Research Article

Analytical Model for the End-Bearing Capacity of Tapered Piles Using Cavity Expansion Theory

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On the basis of evidence from model tests on increasing the end-bearing behavior of tapered piles at the load-settlement curve, this paper proposes an analytical spherical cavity expansion theory to evaluate the end-bearing capacity. The angle of tapering is inserted in the proposed model to evaluate the end-bearing capacity. The test results of the proposed model in different types of sands and different relative densities show good effects compared to conventional straight piles. The end-bearing capacity increases with increases in the tapering angle. The paper then propounds a model for prototypes and real-type pile tests which predicts and validates to evaluate the end-bearing capacity.

1. Introduction

The pile end-bearing capacity in sands is not only affected by the compressibility of soil, shear stiffness, and strength, but also by the angle of tapering of the pile. Very few researchers have noticed the effects of tapering angle in end-bearing resistance when penetrated downward in a frictional mode [1]. The shape of the pile type changes from a straight-sided cylindrical shape to a conical type, while the weight and volume of the pile appendages to the ground and mechanism also change. Therefore, the tapering effects must be taken into consideration when evaluating the end-bearing capacity of tapered piles [2, 3]. There are two main methods to evaluate the end-bearing capacity: a semiempirical method using SPT-N values and a theoretical approach based on geomechanical considerations. Out of all the theoretical methods of geomechanics, the cavity expansion theory is particularly popular among geotech experts [4–6]. Yasufuku et al. derived successfully an evaluation technique for the end-bearing capacity in nondisplacement straight cylindrical piles using a spherical cavity expansion theory for closed solutions [7, 8]. This research asserts the use of the spherical cavity expansion theory as a way of evaluating the end-bearing capacity of tapered piles based on axial cylindrical model tests. The tapering angle is inserted in the analytical

model to evaluate the ultimate end-bearing capacity of the spherical cavity expansion theory previously postulated by Yasufuku et al. [7, 8]. The proposed model then referred to predict and validate other prototypes and real-type pile reference data to estimate the end-bearing capacity.

2. Methodology and Evidence from Test Results

Chromium-plated three steel piles—one straight (S) and two taper-shaped (T-1 and T-2)—were used for model tests (Table 1) on modeled ground, having relative densities of 60% and 80%, respectively (Table 2). The model piles have equal lengths and the same tip diameters. The model chamber has dimensions of 1000 mm height and a diameter of 750 mm, as shown by Figure 1(a).

Japanese Industrial System (JIS A 1224) and Japanese Geotechnical Standards (JGS 0161) [9] were used to determine the maximum and minimum dry densities of sands so as to determine the relative densities (Table 2). A multiple sieving technique was used to prepare model ground at an appropriate relative density by height of fall and nozzle diameter. Changes in nozzle area and/or diameter of multiple sievings determine the relative density of the ground when particles fall at a specific height and vice versa [10]. In this

FIGURE 1: (a) Pile load chamber and (b) cast-in-place pile set up and loading mechanism (figure not to scale).

method, the height of falling was determined by keeping a constant nozzle area. Then, two sands, K-7 and Toyoura (TO) sands, were allowed to fall through multiple sievings from the heights of 1400 mm and 700 mm, respectively. The pile was installed at a height of 710 mm from the bottom, and soil was further poured in up to a height of 930 mm, as shown by Figure 1(b). Following this, an overburden pressure (σ_v) of 50 kPa was furnished vertically through the upper plate, and the pile was installed up to 200 mm at a rate of 5 mm/min to facilitate in cast-in-place pile condition. A minimum of fifteen hours is required for stress relaxation after installation. Afterwards, a pile loading test was performed up to 0.4 settlement ratio. In the loading test, load cells were installed inside the pile tip and at the pile head, which were connected with the cord. During loading, the load cell at the pile tip can measure directly the end-bearing capacity of the pile.

