Research Article

The Analysis of Stress Waves at a Junction of Beam and String

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In the bridge engineering, there are some problems about the dynamics that traditional theory cannot solve. So, the theory about stress waves is introduced to solve the related problems. This is a new attempt that the mechanic theory is applied to practical engineering. The stress wave at a junction of the structure composed of beams and strings is investigated in this paper. The structure is studied because the existence of a soft rope makes the transmission of the force in the bridge structure different from the traditional theory, and it is the basis for further research. The equilibrium equations of the displacement and the internal force are built based on the hypothesis. The fast Fourier transform (FFT) numerical algorithm is used to express an incident pulse of arbitrary shape. The analytical solutions are substantiated by comparing with the finite element programs. The conclusion that if the cross section of the string is relatively small, then the energy density of the structure is relatively large, which is disadvantageous to the structure, can be obtained from this paper.

1. Introduction

Complex structures, such as arch bridge, suspension bridge, and cable-stayed bridge, are used increasingly in the world. Different from the traditional theory, the vehicle load is regarded as the vibration load in the bridge structure. The theory about stress waves is introduced to study the influence of the vibration load. The propagation speed of longitudinal waves and transverse waves in the concrete and steel is in the range of 1000 m/s to 5000 m/s, and in those large structures, the time scale of vehicle load and stress wave in balance are in the same order of magnitude. So, different from the traditional theory of structural dynamics that ignores stress wave propagation and interaction before the static equilibrium, the spread of the stress process is what we concern. Investigation of stress waves at a junction composed of beams and strings is the most important and foundational work.

Lee and Kolsky [1] investigate the generation of stress pulses at the junction of two noncollinear rods. They used the simple one-dimensional theory to describe the propagation of longitudinal waves and used the Timoshenko equation for flexural propagation. The analytic relations

between the incident pulse and the four generated pulses are derived both for an incident longitudinal pulse and an incident flexural pulse. Atkins and Hunter [2] study the propagation of longitudinal elastic waves around right-angled corners in rods of square cross section. In their work, the rod is assumed to behave as a rigid body, and it was demonstrated that longitudinal elastic waves in rods of square cross section are transmitted in part around right-angled corners. Doyle [3] used the fast Fourier transform algorithm that allows the force history to be determined to establish a strain relation for the transverse impact of beams.

Mace [4] studied wave reflection and transmission in beams. In his paper, it is seen that incident near fields can give rise to substantial propagation components. Doyle and Kamle [5, 6] had taken an experimental study of the reflection and transmission of flexural waves at an arbitrary T-joint. It could be obtained from their study that the dynamic response is not sensitive to particular values of the joint model, and it depends primarily on the member arrangement and only secondarily on the particular shape and mass of the joint. Horner and White [7] predicted vibrational power transmission through bends and joints

in beam-like structures. They built some models which determine the wave type which carries most power in each section of the system. By establishing the wave type, it is then possible to apply the most suitable vibration control technique. Sack [8] studied transverse oscillations in travelling strings. He found that the frequency response to a simple harmonic disturbance and the envelope of the oscillations are similar to those of a clamped string, but the phase of oscillation varies from point to point so that the instantaneous configuration of the string is not a sine curve. Little and Heywoodb [9] had described the axial stress waves in an elastic-perfectly-plastic bar. Different from the classical elastic solution, the load is greater than half the yield load in their study. The paper describes and verifies the model and presents the predictions that it gives for various levels of load. Chen [10] had thoroughly reviewed the research of transverse vibrations of axially moving strings and their control. Chen presents the governing equations with large amplitude and reviews progress on discretized or direct approximate analytical analyses and numerical approaches based on the Galerkin method or the finite difference method. Guo et al. [11] investigated the elastic wave and energy propagation in angled beams. Experimental and numerical methods were taken. It could demonstrate that, after several reflections from and transmission across the bends, energy is progressively smeared throughout the entire beam and does not concentrate at any particular segment. Li et al. [12] developed a fractal damage joint model based on the fractal damage theory to investigate the transmission and reflection of stress waves across joints. The fractal geometrical characteristics of joint surfaces are investigated by using a laser profilometer to scan the joint surfaces in their study. Lancioni et al. [13] studied the scattering of seismic waves induced by the presence of underground cavities in homogeneous soils by using the numerical method. Singh and Goya [14] established the governing equations of a transversely isotropic microstretch material, which are specialized in x - z plane. Plane wave solutions of these governing equations result in a biquadratic velocity equation. They found that, for a specific material, numerical simulation in the presence as well as in the absence of microstretch shows that the coupled longitudinal displacement (CLD) wave is the fastest wave and the coupled transverse microrotational (CTM) wave is the observed slowest wave. Piccolroaza et al. [15] presented a novel analysis of Floquet–Bloch flexural waves in a periodic lattice-like structure consisting of flexural beam ligaments. A special feature of this structure is in the presence of the rotational inertia, which is commonly neglected in conventional models of the Euler–Bernoulli type. Numerical examples were taken to illustrate directional localization, negative refraction, localization at an interface, and neutrality for propagating plane waves across a structured interface for a frequency range corresponding to a Dirac cone.

