Flow of MHD Carreau fluid in a curved channel

Saima Noreen^{a,*}, Tasawar Hayat^b and Ahmed Alsaedi^c

^a*Department of Mathematics, Comsats Institute of Information Technology, Attock, Pakistan*

^b*Department of Mathematics, Quaid-I-Azam University, Islamabad, Pakistan*

^c*Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia*

Abstract. Analysis has been made for the curvature effects on the MHD peristaltic flow of an incompressible Carreau fluid in a channel. The flow problem is first reduced in the wave frame of reference and then solved after employing the long wavelength and low Reynolds number approximations. Expressions of stream function, pressure gradient, magnetic force function, induced magnetic field and current density are derived and then examined for various parameters of interest.

Keywords: Curved channel, induced magnetic field, Peristaltic flow

1. Introduction

Peristalsis flows have vital role in physiology and engineering. Several authors have investigated such flows under various aspects since the seminal researches by Latham [1] and Shapiro et al. [2]. The studies on this topic have been undertaken both in the symmetric and asymmetric channels (see for example the attempts [3–10] and several refs. therein).

Presently the concept of magnetic field has importance in magnetotherapy, magnetic resonance imaging and magnetic devices. The motion of conductive fluid across the magnetic field induces currents, which thereby effects the propagating field. In fact the fluid flow is influenced by Lorentz force [11]. Motivated by such applications, some recent workers [12–17] in the field discussed the peristaltic flow in the absence of an induced magnetic field. On the other hand, Vishnyakov and Pavlov [18] investigated the peristaltic flow of viscous fluid in the presence of an induced magnetic field.

Mekheimer [19, 20] analyzed the induced magnetic field effects on flow of couple stress and micropolar fluid in a symmetric channel. Hayat et al. [21–25] put forward such analysis by examining the peristaltic transport of third order, fourth grade and Phan-Thien-Tanner fluids.

Peristaltic transport in curved channel is another important area of research which is not yet given a due attention. All the above mentioned studies have been undertaken in straight channels/tubes. Flow over curved channels is relevant to many industrial, biological and environmental applications. Examples are the spreading of pollution in a fjord and the wear experienced by pipes going through rough oceanic terrain. It is known that shape of most of the physiological tubes and glandular ducts is curved. Laser guiding in curved plasma channels have important applications such as an efficient circular x-ray laser medium, optical synchrotrons, laser accelerators and harmonic generators. Flow characteristics during flood associated with sediments transport is an application of curved geometry from engineering point of view. Micro heat exchangers involve curved channels where goal is often to increase the heat flux while limiting the pressure drop.

*Corresponding author: Saima Noreen, Department of Mathematics, Comsats Institute of Information Technology, Attock 43600, Pakistan. Tel.: +92 51 90642172, Fax: +92 51 2601171. E-mail: laurel.lichen@yahoo.com.

Ali et al. [26] discussed the peristaltic flow of viscous fluid in a curved channel. Later on, they [27] extended the analysis by considering the heat transfer characteristics. Very recently Ali et al. [28] studied the peristaltic motion of third grade fluid in a curved channel.

This communication studies the induced magnetic field effects on the peristaltic flow of Carreau fluid in the curved channel. Section two deals with the modeling of governing equations. Solution to the problem and discussion are presented in the sections three and four respectively. Last section deals with the concluding remarks.

2. Problem statement

We consider a channel which is coiled just like a circle with radius R^* , half width a and centre O as shown in Fig. 1. An incompressible Carreau fluid fills the space in a curved channel. A sinusoidal wave of velocity c and amplitude b propagates on the flexible channel walls. The fluid in the channel flows due to the contraction and expansion of the walls of the channel. The inertial effects are taken small. We choose coordinates $(\bar{R}, \bar{X})$ with $\bar{X}$ in the direction of wave propagation and $\bar{R}$ transverse to it.

An external magnetic field of strength H_0^* ($= H_0 \frac{R^*}{R^* + \bar{R}}$) acts in the radial direction (H_0 is the constant magnetic field). This results in an induced magnetic field $\mathbf{H}(\bar{h}_{\bar{r}}(\bar{R}, \bar{X}, \bar{t}), \bar{h}_{\bar{x}}(\bar{R}, \bar{X}, \bar{t}), 0)$ and therefore the total magnetic field becomes $\mathbf{H}^+(H_0^* + \bar{h}_{\bar{r}}(\bar{R}, \bar{X}, \bar{t}), \bar{h}_{\bar{x}}(\bar{R}, \bar{X}, \bar{t}), 0)$. The wall surface is described as follows:

$$\bar{h}(\bar{X}, \bar{t}) = a + b \sin\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right) \text{ upper wall, (1)}$$

where λ indicates the wavelength, a the channel half width, $\bar{t}$ the time and b the wave amplitude. If $\bar{V}$ and $\bar{U}$ are velocity components along the radial ($\bar{R}$) and axial directions ($\bar{X}$) respectively in the fixed frame, then the definition of velocity field $\mathbf{V}$ is

$$\mathbf{V} = [\bar{V}(\bar{R}, \bar{X}, \bar{t}), \bar{U}(\bar{R}, \bar{X}, \bar{t}), 0]. \quad (2)$$