Figure 2 shows evidence obtained from model test results of straight and tapered piles of K-7 and TO sands. These tests show that when the tapering angle increases, the pile tip resistance increases for both sands at a normalized settlement ratio. Here, S and D are considered as settlement and the pile tip diameter, respectively. The normalized total end-bearing capacity has been plotted by dividing mean effective stress σ' which is the mean of lateral and vertical stresses (i.e., $\sigma' = (\sigma_0 + \sigma_v)/2$). For normally consolidated soil, the lateral stress can be determined by selecting a 0.5-value in the relation between lateral and overburden stresses (i.e., $\sigma_0 = K_0 \sigma'_v$), where K_0 is the coefficient of earth pressure at rest relating with dry unit weight of the ground and its depth. Figure 3 shows the normalized total end-bearing capacity of sands at different relative densities. The result adds tapering effects to the end-bearing mechanism by increasing the tapering angle. This evidence obtained through small model tests explains the merits over tapered piles. Given this, it is necessary to

FIGURE 2: End-bearing resistance of K-7 and TO sands.

develop an analytical model for evaluating the end-bearing capacity, and it is imperative to check the validity and liability of verifying with prototypes and real pile references, which will be discussed in the next section.

3. Analytical Model to Evaluate End-Bearing Capacity

The compressibility of soil, shear stiffness, and strength all affect the pile end-bearing capacity in sands. Compressibility is believed to differ greatly, ranging from incompressible silica sands to highly compressible carbonate sands for different types of soils. A spherical cavity expansion solution

TABLE 1: Geometrical configuration of different types of piles.

Types of model piles	Naming	L mm	D_t mm	D mm	α °	FRP reinforcement direction	Modulus of elasticity (GPa)
Model steel piles	S	500	25	25	0.00	na	2
	T-1	500	35	25	0.70	na	2
	T-2	500	45	25	1.40	na	2
Prototype FRP piles	FC	1524	168.3	168.3	0.00	na	31.86
	T-3	1524	170.0	198.0	0.53	0°	33.20
	T-4	1524	159.0	197.0	0.71	0°	33.15
	T-5	1524	155.0	215.0	1.13	0°	33.15

Note: L : length of pile; D_t : diameter at the pile head; D : pile tip diameter; FRP: fiber-reinforced polymer; α : angle of tapering; na: not applicable.

TABLE 2: Geotechnical properties of different sands.

Descriptions	TO	K-7	Fanshawe brick sand (Sakr et al.) [23–25]
Density of particles, (g/cm ³) ρ_s	2.65	2.62	2.68
Maximum density, (g/cm ³) $\rho_{\max}$	1.64	1.60	1.772
Minimum density, (g/cm ³) $\rho_{\min}$	1.34	1.19	1.466
Density at I_D 80%, (g/cm ³) ρ_{80}	1.58	1.52	na
Density at I_D 60%, (g/cm ³) ρ_{60}	1.52	1.43	na
Maximum void ratio, $e_{\max}$	0.98	1.20	0.794
Minimum void ratio, $e_{\min}$	0.62	0.64	0.484
Void ratio at I_D 90%, e_{90}	na	na	0.68
Void ratio at I_D 80%, e_{80}	0.68	0.73	na
Void ratio at I_D 60%, e_{60}	0.74	0.83	na
Effective grain size, (mm) D_{10}	na	na	0.14
Mean grain size, (mm) D_{50}	na	na	0.26
Uniformity coefficient, U_c	1.40	4.0	2.143
Coefficient of curvature, U'_c	0.86	1.21	0.905
Percent fines, (%) F_c	1.10	14	na
Peak stress, (deg)° ϕ	42.00	47.00	37.00
Critical stress state, (deg)° ϕ'_{cv}	32.00	34.00	31.00 (assumed)

has been incorporated to estimate the end-bearing capacity of cylindrical straight piles in closed forms by Yasufuku et al. [7, 8]. On the basis of this estimation technique, this model is advanced by introducing the angle of tapering to evaluate the end-bearing capacity of tapered piles. Figures 4(a) and 4(b) explain a modified failure mechanism using a spherical cavity expansion theory which was initially postulated by Yasufuku et al. [7, 8] for frictional soils with cavity expansion pressure p_u proposed by Vesic [4] to determine the ultimate bearing capacity q_{pcal} . In the modified mechanism of tapered piles, it is assumed that a rigid cone of soil exists beneath the pile tip with the angle $\Psi' (= \pi/4 + \phi'/2 + \alpha)$ and that outside the conical region, the zone is subjected to isotropic stress which is equal to the cavity expansion pressure p_u . In addition, the active earth pressure conditions $\sigma_A [= q_{pcal} \{ (1 - \sin(\phi'_{cv} + 2\alpha)) / (1 + \sin(\phi'_{cv} + 2\alpha)) \}]$ are considered to exist immediately beneath the pile tip along AC plane. Then, at point B, the moment is created as shown by Figure 4 for the cavity expansion pressure p_u , ultimate end-bearing pressure q_{pcal} , and the active earth pressure σ_A [3]. Then, the ultimate