The previous studies focused on the distribution of the stress wave in the junction of the beams. According to the need of practical engineering, the distribution of the stress wave that was produced by transverse impact in the junction of the beam and string is the focus of the study in this paper.

2. Theoretical Analysis

2.1. Wave Propagation through the Junction. We will here consider the reflection of a stress pulse at the boundary of beams and strings of different diameters and materials (Figure 1(a)). There is an angle β between two beams and α between beam 1 and string 2. We call the flexural displacement due to the incident wave v_0 . We denote the displacement of the reflected longitudinal wave by u_1 , the lateral displacement produced by the flexural wave by v_1 in beam 1, the longitudinal displacement in string 2 by u_2 , the lateral displacement in string 2 by v_2 , the longitudinal displacement in beam 3 by u_3 , and the lateral displacement in beam 3 by v_3 . The joint is modeled as a rigid body, and the internal forces are shown in Figure 1(b).

Continuity of the displacement of beam 1 and string 2 leads to

$$u_1 = u_2 \cos \alpha - v_2 \sin \alpha, \quad (1)$$

$$v_0 - v_1 = u_2 \sin \alpha + v_2 \cos \alpha. \quad (2)$$

Continuity of the displacement of beams 1 and 3 leads to

$$u_1 = u_3 \cos \beta - v_3 \sin \beta + \frac{L}{2} \sin \beta \frac{\partial v_3}{\partial x_3}, \quad (3)$$

$$v_0 - v_1 = u_3 \sin \beta + v_3 \cos \beta - \frac{L}{2} (1 + \cos \beta) \frac{\partial v_3}{\partial x_3}, \quad (4)$$

$$\frac{\partial v_0}{\partial x_1} - \frac{\partial v_1}{\partial x_1} = -\frac{\partial v_3}{\partial x_3}. \quad (5)$$

The balance of the forces at the junction leads to

$$E_1 I_1 \frac{\partial^2 (v_0 - v_1)}{\partial x_1^2} - \frac{L}{2} E_1 I_1 \frac{\partial^3 v_1}{\partial x_1^3} \quad (6)$$

$$= E_3 I_3 \frac{\partial^3 v_3}{\partial x_3^3} + \frac{L}{2} E_3 I_3 \frac{\partial^3 v_3}{\partial x_3^3} - I_j \frac{\partial v_1}{\partial x_1},$$

$$E_1 A_1 \frac{\partial u_1}{\partial x_1} = E_2 A_2 \frac{\partial u_2}{\partial x_2} \cos \alpha + E_3 A_3 \frac{\partial u_3}{\partial x_3} \cos \beta \quad (7)$$

$$+ E_3 I_3 \frac{\partial^3 v_3}{\partial x_3^3} \sin \beta - m_j \frac{\partial^2 u_1}{\partial t^2},$$

$$E_1 I_1 \frac{\partial^3 (v_0 - v_1)}{\partial x_1^3} = E_2 A_2 \frac{\partial u_2}{\partial x_2} \sin \alpha - E_3 A_3 \frac{\partial u_3}{\partial x_3} \sin \beta \quad (8)$$

$$- E_3 I_3 \frac{\partial^3 v_3}{\partial x_3^3} \cos \beta - m_j \frac{\partial u_2}{\partial t},$$

where $E_1 I_1$ and $E_3 I_3$ are bending stiffness of beams 1 and 3. $E_1 A_1$, $E_2 A_2$, and $E_3 A_3$ are axial stiffness of beams 1 and 3 and string 2. L is a parameter characterizing the size of the joint. I_j and m_j are the moment of inertia and quality of the joint.

