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Research Article

Correction of Line-Sampling Bias of Rock Discontinuity Orientations Using a Modified Terzaghi Method

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The Terzaghi method is widely used to correct the line-sampling bias of rock discontinuity orientations. The method includes four procedures, one of which is meshing the stereographic projection diagram into cells. The method is based on the bias-compensatory factor, $1/\sin\theta$, where θ is the angle between the scanline and the discontinuity defined at each cell center. This paper presents a modified Terzaghi method that eliminates meshing, thereby reducing the method to three steps that (1) count the frequencies, (2) weigh the frequencies by the bias-compensatory factor, and (3) round the weighed frequencies to the nearest integer. Due to the elimination of the mesh, the counting object has changed to the frequency at each pole, and θ in the bias-compensatory factor is redefined as the angle between the scanline and the discontinuity at each pole. The applicability of the redefined bias-compensatory factor is verified through a mathematical logical deduction. The accuracy of the conventional and the modified Terzaghi methods are compared using a case study in Wenchuan, China, revealing improved accuracy for the latter.

1. Introduction

Many important engineering structures are built on or in rock, such as tunnels, dam foundations, bridge foundations, tall buildings, etc. Rock is an inhomogeneous natural material owing to the presence of discontinuities, defined as any significant mechanical break or fracture of negligible tensile strength [1]. Discontinuities include bedding planes, faults, fissures, fractures, joints, etc. The orientation of discontinuities is known to greatly affect the kinematical and mechanical behavior of rock [2–5]. These orientations can be observed on rock exposures such as outcrops, tunnel faces, pit faces, and drillhole faces. A common observational method is line sampling down a borehole or along a scanline on an exposure. However, such sampling biases the apparent abundance of the orientations, because the probability that a particular discontinuity is intersected by a scanline depends on its orientation [6, 7]. To correct for this bias, Terzaghi [8] proposed a well-known method that weighs the observed frequencies by a bias-compensatory factor, which is the reciprocal of the sine of the intersection angle between the scanline and discontinuity. A detailed description of this method is available in Einstein and Beacher [9, 10]. The Terzaghi method has been widely used by researchers, including Goodman [11], Park [12], and Fouché and Diebolt [13].

Tang et al. [14] found that the Terzaghi method involves an error, even if optimizing countermeasures are applied. Unfortunately, the origin of the error is unknown, but the meshing procedure is suspected to be a significant component.

This paper presents a modified Terzaghi method that eliminates the meshing procedure. Following a brief introduction of the conventional Terzaghi method, we describe this modified method and its internal procedures.

Owing to mesh elimination, some procedural modifications are required. Second, the bias-compensatory factor had been reobtained in the case of the presence of mesh [8], so we determine whether this factor is still applicable in the no-mesh case, using a mathematical logical deduction. Finally, we apply both the conventional and the modified Terzaghi methods to a test case and compare the accuracy of the two results.

2. Modified Terzaghi Method

2.1. Procedures. The procedures of the conventional Terzaghi method [8] are (Figure 1(a)) as follows:

(1) Subdivide the projection net into cells.

(2) Count the frequencies lying in each cell.

(3) Weigh the frequencies by the bias-compensatory factor $1/\sin\theta$, where θ is the intersection angle between the scanline and the discontinuity defined at each cell center. Since the orientation is composed of two elements, that is, the dip direction and dip angle, the weighing should be executed for dip direction and dip angle, respectively.

(4) Because the frequencies defined in this manner must be integers, the weighed frequencies that are rounded to the nearest integer.

The procedures of the modified Terzaghi method are (Figure 1(b)) as follows:

(1) Count the frequencies at each pole. Owing to mesh elimination, the counting objects have changed to the frequencies at each pole.

(2) Weigh the frequencies by the bias-compensatory factor $1/\sin\theta$, where θ is redefined as the angle between the scanline and the discontinuity at each pole.

(3) Round the weighed frequencies to the nearest integer.

2.2. Applicability of the Redefined Bias-Compensatory Factor. The kernel of the modified Terzaghi method is the bias-compensatory factor. This factor is known to be $1/\sin\theta$ for the conventional Terzaghi method [8], but its applicability to the modified Terzaghi method with redefinition remains unknown. In this section, a mathematical deduction is used to verify its applicability.