bearing capacity q_{pcal} of the tapered pile can be addressed with the formula

$$q_{pcal} = \frac{1}{1 - \sin(\phi'_{cv} + 2\alpha)} p_u. \quad (1)$$

The cavity expansion pressure p_u on the basis of Vesic theory [4] can be written as

$$p_u = F_q \frac{(1 + 2K_0)}{3} \sigma'_v, \quad (2)$$

where

$$F_q = \frac{3(1 + \sin \phi'_{cv})}{(3 - \sin \phi'_{cv})} [I_{rr}]^{(4 \sin \phi'_{cv})/3(1 + \sin \phi'_{cv})}, \quad (3)$$

$$I_{rr} = \frac{I_r}{1 + I_r \Delta_{av}}, \quad I_r = \frac{3G}{(1 + 2K_0) \sigma'_v \tan \phi'_{cv}},$$

where F_q , I_{rr} , and I_r are known as dimensionless spherical cavity expansion factors, reduced rigidity index, and rigidity

FIGURE 3: Normalized total end-bearing capacities of K-7 and TO sands.

FIGURE 4: (a) Concept of modified failure mechanism around the tapered pile tip in cavity expansion solution and (b) geometry of calculation procedure to find ultimate end-bearing capacity of tapered pile.

index, respectively, which are associated with friction angle ϕ' , shear stiffness G , and average volumetric strain Δ_{av} for the plastic zone around a cavity, with the coefficient of earth pressure at rest K_0 and overburden pressure σ'_v . I_r provides a ratio of shear stiffness to strength, and I_{rr} is a parameter in lieu of the soil compressibility, in which shear stiffness, shear strength, and average volumetric strain are combined. Proceeding, σ'_v is basically calculated as an averaged unit weight $\gamma_{av}(= \gamma_{av}z)$. On the basis of the theoretical relation to plastic equilibrium, Ochiai [11] derived the critical state friction angle ϕ'_{cv} as

$$K_0 = 1 - \sin \phi'_{cv}. \quad (4)$$

Asserting ϕ'_{cv} is effective and rational in practical applications as a strength parameter [12–15], the friction angle

guarantees the minimum shear strength under the same initial conditions and soil density, with initial fabric and confining pressure considered to be independent. The value of ϕ'_{cv} of sands is almost equal to the maximum friction angle under the high confining stress, which will be mobilized below the pile tip [16, 17]. Therefore, the ϕ'_{cv} is recommended for estimating the pile end-bearing capacity of sands. At the same time, an empirical equation $G = 7.0N^{0.72}$ is used to predict the G -value in the strain level of 10^{-3} from the measured N -value [18], in MPa. The penetration resistance is presumed to increase with the square of the relative density and to be directly proportional to the effective overburden pressure and inversely proportional to the void ratios [19] such that

$$N = \frac{9I_D^2}{(e_{\max} - e_{\min})^{1.7}} \left\{ \frac{\sigma'_v}{98} \right\}^{0.5}, \quad (5)$$

where I_D is relative density and $e_{\max}$ and $e_{\min}$ are maximum and minimum void ratios. Rearranging values of N from (5), the empirical correlation can be expressed as

$$G = 7.0 \left\{ \frac{9I_D^2}{(e_{\max} - e_{\min})^{1.7}} \left\{ \frac{\sigma'_v}{98} \right\}^{0.5} \right\}^{0.72}. \quad (6)$$

The empirical equation $\Delta_{av} = 50(I_r)^{-1.8}$ is a function of I_r in a simple manner [6]. Rearranging from (3), a function of G , K_0 , σ'_v , and ϕ'_{cv} can be written in this form

$$\Delta_{av} = 50 \left\{ \frac{((1 + 2K_0)/3)\sigma'_v \tan \phi'_{cv}}{G} \right\}^{1.8}. \quad (7)$$

