peristaltic transport in an inclined channel with mass transfer and chemical reaction

T. Hayat, Humaira Yasmina, S. Asghar and Awatif A. Hendi

Abstract. An analysis is carried out for the peristaltic flow in an inclined asymmetric channel when no-slip condition does not hold. The whole analysis has been carried out in the presence of mass transfer and chemical reaction. The channel asymmetry is generated because of peristaltic wave train on the walls through different amplitudes and phases. Long wavelength and low Reynolds number assumption is adopted in the whole mathematical analysis. Expressions for the stream function and longitudinal pressure gradient have been developed. Numerical integration is performed for the analysis of pressure rise per wavelength. Longitudinal velocity, pumping and trapping phenomena are analyzed in detail via plots.

Keywords: Slip condition, nonlinear analysis and inclined asymmetric channel

1. Introduction

Peristaltic flows have attracted the recent researchers because of their relevance in many engineering and physiological processes. To be more specific such processes include urine transport from kidney to bladder, chyme movement in the gastrointestinal tract, food stuffs through esophagus, spermatocoria in the ductus afferents of male reproductive tract, vasomotion of small blood vessels etc.

The peristaltic flows also have indispensable role in heart lung machines, sanitary and corrosive fluids transport and locomotion. Latham [1] carried out the experimental work on peristaltic transport initially. Since then a rather ample literature is dedicated to peristaltic flows under various aspects. Kothandapani and Srinivas [2] have discussed the peristaltic transport of Jeffrey fluid under the effect of magnetic field in an asymmetric channel. In ref. [3], they discussed the effects of heat and mass transfer on peristaltic flow with compliant walls. Abd elmaboud and Mekheimer [4] studied the peristaltic motion of second order fluid in a porous medium. Mekheimer and Abd elmaboud [5] examined the endoscopic effects on the peristalsis of couple stress fluid. Effect of heat transfer on the MHD peristaltic flow of Newtonian fluid in a vertical annulus was studied by Mekheimer and Abd elmaboud [6]. The blood flow in a tapered artery in the presence of magnetic field is analyzed by Hayat et al. [8]. Hayat and Abbasi [9] described the peristaltic flow with variable viscosity. Tripathi et al. [10] examined the peristaltic flow of fractional viscoelastic
fluid. Pandy and Tripathi [11] analyzed the influence of magnetic field on the peristaltic flow of viscous fluid in a finite length cylindrical tube. Peristaltic flow in an inclined channel with slip effect is described by Srini-vas and Muthuraj [12]. Ali et al. [13] pointed out the slip effects on the peristalsis in MHD viscous fluid having variable viscosity. The peristaltic flow of Jeffrey fluid in an asymmetric channel with slip and induced magnetic field is studied by Nadeem and Akram [14]. Hayat et al. [15] examined the influence of heat transfer on the peristaltic flow with slip condition. Note that the slip flow is significant when the fluid has adhesion loss at the wetted wall making the fluid slide along the wall. Derek et al. [16] have experimentally shown that the fluid possesses non-continuum features such as slip flow when the molecular mean free path length of the fluid is comparable to the distance between the plates as in nanochannels/microchannels [17].

The present paper discusses the peristaltic flow of fourth grade fluid in an inclined asymmetric channel with mass transfer and chemical reaction. Analysis has been carried out in the presence of slip condition. In fact, the processes with mass transfer and chemical reaction effects are quite significant in chemical equipment. The mass transfer has indispensable role in several industrial applications including the polymer production and manufacturing of ceramics/glassware. In view of such motivation, the present study is arranged as follows. Next section deals with the problem statement. Solution of the arising nonlinear problem is presented in section three for small Debo-rah number and long wavelength analysis. Results and discussion are given in section four whereas section five consists of main conclusions.

2. Governing problem

Consider peristaltic flow of an incompressible fourth grade fluid through an asymmetric channel inclined at an angle $\alpha$ to the horizontal (see Fig. 1). We select $\bar{X}$ and $\bar{Y}$ axes along and perpendicular to the channel walls respectively. The flow induced is because of sinusoidal waves propagating with speed $c$. The wave shapes are

![Fig. 1. Geometry of the problem.](image-url)
\[ \dot{H}_1(\bar{x}, \bar{t}) = d_1 + a_1 \sin \left( \frac{2\pi}{\lambda}(\bar{x} - \bar{c}) \right), \text{ upper wall}, \]
\[ \dot{H}_2(\bar{x}, \bar{t}) = -d_2 - a_2 \sin \left( \frac{2\pi}{\lambda}(\bar{x} - \bar{c}) + \phi \right), \text{ lower wall}. \]  

(1)

In above expressions \( a_i (i = 1, 2) \) are the wave amplitudes, \( \lambda \) the wavelength, \( d_1 + d_2 \) is the width of the channel, the phase difference \( \phi \) varies in the range \( 0 \leq \phi \leq \pi \) (\( \phi = 0 \) corresponds to symmetric channel with waves out of phase and \( \phi = \pi \) the waves are in phase) and further \( a_1, a_2, d_1, d_2 \) and \( \phi \) obey the following relation
\[ a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \leq (d_1 + d_2)^2. \]  

(2)

We further have \( C_1 \) and \( C_2 \) as the respective concentration fields at the lower and upper walls respectively.

\[ \bar{S} = \mu \bar{A}_1 + a_1 \bar{A}_2 + a_2 \bar{A}_3 + \beta_1 \bar{A}_4 + \beta_2 \bar{A}_5 \bar{A}_1 + \bar{A}_6 \bar{A}_2 \]
\[ + \beta_3 \bar{A}_7 \bar{A}_1 + \gamma_1 \bar{A}_8 + \gamma_2 \bar{A}_9 \bar{A}_1 + \gamma_3 \bar{A}_3^2 \]
\[ + \gamma_4 \bar{A}_4 \bar{A}_2 + \gamma_5 \bar{A}_6 \bar{A}_3 + \gamma_6 \bar{A}_7 \bar{A}_2 \]
\[ + \gamma_7 \bar{A}_7 \bar{A}_3 + \gamma_8 \bar{A}_8 \bar{A}_2 + \gamma_9 \bar{A}_8 \bar{A}_3 \]
\[ + \gamma_10 \bar{A}_9 \bar{A}_1 \bar{A}_1, \]  

