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Research Article

Calculation of Capillary Rise Height of Soils by SWCC Model

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The maximum capillary rise height of soil is a complex system which is mainly determined by the distribution characteristics of soil pores. The tests of the rising height of capillary water on 8 kinds of soils by the method of vertical tube are widely conducted to measure the maximum capillary rise height. Based on the BCC model and principles of thermodynamics, the soil-water characteristic curve test is designed for the purpose of calculating the pore distribution of soil samples. A new method for calculating the maximum capillary rise height of soil is proposed by the author by using the distribution function of the soil pore. The coefficient β which reflects the relationship between the maximum capillary rise and the average pore radius of soils is utilized during the calculation process, and then the reference range of β for different soils is obtained according to series of experiments corresponding. The proposed calculation method offers an effective way to calculate the maximum capillary rise height, which can be applied to analyze the capillary effect area of relevant engineering problems.

1. Introduction

Capillary-driven liquid flow is the main transport mechanism in the soil system of which the water erodes continuously by capillary rise from a lower elevation to higher elevation. Such capillary rise phenomena lead to an increase of the saturation of the soil, which will not only decrease the strength of the soil but also alter the elastic modulus of substructure soils, thereby leading to the corresponding changes in stress and strain response under the external load, for example, the traffic load. Therefore, the defect of the roadbed is closely related to capillary rise erosion. Seasonally frozen ground has always been an important problem in highway construction and channel slope at high latitudes. In the research of frost heaving zone, it is inevitable to determine the height of capillary rise of the substructure. For channel slope and embankment close to the riverside, capillary action path is shortened as well as the effect of capillary action on the supply of water is accelerated. As a result, these infrastructures are more prone to defect and frost heave in such particular areas because of capillary rise. To sum up, study of capillary rise, particularly the maximum capillary rise height, is of great significance to the design of the substructure and channel because the maximum capillary rise height is tightly connected to the strength reduction region and frost area.

A series of studies have been carried out on the capillary rise; LU offered a complete analytical solution for the relationship between the rate and time of capillary rise in soils [1]. However, in the equation, the maximum height of capillary rise h_c is a known soil parameter. In other words, the accurate determination of the maximum height of capillary rise is the prerequisite to calculate the rate of capillary rise. The maximum height of capillary rise has an important influence on the overall engineering behavior of unsaturated soils and is a highly complex system of both the soil and pore water properties. It is extremely difficult to calculate the maximum capillary height accurately in real soils. In order to overcome this, scholars have done a lot of studies and have put forward some empirical formulas which established the correlation between the maximum capillary height and certain measured soil parameters. The earliest formula was proposed by Lane and Washburn [2, 3], after conducting the capillary rising test for 8 kinds of different soils, and the result shows that the maximum capillary height is linear with $\ln D_{10}$:

$$h_c = -990(\ln D_{10}) - 1540. \quad (1)$$

On the basis of (1), Peak and Hansen put forward another empirical formula [4]:

$$h_c = \frac{C}{eD_{10}}. \quad (2)$$

In this formula, 10% particle size d_{10}, void ratio e, and the coefficient $C\,(C = 3 \times 10^{-5}\,\mathrm{m}^2 \sim 8 \times 10^{-5}\,\mathrm{m}^2)$ are used to calculate the maximum capillary rise height. However, the coefficient C is difficult to choose because the range of C is relatively large, and the value of C depends hugely on the real condition of soil. Kumar and Malik by carrying out indoor tests summarized as follows [5]:

$$h_c = h_a + 134.84 - 5.16\sqrt{r}, \quad (3)$$

where h_a is the height corresponding to air-entry pressure value of soils and r is the equivalent of capillary radius of soils. In this formula, the unit of r is μm, so it has little influence on the result of the calculation. Therefore, h_a is the only key computational parameter which can be determined by the soil-water characteristic curve. However, it requires an instrument to control the matric suction very precisely especially when the suction is less than 1 kPa [6].

We summarized that according to previous research results, using the micropore distribution of soils and using the pore radius parameter instead of the gradation parameter and void ratio are effective methods to accurately calculate the maximum capillary rise height. In this paper, a new method is proposed to calculate the maximum capillary rising height by using the radius of microscopic pore distribution. A large number of capillary rise tests as well as soil-water characteristic (SWCC) tests have been done for different kinds of soils, from which the parameters of different soils required for the calculation are obtained, and the feasibility of this method will be verified.