Equation (7) measures the average volumetric strain Δ_{av} for a plastic zone around a cavity, which increases with increasing overburden pressure in the ground. The rearrangement of this equation addresses the evaluation of the ultimate pile end-bearing capacity as follows:

$$q_{pcal} = \frac{A'}{1 - \sin(\phi'_{cv} + 2\alpha)} \left\{ \frac{G/\sigma'_v}{B' + D'(G/(\sigma'_v))^{-0.8}} \right\}^{C'} \sigma'_v, \quad (8)$$

where

$$\begin{aligned} A' &= \frac{3(1 + \sin \phi'_{cv})}{(3 - \sin \phi'_{cv})} \left(\frac{1 + 2K_0}{3} \right), & B' &= \left(\frac{1 + 2K_0}{3} \right) \tan \phi'_{cv}, \\ C' &= \frac{4 \sin \phi'_{cv}}{3(1 + \sin \phi'_{cv})}, & D' &= 50 \left\{ \left(\frac{1 + 2K_0}{3} \right) \tan \phi'_{cv} \right\}^{1.8}. \end{aligned} \quad (8a)$$

The Kondner types of hyperbolic curves are useful for predicting the load settlement curves of nondisplacement piles in virgin loading [20–22]. A simple hyperbolic function is assumed for estimating the relationship between the applied pile tip stress, q_{cal} , and the corresponding normalized pile tip settlement, S/D , as

$$q_{cal} = \frac{S/D}{n + m(S/D)}, \quad (9)$$

where n and m are experimental parameters corresponding to the inverse values of suitable initial shear stiffness and ultimate pile stress, respectively.

When introducing reference displacement $(S/D)_{\text{ref}} = 0.25$, presented by Hirayama [22], which is empirically derived for nondisplacement piles in sands, expressed as the normalized settlement S/D required to mobilize the half of the ultimate end-bearing capacity q_{pcal} , the inverse of the initial shear stiffness is expressed such that

$$q_{\text{cal}} = \frac{S/D}{\{0.25 + S/D\}} q_{\text{pcal}}. \quad (10)$$

4. Validity of the Model

The validity of the model was checked after verifying it with a small model and prototype pile materials, including with different sources of data. The parameters of TO sand K-7 sand and pile materials have been taken from Manandhar et al. [1] and Fanshawe brick sand from Sakr et al. [23–25]. This research employs a cylindrical fiber-reinforced polymer (FRP) FC pile and a three-tapered FRP composite tapered pile, which is an off-the-shelf pipe with an average diameter of 162.4 mm and a ply angle of 55° . These fabricated FRP tapered piles using six layers of glass filament wound (GFW) were placed at ply angles 0° (parallel to pile axis) and 90° (hoop layer) [23] (Table 1). The pile installation of Fanshawe brick sand is confined to a low pressure (LP) with an initial radial stress of 30 kPa and a vertical pressure of 60 kPa, as well as a high pressure (HP) with an initial radial stress of 60 kPa and a vertical pressure of 120 kPa at different depths and mobilized up to 0.4 normalized settlement ratios. The fundamental geotechnical properties of sand are set out in Table 2. The vertical pressure can be obtained by using the simple formula discussed in the previous section. Figure 5 shows there is increase in total end-bearing capacity when increasing the tapering angles in the closed form cavity expansion solution. Larger total end-bearing resistance is reflected by high density ground. Similarly, high end-bearing capacity has been obtained at a high radial stress at the same density ground, as shown by Figure 6.

Further, the total end-bearing capacity measured in kN is

$$P_B = q_{\text{cal}} \pi r_b^2, \quad (11)$$

where r_b is the radius of the pile tip at the middle point of the embedded pile section.

The following arrangement can be obtained by substituting the value of q_{cal} using (1), (2), (10), and (11) into (10):

$$P_B = \frac{S/D}{\{0.25 + S/D\}} \frac{1}{1 - \sin(\phi'_{\text{cv}} + 2\alpha)} \times A' \left\{ \frac{G/\sigma'_v}{B' + D'(G/\sigma'_v)^{-0.8}} \right\}^C \sigma'_v \pi r_b^2 \quad (12)$$

Assume $(P_B)_{\alpha=0}$ for the total end-bearing capacity of straight piles. When the total end-bearing capacity of tapered piles is normalized with the total end-bearing capacity of

FIGURE 5: Total end-bearing capacity of TO and K-7 sands of different piles at normalized settlement ratio.