(3)

The flow situation in absence of body forces can be described by the following equations
\[ \text{div} \, \bar{V} = 0, \]  

(4)

\[ \rho \frac{\partial \bar{V}}{\partial \bar{t}} = -\text{grad} \, \bar{p} + \text{div} \, \bar{S}, \]

\[ \bar{A}_1 = (\text{grad} \, \bar{V}) + (\text{grad} \, \bar{V})^T, \]
\[ \bar{A}_e = \frac{d\bar{A}_{e-1}}{d\bar{t}} + \bar{A}_{e-1} (\text{grad} \, \bar{V}) + (\text{grad} \, \bar{V})^T \bar{A}_{e-1}, n > 1. \]  

(5)

\[ \frac{\partial \bar{C}}{\partial \bar{t}} + U \frac{\partial \bar{C}}{\partial \bar{x}} + V \frac{\partial \bar{C}}{\partial \bar{y}} = D \left[ \frac{\partial^2 \bar{C}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \right] - \kappa \bar{C}, \]  

(6)

where \( \bar{V} \) is the velocity, \( C \) the concentration of the fluid, \( \bar{p} \) the pressure, \( \rho \) density of the fluid, \( \frac{\partial}{\partial \bar{t}} \) is the material time derivative, \( \bar{S} \) the extra stress tensor, \( D \) the concentration parameter, \( k_1 \) the chemical reaction parameter and \( \bar{t} \) refers the dimensional quantity.

The velocity \( \bar{V} \) and extra stress tensor \( \bar{S} \) in a fourth grade fluid can be written as
\[ \bar{V} = (\bar{U}, \bar{X}, \bar{Y}, \bar{V}(\bar{X}, \bar{Y}, \bar{t}, 0))^T, \]  

(7)

where \( \mu \) denotes the coefficient of shear viscosity and \( \alpha_i (i = 1, 2), \beta_j (j = 1 - 3), \gamma_k (k = 1 - 8) \) are the material constants. When \( \gamma_k \)'s are zero then we have the third grade fluid. For \( \beta_j \)'s and \( \gamma_k \)'s equal to zero, we get the expression for second grade fluid and \( a_1 = \beta_1 = \gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \gamma_8 = \gamma_9 = \gamma_10 = 0 \) corresponds to the viscous fluid case.

The definition of Rivlin-Ericksen tensors are as follows:

\[ \bar{A}_1 = (\text{grad} \, \bar{V}) + (\text{grad} \, \bar{V})^T, \]
\[ \bar{A}_e = \frac{d\bar{A}_{e-1}}{d\bar{t}} + \bar{A}_{e-1} (\text{grad} \, \bar{V}) + (\text{grad} \, \bar{V})^T \bar{A}_{e-1}, n > 1. \]  

(8)

If \( (U, V) \) and \( (\bar{u}, \bar{v}) \) are the velocity components in the laboratory and wave frames then the relations between these frames are
\[ \bar{x} = \bar{X} - \bar{c} \bar{t}, \bar{y} = \bar{Y}, \bar{u}(\bar{x}, \bar{y}) = \bar{U}(\bar{X}, \bar{Y}) - \bar{c}, \bar{v}(\bar{x}, \bar{y}) = \bar{V}(\bar{X}, \bar{Y}) \bar{t}. \]  

(9)

Using above transformations and introducing the following dimensionless variables

\[ x = \frac{2\pi x}{\lambda}, \quad y = \frac{y}{d_1}, \quad \bar{u} = \frac{\bar{u}}{c}, \quad \bar{v} = \frac{\bar{v}}{\bar{c}}, \quad \bar{p} = \frac{2\pi^2 \bar{p}}{c \mu \lambda}, \]
\[ h_1 = \frac{h_1}{d_1}, \quad h_2 = \frac{h_2}{d_1}, \quad \mu = \frac{\mu}{\mu \lambda}, \]
\[ \bar{S} = \frac{d_1}{\mu \bar{c}}, \quad \bar{c} = \frac{C - C_0}{C_1 - C_0}. \]  

(10)
we obtain

\[
\delta \text{Re} \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\delta \psi}{\partial x} \frac{\partial}{\partial y} \right) \psi \right] = \frac{\partial S_{\text{Ex}}}{\partial y} + \delta \text{Re} \sin \alpha \frac{u}{Fr} \delta \text{Re} \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\delta \psi}{\partial x} \frac{\partial}{\partial y} \right) \psi \right] = \frac{\partial S_{\text{Ex}}}{\partial y} + \delta \text{Re} \sin \alpha \frac{u}{Fr}.
\]  

(11)

\[
-\delta \text{Re} \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\delta \psi}{\partial x} \frac{\partial}{\partial y} \right) \sigma \right] = \frac{1}{Sc} \left[ \frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} \right] \sigma - \gamma \sigma = C^*.
\]

(13)

where continuity equation is automatically satisfied and

\[
S = A_1 + 2A_2 + x_2A_1^2 + x_1A_3 + \epsilon_1 x_1A_3 + \epsilon_2 (A_2A_1 + A_1A_2)
\]

\[
+ \epsilon_3 (A_2A_3) + \epsilon_4 A_2A_3 + \epsilon_5 (\epsilon_6 A_1A_3) + \epsilon_6 (A_2A_3)A_1 + \epsilon_7 (A_2A_3)A_2 + \epsilon_8 (A_2A_3)A_3.
\]

In the above equations \( u = \frac{\delta}{E} \), \( v = -\frac{\delta}{E} \), \( \psi \) as the stream function. Further, the wave number \( \delta \), the Reynolds number (Re), the Froud number \( (Fr) \), the Schmidt number \( (Sc) \), the chemical reaction parameter \( (\gamma) \) and the dimensionless material parameters \( \lambda_i = 1, 2, \ldots \) are

\[
\delta = \frac{2\pi d_1}{\lambda_i}, \quad \text{Re} = \frac{\rho c d_1}{\mu}, \quad Fr = \frac{c^2}{g d_1}, \quad Sc = \frac{\mu}{\rho d_1},
\]

\[
\epsilon_i = \frac{\mu c^2}{\rho d_1^2}, \quad \xi_i = \frac{\mu c^2}{\rho d_1^2}, \quad \eta_i = \frac{\mu c^2}{\rho d_1^2}, \quad \xi_1 = \frac{\mu c^2}{\rho d_1^2}, \quad \eta_1 = \frac{\mu c^2}{\rho d_1^2}, \quad \eta_7 = \frac{\mu c^2}{\rho d_1^2}, \quad \eta_8 = \frac{\mu c^2}{\rho d_1^2}.
\]

(15)

with \( C^* = \frac{\delta c d_1^2}{\rho d_1} \).