2.2. Characterization of the Wave. In the paper by Doyle and Kamle [5, 6], the fast Fourier transform (FFT) numerical algorithm was used to express an incident pulse of arbitrary shape. The FFT is defined as

FIGURE 1: The displacement and stress of the three-member intersection. (a) The displacement of the components. (b) The internal forces of the junction.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-2\pi i k n / N}, \quad n, k = 0, 1, 2, \dots, N-1, \quad (9)$$

where $x(n)$ is the value of the signal at time $(n-1)\Delta t$ and $X(k)$ is the FFT at frequency $k/N\Delta t$. Different from the previous study, the Timoshenko beams are used in this paper. The displacement can be written as

$$v = [Ae^{-ik_1x} + Be^{-k_2x}]e^{i\omega t}, \quad (10)$$

where

$$k_1 = k \left[\sqrt{1 + \frac{1}{4} \left(\frac{\omega}{kc_s} \right)^4} + \frac{1}{2} \left(\frac{\omega}{kc_s} \right)^2 \right]^{1/2}, \quad k = \left[\frac{\rho A}{EI} \right]^{1/4} \sqrt{\omega},$$

$$k_2 = k \left[\sqrt{1 + \frac{1}{4} \left(\frac{\omega}{kc_s} \right)^4} - \frac{1}{2} \left(\frac{\omega}{kc_s} \right)^2 \right]^{1/2}, \quad c_s = \sqrt{\frac{G}{\rho}}. \quad (11)$$

The displacements and strains can be written as

$$v(x, t) = \sum_{n=0}^N [A_n e^{-ik_1x} + B_n e^{k_2x}] e^{i\omega t},$$

$$e(x, t) = -\frac{h}{2} \frac{\partial^2 v}{\partial x^2} = \frac{h}{2} \sum_{n=0}^N [A_n k_1^2 e^{-ik_1x} - B_n k_2^2 e^{k_2x}] e^{i\omega t}, \quad (12)$$

where A_n , B_n , and k_1 , k_2 are understood to be frequency dependent. If the FFT is taken of the strain history, then it can be approximated with a finite set of frequency components, that is,

$$e(x, t) = \sum_{n=0}^N \hat{e}(x, \omega_n) e^{i\omega_n t}, \quad (13)$$

where $\hat{e}(x, \omega_n)$ is the FFT component at frequency ω_n of the strain history at x . If the strain is sampled at two locations, then A_n and B_n at discrete frequency can be solved as follows:

$$A_n = \frac{1}{k_{2n}^2} \frac{(\hat{e}(x_1, \omega_n) e^{-k_{2n}x_1} - \hat{e}(x_2, \omega_n) e^{-k_{2n}x_2})}{Q},$$

$$Q = \frac{k_{1n}^2}{k_{2n}^2} (e^{-ik_{1n}x_1 - k_{2n}x_1} - e^{-ik_{1n}x_2 - k_{2n}x_2}),$$

$$B_n = \frac{1}{k_{1n}^2} \frac{(\hat{e}(x_1, \omega_n) e^{ik_{1n}x_1} - \hat{e}(x_2, \omega_n) e^{ik_{1n}x_2})}{L},$$

$$L = \frac{k_{2n}^2}{k_{1n}^2} (e^{k_{2n}x_2 + ik_{1n}x_1} - e^{k_{2n}x_1 + ik_{1n}x_2}). \quad (14)$$

This is different from that reported by Doyle and Kamle [5, 6]. They assumed that the accuracy of the simpler Bernoulli-Euler model was up to the requirements of the study. This is generally true when the wavelength is larger than the size of the beam section. However, if the wavelength and the size of the beam section are approximately equal, then the Timoshenko model can get more accurate results.