The present flow situation is governed by the following fundamental equations:

$$\nabla \cdot \mathbf{V} = 0, \quad (3)$$

$$\begin{aligned} \rho \frac{d\mathbf{V}}{dt} &= \text{div } \mathbf{T} + \mu_e (\nabla \times \mathbf{H}^+) \times \mathbf{H}^+ \\ &= \text{div } \mathbf{T} + \mu_e \left[(\mathbf{H}^+ \cdot \nabla) \mathbf{H}^+ - \frac{\nabla \mathbf{H}^{+2}}{2} \right], \end{aligned} \quad (4)$$

$$\frac{d\mathbf{H}^+}{dt} = \nabla \times (\mathbf{V} \times \mathbf{H}^+) + \frac{1}{\varsigma} \nabla^2 \mathbf{H}^+, \quad (5)$$

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{H} = 0, \quad (6)$$

$$\nabla \times \mathbf{E} = -\mu_e \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \times \mathbf{H} = \mathbf{J}, \quad (7)$$

$$\mathbf{J} = \sigma (\mathbf{E} + \mu_e (\mathbf{V} \times \mathbf{H})). \quad (8)$$

In the above equations $\varsigma = \sigma \mu_e$ denotes the magnetic diffusivity, σ the electrical conductivity, μ_e the magnetic permeability, ρ the density, d/dt the material derivative and $\mathbf{J}$, $\mathbf{E}$ and $\mathbf{H}$ as the current density, the electric field and the magnetic field respectively.

The Cauchy stress tensor $\mathbf{T}$ in Carreau fluid is

$$\mathbf{T} = -\bar{p}\mathbf{I} + \bar{\tau} \quad (9)$$

$$\bar{\tau} = - \left[\eta_\infty + (\eta_0 - \eta_\infty) \left(1 + (\Gamma \bar{\gamma})^2 \right)^{\frac{n-1}{2}} \right], \quad (10)$$

$$\bar{\gamma} = \sqrt{\frac{1}{2} \sum_i \sum_j \bar{\gamma}_{ij} \bar{\gamma}_{ji}} = \sqrt{\frac{1}{2} \Pi} = \sqrt{\frac{\text{trac} A_1^2}{2}} \quad (11)$$

where p is the pressure, $\mathbf{I}$ the identity tensor, $\bar{\tau}$ the extra stress tensor, η_∞ the infinite shear rate viscosity, η_0 the zero shear-rate viscosity, Γ the time constant, n the dimensionless power law index, Π the second invariant of strain-rate tensor and Equation (10) for $\eta_\infty = 0$ becomes

$$\bar{\tau} = - \left[\eta_0 \left(1 + (\Gamma \bar{\gamma})^2 \right)^{\frac{n-1}{2}} \right] \bar{\gamma}, \quad (12)$$

$$\bar{\mathbf{A}}_1 = \bar{\mathbf{L}} + \bar{\mathbf{L}}^T, \quad \bar{\mathbf{L}} = \text{grad } \bar{\mathbf{V}},$$

and for $n = 1$ or $\Gamma = 0$ we have the case just like viscous fluid.

In fixed frame we have the following set of equations

$$\frac{\partial}{\partial \bar{R}} \{ (R + R^*) V \} + R^* \frac{\partial U}{\partial \bar{X}} = 0, \quad (13)$$

$$\begin{aligned}
& \rho \left[\frac{\partial \bar{V}}{\partial t} + V \frac{\partial \bar{V}}{\partial \bar{R}} + \frac{R^* \bar{U}}{R^* + \bar{R}} \frac{\partial \bar{V}}{\partial \bar{X}} - \frac{\bar{U}^2}{R^* + \bar{R}} \right] \\
&= -\frac{\partial \bar{p}}{\partial \bar{R}} + \frac{1}{R^* + \bar{R}} \frac{\partial}{\partial \bar{R}} \left\{ (R^* + R) \bar{\tau}_{\bar{R}\bar{R}} \right\} \\
&+ \frac{R^*}{R^* + \bar{R}} \times \frac{\partial \bar{\tau}_{\bar{X}\bar{R}}}{\partial \bar{X}} - \frac{\bar{\tau}_{\bar{X}\bar{X}}}{\bar{R} + R^*} - \frac{\mu_e}{2\rho} \left(\frac{\partial H^{+2}}{\partial \bar{r}} \right) \\
&+ \frac{\mu_e}{\rho} \times \left[\left(H_0 \frac{R^*}{R^* + \bar{R}} + \bar{h}_r \right) \frac{\partial \bar{h}_r}{\partial \bar{R}} \right. \\
&\left. + \bar{h}_{\bar{x}} \frac{R^*}{R^* + \bar{R}} \frac{\partial \bar{h}_r}{\partial \bar{X}} - \frac{\bar{h}_x^2}{R^* + \bar{R}} \right], \quad (14)
\end{aligned}$$

