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Research Article

The Unified Strength Theory for Plastic Limit Load Analysis of Vertical Shaft Lining

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Plastic limit analysis is a significant application for structural design and failure prediction, implying that the potential bearing capacity of material can be fully considered. Meanwhile, the failure process of structures usually occurs in the deformation of the plastic stage. Hence, it is important to investigate the mechanical properties of structures in the plastic state through proper mechanical models and principles. In this paper, the analytical stress expression and formulas for computing plastic limit load have been deduced based on the unified strength theory. Additionally, the relationships between plastic limit load of the shaft lining and the strength differential (SD) of structural material, the geometrical characteristic (R_0/R) and the intermediate principal shear stress (b) have been discussed. Finally, the method of the plastic limit load is adopted for the safety evaluation of the shaft lining through a practical case.

1. Introduction

Coal resource still plays an important role in the economic growth of China currently, and massive coalmines have been rapidly constructed for coal excavation in the last century. For those deep mines, the protected structures called the shaft lining for transporting labors and minerals become a kind of necessary infrastructure.

However, in the past few decades, a great loss of properties and safety accidents has been induced as a result of instability or fracture of the shaft lining. Accordingly, the structural design and safety evaluation become definitely vital issues for the shaft lining. Recently, the explorations on the external load estimation, computational methods, and related topics of the shaft lining have been carried out. For instance, for the quick estimation of the stress acting on the lining extrados, the convergence-confinement method (CCM) has been used to calculate both the radial displacements associated with the construction of the shafts and the equivalent loads [1]. In order to resist ground pressure to a shaft lining in creeping rock salt strata, a deformation zone filled with a compressible material is employed between the excavation and permanent lining [2]. From the perspective of the shaft material, it is reported that high-strength-reinforced concrete [3] and fiber-reinforced concrete (FRC) can also be applied to the construction of the vertical shaft lining [4].

In actual, the mechanical mechanism is the core work of the shaft lining, and the proper mechanical principles are very significant for practical applications and numerical simulations. Nevertheless, the previous plastic mechanical models of the shaft lining were built based on the classical yield criterion like the Coulomb–Mohr criterion or the Hoek–Brown criterion [5, 6], which overlooked the SD effect and the intermediate principal stress of researched objects. Therefore, as the absence of related investigation, the unified strength theory is adopted for plastic limit load analysis of the shaft lining, which takes the effects of intermediate principal stress and SD effect of material on the material strength into account [7–9], and can be used to analyze the plastic limit of the shaft lining which is verified by a case study.

2. Plastic Limit Analysis Equation of Shaft Lining

2.1. Unified Strength Theory. It is reported that many existing strength criteria and some new strength criteria can be derived from the unified strength theory [8, 9]. The contents of unified strength theory are presented as

$$
\begin{aligned}
&F = \sigma_1 - \frac{\alpha}{1+b}\left(b\sigma_2 + \sigma_3\right) = \sigma_s, && \text{when } \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \\
&F' = \frac{1}{1+b}\left(\sigma_1 + b\sigma_2\right) - \alpha\sigma_3 = \sigma_s, && \text{when } \sigma_2 \ge \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha} \\
&\alpha = \frac{\sigma_t}{\sigma_c}, && 0 \le \alpha \le 1 \\
&b = \frac{(1+\alpha)\tau_s - \sigma_t}{\sigma_t - \tau_s}, && 0 \le b \le 1
\end{aligned}
\quad (1)
$$

where σ_1, σ_2, and σ_3 are the first, second, and third principal stress. α represents the yield-to-tensile strength ratio, and b is an influence coefficient which reflects the effect of intermediate principal stress on the material failure. It can be found that the unified strength theory considers the strength differential of materials and also the effect of intermediate principle shear stress on material failure by different values of parameters α and b.