For the conventional Terzaghi method, the weighing procedure is obtained by multiplying the bias-compensatory factor, $1/\sin\theta$, by the observed frequency defined in the cell [8]. This step can be formulated as follows:

$$P_{\mathrm{A}} = \frac{\int_{\alpha}^{\alpha+\Delta\alpha}\int_{\beta}^{\beta+\Delta\beta} p_{\mathrm{AB}}(\alpha,\beta)\,d\alpha\,d\beta}{\sin\theta} = \frac{P}{\sin\theta}, \tag{1}$$

where α is the dip direction, β is the dip angle, $\Delta\alpha$ is the cell size on dip direction, $\Delta\beta$ is the cell size on dip angle (Figure 2), P_{A} is the corrected frequency in cell, $p_{\mathrm{AB}}(\alpha,\beta)$ is the joint probability density that the dip direction and dip angle are intersected by scanline, and P is the observed frequency in cell.

For the modified Terzaghi method, the weighing procedure is to multiply $1/\sin\theta$ by the observed frequency at each orientation pole. Such a pole, in calculus, can be regarded as a special cell, that is, an infinitesimal cell around the pole ($\Delta\alpha \longrightarrow 0^{\circ}$, $\Delta\beta \longrightarrow 0^{\circ}$). In this case, P and P_{A} in Equation (1) become the observed frequency and the corrected frequency at the orientation pole, respectively. Therefore, despite a slight redefinition of θ, the bias-compensatory factor can be also used for the modified Terzaghi method.

3. Application of the Modified Terzaghi Method

The use of the modified Terzaghi method is illustrated and its accuracy is tested against the conventional Terzaghi method, using a real example of bedding orientation observations in a roadcut in China. The study area is near Yingxiu town in Wenchuan, Sichuan Province, and located only about 1,800 m east of the epicenter of the 2008 Wenchuan Earthquake (Figure 3). The particular roadcut is 11 m long, 5 m wide, and 6 m high and consists of Upper Triassic lithic arkose of the Xujiahe Formation. The rock has two discontinuity sets, one of which is the bedding plane.

A scanline with the trend/plunge of 108/15° was fixed to observe the bedding planes on this outcrop (Figure 4). Table 1 lists 55 observed orientations, whose pole diagram is shown in Figure 5.

First, the sampling bias of the observed orientations was corrected according to the conventional and the modified Terzaghi methods. A cell size of $2^{\circ}\times 2^{\circ}$ was selected for the conventional Terzaghi method because Tang [15] concluded that this size optimizes the accuracy. The result corrected by the conventional Terzaghi method is shown in Figure 6(a) and the result corrected by the modified Terzaghi method is shown in Figure 6(b). In addition, the volumetric abundance, diameter, and aperture were calculated, with the results listed in Table 2.

Next, a three-dimensional model of the rock was constructed by discrete fracture network modeling, as described by Xu and Dowd [16], Brzovic and Villaescusa [17], and Grenon and Hadjigeorgiou [18]. Multiplying the volumetric abundance ($10\,\mathrm{m}^{-3}$) by the volume of the simulated zone ($400\,\mathrm{m}^{3}$), in Table 2, results in a total of 4,000 discontinuities. Then, pseudorandom numbers of these 4,000 discontinuities were generated for the five elements, namely the X-coordinate, Y-coordinate, Z-coordinate, diameter and aperture. These pseudorandom numbers are not listed here because of space limitations. After entering the pseudorandom numbers and the corrected orientation data into modeling software such as OpenGL or AutoCAD, two models can be built corresponding to the conventional and the modified Terzaghi methods (Figure 7). A scanline with the same orientation as the field scanline was applied to the model outcrop and the discontinuities that are intersected by