$$\begin{aligned}
& \rho \left[\frac{\partial \bar{U}}{\partial t} + \bar{V} \frac{\partial \bar{U}}{\partial \bar{R}} + \frac{R^* \bar{U}}{R^* + \bar{R}} \frac{\partial \bar{U}}{\partial \bar{X}} - \frac{\bar{U} \bar{V}}{R^* + \bar{R}} \right] \\
&= -\frac{R^*}{R^* + \bar{R}} \frac{\partial p}{\partial \bar{X}} \\
&+ \frac{1}{(R^* + R)^2} \frac{\partial}{\partial \bar{R}} \left\{ (R^* + R)^2 \bar{\tau}_{\bar{X}\bar{R}} \right\} \\
&+ \left(\frac{R^*}{R^* + \bar{R}} \right) \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\mu_e}{2\rho} \frac{R^*}{R^* + \bar{R}} \left(\frac{\partial H^{+2}}{\partial \bar{x}} \right) \\
&+ \frac{\mu_e}{\rho} \left[\left(\frac{\bar{h}_x}{R^* + \bar{R}} + \frac{\partial \bar{h}_x}{\partial \bar{R}} \right) \right. \\
&\left. \left(H_0 \frac{R^*}{R^* + \bar{R}} + \bar{h}_r \right) + \frac{R^*}{R^* + \bar{R}} \bar{h}_{\bar{x}} \frac{\partial \bar{h}_x}{\partial \bar{X}} \right], \quad (15)
\end{aligned}$$

$$\begin{aligned}
\bar{\tau}_{\bar{R}\bar{R}} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial \bar{V}}{\partial \bar{R}} \right)^2 + \left(\frac{\partial \bar{U}}{\partial \bar{R}} \right. \right. \right. \\
& \left. \left. - \frac{\bar{U}}{\bar{R} + R^*} + \frac{R^*}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} \right)^2 \right\} \right] \left(2 \frac{\partial \bar{V}}{\partial \bar{R}} \right), \quad (16)
\end{aligned}$$

$$\begin{aligned}
\bar{\tau}_{\bar{X}\bar{R}} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial \bar{V}}{\partial \bar{R}} \right)^2 \right. \right. \\
& \left. \left. + \left(\frac{\partial \bar{U}}{\partial \bar{R}} - \frac{\bar{U}}{\bar{R} + R^*} + \frac{R^*}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} \right)^2 \right\} \right] \\
& \left(\frac{\partial \bar{U}}{\partial \bar{R}} - \frac{\bar{U}}{\bar{R} + R^*} + \frac{R^*}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} \right), \quad (17)
\end{aligned}$$

$$\begin{aligned}
\bar{\tau}_{\bar{X}\bar{X}} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial \bar{V}}{\partial \bar{R}} \right)^2 \right. \right. \\
& \left. \left. + \left(\frac{\partial \bar{U}}{\partial \bar{R}} - \frac{\bar{U}}{\bar{R} + R^*} + \frac{R^*}{\bar{R} + R^*} \frac{\partial \bar{V}}{\partial \bar{X}} \right)^2 \right\} \right] \\
& \left(-2 \frac{\partial \bar{V}}{\partial \bar{R}} \right). \quad (18)
\end{aligned}$$

The transformations between the fixed frame $(\bar{R}, \bar{X})$ and wave frame $(\bar{r}, \bar{x})$ are

$$\begin{aligned}
\bar{x} &= \bar{X} - c\bar{t}, \quad \bar{r} = \bar{R}, \\
\bar{u} &= \bar{U} - c, \quad \bar{v} = \bar{V}, \quad (19)
\end{aligned}$$

and hence Equations. (13)–(18) are reduced to the following expressions

$$\frac{\partial \bar{v}}{\partial \bar{r}} + \frac{R^*}{\bar{r} + R^*} \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\bar{v}}{r + R^*} = 0, \quad (20)$$

$$\begin{aligned}
& \rho \left[-c \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{R^* (\bar{u} + c)}{R^* + \bar{r}} \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{(\bar{u} + c)^2}{R^* + \bar{r}} \right] \\
&= -\frac{\partial \bar{p}_m}{\partial \bar{r}} + \frac{1}{R^* + \bar{r}} \frac{\partial}{\partial \bar{r}} \left\{ (R^* + R) \bar{\tau}_{rr} \right\} \\
&+ \left(\frac{R^*}{R^* + \bar{r}} \right) \frac{\partial \bar{\tau}_{xr}}{\partial \bar{x}} - \frac{\bar{\tau}_{xx}}{\bar{r} + R^*} - \frac{\mu_e}{2} \left(\frac{\partial H^{+2}}{\partial \bar{r}} \right) \\
&+ \mu_e \left[\left(H_0 \frac{R^*}{R^* + \bar{r}} + \bar{h}_{\bar{r}} \right) \frac{\partial \bar{h}_{\bar{r}}}{\partial \bar{r}} \right. \\
&\left. + \bar{h}_{\bar{x}} \frac{R^*}{R^* + \bar{r}} \frac{\partial \bar{h}_{\bar{r}}}{\partial \bar{x}} - \frac{\bar{h}_{\bar{x}}^2}{R^* + \bar{r}} \right], \quad (21)
\end{aligned}$$