2.2. Deduction of Plastic Limit Load of Shaft Lining. The long-term consolidation of surrounding soil with a large thickness causes an additional vertical force to the shaft lining, leading to the three-dimensional stress: σ_z, σ_θ, and σ_r (Figure 1). According to the stress state ($\sigma_z \le \sigma_\theta \le \sigma_r$) of the shaft lining, the first, second, and third principle stresses are equal to σ_r, σ_θ, and σ_z, respectively, and the second principle stress is not more than $(\sigma_r + \alpha\sigma_z)/(1+\alpha)$due to the values of α is lower than 1. Based on (1), the following equation can be obtained:

$$
\begin{aligned}
&F = \sigma_r - \frac{\alpha}{1+b}\left(b\sigma_\theta + \sigma_z\right) = \sigma_s, && \text{when } \sigma_\theta \le \frac{\sigma_r + \alpha\sigma_z}{1+\alpha} \\
&F' = \frac{1}{1+b}\left(\sigma_r + b\sigma_\theta\right) - \alpha\sigma_z = \sigma_s, && \text{when } \sigma_2 \ge \frac{\sigma_r + \alpha\sigma_z}{1+\alpha}.
\end{aligned}
\quad (2)
$$

The equilibrium equation in the spatial axis-symmetrical question can be expressed by

$$
\begin{aligned}
&\frac{\partial\sigma_r}{\partial r} + \frac{\partial\sigma_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} + K_r = 0, \\
&\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{zr}}{\partial r} + \frac{\tau_{rz}}{r} + Z = 0,
\end{aligned}
\quad (3)
$$

where K_r and Z are the body force in the direction of axis X and axis Z, respectively. Combining the given condition,

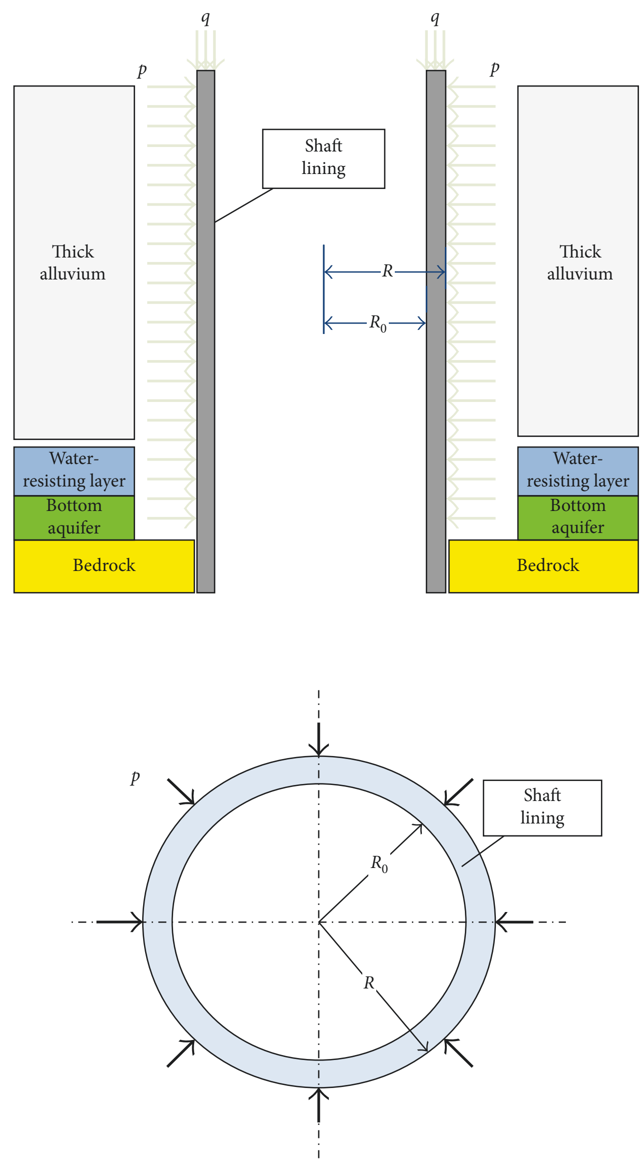

Figure 1: Mechanical model of the shaft lining.

$$
\begin{aligned}
&\sigma_z = \text{const}, \\
&\tau_{zr} = \tau_{rz} = K_r = Z = 0.
\end{aligned}
\quad (4)
$$

So, (3) is written as (5):

$$r\frac{\partial\sigma_r}{\partial r} + \sigma_r - \sigma_\theta = 0. \quad (5)$$

By (2), the yield criteria can be given as

$$\sigma_\theta = \frac{1+b}{\alpha b}\left(\sigma_r - \sigma_\theta\right) - \frac{1}{b}\sigma_z. \quad (6)$$