Finally, the incident and transmitted displacements are expanded in the forms:

$$u_1 = \sum_{n=0}^N D_{1n} e^{i\omega(t-x_1/c_0)}, \quad (15)$$

$$v_1 = \sum_{n=0}^N [D_{2n} e^{-ik_{11}x_1} + D_{3n} e^{-k_{21}x_1}] e^{i\omega t}, \quad (16)$$

$$u_2 = \sum_{n=0}^N D_{4n} e^{i\omega(t-x_2/c_2)}, \quad (17)$$

$$v_2 = \sum_{n=0}^N D_{5n} e^{i\omega(t-x_2/c'_2)}, \quad (18)$$

$$u_3 = \sum_{n=0}^N D_{6n} e^{i\omega(t-x_3/c_3)}, \quad (19)$$

FIGURE 2: A cycle of cosine load (a) and the finite element of the model (b).

$$v_3 = \sum_{n=0}^N [D_{7n} e^{-ik_{13}x_3} + D_{8n} e^{-k_{23}x_3}] e^{i\omega t}, \quad (20)$$

where c_2 and c'_2 are velocities of the longitudinal and flexural waves in string 2. The hypotheses that v_2 is small and perpendicular to the x -axis at any time are taken in this paper. So, the tension T of the string does not change over time. c_2 and c'_2 can be expressed in the forms:

$$c_2 = \sqrt{\frac{E_2}{\rho_2}}, \quad (21)$$

$$c'_2 = \sqrt{\frac{T}{\rho_2}}.$$

c_3 in (19) is the velocity of the longitudinal wave and it can be found using the following equation:

$$c_3 = \sqrt{\frac{E_3}{\rho_3}}. \quad (22)$$

For convenience, the joint is located at $x = 0$. From (1)–(8), the eight unknowns $D_{1n}, D_{2n}, \dots, D_{8n}$ can be determined in terms of the coefficients A_n and B_n which are given by the initial conditions. Due to the limitation of the space, the expression of the coefficients will not be given in this article.

3. Numerical Validation of the Cosine Wave

According to the theory of Fourier transform, the arbitrary wave produced by vehicle vibration load can be explained as

a superposition of the different frequency and amplitude cosine waves. In order to simplify calculation, consider a beam-string system subjected to a cosine wave. The length of the string and beam is 1 m. A square whose side length is 25 mm is used as a beam cross section, and a circle whose radius is 1 mm is used as a string cross section. The top of the string and the right of the beam are fixed. The load is applied in the left side of the beam. In order to simplify the process of calculation, a cycle of cosine wave is calculated in this article, as shown in Figure 2(a). The circular frequency and the amplitude of the wave are 2000π and 3 mm. For convenience of the calculation, two beams adopt the same material and cross section. The beam is made of the steel with Young's modulus $E = 207,800 \text{ N/mm}^2$, Poisson's ratio $= 0.3$, and density $= 7.8e^{-9} \text{ t/mm}^3$. The string is made of the steel with Young's modulus $E = 195,000 \text{ N/mm}^2$, Poisson's ratio $= 0.3$, and density $= 7.8e^{-9} \text{ t/mm}^3$.

The angle α changed with the different mechanical system of the bridge. To investigate the angle α between the beam and string on the distribution of the force in the joint, three angles corresponding to $\pi/2$, $\pi/3$, and $\pi/6$ are investigated, respectively. The angle β is kept as zero. Figure 2(b) shows the finite element of the model when $\alpha = \pi/2$. The right end of the beam and the left end of the cable are consolidated. The load is applied to the left of the beam. Because the section size of the string is much smaller and the string is always tensioned under the cosine load, a two-node linear Timoshenko beam element (B31) was used and each bent beam and the string were discretized into 100 elements of uniform size.

Figures 3(a)–3(c) show the difference between the two methods (theoretical analysis and finite element analysis by Abaqus programs) in three locations (the left and right of the joint and the string). We choose the half cycle when

FIGURE 3: The internal force by two methods ($\alpha = \pi/2$). (a) Reflected wave. (b) The string. (c) The transmission wave.

the string bears the pulling force to discuss. The phase is ignored because it is not our concern, and the time domain is from 0 to 0.5 ms. It can be seen from Figure 3 that the amplitudes of Q_1 , N_2 , and Q_3 are 23940 N, 405 N, and 23220 N by theoretical analysis and 20836 N, 328 N, and 20128 N by finite element analysis. It is noticed that the amplitude of the internal force by finite element analysis is slightly smaller than that by theoretical analysis. This is not surprising because of the loss in the process of the wave propagation and the error of the algorithm. It can be seen from the results of the comparison

that the analytical solutions are in good agreement with finite element analysis prediction.