$$\begin{aligned}
& \rho \left[-c \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{R^* (\bar{u} + c)}{R^* + \bar{r}} \frac{\partial \bar{u}}{\partial \bar{x}} - \frac{(\bar{u} + c) \bar{v}}{R^* + \bar{r}} \right] \\
&= -\frac{R^*}{R^* + \bar{r}} \frac{\partial \bar{p}_m}{\partial \bar{x}} + \frac{1}{(R^* + \bar{r})^2} \frac{\partial}{\partial \bar{r}} \left\{ (R^* + \bar{r})^2 \bar{\tau}_{xr} \right\} \\
&+ \left(\frac{R^*}{R^* + \bar{r}} \right) \frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}} - \frac{\mu_e}{2} \frac{R^*}{R^* + \bar{r}} \left(\frac{\partial H^{+2}}{\partial \bar{x}} \right) \\
&+ \mu_e \left[\left(\frac{\bar{h}_{\bar{x}}}{R^* + \bar{r}} + \frac{\partial \bar{h}_{\bar{x}}}{\partial \bar{r}} \right) \right. \\
&\left. \left(H_0 \frac{R^*}{R^* + \bar{r}} + \bar{h}_{\bar{r}} \right) + \frac{R^*}{R^* + \bar{r}} \bar{h}_{\bar{x}} \frac{\partial \bar{h}_{\bar{x}}}{\partial \bar{x}} \right], \quad (22)
\end{aligned}$$

$$\begin{aligned} \bar{\tau}_{rr} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial v}{\partial r} \right)^2 \right. \right. \\ & \left. \left. + \left(\frac{\partial u}{\partial r} - \frac{(u+c)}{r+R^*} + \frac{R^*}{r+R^*} \frac{\partial v}{\partial x} \right)^2 \right\} \right] \\ & \left(2 \frac{\partial v}{\partial r} \right), \end{aligned} \quad (23)$$

$$\begin{aligned} \bar{\tau}_{rx} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial v}{\partial r} \right)^2 \right. \right. \\ & \left. \left. + \left(\frac{\partial u}{\partial r} - \frac{(u+c)}{r+R^*} + \frac{R^*}{r+R^*} \frac{\partial v}{\partial x} \right)^2 \right\} \right] \\ & \left(\frac{\partial u}{\partial r} - \frac{(u+c)}{r+R^*} + \frac{R^*}{r+R^*} \frac{\partial v}{\partial x} \right), \end{aligned} \quad (24)$$

$$\begin{aligned} \bar{\tau}_{xx} = & -\eta_0 \left[1 + \frac{(n-1)}{2} \Gamma^2 \left\{ 4 \left(\frac{\partial v}{\partial r} \right)^2 \right. \right. \\ & \left. \left. + \left(\frac{\partial u}{\partial r} - \frac{(u+c)}{r+R^*} + \frac{R^*}{r+R^*} \frac{\partial v}{\partial x} \right)^2 \right\} \right] \\ & \left(-2 \frac{\partial v}{\partial r} \right). \end{aligned} \quad (25)$$

We now define

$$\begin{aligned} x &= \frac{\bar{x}}{\lambda}, \quad r = \frac{\bar{r}}{a}, \quad t = \frac{c\bar{t}}{\lambda}, \\ p &= \frac{a^2 \bar{p}}{c\lambda\eta_0}, \quad M^2 = Re S^2 R_m, \quad \delta = \frac{a}{\lambda} \\ We &= \frac{\Gamma c}{a}, \quad \tau_{ij} = \frac{a \bar{\tau}_{ij}}{c\eta_0}, \quad u = \frac{\bar{u}}{c}, \\ v &= \frac{\bar{v}}{c}, \quad k = \frac{R^*}{a}, \quad E = \frac{-\bar{E}}{cH_0\mu_e}, \\ Re &= \frac{ca\rho}{\eta_0}, \quad R_m = \sigma\mu_e ac, \\ S &= \frac{H_0}{c} \sqrt{\frac{\mu_e}{\rho}}, \quad \phi = \frac{\bar{\phi}}{H_0 a}, \quad u = -\frac{\partial \psi}{\partial r}, \\ \bar{h}_{\bar{x}} &= -\bar{\phi}_{\bar{r}}, \quad \bar{h}_{\bar{r}} = \frac{R^*}{R^* + \bar{r}} \bar{\phi}_{\bar{x}}, \\ p_m &= p + \frac{1}{2} Re \delta \frac{\mu_e (H^+)^2}{\rho c^2}, \quad h_x = -\frac{\partial \phi}{\partial r}, \end{aligned} \quad (26)$$