Substituting (6) into (5) gives

$$r\frac{\partial\sigma_r}{\partial r} + \frac{(\alpha-1)b-1}{\alpha b}\sigma_r + \frac{1}{b}\sigma_z + \frac{b+1}{\alpha b}\sigma_s = 0. \quad (7)$$

And (7) is equal to (8) as

$$r\frac{\partial \sigma_r}{\partial r}+M\sigma_r+N=0. \tag{8}$$

where $M=(\alpha-1)b-1/\alpha b$, $N=(1/b)\sigma_z+(b+1/\alpha b)\sigma_s$, and $\sigma_z=(P/A)=(P/\pi(R^2-R_0^2))=\text{const}$.

Solving (8) by substituting $r=e^t$, σ_r is obtained as

$$\sigma_r=Cr^{-M}-\frac{N}{M}. \tag{9}$$

The stress field is stated by

$$\begin{aligned}\sigma_r&=Cr^{-M}-\frac{N}{M},\\ \sigma_z&=\frac{P}{\pi(R^2-R_0^2)},\\ \sigma_\theta&=\frac{1+b}{\alpha b}(\sigma_r-\sigma_s)-\frac{1}{b}\sigma_z.\end{aligned} \tag{10}$$

Considering the boundary condition when $r=R$, $\sigma_r=q$, the stress of the shaft lining can be expressed as (11) by substituting M and N into (10):

$$\begin{aligned}\sigma_r&=\left(\frac{R}{r}\right)^{((\alpha-1)b-1/\alpha b)}\left[q+\frac{\alpha\sigma_z+(b+1)\sigma_s}{\alpha b-b-1}\right]-\frac{\alpha\sigma_z+(b+1)\sigma_s}{\alpha b-b-1},\\ \sigma_z&=\frac{P}{\pi(R^2-R_0^2)},\\ \sigma_\theta&=\frac{1+b}{\alpha b}(\sigma_r-\sigma_s)-\frac{1}{b}\sigma_s,\end{aligned} \tag{11}$$

when $r=R_0$ and $\sigma_r=0$, the following equation can be obtained:

$$\left(\frac{R}{R_0}\right)^M\left(q+\frac{N}{M}\right)-\frac{N}{M}=0. \tag{12}$$

Substituting $N=(1/b)\sigma_z+(b+1/\alpha b)\sigma_s$ into (12), the relationship between q and σ_z can be given as

$$q=\left(\frac{1}{Mb}\sigma_z+\frac{1+b}{\alpha bM}\sigma_s\right)\left[\left(\frac{R_0}{R}\right)^M-1\right]. \tag{13}$$

The relationship between P and q is presented as

$$q=\left(\frac{1}{Mb\pi(R^2-R_0^2)}P+\frac{1+b}{\alpha bM}\sigma_s\right)\left[\left(\frac{R_0}{R}\right)^M-1\right]. \tag{14}$$

2.3. The Envelope of Plastic Limit Load of Shaft Lining. In the above sections, we have deduced the relationship between q and σ_z based on unified strength theory. In this part, the relation between q and σ_z will be investigated further through dimensionless parameters.

As $\sigma_z=(P/A)=(P/\pi(R^2-R_0^2))$ and $M=((\alpha-1)b-1/\alpha b)$ (14) can be presented as

$$q=\left(\frac{\alpha}{\alpha b-b-1}\sigma_z+\frac{1+b}{\alpha b-b-1}\sigma_s\right)\left[\left(\frac{R_0}{R}\right)^{((\alpha b-b-1)/\alpha b)}-1\right]. \tag{15}$$

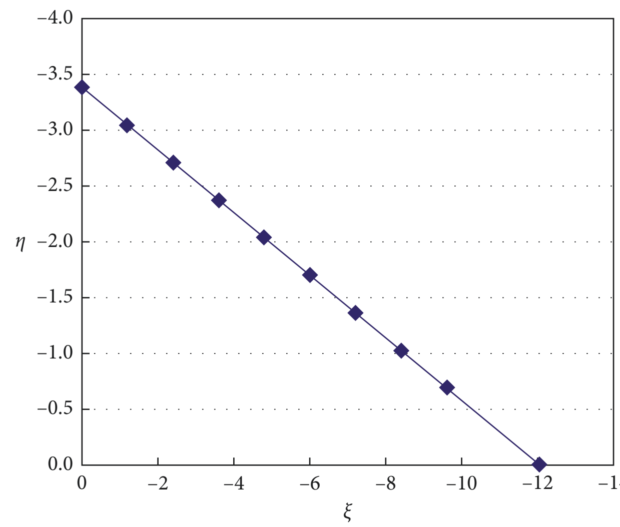

Figure 2: Envelope line of plastic limit loads of the shaft lining.