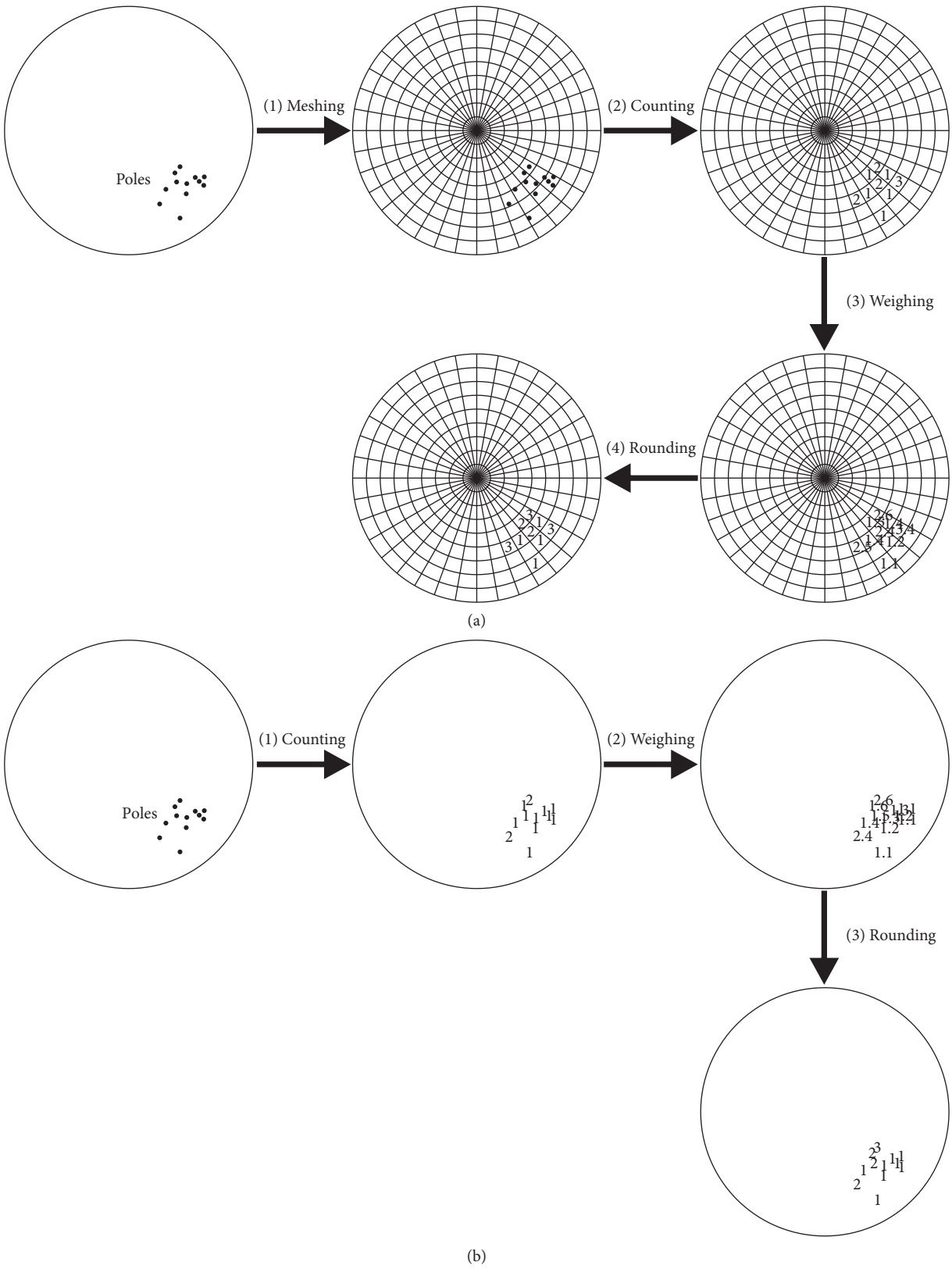


Figure 1: Procedures of (a) the conventional Terzaghi method and (b) the modified Terzaghi method.

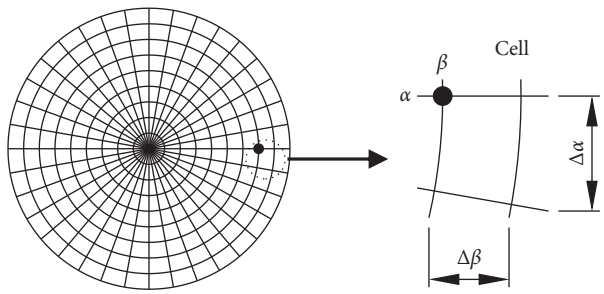


Figure 2: Mesh of projection net.