From Equations. (11 and 12) we can write

\[
\delta \text{Re} \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial}{\partial x} + \frac{\delta \psi}{\partial x} \frac{\partial}{\partial y} \right) \nabla^2 \psi \right] = \left[ \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) S_{\text{Ex}} \right] + \delta \left[ \frac{\partial^2 \psi}{\partial y^2} (S_{\text{Ex}} - S_{\text{Re}}) \right].
\]

(16)

The dimensional volume flow rate in fixed frame is defined by the following expression

\[
Q = \int_{\bar{X}(\xi)}^{\bar{X}(\xi)} \bar{U}(\bar{X}, \bar{Y}, \bar{T}) d\bar{Y}.
\]

(18)

The above expression in wave frame reduces to

\[
q = \int_{\bar{X}(\xi)}^{\bar{X}(\xi)} \bar{U}(\bar{X}, \bar{Y}) d\bar{Y}.
\]

(19)

Moreover Equations (9, 18 and 19) give

\[
Q = q + c \epsilon_8 x_1 - c \epsilon_8 x_1.
\]

(20)

The time-mean flow over a period \( T \) is defined by

\[
\bar{Q} = \frac{1}{T} \int_0^T Q d\bar{Y}.
\]

(21)

Invoking Equation (20) into Equation (21) and then performing integration we can express that

\[
\bar{Q} = q + cd_1 + cd_2.
\]

(22)

Introducing the dimensionless mean flow rates \( \theta \)

(in fixed frame) and \( F \) (in laboratory frame) by the relations
\[ \bar{\theta} = \frac{Q}{cd}, \quad F = \frac{q}{cd} \]  
(23)

we get from Equation (22) the following expression
\[ \bar{\theta} = F + 1 + d, \]  
(24)

where
\[ F = \int_{b_1(x)}^{b_2(x)} \frac{\partial \phi}{\partial y} dy = \psi(b_1(x)) - \psi(b_2(x)), \]  
(25)

and the dimensionless forms of \( b_1 \) (i = 1, 2) are
\[ b_1(x) = 1 + a \sin(x), \quad b_2(x) = -d + a \sin(x), \]  
(26)

with \( a = a_1/d_1, \quad b = a_2/d_1, \quad d = d_2/d_1 \) and \( \phi \) satisfies the following relation
\[ a^2 + b^2 + 2ab \cos \phi \leq (1 + d)^2. \]  
(27)

The dimensionless slip conditions in wave frame are
\[ \psi = \frac{F}{2}, \quad \frac{\partial \phi}{\partial y} + \beta S_{xy} = -1, \quad \sigma + \frac{\partial \psi}{\partial y} = 0, \quad \text{at} \, y = b_1(x), \]  
\[ \psi = \frac{-F}{2}, \quad \frac{\partial \phi}{\partial y} - \beta S_{xy} = -1, \quad \sigma - \frac{\partial \psi}{\partial y} = 1, \quad \text{at} \, y = b_2(x), \]  
(28)

where \( \beta = \mu \beta/d_1 \) and \( \beta_1 = \mu \beta_1/d_1 \) are the dimensionless velocity and concentration slip parameters, \( \beta \) and \( \beta_1 \) are the dimensional velocity and concentration slip parameters and \( S_{xy} \) is given through Equation (14). The resulting problem after adopting long wavelength and low Reynolds number procedure becomes:
\[ \frac{d^2 S_{xy}}{dx^2} = 0, \]  
(29)
\[ \frac{dp}{dx} = \frac{\sigma \partial S_{xy}}{\bar{p}}, \quad \text{Re} \sin \alpha, \]  
(30)
\[ \frac{1}{3\varepsilon} \left( \frac{\partial^2 \sigma}{\partial y^2} \right) - \gamma \sigma - C^* = 0, \]  
(31)

with
\[ \gamma = M_1 \psi^4 + M_2 \psi^3 + M_3 \psi^2 + M_4 \psi + M_5, \]  
(32)
\[ \psi_1 = M_1 \psi^4 + M_2 \psi^3 + M_3 \psi^2 + M_4 \psi + M_5, \]  
(33)
\[ \psi_2 = N_1 \psi^4 + 2N_2 \psi^3 + N_3 \psi^2 + N_4 \psi + N_5, \]  
(34)
\[ u_1 = 3R_1 \psi^2 + 2R_2 \psi + R_3, \]  
(35)
\[ u_2 = 7N_1 \psi^4 + 6N_2 \psi^3 + 5N_3 \psi^2 + 4N_4 \psi + 3N_5 \psi^2 + 2N_6 \psi + N_7, \]  
(36)
\[ \frac{dp}{dx} = -12(F_0 + h_1 - h_2) \frac{1}{(h_1 - h_2)^2 (h_1 - h_2 + 6\beta)} + 12G. \]  
(37)
\[
\frac{dp_1}{dx} = -60F_1 + \left( \frac{dp_0}{dx} - 12G \right)^2 L_{10} + \left( \frac{dp_0}{dx} - 12G \right) R_{11} L_{20} + \left( \frac{dp_0}{dx} - 12G \right) S_{2} L_{21},
\]

(41)

\[
\frac{dp_2}{dx} = -420F_2 + \left( \frac{dp_0}{dx} - 12G \right)^2 L_{37} + \left( \frac{dp_0}{dx} - 12G \right) S_{1} L_{38} + S_{2} L_{39} + S_{2}^2 L_{40},
\]

(42)

where \( G = \frac{(\text{Re} \sin \alpha)}{(12 \text{Fr})} \) and the different quantities appearing in the above expressions have been presented in the Appendix A.