$$v = \delta \frac{k}{k+r} \frac{\partial \psi}{\partial x}, \quad h_r = \delta \frac{k}{k+r} \frac{\partial \phi}{\partial x}, \quad (27)$$

where ψ is the stream function, ϕ is the magnetic force function, p_m the total pressure (sum of ordinary and magnetic pressures), E the electric field strength and δ , We , Re , R_m , S and M are the wave, Weissenberg, Reynolds, magnetic Reynolds, Stommer's and Hartman numbers respectively. Equation (20) is satisfied identically in view of Equation (26) and other Equations yield

$$\begin{aligned} & \frac{Re \delta}{k+r} \left[-\delta^2 k \frac{\partial^2 \psi}{\partial x^2} + \frac{\delta^2 k^2}{k+r} \left\{ \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial r \partial x} \right. \right. \\ & \left. \left. + \left(1 - \frac{\partial \psi}{\partial r} \right) \frac{\partial^2 \psi}{\partial x^2} \right\} - \left(1 - \frac{\partial \psi}{\partial r} \right)^2 \right] \\ &= -\frac{\partial p_m}{\partial r} - \frac{\delta}{k+r} \\ & \left[\frac{\partial}{\partial r} (r+k) \tau_{rr} - \delta k \frac{\partial}{\partial x} \tau_{rx} - \frac{\tau_{xx}}{r+k} \right] \\ &+ \frac{Re \delta S^2}{k+r} \left[\frac{k^2 \delta}{k+r} \left(1 + \delta \frac{\partial \phi}{\partial x} \right) \frac{\partial^2 \phi}{\partial r \partial x} \right. \\ & \left. - \frac{k^2 \delta^2}{k+r} \frac{\partial \phi}{\partial r} \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial r^2} \right], \end{aligned} \quad (28)$$

$$\begin{aligned} & \frac{Re \delta (k+r)}{k} \left[\frac{\partial^2 \psi}{\partial x \partial r} + \frac{k}{k+r} \left\{ \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial r \partial x} \right. \right. \\ & \left. \left. + \left(1 - \frac{\partial \psi}{\partial r} \right) \frac{\partial^2 \psi}{\partial x \partial r} \right\} \right. \\ & \left. - \frac{k}{(r+k)^2} \left(1 - \frac{\partial \psi}{\partial r} \right) \frac{\partial \psi}{\partial x} \right] \\ &= -\frac{\partial p_m}{\partial x} + \frac{1}{k(r+k)} \frac{\partial}{\partial r} \left((r+k)^2 \tau_{rx} \right) - \delta \frac{\partial}{\partial x} \tau_{xx} \\ &+ \frac{Re (k+r) S^2}{k} \left[\frac{k}{k+r} \left(1 + \delta \frac{\partial \phi}{\partial x} \right) \right. \\ & \left. \left(\frac{1}{k+r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} \right) - \frac{k \delta}{k+r} \frac{\partial \phi}{\partial r} \frac{\partial^2 \phi}{\partial x \partial r} \right], \end{aligned} \quad (29)$$

$$E = \frac{k}{k+r} \frac{\partial \psi}{\partial r} - \delta \frac{k}{k+r} \frac{\partial \psi}{\partial x} \frac{\partial \phi}{\partial r} + \frac{1}{R_m} \nabla^2 \phi, \quad (30)$$

$$\nabla^2 = \left[\frac{\partial^2}{\partial r^2} + \frac{1}{k+r} \frac{\partial}{\partial r} + \delta^2 \frac{k^2}{(k+r)^2} \frac{\partial^2}{\partial r^2} \right].$$

Adopting long wavelength and low Reynolds number considerations we arrive at

$$\frac{\partial p}{\partial x} = -\frac{1}{k(r+k)} \frac{\partial}{\partial r} \left[(k+r)^2 \tau_{rx} \right] + \text{Re } S^2 \left[\frac{1}{k+r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} \right], \quad (31)$$

$$E = \frac{k}{k+r} \frac{\partial \psi}{\partial r} + \frac{1}{R_m} \left[\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{k+r} \frac{\partial \phi}{\partial r} \right], \quad (32)$$

$$\tau_{rx} = - \left[1 + \frac{(n-1)}{2} \text{We}^2 \left\{ \psi_{rr} - \frac{1}{k+r} (1 - \psi_r) \right\}^2 \right] \left(-\psi_{rx} + \frac{1}{k+r} (1 - \psi_r) \right) \quad (33)$$

where Equation (28) results in that $p \neq p(r)$. The subjected conditions are

$$\begin{aligned} \Psi = -\frac{F}{2}, \quad \frac{\partial \Psi}{\partial r} = 1, \quad \phi = 0 \quad \text{at} \quad y = h, \\ \Psi = \frac{F}{2}, \quad \frac{\partial \Psi}{\partial r} = 1, \quad \phi = 0 \quad \text{at} \quad y = -h, \end{aligned} \quad (34)$$

where the dimensionless time mean flow rate F in the wave frame is related to the dimensionless time mean flow rate θ in the laboratory frame by the following relations

$$\theta = F + 2, \quad F = - \int_{-h}^h \frac{\partial \Psi}{\partial r} dr. \quad (35)$$

Fig. 1. Geometry of flow problem.