Taking $\eta=q/\sigma_s$ and $\xi=\sigma_z/\sigma_s$, (15) is been written as

$$\eta=\left(\frac{\alpha}{\alpha b-b-1}\xi+\frac{1+b}{\alpha b-b-1}\right)\left[\left(\frac{R_0}{R}\right)^{((\alpha b-b-1)/\alpha b)}-1\right]. \tag{16}$$

For the actual geometrical characteristics and material properties of the shaft lining, the following equation can be obtained by $\alpha=0.166$, $b=1.0$, and $R_0/R=0.88$:

$$\eta=-0.2811\xi-3.3866. \tag{17}$$

Accordingly, the plastic limit load envelope of the shaft lining is shown in Figure 2.

The qualification of q and σ_z is decided by the relation of three-dimensional stresses, whose dynamics can be used to justify whether the shaft ling turns into ultimate state or not. When the point is located in the area below the envelope, it is indicated that the shaft lining has not come to the plastic ultimate state. When the point (η,ξ) appears above the envelope line, it can be inferred that the shaft lining has came to the plastic limit.

3. Analysis of Plastic Limit Loads of Shaft Lining

In Section 2.3, the envelope line of the plastic limit has been given by dimensionless parameters, which can be adopted to justify the stress state of the shaft lining. As we can see from (16), η and ξ are definitely influenced by the parameters like α, b, and R_0/R, so it is of significance to study the noted relation for instruction of vertical shafts.

3.1. The Influence of SD Effect on the Plastic Limit Load of Shaft Lining. One of the advantages of unified strength theory is which it considers the SD effect of various materials by arranging different values of α. It is necessary to carry out the plastic limit analysis recognizing the difference in compressed and tensile properties of engineering

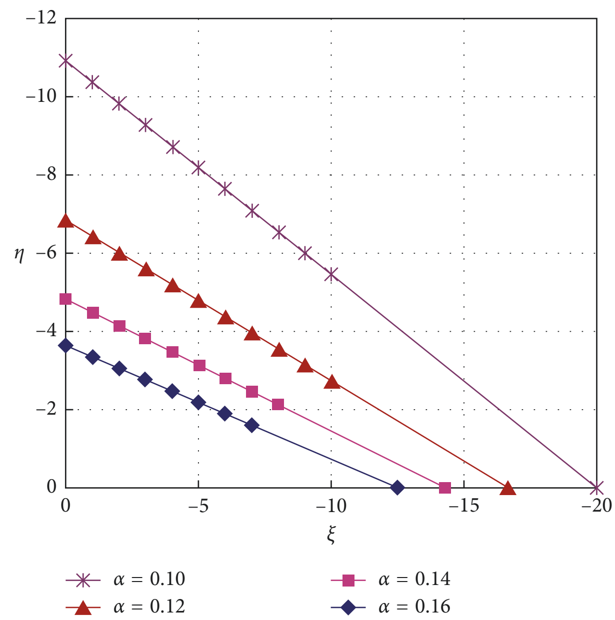

Figure 3: Plastic limit load envelopes of the shaft lining considering the SD effect.