The non-dimensional pressure rise per wavelength at zeroth, first and second orders are

\[
\Delta P_{d1} = \int_0^{2\pi} \frac{dp_0}{dx} \, dx, \quad \Delta P_{d1} = \int_0^{2\pi} \frac{dp_1}{dx} \, dx, \quad \Delta P_{d1} = \int_0^{2\pi} \frac{dp_2}{dx} \, dx.
\]

(43)

The perturbation expressions of \( \psi \), \( \Delta P \) and \( dp/dx \) upto \( O(\Gamma^2) \) are denoted by \( \psi^{(2)} \), \( \Delta P^{(2)} \) and \( dp^{(2)}/dx \). These can be written as

\[
\psi^{(2)} = \psi_0 + \Gamma \psi_1 + \Gamma^2 \psi_2.
\]

(44)
Fig. 6. Effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\lambda$ for $\alpha = 0.7, b = 1.2, d = 2, Re = 10, \Gamma = 0.01, \beta = 0.1, \sin \alpha = 0.6$ and $Fr = 1$.

Fig. 7. Effect of width of the channel $d$ on variation of $\Delta p^{(2)}$ with $\theta$ for $\alpha = 0.2, b = 0.4, \phi = \pi/6, Re = 10, \Gamma = 0.01, \beta = 0.1, \sin \alpha = 0.2$ and $Fr = 1$.

Fig. 8. Effect of Deborah number $\Gamma$ on variation of $\Delta p^{(2)}$ with $d$ for $\alpha = 0.3, b = 0.2, \theta = -0.001, Re = 10, \beta = 0.1, Fr = 1$, $\sin \alpha = 0.2$ and $\phi = \pi/2$.

Fig. 9. Effect of inclined angle $\alpha$ on variation of $\Delta p^{(2)}$ with $a$ for $\theta = 0.0001, b = 0.4, d = 1, Re = 10, \Gamma = 0.02, Fr = 1, \beta = 0.1$ and $\phi = \pi/2$.

Fig. 10. Effect of Froud number $Fr$ on variation of $\Delta p^{(2)}$ with $b$ for $\alpha = 0.5, \theta = 0.0001, d = 1.5, Re = 10, \Gamma = 0.02, \beta = 0.1, \sin \alpha = 0.2$ and $\phi = \pi/4$.

Fig. 11. Effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $Re$ for $\alpha = 0.5, b = 0.7, d = 1.5, \theta = 0.0001, \Gamma = 0.02, \beta = 0.1, \sin \alpha = 0.2$ and $Fr = 1$. 
\[ \Delta P^{(2)} = \Delta P_{h_0} + \Gamma \Delta P_{h_1} + \Gamma^2 \Delta P_{h_2}. \]  
\[ \frac{dp^{(2)}}{dx} = \frac{dp_0}{dx} + \Gamma \frac{dp_1}{dx} + \Gamma^2 \frac{dp_2}{dx}. \]  

It is worth pointing to note that solution expression arising integral is not solvable analytically. Hence velocity

\[ \sigma(y) = \left[ e^{-\beta \frac{2+2\gamma}{2}} \left( 1 + \beta \sqrt{Sc + \gamma} \right)^2 + e^{\beta \frac{2+2\gamma}{2}} \left( 1 - \beta \sqrt{Sc + \gamma} \right)^2 \right] \]

\[ \times \left[ (1 + \beta \sqrt{Sc + \gamma}) + e^{\beta \gamma} (1 + \beta \sqrt{Sc + \gamma}) \right] \]

\[ \times \left[ e^{\beta \gamma} (1 + \beta \sqrt{Sc + \gamma}) - e^{-\beta \gamma} (1 + \beta \sqrt{Sc + \gamma}) \right] \]

\[ \times \left[ \left( 1 + \beta \sqrt{Sc + \gamma} \right)^2 - e^{\beta \gamma} \left( 1 + \beta \sqrt{Sc + \gamma} \right) \right] \]

4. Results and discussion

4.1. Pumping characteristics

Our intention in this subsection is to discuss the behaviors of pressure rise and streamlines for embedded flow parameters. Note that the definition of pressure rise involves integration of \( dp^{(2)}/dx \). The arising integral is not solvable analytically. Hence

\[ \text{MATHEMATICA} \text{ is used to perform the integration.} \]

\[ \text{The variations of } \Delta \rho^{(2)} \text{ against the flow rate } \theta^{(2)} \text{ for fixed values of involved parameters have been sketched in the Figs. 2–7. Figure 2 represents} \]

\[ \text{the variation of dimensionless pressure rise up to second order } (\Delta \rho^{(2)}) \text{ with the second order dimensionless mean flow rate } (\theta^{(2)}) \text{ for various values of } \beta. \]

\[ \text{This figure illustrates that peristaltic pumping rate decreases by increasing } \beta. \]

\[ \text{It is interesting to note that in copumping the pumping rate increases} \]

\[ \text{by increasing } \beta. \]

\[ \text{Figure 3 displays the dimensionless pressure gradient to second-order } \Delta \rho^{(2)} \text{ for different values of Deborah number } \Gamma. \]

\[ \text{A linear relationship is noticed between the pressure gradient and the flow rate in a viscous fluid i.e. when } \Gamma = 0. \]

\[ \text{As expected the pumping curves are non-linear when } \Gamma \neq 0. \]

\[ \text{Therefore, for } \Delta \rho^{(2)} > 30, \text{ increasing } \Gamma \text{ gives a better pumping performance and for } \Delta \rho^{(2)} = 30, \text{ there is no difference} \]

\[ \text{between the viscous and fourth-grade fluids on pumping as the pumping curves coincide with each other.} \]
Fig. 14. Effect of the Froude number $Fr$ on the variation of longitudinal velocity $u$ plotted for $a = 0.3$, $b = 0.5$, $d = 0.9$, $Re = 0.7$, $\phi = \pi/4$, $\Gamma = 0.15$, $\alpha = \pi/8$, $\beta = 1.5$, $x = -\pi/3$, $\beta = 0.02$ and $dp/dx = 1$.

Fig. 15. Effect of the Reynold number $Re$ on the variation of longitudinal velocity $u$ plotted for $a = 0.3$, $b = 0.5$, $d = 0.9$, $Fr = 1.2$, $\phi = \pi/4$, $\Gamma = 0.15$, $\alpha = \pi/8$, $\beta = 1.5$, $x = -\pi/3$, $\beta = 0.02$ and $dp/dx = 1$.