3. Solution expressions

Expanding ψ , F , p , ϕ and τ_{rx} in terms of perturbation quantity We^2 and using in Equations. (30–34) and then solving the resulting zeroth and first order systems and employing

$$F_0 = F - \text{We}^2 F_1, \quad (36)$$

we finally obtain

$$\begin{aligned} \psi(r) = & C_1 - \frac{k+r}{b^2} + C_2 (k+r)^2 \\ & + (r+k) (C_3 \cos(b \ln(r+k)) \\ & + C_4 \sin(b \ln(r+k))) \\ & + \text{We}^2 [A_1 + A_2 (r+k)^2 \\ & + (r+k) (A_3 \cos(b \ln(r+k)) \\ & + A_4 \sin(b \ln(r+k))) - \frac{1}{2(r+k)} \frac{(n-1)}{2} \\ & \{L_0 + L_1 \ln(r+k) \sin(b \ln(r+k)) \\ & + \cos(b \ln(r+k)) \\ & + L_3 \times \cos(3b \ln(r+k)) L_2 \ln(r+k) \\ & + L_4 \sin(3b \ln(r+k)) + \cos(2b \ln(r+k)) \\ & + L_6 \sin(2b \ln(r+k)) L_5 \}], \end{aligned} \quad (37)$$

$$\begin{aligned} \phi(r) = & B_1 + B_2 \ln(r+k) \\ & - \frac{1}{4} R_m \text{We}^2 L_0 (\ln(r+k))^2 \\ & + \frac{1}{4} R_m \left(E + \frac{1 + \text{We}^2}{b^2} \right) L_0 (r+k)^2 \\ & - \frac{2R_m}{4b^2} (C_2 + \text{We}^2 A_2) (r+k)^3 \\ & - \frac{L_1 \text{We}^2 R_m}{b^2} \sin(b \ln(r+k)) \\ & - \cos(b \ln(r+k)) \times \frac{L_1 \text{We}^2 R_m}{b^2} \\ & + \frac{\text{We}^2 R_m}{4b^2} (2bL_5 + L_6) \sin(2b \ln(r+k)) \\ & + \frac{\text{We}^2 R_m}{4b^2} (-2bL_6 + L_5) \\ & \times \cos(2b \ln(r+k)) \end{aligned}$$

$$\begin{aligned}
& + \frac{We^2 R_m}{9b^2} (3bL_3 + L_4) \sin(3b \ln(r+k)) \\
& + \frac{We^2 R_m}{9b^2} (-3bL_3 + L_4) \cos(3b \ln(r+k)) \\
& + \frac{We^2 R_m}{b^2} (bL_1 - L_2) \left(-\ln((r+k) \right. \\
& \quad \left. \cos(3b \ln(r+k)) + \frac{2}{b} \sin(b \ln(r+k))) \right) \\
& + \left(\ln(r+k) \sin(b \ln(r+k)) \right. \\
& \quad \left. + \frac{2}{b} \cos(b \ln(r+k)) \right) (L_1 + bL_2) \\
& - \frac{R_m (r+k)^2}{b^4 + 8b^2 + 16} \left((4-b^2) \cos(b \ln(r+k)) \right. \\
& \quad \left. + \frac{4}{b} \sin(b \ln(r+k)) \right) \\
& \quad \left(C_3 + bC_4 + We^2 (A_3 + bA_4) \right) \\
& - \frac{R_m (r+k)^2}{b^4 + 8b^2 + 16} (C_4 - bC_3 \\
& \quad + We^2 (A_4 - bA_3)) \\
& \quad \left((4-b^2) \sin(b \ln(r+k)) \right. \\
& \quad \left. - \frac{4}{b} \cos(b \ln(r+k)) \right), \\
& b = \sqrt{k^2 M^2 - 1}, \\
& \frac{dp}{dx} = -\frac{1}{k(k+r)} \frac{\partial}{\partial r} \left[(k+r)^2 \left(\frac{\partial^2 \psi}{\partial r^2} \right. \right. \\
& \quad \left. \left. + \frac{1}{k+r} \left(1 - \frac{\partial \psi}{\partial r} \right) + \frac{(n-1)}{2} \left\{ \frac{\partial^2 \psi_0}{\partial r^2} \right. \right. \right. \\
& \quad \left. \left. \left. + \frac{1}{k+r} \left(1 - \frac{\partial \psi_0}{\partial r} \right) \right\}^3 \right] \right. \\
& \quad \left. + M^2 \left[E - \frac{k}{k+r} \frac{\partial \psi}{\partial r} \right], \tag{39}
\end{aligned}$$