2. Theoretical Framework

2.1. Capillary Rise Equilibrium Equation in Soil System. In a single capillary when water column reaches the maximum height, the gravity of the water column is balanced by the surface tension along water-solid interface, as shown in Figure 1. The balance equation is as follows:

$$h_c \rho_w g \pi r^2 = T_S \pi d \cos\alpha, \quad (4)$$

where α is the liquid-solid contact angle; ρ_w is the density of water; r is the radius of the capillary channel ($r = d/2$); and T_S is the surface tension of water.

For soils, we regard it as homogenous porous media, which means the pores inside are similar in diameter and the dispersion is small. In the interior of soils, tiny pores which are connected with each other fulfill the precondition of capillary rise [7, 8]. The total capillary gas-liquid area A in each cross section can be expressed as follows:

$$A = \phi_A \pi \left(\int_0^\infty r f(r)\, dr \right)^2, \quad (5)$$

where ϕ_A is the number of gas-liquid interfaces in total capillary gas-liquid area A of each cross section and $f(r)$ is pore size distribution (PSD) function in the porous system of soils [9, 10]. When we established the gravity-tension balance equations assuming that soil pores are interconnected with each other, the capillary migration path of water is not like a single capillary tube, meaning that channel interleaving was bent and most of the gas-liquid interfaces on the same section are connected. As a simplified model, we argue that the total capillary route is equal to the maximum capillary rise height h_c multiplied by a path retardation coefficient β; thereafter, the capillary equilibrium equation for any gas-liquid interface can be modified from (4) to

$$h_c \rho_w g \pi \left[\int_0^\infty r f(r)\, dr \right]^2 \beta = 2T_S \pi \int_0^\infty r f(r)\, dr, \quad (6)$$

$$h_c = \frac{2T_S \cos\alpha}{\rho_\omega g \beta \int_0^\infty r f(r)\, dr}. \quad (7)$$

When $\rho_w = 1\,\mathrm{g/cm}^3$, $g = 980\,\mathrm{cm/m}^2$, $T_S = 72\,\mathrm{mN/m}$, and $\alpha = 0^\circ$ (dehydration process), (7) can be simplified as follows:

$$h_c = \frac{0.15}{\beta \int_0^\infty r f(r)\, dr}. \quad (8)$$

2.2. Testing Method of PSD $f(r)$. One effective way to study the pore characteristics of homogenous porous media is to establish the microscopic pore structure model. Many conventional models for pore characteristics, liquid distribution, flow, and transport in porous media are based on representing pore space geometry as a bundle of cylindrical capillaries (BCC model) [11, 12]. In the literature, there have been some standardized experimental methods for directly measuring pore size distribution (MIP, mercury intrusion porosimetry; BM, bubble method), and these methods are based on the basic theory of the BCC model. In BCC model, according to the Young–Laplace equation, the relationship between the pore radius r and the suction S (or capillary pressure) is represented as follows:

$$S = 2T_S \cos\frac{\alpha}{r}. \quad (9)$$

Based on the assumption of pore local equilibrium, the balance of soil is achieved in a given suction S^* inside the pore while one part of the pore with the pore radius greater than r^* ($r^* = 2T_s \cos\alpha / S^*$) is filled with air and the other one with pore radius less than r^* is filled with water. By defining $g(r)$ as pore size distribution function, $g(r)\,dr$ represents the percentage of void volume with pore radius ranging from r to $r + dr$ under the unit soil volume. When the suction of the soil reaches S, the maximum pore radius which is filled with water becomes $C/S\,(C = 2T_S \cos\alpha)$. Therefore, the relationship between soil volumetric water content and pore distribution can be expressed as follows:

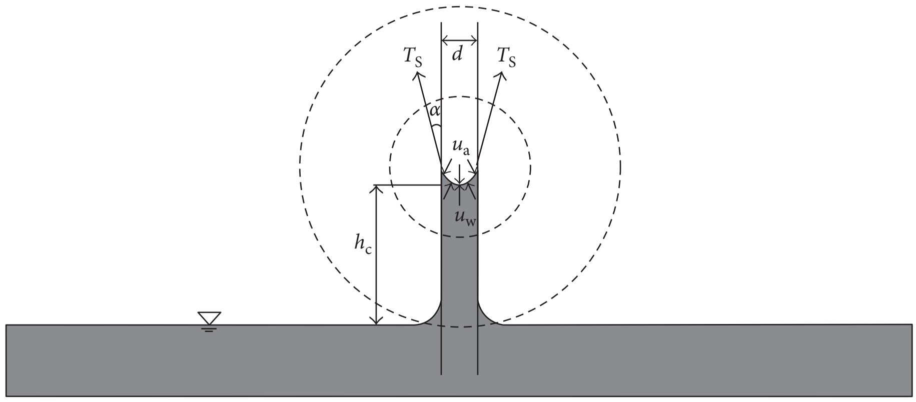

Figure 1: Mechanical equilibrium of capillary rise in the small diameter tube.

$$d\theta = g(r)\, dr. \tag{10}$$

In this formula, θ indicates the volumetric water content of soil when soil suction equals S and $d\theta$ represents the ratio of pore water volume with soil particle radius between r and $r + dr$ to the total pore water volume under a well-saturated condition. In fact, in this formula, the volume of water being evacuated is used as an alternative to that of void in the soil. When the calculated radius is in the range of the whole pore medium, $g(r)$ can be equal to $f(r)$.

In this paper, the pore size distribution of soil was measured, and the methods mentioned above have some limitations when being applied. Taking MIP as an example, sample preparation for MIP is difficult, and under high pressure, liquid mercury will jeopardize soil particles, resulting in the deformation of soil skeleton and pore during the test. Another important reason which restricts the widespread use of conventional methods is the low popularity of test instruments as well as high test costs, which forced us to look for other convenient testing methods. In this case, mercury is replaced by water, and the soil-water characteristic curve is utilized instead of the mercury injection curve. Pores among soil particles rather than the internal pore of the particle are mainly considered with regard to the soil capillary phenomenon and the pore water flow problem. When the soil-water characteristic curve is used, the pressure can be increased to 2 MPa, which means the pore radius can be attached to 1000 Å correspondingly. In other words, it can obtain the exact pore distribution of the above 1000 Å, allowing us to have enough pore radius distribution data to analyze soil capillary phenomenon.

2.3. Basic Theories and Calculation Method. The pore of soil can be regarded as a capillary in the Kelvin formula without violating original characteristics of soil system. Radius, contact angle between soil particles and pore water, and surface tension are denoted by r, α, and T_S, respectively. When using measurable physical quantities, Kelvin formula is expressed as

$$\frac{2T_S v_\omega \cos\alpha}{r} = -RT \ln \frac{u_{v1}}{u_{v0}} = \mu_1 - \mu_0, \tag{11}$$

where form of matric suction is described as

$$r = \frac{2T_S \cos\alpha}{u_a - u_w}, \tag{12}$$

where $R = 8.314$ J/(mol·K) is the universal gas constant; $T = 298$ K is the thermodynamic temperature; μ_{vo} (kPa) is the free water saturated vapor pressure in equilibrium state under the condition of temperature T. μ_{v1} (kPa) is the vapor pressure value of solution at the state of balance; $v_m = 18 \times 10^{-6}$ m^3/mol is the partial molar volume of water vapor. $u_a - u_w$ is the matric suction, u_{v1}/u_{v0} is defined as relative humidity (RH) and can be obtained from (13) which is another expression of (11):

$$RH = \exp\left(-\frac{(u_a - u_w) v_w}{RT}\right). \tag{13}$$

When the relative humidity or matric suction is in the increment of Step i, the volume change of the pore volume of soil mass which is replaced by the gas can be calculated as

$$\nabla v_p^i = \frac{\nabla w^i}{\rho_w}. \tag{14}$$

Kelvin radius can be estimated by (12), while the actual pore radius is expressed as

$$r_p^i = r_k^i + t^i, \tag{15}$$

where the thickness of adsorbed water film t^i corresponding to RH or matric suction at Step i can be expressed by Halsey equation as [13]

$$t^i = \tau \left[-\frac{5}{\ln(RH^i)}\right]^{1/3}, \tag{16}$$

where τ is the effective diameter of adsorbed water molecules. Assuming that the cross-sectional area occupied by the water molecules is approximately 10.8 Å and the Avogadro constant N_A is 6.02×10^{23}/mol, the value of τ can be calculated as

$$\tau = \frac{v_w}{AN_A} = 2.77\text{Å}. \tag{17}$$

Table 1: Main physical indexes of fine-grained soils.