Fig. 16. Effect of $\sin \alpha$ on the variation of longitudinal velocity $u$ plotted for $a = 0.3$, $b = 0.5$, $d = 0.9$, $Fr = 1.3$, $\phi = \pi/4$, $\Gamma = 0.0$, $\alpha = -\pi/3$, $\beta = 0.02$ and $dp/dx = 1$.

Fig. 17. Variation of $Sc$ on $\sigma$ when $a = 0.2$, $b = 0.8$, $d = 1$, $\beta_1 = 0.01$, $\phi = \pi/4$, $s = \pi$, $C^* = 1$, $y = 1$.

Fig. 18. Variation of $\gamma$ on $\sigma$ when $a = 0.2$, $b = 0.8$, $d = 1$, $\beta_1 = 0.01$, $\phi = \pi/4$, $s = \pi$, $C^* = 3.2$, $Sc = 2.2$.

Fig. 19. Variation of $\beta_1$ on $\sigma$ when $a = 0.2$, $b = 0.8$, $d = 1$, $\Delta = 1.5$, $\phi = \pi/4$, $s = \pi$, $C^* = 1.5$, $y = 1$. 
Fig. 20. Streamlines for $a = 0.4, b = 0.4, d = 1, Re = 10, Fr = 1, \phi = \pi/6, \beta = 0.01, \sin u = 0.2, dp^2/dx = 1, \theta = 1.4$ with different $\Gamma$
(a) $\Gamma = 0.00$, (b) $\Gamma = 0.1$, (c) $\Gamma = 0.2$, (d) $\Gamma = 0.3$.

Fig. 21. Streamlines for $a = 0.5, b = 0.7, d = 1, Re = 10, Fr = 0.6, \beta = 0.1, \phi = 1.4, dp^2/dx = 2, u = 0.2, \Gamma = 0.01$ with different $\phi$
(a) $\phi = 0$, (b) $\phi = \pi/4$, (c) $\phi = \pi/2$, (d) $\phi = \pi$. 
Fig. 22. Streamlines for $a = 0.5$, $b = 0.7$, $d = 1$, $Re = 10$, $Fr = 1$, $\beta = 0.1$, $\theta = 0.5$, $dp^{(2)}/dx = 1.2$, $\psi = \pi/6$, $\Gamma = 0.01$ with different $\sin \alpha$
(a) $\sin \alpha = 0$, (b) $\sin \alpha = 0.4$, (c) $\sin \alpha = 0.8$, (d) $\sin \alpha = 1.2$.

Fig. 23. Streamlines for $a = 0.5$, $b = 0.7$, $d = 2$, $Re = 10$, $\alpha = 0.3$, $\beta = 0.1$, $\theta = 1.35$, $dp^{(2)}/dx = 1.2$, $\psi = \pi/6$, $\Gamma = 0.1$ with different $Fr$
(a) $Fr = 1$, (b) $Fr = 2$, (c) $Fr = 3$, (d) $Fr = 4$. 
While for an appropriately chosen $\Delta p^{(2)} < 30$, the pumping rate decreases with an increase in Deborah number $\Gamma$. The effect of inclined angle $\alpha$ on pumping characteristics is plotted in Fig. 4. It is observed that the pumping rate and the pressure rise increase by increasing $\sin \alpha$. The variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise increase by increasing $\sin \alpha$. The variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise increase by increasing $\sin \alpha$. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases. The effect of phase difference $\phi$ on variation of $\Delta p^{(2)}$ with $\theta^{(2)}$ for different values of Froud number $Fr$ is shown in Fig. 5. It is found that the pumping rate and the pressure rise decrease when Froud number $Fr$ increases.
to a decrease in the magnitude of $u$ at the boundaries of the channel. However at the centre of the channel the magnitude of $u$ increases when $\beta$ increases. Figures 13 and 14 witness that there is a decrease in the magnitude of $u$ at the boundaries of the channel when $\Gamma$ and $Fr$ are increased. At the centre of channel, an increase in the magnitude of $u$ is noticed with an increase in $\Gamma$ and $Fr$. Figure 15 depicts that an increase in Re causes an increase in the magnitude of $u$ at the boundaries of the channel where at the centre of the channel, the magnitude of $u$ decreases by increasing Re. Figure 16 shows that an increase in $\sin \alpha$ leads to a decrease in the magnitude of $u$ at the boundaries of the channel. For channel centre there is an increase in the magnitude of $u$ with an increase in $\sin \alpha$.

4.3. Concentration field

Figures 17 and 19 represents the concentration distribution for the different parameters. These figures display concentration distribution for various values of $Sc$ and $\beta_1$. It is noticed that concentration field decreases with an increase in $Sc$ and $\beta_1$. Figure 18 shows the concentration field increases in view of increasing values of chemical reaction parameter $\gamma$.

4.4. Trapping

The formation of an internally circulating bolus of fluid by closed streamlines is called trapping and this trapped bolus is pushed ahead along with the peristaltic wave. The effect of $\Gamma$ on trapping is illustrated in Fig. 20. It is worthwhile mention that the size of trapped bolus decreases by increasing $\Gamma$ and disappears when $\Gamma = 0.2$. The streamlines for different $\phi$ are shown in Fig. 21. It is clear that the bolus appearing at the center region for $\phi = 0$ and it moves towards left and decreases in size as $\phi$ increases and disappears when $\phi = \pi$. The streamlines for different $\alpha$ are shown in Fig. 22. The trapped bolus does not exist for $\sin \alpha = 0$, but the trapping exists when $\sin \alpha \neq 0$ and the bolus volume increases by increasing $\sin \alpha$. The streamlines for different $Fr$ have been displayed in Fig. 23. It is observed that the size of trapped bolus decreases with increasing $Fr$ and disappears when $Fr = 3$. Effect of $\beta$ on the trapping is plotted in Fig. 24. Here the size of trapped bolus decreases with increasing $\beta$ and disappears for $\beta = 0.9$.

5. Concluding remarks

In this work the effects of slip condition on the peristaltic flow of fourth grade fluid in an inclined asymmetric channel with chemical reaction are analyzed subject to long wavelength and low Reynolds number approximation. The effects of slip parameters $\beta$ and $\beta_1$ on pumping characteristics, longitudinal velocity, trapping and concentration are discussed in detail. The main points can be summed up as follows.