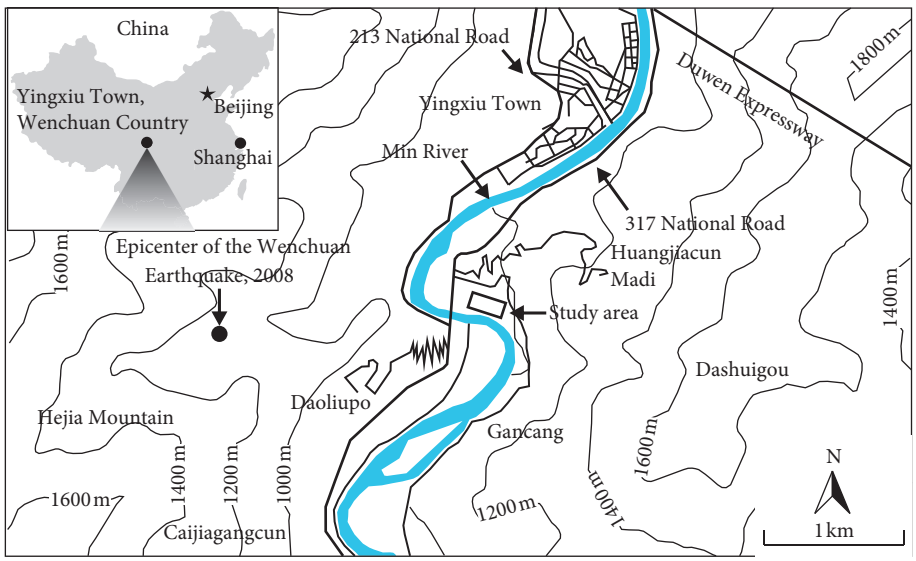


Figure 3: Map of the Wenchuan area showing the location of the study area. The blue line is the Min River, the thick black lines denote the 213 National Road and the 317 National Road, the medium black lines are rural roads, and the thin black lines are topographic contours as labeled.

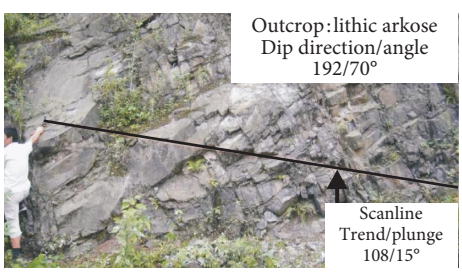


Figure 4: Photograph of outcrop and the scanline used for observation.

this scanline were then "measured." The quantity of these measured discontinuities is set equal to the number of observed discontinuities in the field. To distinguish between these measured discontinuities and the real discontinuities observed in the field, the former are named "modeled" discontinuities. Figure 8 shows the modeled discontinuity orientations.

Third, the distribution difference between the observed and the modeled orientations was tested by the Kolmogorov–Smirnov two-sample test. This nonparametric hypothesis test evaluates the difference between the cumulative distribution functions of two sample data vectors. The test can be executed by the software Statistical Product and Service Solutions and returns an asymptotic significance to characterize the difference. The significance ranges from 0 to 1; the higher the significance, the lower the difference. More information about this test is given in Özcomak et al. [19] and Al-Labadi and Zarepour [20].

The test returned two significances corresponding to the dip direction and dip angle. These two significances were combined and their average is 0.686 for the conventional Terzaghi method and 0.988 for the modified Terzaghi method. It is apparent that the latter value is much higher than the former value, demonstrating that the modified Terzaghi method is more accurate than the conventional method, even if the optimal cell size of $2^\circ \times 2^\circ$ is chosen for the conventional method.

4. Discussion

Tang [14] presented the accuracy of the conventional Terzaghi method in two cases:

Table 1: Data of observed orientations.

Discontinuity	Dip direction/angle (°)	Discontinuity	Dip direction/angle (°)
1	128/73	29	139/66
2	134/70	30	138/65
3	135/70	31	135/70
4	133/71	32	137/72
5	130/75	33	137/77
6	133/71	34	129/74
7	132/74	35	145/78
8	145/72	36	138/70
9	133/76	37	130/69
10	131/67	38	133/71
11	135/74	39	136/70
12	141/72	40	132/70
13	138/75	41	134/72
14	130/69	42	135/77
15	127/73	43	127/69
16	137/80	44	129/73
17	138/73	45	129/71
18	132/78	46	144/72
19	137/73	47	139/71
20	131/73	48	142/75
21	131/69	49	137/76
22	132/71	50	143/74
23	137/69	51	144/71
24	132/79	52	141/79
25	138/74	53	144/73
26	131/77	54	141/75
27	132/73	55	140/80
28	133/78	—	—

