Effect of hematocrit on wall shear stress for blood flow through tapered artery

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Abstract. The purpose of this study to show the effects of Hematocrit (Red blood cells), height of stenosis, porous parameter and velocity of blood on wall shear stress of the flow of blood through tapered artery. The study reveals that wall shear stress reduces for increasing Hematocrit percentage. It is also observed that wall shear stress increases as stenosis height and porous parameter increase whereas it decreases with the increasing values of velocity of blood and slope of tapered artery.

Keywords: Wall shear stress, hematocrit, porous parameter, velocity of blood

1. Introduction

Atherosclerosis is a dangerous disease and is caused due to the abnormal growth in lumen of arterial wall. It is a condition in which an artery wall thickens as a result of the accumulation of fatty materials such as cholesterol. The inner lining of the artery, called the endothelium, can be damaged due to high cholesterol and triglyceride levels, toxic substances in cigarette smoke, high sugar levels, and other factors in the blood. High blood pressure can also cause damage to the inner lining of an artery. Once the blood vessel is damaged, atherosclerosis begins and a plaque forms. Biswas and Chakraborty [1] investigated pulsatile flow of blood through a tapered artery in the presence of a mild stenosis. They studied the influence of Hematocrit and a velocity slip at wall, in blood flow through stenosed arteries. Bali and Awasthi [2] have studied the flow of blood in a stenosed artery under the influence of an external magnetic field by considering the viscosity of blood as a function of Hematocrit and radial distance.

Verma and Parihar [3] proposed the effects of Hematocrit on blood flow in an artery with multiple mild stenosis. In their theoretical modeling, blood viscosity is assumed to behave as a linear function of Hematocrit. A two-phase macroscopic model of blood has been considered by Srivastava and Rastogi [4] for studying the effects of Hematocrit on the impedance and wall shear stress, during stenosed artery catheterization. Verma and Parihar [5] developed a mathematical model to study the effect of stenosis and Hematocrit on flow rate, wall shear stress and resistance parameter through tapered artery under stenotic conditions by considering laminar flow, rigid walls and Newtonian fluid. Mishra and Verma [6] developed a mathematical model to study the effect of porous parameter and height of stenosis on the wall shear stress. The blood flowing through the tapered artery under the porous medium is considered to be Newtonian. They observed that as the height of stenosis and porous parameter is increased, the wall shear stress also increases, but as the velocity of blood is increased, the wall shear stress decreases. Alizadehrad et al. [7] investigated the deformation of red blood cells in micro-vessels numerically for various vessel diameters, hematocrits, and shear rates. They simulated blood flow in circular

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channels with diameters ranging from 9 to 50 μm , hematocrits from 20% to 45%, and shear rates from 20 to 150 s^{-1} using a particle-based model with parallel computing. Brookshier and Tarbell [8] studied that for a fixed oscillatory flow waveform (Poiseuille peak shear rate = 168 s^{-1} ; mean shear rate 84 s^{-1}), increases in hematocrit produced a decrease in the peak wall shear rate in both the straight and curved artery models and a corresponding decrease in wall shear rate

$$R(z) = \begin{cases} R_1 - m(z + l) & 0 \leq z \leq d \\ R_1 - m(z + l) - \frac{h \cos \varphi}{2} \left\{ 1 + \cos \left(\frac{\pi z}{l_0} \right) \right\} & d \leq z \leq d + l_0 \\ R_1 - m(z + l) & d + l_0 \leq z \leq l \end{cases} \quad (1)$$

reversal on the inside wall of the curved artery model. Zheng et al. [9] investigated the effects of acute tachycardia on systemic pressure and blood flow of peripheral arteries using experimental measurements and hemodynamic analysis. They observed that the initial fall of blood flow due to pressure drop led to transient flow reversal and negative wall shear stress. Wenjuan and Jungfeng [10] simulated red blood cells flowing in micro-vessel to examine the induced wall shear stress variation. They observed a typical peak-valley-peak structure and analyzed in terms of its magnitude, spatial influencing range, and temporal elapsed duration. They also investigated the effects of red cell deformability, micro-vessel size, and flow velocity. Akabar et al. [11] examined effect of heat and mass transfer on the blood flow through a tapered artery with stenosis. They foemed the series solutions for the velocity, temperature, concentratuion, resistance impedance, wall shear stress and shearing stress at the stenosis throat. Akabar et al. [12] studied the blood flow through a tapered artery with a stenosis by considering axially non-symmetric but radially symmetric mild stenosis on blood flow characteristics, assuming the flow is steady and blood is treated as Williamson fluid. They also carried out the effects of heat and mass transfer. Perturbation solutions have been calculated for velocity, temperature, concentration, resistance impedance, wall shear stress and shearing stress at the stenosis throat by them.