- The peristaltic pumping rate decreases by increasing $\beta$. However in copumping, the pumping rate increases by increasing $\beta$.
- The maximum pressure against which the peristaltic work as a pump decreases by increasing $\beta$.
- The size of trapped bolus above (below) the centre line increases (decreases) by increasing $\beta$.
- The magnitude of the longitudinal velocity decreases at the boundaries by increasing $\beta$. However it increases by increasing $\beta$ at the centre of channel.
- With an increase in chemical reaction parameter $\gamma$, the concentration field increases.
- Concentration field decreases with an increase in $Sc$ and concentration slip parameter $\beta_1$. 
Here we provide the quantities appearing in the flow analysis.

\[ R_1 = \frac{-2(F_0 + h_1 - h_2)}{(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} \]

\[ R_2 = \frac{3(F_0 + h_1 - h_2)(h_1 + h_2)}{(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} \]

\[ R_3 = \frac{-h_1^3 - 6h_2h_1h_2 - 3h_2^2h_1 + 3h_1^2h_2 + h_2^3 + 6F_0(h_1 - h_2)\beta}{(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} \]

\[ R_4 = \frac{-(h_1 + h_2)(2h_1h_2(h_1 - h_2) + F_0(h_1^2 - 4h_1h_2 + h_2^2 + 6(h_1 - h_2)\beta))}{2(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} \]

\[ M_1 = \frac{-1}{10} \left( \frac{d\mu}{dx} - 12G \right)^3 \]

\[ M_2 = \frac{-20F_1 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2}{10(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} \]

\[ M_3 = \frac{L_4 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_0L_5 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_0L_6 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_3 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_4}{5(h_1 - h_2)^2(h_1 - h_2 + 2\beta)(h_1 - h_2 + 6\beta)} \]

\[ M_4 = \frac{L_4 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_0L_i + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_1 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_3 + \left( \frac{d\mu}{dx} - 12G \right)^2 L_1}{10(h_1 - h_2)^2(h_1 - h_2 + 2\beta)(h_1 - h_2 + 6\beta)} \]

\[ M_5 = \frac{L_4 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_0L_1 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_1 + \left( \frac{d\mu}{dx} - 12G \right)^2 R_2L_1 + \left( \frac{d\mu}{dx} - 12G \right)^2 L_1}{10(h_1 - h_2)^2(h_1 - h_2 + 2\beta)(h_1 - h_2 + 6\beta)} \]

\[ L_1 = (h_1 - h_2)^3(h_1^2 - h_2 + 2\beta) \]

\[ L_2 = 20(h_2^3 - h_2)(h_1 + h_2) \]

\[ L_3 = -240(h_1 - h_2)^2\beta \]

\[ L_4 = 15F_1(h_1 + h_2)(h_1 - h_2 + 2\beta) \]

\[ L_5 = 120(h_1 - h_2)^3(h_1 + h_2)\beta \]

\[ L_6 = -(h_1 - h_2)^3(h_1 + h_2)(h_1^2 + 3h_1(h_2 - \beta) + h_2(h_2 + 3\beta)) \]

\[ L_7 = 5(h_1 - h_2)^3[(h_1 - h_2)(h_1^2 + 4h_1h_2 + h_2^2) + 6h_1^2 + h_2^2][h_1(h_2 - \beta)] \]

\[ L_8 = -80(h_1 - h_2)^2\beta(h_1 - h_2 + 6\beta) \]

\[ L_9 = -60F_1(h_1 - h_2 + 2\beta)(h_1^2 - h_2 + h_2(2h_1 - h_1 - h_2)\beta) \]

\[ L_{10} = 160(h_1 - h_2)^2(h_1 + h_2)^2(h_1 - h_2 + 6\beta) \]

\[ L_{11} = 20(h_1 - h_2)^2(h_1 - h_2)^2(h_1)(h_1h_2 - h_1 - h_2 + h_1h_2 + h_2h_2 + 6h_1^2 + h_2^2) \]