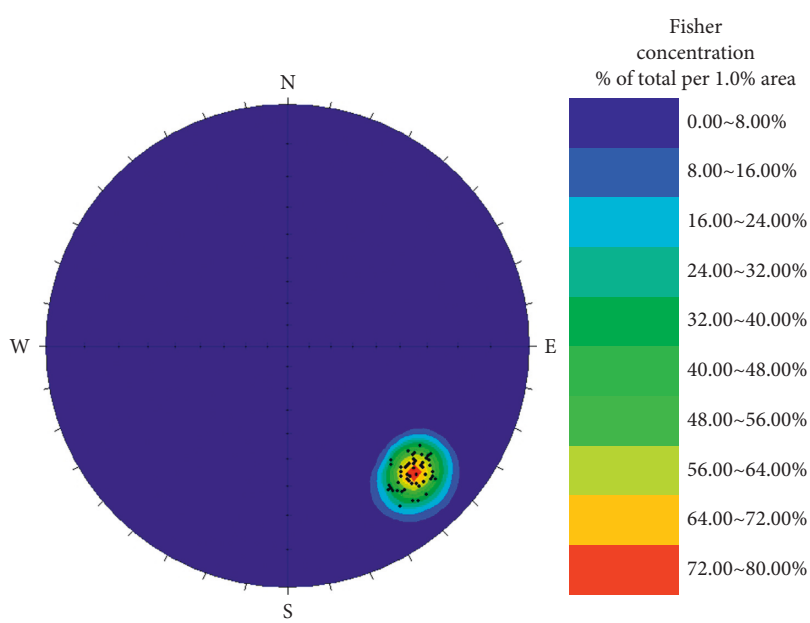


Figure 5: Pole diagram of 55 observed orientations. This diagram was plotted by the software Dips. This is an equal-angle projection of the upper hemisphere, showing equal-density rings shaped by Fisher concentrations.