All the consequences obtained in this paper point out that the development of stenoses is a severe problem in the human artery. The irregular growth of stenoses results the blood pressure, heart attack and brain hemorrhage troubles. It can not be ignored in modeling of problems of blood flow in human arteries. A mathe-

matical model can be developed for the deformation of stenoses with aid of drug release models in human arteries. The nearby analysis will be very valuable in study of above mentioned problems in stenosed artery.

2. Mathematical formulation

Physical model of tapered artery with stenosis

$R(z)$ = Effective radius of tapered artery
 R_1 = Radius of Tapered Artery
 φ = Angle of Tapering
 $H = h \cos \varphi$ = The Height of stenosis in tapered artery
 l_0 = length of stenosis
 $m = \tan \varphi$ = Slope of tapred vessel

For a Newtonian fluid

$$\tau = rG/2 \quad (2)$$

From Darcy's Law

$$u = \frac{-KG}{\mu} \quad (3)$$

From equation (2) and (3), we have

$$\tau = -\frac{\mu u}{2K} r$$

To find the effect of Hematocrit on the wall shear stress, the relation of blood viscosity and the Hematocrit is given by

$$\mu = \mu_p (1 + 2.5H)$$

The wall shear stress through the artery

$$\tau = -\frac{\mu_p (1 + 2.5H) u}{2K} \int_0^l R(z) dz$$

Fig. 1. Physical model of tapered artery.

$$\begin{aligned} \tau = & -\frac{\mu_p(1+2.5H)u}{2K} \left[\int_0^d R_1 - m(z+l) dz + \right. \\ & \int_d^{d+l_0} R_1 - m(z+l) - \frac{h \cos \phi}{2} \left\{ 1 + \cos \left(\frac{\pi z}{l_0} \right) \right\} dz \\ & \left. + \int_{d+l_0}^l R_1 - m(z+l) dz \right] \\ \tau = & -\frac{\mu_p(1+2.5H)}{4K} [a + b + c] \end{aligned}$$

where

$$\begin{aligned} a &= \left[(R_1 - ml)^2 - \{R_1 - m(d+l)\}^2 \right] \\ b &= \left[\{R_1 - h \cos \phi - m(d+l)\}^2 \right. \\ & \quad \left. - \{R_1 - m(d+l_0+l)\}^2 \right] \\ c &= \left[\{R_1 - m(d+l_0+l)\}^2 - (R_1 - 2ml)^2 \right] \end{aligned}$$

3. Results and discussion

From graphs (1), (2) and (3), it is observed that as the height of stenosis $H = h \cos \phi$ and porous parameter increase, wall shear stress also increases but as the velocity of blood increases, wall shear stress decreases. Further that as the hematocrit percentage increase wall shear stress diminishes in the present situation. Therefore, by adjusting the percentage of hematocrit, one

Graph 1. Variation in wall shear stress with stenosis height for different Hematocrit percentage.

Graph 2. Variation in wall shear stress with porous parameter for different hematocrit percentage.

Graph 3. Variation in wall shear stress with velocity of blood for different hematocrit percentage.

Graph 4. Variation in wall shear stress with stenosis height for different slopes of tapered artery.

can control the wall shear stress to maintain the arterial diseases under stenotic condition.

It is also noted that from the graph (4) that the wall shear stress decreases with an increase in slope of tapered vessel.

4. Conclusion

Wall shear stress plays an important role in long-term maintenance of the structure and function of

the blood vessel. Under normal conditions, shear stresses maintain their direction and their magnitude within a range of values that impedes atherogenesis, thrombosis, adhesion of leukocytes, smooth muscle proliferation and endothelial apoptosis. Changes in shear stress magnitude activate cellular proliferation mechanisms as well as vascular remodelling processes. More specifically, a high grade of shear stress increases wall thickness and expands the artery diameter so that shear stress values return to their normal values. In contrast, low shear stress induces a reduction in diameter.

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