Fig. 2. (a) The pressure rise ΔP_λ versus flow rate θ for $E = 0$, $n = 0.398$, $\alpha = 0.4$, $We = 0.002$ and $M = 1.9$. (b) The pressure rise ΔP_λ versus flow rate θ for $E = 0$, $k = 3.2$, $\alpha = 0.6$, $We = 0.002$, $M = 1.9$ and $k = 3.2$. (c) The pressure rise ΔP_λ versus flow rate θ for $E = 0$, $k = 3.2$, $\alpha = 0.6$, $n = 0.398$, $M = 1.9$ and $k = 3.2$. (d) The pressure rise ΔP_λ versus flow rate θ for $E = 0$, $k = 3.2$, $\alpha = 0.6$, $n = 0.298$, $We = 0.002$ and $k = 3.2$.

and the involved C_i ($i = 1 - 4$), A_i ($i = 1 - 4$), B_i ($i = 1, 2$), L_i ($i = 1 - 6$) and l_i ($i = 1 - 7$) can be computed algebraically. Axial induced magnetic field h_x , current density J_z and pressure rise ΔP_λ can be computed by following relations

$$h_x = -\frac{\partial \phi}{\partial r}, \quad J_z = -\frac{\partial h_x}{\partial r},$$

$$\Delta P_\lambda = \int_0^1 \left(\frac{dp}{dx} \right)_{r=0} dx. \quad (40)$$

4. Results and discussion

This section explores the influence of M , k , n and R_m on axial velocity u , pressure rise per wavelength ΔP_λ , axial induced magnetic field h_x and current density J_z . Theoretical results are presented for both curved and straight geometries. It is found that zigzag channel outperforms the straight channel.

4.1. Pumping characteristics

Figure 2a plots the effect of curvature parameter k on pressure rise. It has been noticed that the presence of curvature increases ΔP_λ in the pumping region [26]. It is further noted that there is an increase in pressure rise when one moves from curved to straight channel i.e., peristalsis has to work against lesser pressure rise in curved channel than in straight channel. There is also an increase in the free pumping flux when one moves from curved to straight channel. In copumping region, where the pressure assists the flow, mixed behavior of curvature parameter is observed for the fixed values of flow rate.

Figure 2b discusses the variation of n on pressure rise per wavelength. It is observed that pumping curves for Carreau fluid lie below the curves for Newtonian fluid and these curves tend to coincide with the curves for Newtonian fluid as n increases. Moreover, for some fixed values of flow rate there is no difference between the Newtonian and Carreau fluids. In Fig. 2c we noticed

Fig. 3. (a) The axial velocity u versus r for $\alpha = 0.6$, $We = 0.02$, $\theta = 2.8$, $x = 0.6$, $M = 1.9$ and $n = 0.398$. (b) The axial velocity u versus r for $\alpha = 0.6$, $We = 0.1$, $\theta = 2.8$, $x = 0.6$, $M = 1.9$ and $k = 3.2$. (c) The axial velocity u versus r for $\alpha = 0.6$, $n = 0.398$, $\theta = 2.8$, $x = 0.6$, $M = 1.9$ and $k = 3.2$. (d) The axial velocity u versus r for $\alpha = 0.6$, $n = 0.298$, $\theta = 2$, $x = 0.6$, $We = 0.02$ and $k = 3.2$.

Fig. 4. (a) The magnetic force function ϕ versus r for $n = 0.398$, $\alpha = 0.2$, $\theta = -2$, $E = 1$, $x = 0.6$, $We = 0.02$, $R_m = 4$ and $M = 2$. (b) The magnetic force function ϕ versus r for $k = 1.5$, $\alpha = 0.6$, $\theta = -2.8$, $E = 1$, $x = 0.6$, $We = 0.02$, $R_m = 2$ and $M = 1.9$. (c) The magnetic force function ϕ versus r for $k = 2.2$, $\alpha = 0.6$, $\theta = -2.8$, $E = 1$, $x = 0.6$, $n = 0.398$, $R_m = 4$ and $M = 2$. (d) The magnetic force function ϕ versus r for $k = 2.2$, $\alpha = 0.6$, $\theta = -2.8$, $E = 1$, $x = 0.6$, $n = 0.398$, $R_m = 4$ and $M = 2$. (e) The magnetic force function ϕ versus r for $k = 2$, $\alpha = 0.6$, $\theta = -2.8$, $E = 1$, $x = 0.6$, $n = 0.398$, $We = 0.02$ and $M = 2.3$.

that in pumping region ($\Delta p_\lambda > 0$, $\theta > 0$), an increase in We decreases the pumping rate. For the case of copumping there is no difference between the Newtonian and Carreau fluids.

The variation of pressure rise Δp_λ against mean flow rate θ for different values of M is depicted in Fig. 2d. Here an increase in M causes an increase in the pressure rise. It is found that Δp_λ in the pumping region ($\Delta p_\lambda > 0$, $\theta > 0$) increases by increasing M for the fixed values of flow rate. However for the

case of copumping ($\Delta p_\lambda < 0$) the flow rate θ decreases when M is increased.