\[ L_{12} = 240(h_1 - h_2)^2\beta(h_1h_2(h_1 - h_2) + 2h_1^2 + h_1h_2 + h_2^2\beta) \]
$$L_{14} = (h_1 - h_2)^2(h_1 h_2 - h_2^2)(4h_1^2 + 7h_1 h_2 + 4h_2^2) - 2(h_1 - h_2)(2h_1^2 + 3h_1 h_2 + 3h_2 h_2 + 2h_2^2)\beta$$
$$+ 12(h_1^2 + h_2^2 h_2 + h_1 h_2^2 + h_1^2 h_2^2)\beta^2.$$  
$$L_{15} = \frac{5F_1(h_1 - h_2)h_1 - h_2 + 2\beta(h_1^2 - 4h_1 h_2 + h_2^2 + 6h_1 - h_2)\beta}{160h_1 h_2(h_1 - h_2)^2(h_1 - h_2 + 6\beta)}.$$  
$$L_{17} = 2h_1(h_1 - h_2)^3\left[-h_1 h_2(h_1 - h_2)^2(h_1 + h_2) + 2(h_1 - h_2)(h_1^2 + h_2^2)\beta - 6h_1 + h_2(h_1^2 + h_2^2)\beta^2\right].$$  
$$L_{18} = 10h_1(h_1 - h_2)^2[16(h_1 - h_2)^2 + 2(h_1 - h_2)(h_1^2 + h_2^2)\beta - 12(h_1^2 + h_2 h_2 + h_2^2)\beta^2].$$  
$$L_{20} = 60(h_1 - h_2)^3(h_1 + h_2).$$  
$$L_{21} = 120(h_1 - h_2)^2.$$  
$$L_{22} = (h_1 - h_2)^2(100h_1^2 M_1 + 50h_2 M_1 + 21M_2) + 9h_1^2(100h_1^2 M_1 + 56h_2 M_2 + 21M_3)$$
$$+ 4h_1(500h_1^2 M_1 + 336h_2 M_2 + 189h_2 M_3 + 70M_4) + h_1(800h_1^2 M_1 + 504h_2 M_2 + 252h_2 M_3 + 70M_4).$$  
$$L_{23} = 840(h_1 - h_2)^2([h_1 - h_2]^2(h_1^3 + h_2^3) + 60h_1 h_2 M_1(h_1 + h_2) + 27M_2(h_1^2 + h_2^2)$$
$$36h_1 h_2 M_2 + 15M_3(h_1 + h_2) - 30M_4\beta].$$  
$$L_{25} = 105F_0(h_1 + h_2)(h_1 - h_2 + 2\beta).$$  
$$L_{26} = -84(h_1 - h_2)^2[(h_1 - h_2)^2(h_1^2 + h_2^2)(3h_1 h_2 + h_1 h_2^2)(h_1 + h_2^2)M_1 + 3(h_1 + h_2^2)M_2\)$$
$$+ (5h_1^2 + 4h_1 h_2 + h_2^2)M_3 - 2(20h_1^2 M_1 + h_1(-40h_1 M_1 + 18M_2) - 3h_1^2(20h_1^2 M_1 + 6h_2^2 M_2 - 5M_3)$$
$$+ 2h_1(20h_1^2 M_1 + 9h_2^2 M_2 - 5M_4) + h_2(20h_1^2 M_1 + 18h_2^2 M_2 + 15h_2 M_3 + 10M_4)\beta].$$  
$$L_{27} = -10(h_1 - h_2)^2(h_1 - h_2)(200h_1 + h_2)(2h_1^2 + 6h_1 h_2 + 5h_2^2 h_2 + 6h_1 h_2^2 + 2h_2 h_1^2 M_1$$
$$+ 25h_1^2 h_2 + h_1 h_2^2\beta(h_1 + h_2^2)M_2 + 3h_1 h_2 + h_2^3)M_3 + 126(h_1 + h_2^2\beta(h_1 + h_2^2)M_2 + 3h_1^2 + h_2 h_2^2 + h_2^3)M_3$$
$$+ 3(5h_1 + 4h_1 h_2 + h_2^2)M_4 - 6(100h_1^2 M_1 + h_1(-300h_1 M_1 + 84M_2)$$
$$- 3h_1^2(100h_1 M_1 + 56h_2 M_2 + 21M_3) + h_1(-4h_1(12h_2 M_2 + 42M_2) + 63M_3)$$
$$- h_2^2(100h_2 M_1 + 252h_2 M_2 + 63h_2 M_3 - 35M_4) + h_2(100h_2 M_1 + 84h_2 M_2 + 63h_2 M_3 + 35M_4)\beta].$$  
$$L_{28} = 840(h_1 - h_2)^2[16(h_1 - h_2)^2(2h_1 + h_2)(h_1^2 + h_2^2)M_1 + (h_1^2 + 4h_1 h_2 + h_2^2)M_2\)$$
$$+ 2(h_1 - h_2)^2(3h_1^2 + h_1^2(h_1 M_1 + h_2^2(h_2 M_1 + h_2)\gamma - h_2(h_2 M_1 + h_2)\gamma\beta - 12M_4\beta^2\gamma].$$  
$$L_{29} = -210F_1(h_1^2 - h_2^2)(h_1 + h_2)\beta - h_2^2\beta.$$  
$$L_{30} = 840(h_1 - h_2)^2[16(h_1 - h_2)^4(4h_1^2 + 7h_1 h_2 + 4h_2^2)M_1 + 2(h_1 + h_2)M_2\)$$
$$- 2(h_1 - h_2)(2h_1^2 M_1 + 3h_2 h_2 M_2 - h_2^2(2h_1 M_1 + M_2) + h_2^3(3h_2 M_1 + M_2)$$
$$+ 3h_1 h_2^3(h_2 M_1 + M_2) - (h_1 + h_2)^2\beta\beta + 12h_1^2(h_1^2 + h_2^2(h_2 M_1 + M_2)$$
$$+ h_2^2(h_2 M_1 + M_2) + M_4) + (h_1 + h_2)^2(h_2^2(h_2 M_1 + M_2) + M_4)\beta^2\gamma].$$
\[ L_{31} = 168(h_1 - h_2)^2\{h_1 h_2(h_1 - h_2)^2(20h_2^4 M_1 + 4h_2(10h_2 M_1 + 3M_2) + h_1(40h_2^2 M_1 + 21h_2 M_1 + 5M_2) \]
\[ + 4h_2(5h_2 M_1 + 3M_2) + 5M_3)) - (h_1 - h_2)(20h_2^4 M_1 + 4h_2(5h_2 M_1 + 3M_2) \]
\[ + h_1^2(20h_2^4 M_1 + 18h_2 M_2 + 5M_3) + h_1^2(4h_2(5h_2 M_1 + 3M_2) + 5M_3) - 5h_2(4h_2 M_1 - 3h_2 M_2) \]
\[ + h_2(20h_1^4 M_1 + 18h_1^2 M_2 + 15h_2 M_3 + 10M_4)\beta + 2(20h_1^4 M_1 + 3h_2^2(10h_1 M_1 + 9M_2) \]
\[ + h_1^2(20h_1^4 M_1 + 18h_1^2 M_2 + 15M_3) + \{h_1^2 + h_1 h_2 + h_2^2\}(40h_2^2 M_1 + 18h_2 M_2 + 15h_2 M_3 + 10M_4)\beta^2\} \]
\[ L_{32} = (h_1 - h_2)^2\{h_1 h_2(h_1 - h_2)^2(800h_1^4 M_1 + 4h_1(425h_1 M_1 + 126h_2 M_2) \]
\[ + 4h_1(500h_1^3 M_1 + 252h_2 M_2 + 63M_3) + 2h_1(400h_1^2 M_1 + 252h_2 M_2 + 126h_2 M_2 + 35M_4) \]