Figure 4 illustrates the effect of α on the internal force of the joint. It is shown in Figures 4(a), 4(b), 4(e), and 4(f) that the effect of α on the shearing forces on both sides of the joint is minor compared to that on the pulling force of the string whatever the analytical method is. It means that the pulling force N_2 exhibits a great relevance to the angle α . When $\alpha = \pi/2$, $\pi/3$, and $\pi/6$, $N_2 = 405$, 352, and 212 N by theoretical analysis and 328, 268, and 148 N by finite element analysis. It can be concluded that the pulling force has a nonlinear

FIGURE 4: Effect of α on the internal force of the joint ($\beta=0$). (a) Q_1 by theoretical analysis. (b) Q_1 by finite element analysis. (c) N_2 by theoretical analysis. (d) N_2 by finite element analysis. (e) Q_3 by theoretical analysis. (f) Q_3 by finite element analysis.

FIGURE 5: The influence of the radius of the string on the energy density ($\alpha = \pi/2, \beta = 0$). (a) The energy density caused by the flexural wave $v_0 - v_1$. (b) The energy density caused by the longitudinal wave u_2 . (c) The energy density caused by the flexural wave v_3 .

relationship with the angle α . It can also be observed that the rate of change of the pulling force N_2 by theoretical analysis is close to that by finite element analysis. This can also prove the accuracy of the theoretical method on the other side.

The energy density is another important research object in this paper. $\alpha = \pi/2, \beta = 0$, and the cosine wave are taken to study the influence of the radius of the string on the energy

density. The energy density can be calculated from mechanical energy as follows:

$$W_{k1} = \frac{1}{2} \xi \Delta x \left(\frac{\partial(v_0 - v_1)}{\partial t} \right)^2 = -\frac{1}{2} \xi \Delta x w^2 (A - D_2 - D_3) (e^{i\omega t})^2, \quad (23)$$

$$W_{p1} = \frac{1}{2} c_0' \xi \Delta x \left(\frac{\partial(v_0 - v_1)}{\partial x_1} \right)^2 = \frac{1}{2} c_0' \xi \Delta x \left(A \frac{i\omega}{c_0'} + D_2 i k_{11} + D_3 k_{21} \right)^2 (e^{i\omega t})^2, \quad (24)$$

$$w_1 = \frac{W_{p1} + W_{k1}}{\Delta V} = -\frac{1}{2} \rho_1 \left[(A - D_2 - D_3) w^2 - c_0' \left(A \frac{i\omega}{c_0'} + D_2 i k_{11} + D_3 k_{21} \right)^2 \right] (e^{i\omega t})^2, \quad (25)$$

FIGURE 6: The influence of α on the energy density ($r = 1$ mm). (a) The energy density by the flexural wave $v_0 - v_1$. (b) The energy density by the longitudinal wave u_2 . (c) The energy density by the longitudinal wave v_3 . (d) The energy density by the longitudinal wave u_1 . (e) The energy density by the flexural wave v_2 . (f) The energy density by the longitudinal wave u_3 .

$$W_{k2} = \frac{1}{2} \xi \Delta x \pi \left(\frac{\partial u_2}{\partial t} \right)^2 = -\frac{1}{2} \rho \Delta x D_4^2 w^2 (e^{i\omega t})^2, \quad (26)$$

$$W_{p2} = \frac{1}{2} c_2^2 \xi \Delta x \left(\frac{\partial u_2}{\partial x_2} \right)^2 = -\frac{1}{2} \rho_2 \Delta x D_4^2 w^2 (e^{i\omega t})^2, \quad (27)$$

$$w_2 = \frac{W_{k1} + W_{p1}}{\Delta V} = -\rho_2 D_4^2 w^2 (e^{i\omega t})^2, \quad (28)$$

$$W_{k3} = \frac{1}{2} \xi \Delta x \left(\frac{\partial v_3}{\partial t} \right)^2 = -\frac{1}{2} \xi \Delta x w^2 (D_7 + D_8)^2 (e^{i\omega t})^2, \quad (29)$$

$$W_{p3} = \frac{1}{2} c_3^2 \xi \Delta x \left(\frac{\partial v_3}{\partial x_3} \right)^2 = \frac{1}{2} c_3^2 \xi \Delta x (D_7 i k_{13} + D_8 k_{23})^2 (e^{i\omega t})^2, \quad (30)$$

$$\begin{aligned} w_3 &= \frac{W_{k3} + W_{p3}}{\Delta V} \\ &= \frac{1}{2} \rho_3 [-(D_7 + D_8)^2 w^2 - c_3^2 (D_7 i k_{31} + D_8 k_{32})^2] (e^{i\omega t})^2, \end{aligned} \quad (31)$$

where W_k , W_p , and w are the kinetic energy, potential energy, and energy density. ξ is the quality of the unit length. ΔV is the unit volume. The subscript numbers 1, 2, and 3 represent the locations on the left side of the joint, the string, and the right of the joint. From (25), (28), and (31), the energy densities w_1 , w_2 , and w_3 can be obtained by the coefficients A , D_2 , D_3 , D_4 , D_7 , and D_8 .