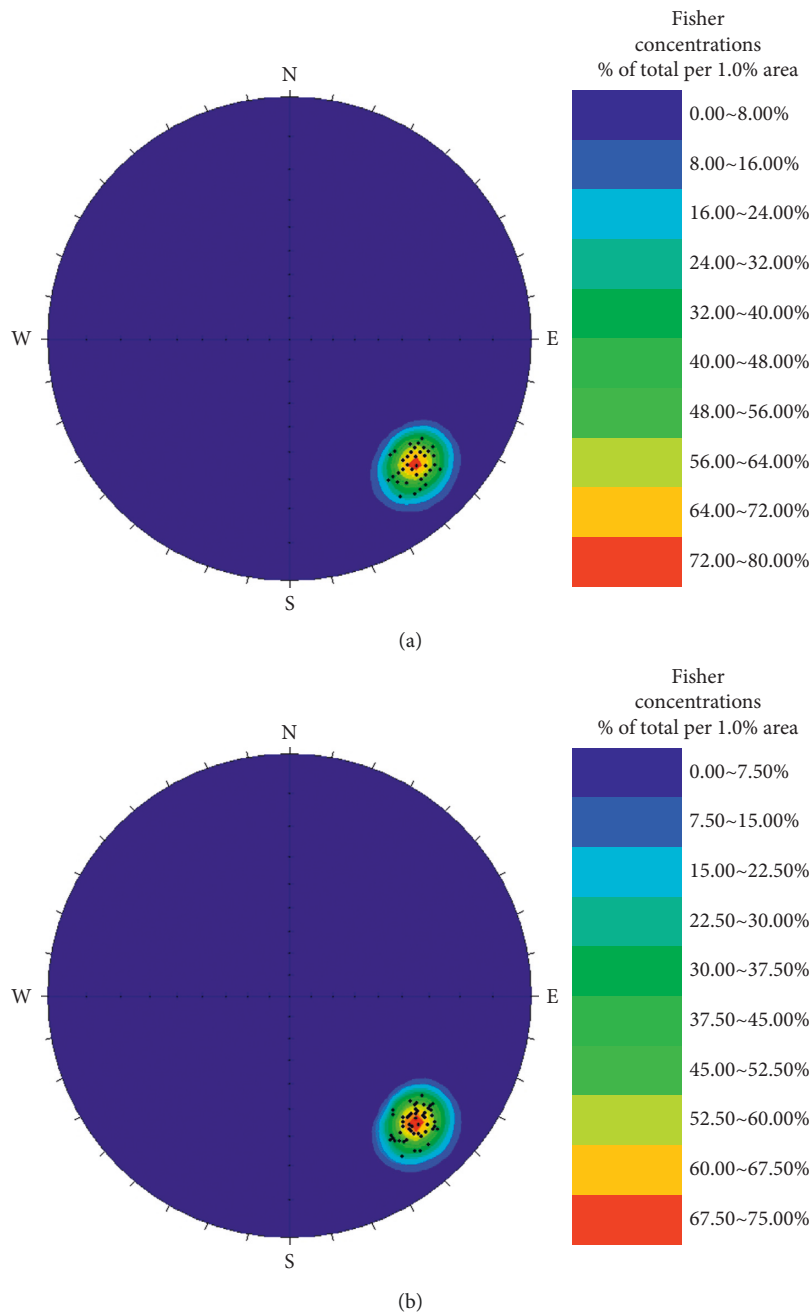


Figure 6: Pole diagram of orientations corrected by (a) the conventional Terzaghi method and (b) the modified Terzaghi method.

Table 2: Volumetric abundance, diameter, aperture, and size of simulated zone.

Volumetric abundance[a] (m^{-3})	Diameter (m)	Aperture (mm)	Simulated zone		
			Length (m)	Width (m)	Height (m)
4	Exp (0.5)[b]	Exp (3.2)	10	10	10

[a]Volumetric abundance = number of discontinuity centers per rock volume. [b]Exp (k) = exponential distribution, where k is the reciprocal of mean.

Case 1. If $n \longrightarrow \infty$, in other words, if the cell size is infinitesimal,

$$\iint_D p_A(\alpha, \beta)\, d\alpha\, d\beta = \frac{1}{k} \lim_{n \longrightarrow \infty} \sum_{i=1}^{n} \frac{P_i}{\sin \theta_{ci}}. \quad (2)$$

Case 2. If n does not approach ∞,

$$\iint_D p_A(\alpha, \beta)\, d\alpha\, d\beta \approx \frac{1}{k} \sum_{i=1}^{n} \frac{P_i}{\sin \theta_{ci}}, \quad (3)$$