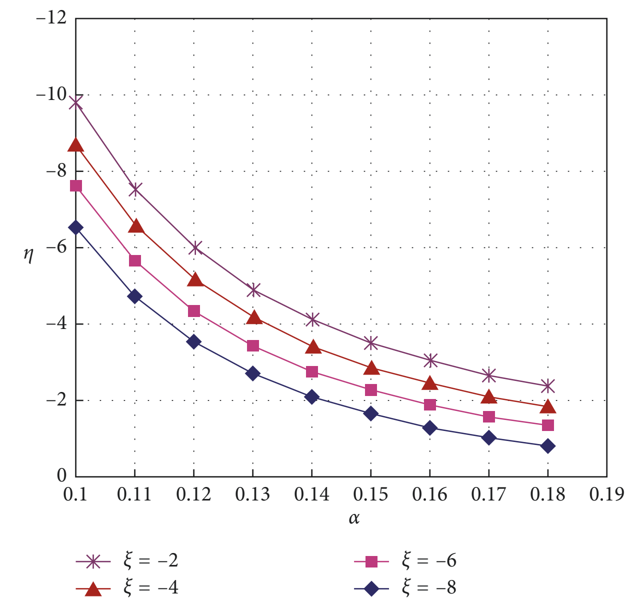

Figure 4: Relationship between plastic limit loads and SD effect considering the effect of ζ.

materials. Figure 3 presents the plastic limit load envelopes of the shaft lining at different tensile strength-compressive strength ratios of 0.10, 0.12, 0.14, and 0.16, and the values of b and R_0/R are consistent with Figure 2. It is noted that the envelopes of the plastic limit load are apparently linked to the values of α. A detailed view can be presented where the greater differential between tensile strength and compressive strength is corresponding to the lower bearing capacity of the shaft lining.

In order to analyze the function between η and α, it is assumed that b and R_0/R are consistent with Figure 2, respectively, and the properties of plastic limit loads of the shaft lining are presented in Figure 4. For the fixed vertical load (ζ is a constant), the plastic limit load η (confining pressure q) decreases in the nonlinear form with the increase of α from 0.1 to 0.18. By comparing the four curves in Figure 4, it can also be found that the plastic limit load (confining pressure q) decreases with the growth of the vertical load, when the parameter α is ensured. So it can be concluded that the plastic limit load of the shaft is decreasing with the increase of depth due to the higher pressure. In other words, the shaft lining in deep alluvium, whose vertical load and confining load are basically constant, is easier to achieve the plastic limit if possessing a larger α.

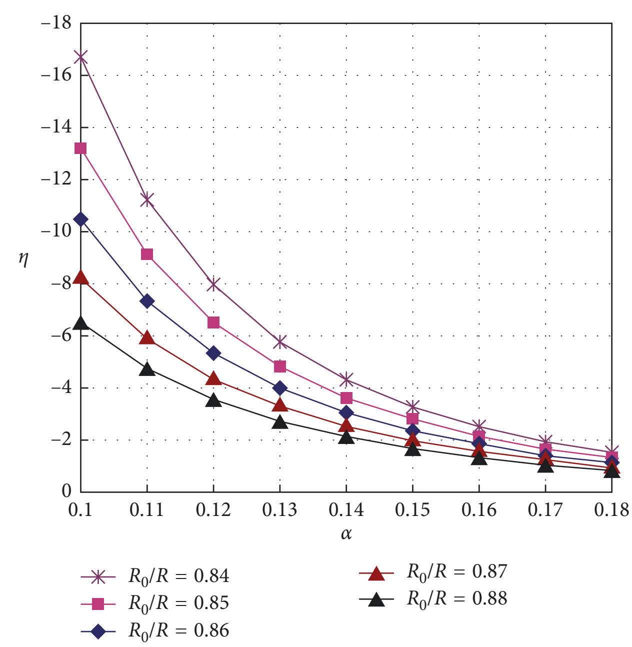

Figure 5: The effect of α on the plastic limit load with different values of R_0/R.

3.2. Influence of Geometrical Characteristic on the Plastic Limit Load. Understanding the effect of α on the plastic limit load with different geometrical features of the shaft lining is very useful for structure design, so the function η for a sequence of α is obtained by different R_0/R, which is shown in Figure 5. It is shown that the plastic limit load of the shaft lining decreases apparently with the increase of α, but the decreasing rate declines gradually. It is also noted that the higher plastic load η is corresponding to the larger R_0/R. In other words, for the same R_0, the plastic limit load can be improved by a thicker shaft lining.

By the fixed parameters $\alpha = 0.166$ and $b = 1.0$, the relationship between plastic load η and geometrical parameters R_0/R considering various levels of vertical load ζ is presented in Figure 6. It can be seen that the plastic limit load η decreases with the increase of the vertical load ζ, and the decreasing degree is negative with the increase of R_0/R.