Figure 5 illustrates the influence of the radius of the string on the energy density. It is shown in Figure 5 that the energy density w_3 is much larger than that in locations 1 and 2. It can be explained that the reflected wave v_1 has an unloading effect on the incident wave v_0 . It also can be observed that the energy density w_3 varies little with the radius of the string from Figure 5(c), and that the energy densities w_1 and w_2 decrease with the increase of the radius of the string from Figures 5(a) and 5(b). Such a discrepancy is attributed to the sensitivity to the tensile strength of the string. As a matter of fact, with the increase of the radius, the string should take on more energy. This can also explain the phenomenon that the energy density w_1 decreases. But the growth rate of the tensile strength of the string is much greater than that of the energy they sustain. This indicates that increasing the rigidity of the string properly can make the structure more reasonable. In practical engineering, the choice of the string will also be limited by other factors. Therefore, meeting the various conditions of the diameter of the string is the best choice.

Referring to (25), the expressions of the energy density w_4 caused by the longitudinal wave u_1 , w_5 caused by the flexural wave v_2 , and w_6 caused by the longitudinal wave u_3 are as follows:

$$\begin{aligned} w_4 &= -\rho_1 D_1^2 w^2 (e^{i\omega t})^2, \\ w_5 &= -\rho_2 D_5^2 w^2 (e^{i\omega t})^2, \\ w_6 &= -\rho_1 D_6^2 w^2 (e^{i\omega t})^2. \end{aligned} \quad (32)$$

Figure 6 plots the influence of α on the energy density when the radius of the string $r = 1$ mm. It can be seen from Figure 6(a) that the energy density w_4 by the flexural wave $v_0 - v_1$ decreases with the increase of the angle α . The energy density w_2 by the longitudinal wave u_2 is opposite to the change in energy density w_1 . The energy density w_3 by the flexural wave v_3 is not influenced by angular change at all. It can be observed from Figures 6(d) and 6(f) that the energy densities w_4 and w_5 by the longitudinal wave reach maximum when the angle α is $\pi/4$. The reason is that the mechanical energy reaches maximum in that angle. The effect of the longitudinal wave on the whole energy is very small because the energy density caused by the longitudinal wave is two orders of magnitude smaller than that caused by the flexural wave. From Figure 6(e), we can obtain that the energy density w_5 caused by the flexural wave v_2 decreases with the increase of the angle. On the contrary, the energy density w_1 increases with the increase of the angle, as shown in Figure 6(b). So, we can come to a conclusion that the flexural wave is the main wave pattern in the string when the angle is relatively small; otherwise, it is the longitudinal wave.

4. Conclusions

The stress waves at a joint of a structure composed of the string and beam are investigated in this paper. The governing equations are established by displacement balance and internal force balance. The fast Fourier transform (FFT) numerical algorithm is used to deal with the incident wave of arbitrary wave type. The unknown coefficients of the reflected wave and transmitted wave are obtained by governing equations. The analytical solutions are compared with the finite element analysis by Abaqus program, and some parameters are analyzed in this paper. We can yield the following conclusions: first, the results of theoretical analysis are slightly larger than those of the finite element analysis, and the error is about 10 percent. Because the wave is dissipative in the finite element analysis, the result is reasonable, and the theoretical analysis is correct. Second, through the analysis of the influence of the angle α on the internal force and energy density, we can come to a conclusion that the flexural wave is the main wave pattern in the string when the angle is relatively small; otherwise, it is the longitudinal wave. The last conclusion is that the tensile strength of the string has a great influence on the stress of the structure. If the tensile strength is relatively small, then the energy density of the string and beam will be relatively large, which is not conducive to the long-term use of the structure. This is important for practical engineering and further study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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