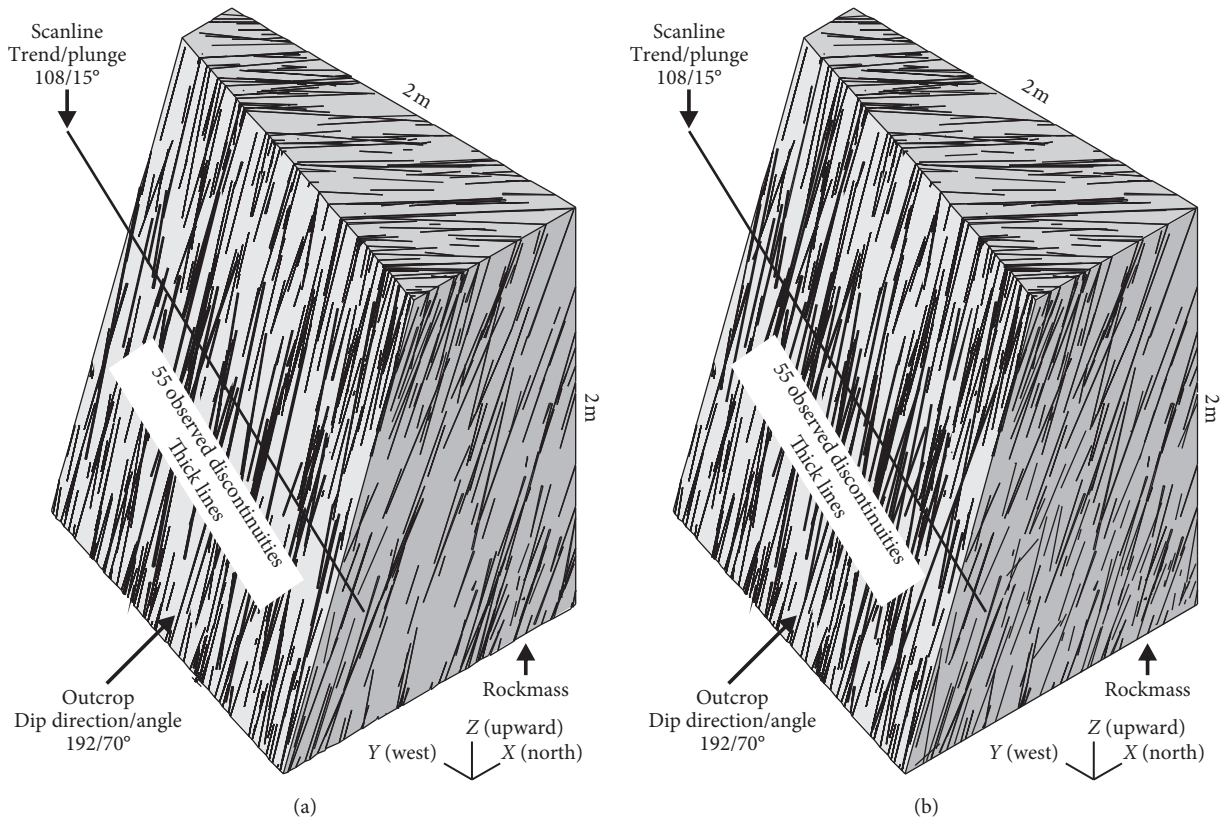
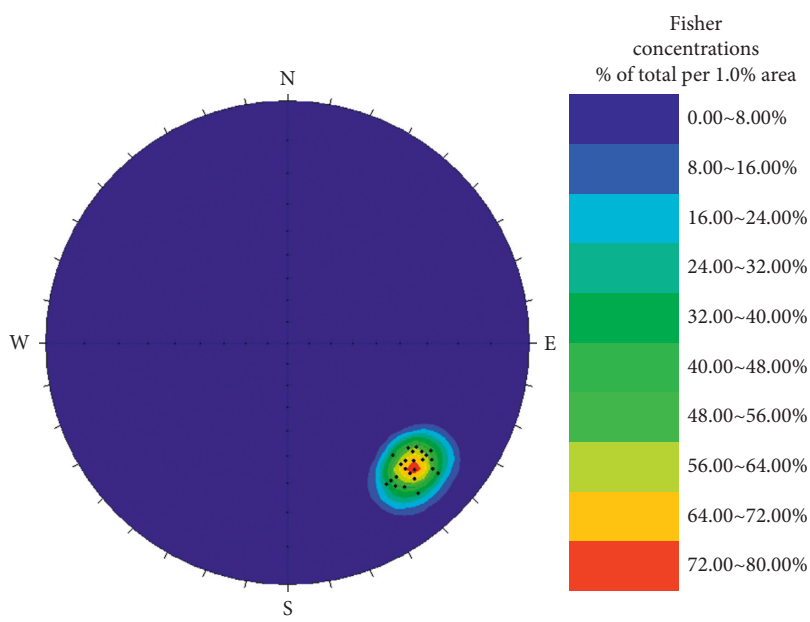
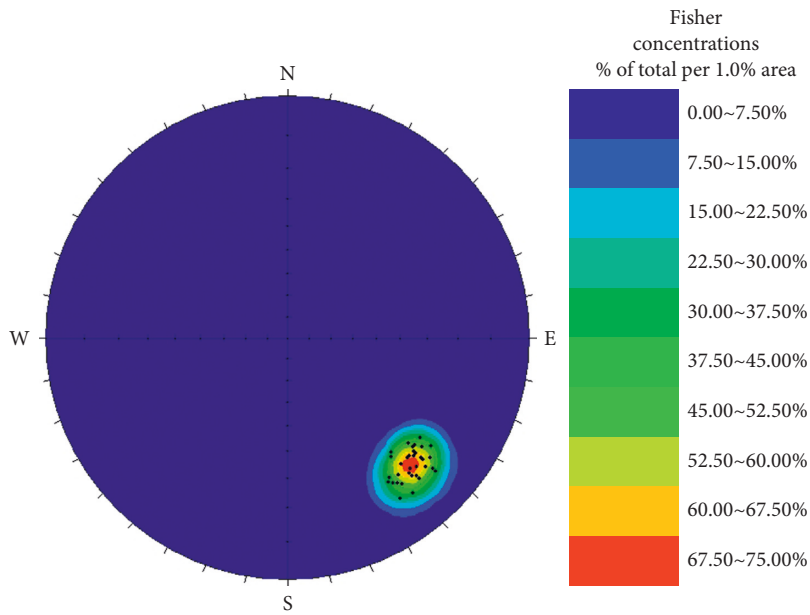


Figure 7: Rock models corresponding to (a) the conventional Terzaghi method and (b) the modified Terzaghi method.