FIGURE 6: Total end-bearing capacity of Fanshawe brick sand of different piles at normalized settlement ratio.

straight piles by taking ratios between them, the following relation is obtained:

$$\frac{P_B}{(P_B)_{\alpha=0}} = \frac{1/(1 - \sin(\phi'_{\text{cv}} + 2\alpha))}{1/(1 - \sin \phi'_{\text{cv}})}. \quad (13)$$

This equation shows the relationship between the angle of internal friction at a critical state and tapering angle as the interdependent function for evaluating the total end-bearing capacity of tapered piles. This verifies that the end-bearing capacity depends on the angle of internal friction at the critical state condition and the tapering angle of piles only. When there is a change in the angle of tapering,

TABLE 3: Pile geometry and soil characteristics from different source papers.

Source paper	No.	Pile geometry		Soil type	Soil characteristics		
		Diameter, d (m)	Length, L (m)		σ'_v (kPa)	$\phi'_{cv,av}$ ($^\circ$)	N_{av} (G) (MPa)
BCP (1B) (1971) [27]	1	0.2	4	Fine sand	60	35 (34–36)	20 (60.5)
BCP (5C) (1971) [27]	2	0.2	11	Dense sand	170	37 (36–38)	48 (133.5)
JGS data (1993) [28]	3	1.5	44.5	Sand	300	35	25 (71.1)
	4	1.5	32	Sand	356	(34–36)	30 (81)
	5	1.5	26.5	Sand	256	35	30 (81)
	6	1.5	22.4	Sand	212	(34–36)	30 (81)
	7				100		— (21.9)
Yasufuku et al. (2001) [8]	8	0.03	—	Quiou sand	200	36	— (42)
	9				400		— (47.0)

FIGURE 7: Normalized end-bearing capacity of K-7 sand at different pile tapering angles.

FIGURE 9: Normalized end-bearing capacity of Fanshawe brick sand and different pile tapering angles.

FIGURE 8: Normalized end-bearing capacity of TO sand at different pile tapering angles.

FIGURE 10: Effect of tapering angle on normalized end-bearing capacity at 0.1 settlement ratio.

the end-bearing capacity will also change independently of overburden pressure, confining pressure, and the shear modulus of soil. Hence, Figures 7, 8, and 9 were plotted to verify this mechanism of normalized ratios of total end-bearing capacity for different sands. There is a clear increase

in the total end-bearing capacity at 0.1 settlement ratios for all types of soils and pile materials.

Further, different tapering angles were asserted to understand the behavior of end-bearing capacity in sands.

FIGURE 11: Calculated and measured end-bearing capacity of different types of piles.

This study employed model tests, the prototype test described by Sakr et al. [23–25] and the real type Rybnikov [26] pile. Rybnikov carried out tests in the Irtysh Pavlodar region of the former Soviet Union and used bored cast-in-place tapered piles. The holes for the piles were drilled with endless screws. Seven different piles were consummated having lengths of 4.5 m each, constituting five tapered piles and two cylindrical piles to comprehend the behavior of tapered piles with respect to straight piles. In this study, only the geometry of pile materials is considered for the analyses, with the assumption of soil properties obtained by TO sand as shown by Table 2.

Figure 10 represents the effects of the tapering angle in four different types of pile materials and soils by dividing the total end-bearing capacity of straight piles to tapered piles. The results show an increase in the end-bearing capacity of almost 10% for the maximum tapered angle, indicating that the end-bearing capacity is affected by the angle of tapering and angle of internal friction only.

Afterwards, the measured and calculated results of the end bearing capacity were plotted to verify the model. The model was verified with the addition of various reference data. The measured and predicted data for the end-bearing capacity, measured in kPa, are shown in Tables 3 and 4,

respectively. Figure 11 shows how the proposed model fits remarkably well and that it is valid for evaluating end-bearing capacity when various data have been obtained. The measured and calculated results were plotted in a 1:1 ratio and proved the validity of the model with the parameters used with different types of pile geometry and sands.

Conclusions

The benefits of tapered piles obtained through evidence from small model tests in the laboratory lead to the development of an analytical model for the estimation of the end-bearing capacity of tapered piles. The assertion of tapering angle in the analytical spherical cavity expansion theory was successfully evaluated for the end-bearing capacity of tapered piles. The proposed models have been validated through the verification of model tests, prototype tests and real type pile tests. The main conclusions drawn using the various proposed models are summarized as follows.

- (1) Parametric studies with the key variable of the tapering angle show that the proposed model effectively sustains the general behavior of tapered piles in evaluating the end-bearing.
- (2) The total end-bearing capacity of tapered piles confines to restrain the failure mode, which increases the end-bearing capacity, and the model is supported by the measured data.

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