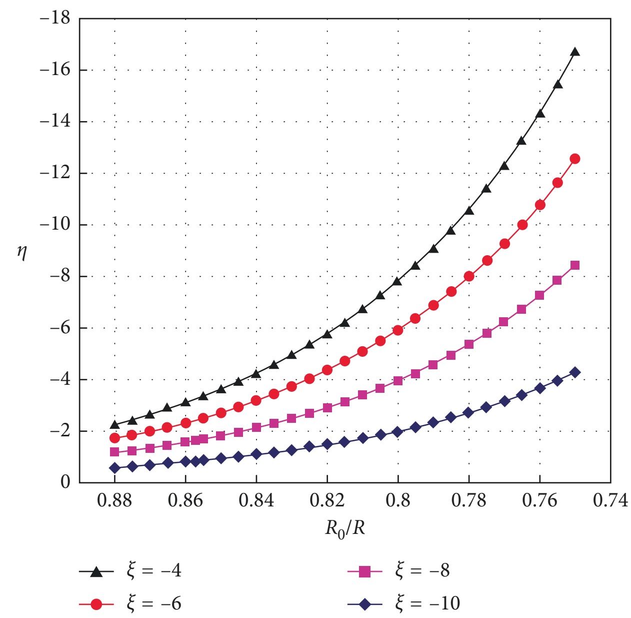

Figure 6: Plastic limit loads of the shaft lining with R_0/R considering the effect of ζ.

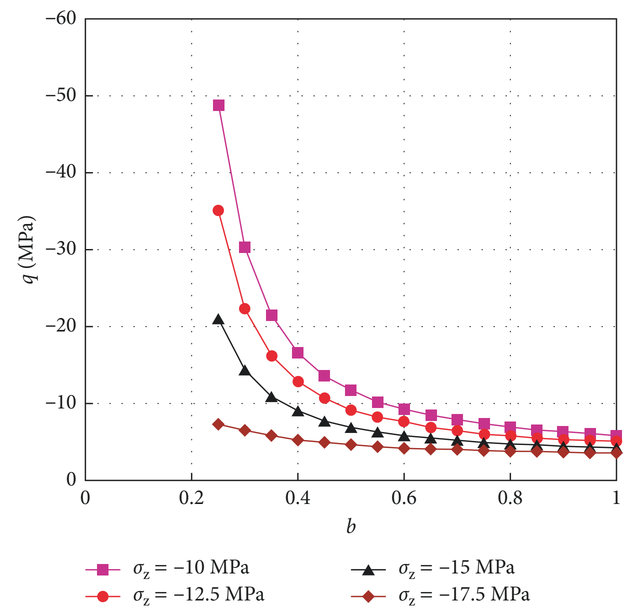

Figure 7: Relationship between plastic limit load q and b under different σ_z.

3.3. Effect of Intermediate Principle Stress on Plastic Limit Load. In order to study the influence of intermediate principle stress on the plastic limit load q and σ_z, by taking $\alpha = 0.166$ and $\sigma_s = 2.5$ MPa, the relationship between σ_z, q, and b will be discussed under a range of R_0/R in this section. According to (15), the following equation is obtained:

$$q = \left[\frac{0.166}{-0.834b-1}\sigma_z + \frac{2.5(1+b)}{-0.834b-1}\right] \times \left[\left(\frac{R_0}{R}\right)^{((-0.834b-1)/0.166b)} - 1\right]. \tag{18}$$

Based on a series of σ_z and fixed $R_0/R = 0.88$, the computing result is resented in Figure 7. Under the given parameters considering the load condition and material characteristic of the shaft lining, it is observed that the plastic limit load q is in stable variation when $0.6 \le b \le 1.0$. In general, the plastic limit load is declining with the raise of σ_z.

Assuming $\sigma_z = -15$ MPa to investigate the effect of b under a series of R_0/R, it is presented that R_0/R has a slight effect on the plastic limit load q when $0.6 \le b \le 1.0$.