\[ + h_1(4h_1(252h_1 M_1 + 252M_2) + 441M_2) + 70M_4) - 2(h_1 - h_2)(400h_1^3 M_1 \]
\[ + 12h_1(252h_1 M_1 + 21M_2) + h_1^2(-600h_1^2 M_1 + 252h_2 M_2 + 126M_2) \]
\[ + h_1^2(-9h_1(425h_1 M_1 + 7M_2 - 21M_2) + 35M_4) \]
\[ + 3h_1 h_2^2(100h_1^2 M_1 + 84h_2^2 M_2 + 63h_1 M_3 + 35M_4) + h_1^2(400h_1^2 M_1 + 252h_2 M_2 + 126h_2 M_2 + 35M_4) \]
\[ - 3h_2(200h_1^2 M_1 + 84h_1 M_3 - 35h_2 M_3)\beta + 12(100h_1^2 M_1 + 4h_1(252h_1 M_1 + 21M_2) \]
\[ + h_1^2(100h_1^2 M_1 + 84h_2 M_2 + 63M_3) + (h_1^2 + h_1 h_2 + h_2^2) \]
\[ \times (100h_1^2 M_1 + 84h_2^2 M_2 + 63h_1 M_3 + 35M_4)\beta^2\} \]
\[ L_{33} = -35F_2(h_1 + h_2)(h_1 - h_2) - 2\beta(h_1^2 - 4h_1 h_2 + h_2^2 + 6h_1 - h_2)\theta \].
\[ L_{34} = 1680h_1 h_2(h_1 - h_2)^2\{-h_1 h_2 h_1 h_2 h_2 h_2^2(2h_1 + h_2)M_1 + M_2) + 2(h_1 - h_2)(2h_1 + h_2)M_1 \]
\[ + h_1^2 + h_2^2 - h_1 h_2 + h_1 h_2 + h_2 h_2)\{M_1 + h_1 (h_1 h_2 + M_2) + h_2 (h_1 h_2 + M_2) + M_1 \}
\[ + h_1 h_2 (h_1 h_2 + M_2) + M_1 + M_2)\}\beta \].
\[ L_{35} = 56h_1 h_2(h_1 - h_2)^2\{-h_1 h_2 h_1 h_2 h_2 h_2^2(20h_1^4 M_1 + 4h_1 h_2 + 3h_2^2 M_1 + 36(h_1 + h_2)M_2 + 15M_3) \]
\[ + 2(h_1 - h_2)(20h_1^4 M_1 - h_1 h_2^2 + 3h_2^2 M_1 + 36(h_1 + h_2)M_2 + 15h_1 h_2 + h_2^2 M_2 \]
\[ - 12(20h_1^4 M_1 + 2h_1(100h_1 M_2 + 9M_2) + h_1 h_2(200h_1^2 M_1 + 18h_2 M_2 + 15M_3) + (h_1 + h_2) \]
\[ \times (200h_1^2 M_1 + 18h_2 M_2 + 15h_2 M_3 + 10M_4)\beta^2\} \]
\[ L_{36} = 2h_1 h_2(h_1 - h_2)^2\{-h_1 h_2 h_1 h_2 h_2 h_2^2(200h_1 M_1 + h_2(2h_1 + h_2 + h_2^2 M_1 \]
\[ + 84h_1^3 M_1 + 4h_1 h_2 + 3h_2^2 M_2 + 126h_1 h_2 M_4 + 35M_5) \]
\[ + 2(h_1 - h_2)(200h_1^2 M_1 - h_1 h_2 + h_1^2 + 2h_2^2 M_2 \]
\[ + 12(h_1 + h_2)M_2 + 35h_1 h_2 + h_2^2 M_2 \]
\[ - 12(100h_1^2 M_1 + 4h_1(252h_1 M_1 + 21M_2) \]
\[ + h_1 h_2(100h_1^2 M_1 + 84h_1 M_2 + 63M_3) + (h_1^2 + h_1 h_2 + h_2^2) \]
\[ \times (100h_1^2 M_1 + 84h_2^2 M_2 + 63h_1 M_3 + 35M_4)\beta^2\} \]
\[ L_{37} = 6(h_1 - h_2)^3\{500h_1^4 M_1 + 16h_1^2(500h_1 M_1 + 21M_2) + 9h_1 h_2(100h_1^2 M_1 + 56h_2 M_2 + 21M_4) \]
\[ + h_1(500h_1^2 M_1 + 336h_2 M_2 + 189h_2 M_3 + 70M_4) + h_1(800h_1^2 M_1 + 504h_2 M_2 + 252h_2 M_3 + 70M_4) \]
\[ L_{38} = 356h_1 h_2^2 - h_2^3\{40M_1(h_1^2 + h_2^2) + 60M_1(h_1^2 h_2 + h_1^2 h_2) \]
\[ + 9h_1(3h_1^2 + 4h_1 h_2 + 3h_2^2) + 15h_1 + h_1 M_1 + 5M_4 \}
\[ L_{39} = 504h_1 h_2^2 - h_2^3\{((3h_1^2 M_1 + 4h_1 h_2 + 3h_2^2 M_1 + 2(h_1 + h_2)M_2 + M_5) \].
\[
N_1 = \frac{-20}{M_1} \left( \frac{dp_1}{dx} - 12G \right)^2 ,
\]
\[
N_2 = \frac{-5}{M_1} \left( \frac{dp_2}{dx} - 12G \right)^2 + 8R_2 \left( \frac{dp_2}{dx} - 12G + 5M_1 R_2 \right) .
\]
\[
N_3 = 8R_2 \left( \frac{dp_2}{dx} - 12G \right)^2 + 8R_2 \left( \frac{dp_2}{dx} - 12G + 5M_1 R_2 \right) ,
\]
\[
N_4 = -M_3 \left( \frac{dp_3}{dx} - 12G \right)^2 - 12R_3 \left( \frac{dp_3}{dx} - 12G + 2M_3 R_3 \right) ,
\]
\[
N_5 = \frac{-70F_2 + \left( \frac{dp_4}{dx} - 12G \right)^2 L_{23} + R_2^2 L_{24} + \left( \frac{dp_4}{dx} - 12G \right) R_2 L_{24} }{35(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} ,
\]
\[
N_6 = \frac{L_{23} + \left( \frac{dp_4}{dx} - 12G \right) R_2 L_{23} + \left( \frac{dp_4}{dx} - 12G \right)^2 L_{23} + R_2^2 L_{24} }{35(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} ,
\]
\[
N_7 = \frac{L_{23} + \left( \frac{dp_4}{dx} - 12G \right) R_2 L_{23} + \left( \frac{dp_4}{dx} - 12G \right)^2 L_{23} + R_2^2 L_{24} }{70(h_1 - h_2)^2(h_1 - h_2 + 6\beta)} .
\]

Acknowledgments

First author appreciates the support of Global Research Network for Computational Mathematics and King Saud University of Saudi Arabia for this research.

References