(a)

Figure 8: Continued.



(b)

Figure 8: Pole diagram of 55 modeled orientations corresponding to (a) the conventional Terzaghi method and (b) the modified Terzaghi method.

where D is a given orientation interval and $p_A(\alpha, \beta)$ is the joint probability density of the dip direction and dip angle in the rockmass.

As mentioned in Section 2.2, the elimination of mesh in the modified Terzaghi method is identical to meshing the projection diagram into infinitesimal cells (Case 1 for Equation (2)). Tang [14] proved that Equation (2) is accurate, while Equation (3) introduces error. Hence, the modified Terzaghi method is a more accurate procedure. In contrast, for the conventional Terzaghi method, the cells can only approach but can never attain an infinitesimal size. Such a mesh can only meet Case 2 and not Case 1, so the conventional Terzaghi method will consequently involve error. This analysis reinforces the result regarding accuracy obtained in Section 3.

As shown in Section 3, the significance of the modified Terzaghi method approaches but does not attain unity, suggesting that factors in addition to meshing contribute error to the Terzaghi method. One inferred source of error is rounding the corrected frequencies to the nearest integer. This issue will be studied in the future.

5. Conclusions

The Terzaghi method is widely used to correct the observed distribution of rock discontinuity orientations. A modified Terzaghi method that eliminates the meshing procedure was developed to improve accuracy. This modified method only includes three procedures that (1) count the frequencies at each pole, (2) weigh the observed frequencies by the bias-compensatory factor, and (3) round the weighed frequencies to the nearest integer. Mesh elimination requires that the counting objects are changed to the frequencies at each pole instead of the frequencies lying in each cell; accordingly, the bias-compensatory factor has become the reciprocal of the sine of the intersection angle between the scanline and the discontinuity at each pole, rather than the reciprocal of the sine of the intersection angle between the scanline and the discontinuity defined at each cell center. A mathematical logical deduction verifies that this slightly redefined bias-compensatory factor can be used in the modified Terzaghi method.

The conventional and the modified Terzaghi methods were both applied to a case study near Yingxiu, in Sichuan, China. The results indicate that the modified Terzaghi method performed more accurately than the conventional method.

Disclosure

Mutasim Adam Mohamed Ez Eldin is presently at the Section of Engineering Geology, Department of Applied Geology, Saudi Geological Survey, Jeddah 21514, Saudi Arabia.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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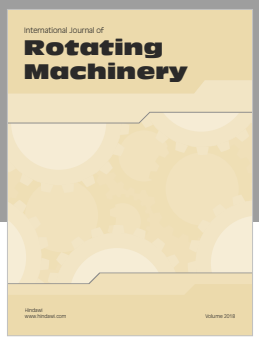

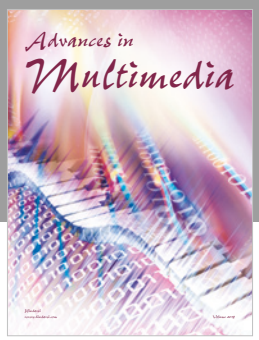

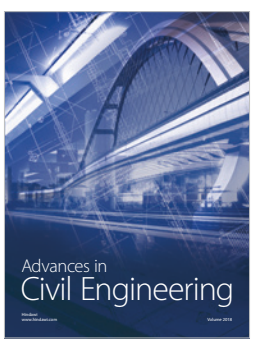

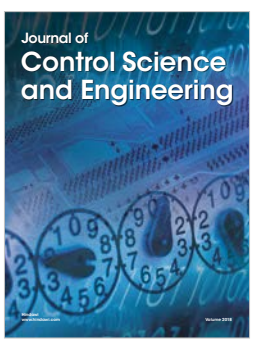

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