4. A Case Study

It is proposed that the plastic limit analysis is more suitable for estimation of ultimate bearing capacity of the shaft lining, which is helpful for structural design and safety evaluation. So in this section, the plastic limit envelope based on unified strength theory will be adopted to justify the mechanical state of the vertical shaft lining based on the practical case. The shaft lining with a thick deep alluvium of about 153 m was built by the drilling method in east China with the thickness of 0.4 m and the diameter of 6.0 m, made of prefabricated-reinforced concrete segment.

The shaft lining fractured on December 2004 and the related plastic analysis are displayed below. Considering the real conditions of the shaft lining, the corresponding parameters are presented as

$$\begin{aligned} \frac{R_0}{R} &= 0.88, \\ \alpha &= 0.166, \\ b &= 1.0, \\ \sigma_s &= 3.0 \text{ MPa}. \end{aligned} \tag{19}$$

By (15), the expression of the plastic limit load can be obtained:

$$q = -0.281\sigma_z - 10.16. \tag{20}$$

The mass of the shaft lining tower is 3800 T, so the corresponding vertical stress caused is $\sigma_{zT} = 4.72$ MPa. The gravity of the shaft lining material can be computed by $\gamma = 25$ kN/m^3, and the vertical stress in the depth of 158 m due to self-gravity is obtained as $\sigma_{zG} = \gamma H = 3.95$ MPa. Combining the characteristic of the surroundings composed of sand and sandy clay, the drainage rate of aquifer is about 6~8 m/a; hence, the vertical additional force in the shaft lining can be calculated by empirical estimation as $f_n = 50$ kPa. Assuming the distribution of vertical additional force is uniform in the shaft lining, the corresponding vertical stress in 158 m is $\sigma_{zF} = 20.98$ MPa. It is noted that the stress caused by vertical additional force is time dependent, which means that the stress state of the shaft lining is mainly dominated by vertical additional force.

Consequently, the total vertical stress is $\sigma_z = \sigma_{zT} + \sigma_{zG} + \sigma_{zF} = 29.65$ MPa. And the confining pressure of surroundings in the depth of 158 m is $p = 2.054$ MPa estimated

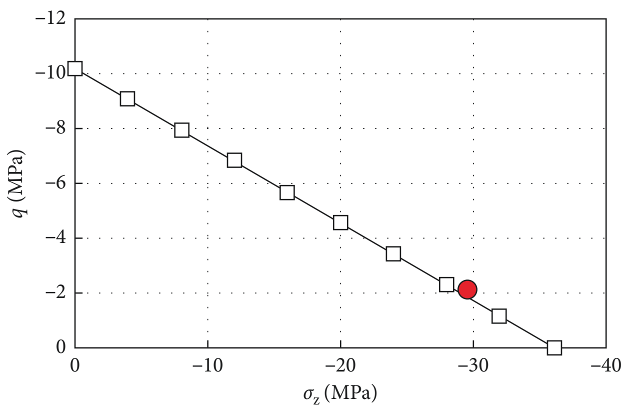

FIGURE 8: Envelope line of the plastic limit load and stress state evaluation of the shaft lining case.

by $p = 0.013$ H. The envelope line of the plastic load of the shaft lining is obtained by (15) and shown in Figure 8, and it can be found that the stress state of the shaft lining is basically on the envelope line. Therefore, the corresponding analysis based on unified strength theory agrees with the calculation of the practical case.

5. Conclusions

In this paper, the formulas of plastic limit loads of the shaft lining have been deduced based on unified strength theory, followed by related discussions. The envelope line of a plastic limit load has been proposed, by which the stress state of the shaft lining can be justified whether entering plastic limit or not. The relationship between the plastic limit load and the factors like SD effect, geometrical characteristic, and intermediate principle stress has been investigated, which is useful for the design of the vertical shaft lining. It is presented that the SD effect has a significant negative influence on the plastic limit of the concrete shaft lining. With the increase of R_0/R, the plastic limit load decreases nonlinearly. Considering the real condition of the shaft lining, the impact of intermediate principle stress on the plastic limit load can be neglected when $0.6 \leq b \leq 1.0$.

As the deformation of the fractured shaft lining has developed into the plastic state, it is suggested to adopt plastic analysis for the safety evaluation of the shaft lining. Based on a practical case, the stress state of the shaft lining has been predicated accurately through the envelope line based on the unified strength theory.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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