4.2. Flow characteristics

A flow proceeding up a curved channel is the same flow, even though its direction varies continuously along the channel. Curved channels provide greater length for the same disc diameter compared to straight. The axial velocity u for various values of curvature

Fig. 5. (a) The axial induced magnetic field h_x versus r for $\alpha = 0.2, \theta = 5, E = -3, x = -0.2, n = 0.898, We = 0.3, R_m = 4$ and $M = 5$. (b) The axial induced magnetic field h_x versus r for $\alpha = 0.3, \theta = -0.2, E = -1, x = -0.2, k = 2.5, We = 0.5, R_m = 2$ and $M = 6.25$. (c) The axial induced magnetic field h_x versus r for $\alpha = 0.3, \theta = -0.6, E = -1, x = -0.2, k = 2.5, n = 0.9, R_m = 2$ and $M = 6.25$. (d) The axial induced magnetic field h_x versus r for $\alpha = 0.2, \theta = 5, E = -3, x = -0.2, k = 2.5, n = 0.898, R_m = 4$ and $We = 0.3$. (e) The axial induced magnetic field h_x versus r for $\alpha = 0.3, \theta = -0.6, E = -1, x = -0.2, k = 2.5, n = 0.9, M = 6.25$ and $We = 0.2$.

parameter k is sketched in Fig. 3a. We observed that profiles are not symmetric about $r = 0$. There is a shift of profiles for smaller values of k (i.e., an increase in the curvature of channel) towards the lower wall. However, symmetry is possible when $k \rightarrow \infty$.

Figure 3b shows that axial velocity increases when the power law index increases for $r = 0$. Figure 3c depicts that maxima in $u(r)$ for Carreau fluid ($We \neq 0$) lies below the maxima of Newtonian fluid ($We = 0$). Hence the velocity in Carreau fluid is smaller than Newtonian fluid.

The variation of Hartman number M on u is shown in Fig. 3d. We notice that velocity profile is not symmetric about the central line of the channel due to curvature. Behavior of M near the walls of channel is quite opposite to that at the centre of channel. At $r = 0$ magnitude of velocity is an increasing function of M .

4.3. Magnetic field characteristics

This discussion yields insights into the mechanisms of blood flow in curved channels having frictionless

Fig. 6. (a) Current density versus J_z versus r for $\theta = 0.5$, $\alpha = 0.3$, $R_m = 1.5$, $E = -1$, $x = -0.2$, $n = 0.9$, $M = 3.3$ and $We = 0.9$. (b) Current density J_z versus r for $\alpha = 0.3$, $\theta = 0.5$, $R_m = 1.5$, $E = -1$, $x = -0.2$, $k = 2.5$, $M = 6.23$ and $We = 0.6$. (c) Current density J_z versus r for $\alpha = 0.3$, $\theta = 0.5$, $R_m = 2$, $E = -1$, $x = -0.2$, $k = 2.5$, $n = 0.9$ and $We = 0.9$. (d) Current density J_z versus r for $\alpha = 0.3$, $\theta = 0.5$, $M = 3.3$, $E = -1$, $x = -0.2$, $k = 2.5$, $n = 0.9$ and $We = 0.9$.

walls exposed to a magnetic field. Blood is a biomagnetic fluid. Figure 4a–e explain the magnetic field characteristics k , n , We and M . One observes that the parabolic profiles for magnetic force function depict a left shift at $r = 0$ due to curvature effects. Magnetic force function is zero at the walls, which is in complete agreement with the imposed boundary condition. The profiles here are increasing function of k , n , We and M near the upper wall of channel. Magnetic force function is decreasing function of R_m (Fig. 4e).

Figure 5a–e discuss the axial induced magnetic field h_x against r for different values of k , n , We , M and R_m . In the half region induced magnetic field is in one direction whereas in the other half it is in opposite direction. Magnitude of h_x increases with We , M and R_m while shows an opposite behavior for k and n .

The current density distribution J_z for various values of k , n , M and R_m is plotted in the Fig. 6a–d. Here the magnitude of current density J_z decreases about the centre of channel by increasing k and n . A shift

in the profiles is observed towards the lower wall. For large values of k symmetry in the profiles is obtained and results of planar channel are deduced. Both M and R_m have increasing effects on the current density distribution (Fig. 6c and d).

5. Concluding remarks

We have discussed the flow characteristics of Carreau fluid in a curved channel. The main observations are as follows.

- Axial velocity for Carreau fluid is smaller than viscous fluid about the centre of channel.
- Pressure rise per wavelength and axial velocity are greater for viscous fluid in comparison to Carreau fluid.
- The tilt in velocity profiles shows that flow is more towards the lower wall.
- Magnitude of induced magnetic field is greater for Carreau fluid in comparison to viscous fluid.

- Symmetry of quantities of interest such as velocity, current density and induced magnetic field at $r = 0$ is disturbed due to curvature. The results of planar channel are recovered when $k \rightarrow \infty$.

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