Soil samples	$\gamma_{d(max)}$	ω_P	ω_L	I_p	Particle size (mm) and content (%)				
					≥0.5 mm	0.25~0.05 mm	0.05~0.01 mm	0.01~0.002 mm	≤0.002 mm
CL	1.88	22.4	41.8	19.4	0.8%	13.4%	21.7%	34.5%	29.6%
CH	1.84	27.9	55.6	27.7	0.3%	9.5%	17.5%	37.1%	35.6%
ML	1.93	18.2	28.3	8.1	3.9%	29.6%	31.3%	25.4%	7.8%
CHE	1.83	28.3	65.2	36.9	0.2%	7.3%	16.5%	34.8%	41.2%

After terminating the calculation of pore radius which corresponds to different suctions, points can be selected in the SWCC to calculate the pore size distribution curve. Considering the nonuniform pore size and inhomogeneous pore distribution as well as the data of pore distribution curve obtained by point-selecting method, r_0 as average pore radius of soil by using weighted average of the statistical theory is defined as

$$r_0 = \frac{\sum v_p r_p^i}{\sum r_p^i}. \quad (18)$$

Similar to the parameters of d_{10}, d_{60}, and C_u found in the particle size distribution curve, the average capillary pore radius can be obtained by pore size distribution curve. According to the definition, r_0 represents the average radius within the calculating range. For example, if suction value ranges from 1 kPa to 10000 kPa, then the r_0 is the average radius of the pore which falls into the corresponding range.

When the calculated aperture range relates to the whole soil (suction value ranges from the air-entry pressure value S_S to the residual suction S_r), we can approximate r_0 as $r_0 = \sum_{i=1}^{n} r_i/\mathrm{n}$, by $\sum_{i=1}^{n} r_i \approx nE(r)$ where

$$E(r) = \int_0^{\infty} r f(r)\, dr. \quad (19)$$

We can easily argue that

$$\int_0^{\infty} r f(r)\, dr \approx r_0. \quad (20)$$

By conducting substitution of (20) to (8), (8) can be simplified as follows:

$$h_c = \frac{0.15}{\beta r_0}. \quad (21)$$

As a parameter, average pore radius r_0 characterizing the pore state can be obtained by SWCC, and path retardation coefficient β as an empirical parameter will be measured by the following tests.

3. Materials and Methods

3.1. Test Materials. The test soils are distinguished as two different kinds of soils: fine-grained soils and coarse-grained soils. Fine-grained soils include low liquid limit clay (CL), high liquid limit clay (CH), low liquid limit silt (ML), and high liquid limit expansive clay (CHE), while coarse-grained soils include silt sand (SS), fine sand (FS), medium sand (MS), and coarse sand (CS). The parameters of the soil samples are obtained by the conventional tests and are shown in Tables 1 and 2.

3.2. Capillary Rise Test. The method of vertical tube (a transparent plexiglass tube with diameter of 4.5 cm and height of 500 cm) is applied when performing soil capillary rise test. 20 capillary plexiglass tubes with 8 kinds of soil samples are tested and the compaction degree of soils is controlled by the total density of soils during the sample loading process. The test results of the steady rising height of capillary water in different soil samples are shown in Table 3.

3.3. SWCC Test. SWCC is mainly determined by pressure plate method, in which four samples of each kinds of soil and the as-fabricated samples were placed in a vacuum saturator for 48 h to ensure the full saturation. PF moisture characteristic curve instrument (DIK-4303) is used for measuring soil-water characteristic curve. First of all, samples are weighted by using electronic balance to ensure that the mass is recorded. And then, samples are put into a determinator (4 samples at a time) which connects a water channel between the instrument and the ceramic plate. The upper cover should be well tightened in case of air leakage. The maximum pressure is set to 1200 kPa, which is controlled by the increment of each step of the 100 kPa, and before applying the pressure, each sample's gravimetric mass is recorded. The suction balance will be achieved if no water flows out from the pipe within 48 hours.

4. Analysis and Calculation

The results of the SWCC test are shown in Figures 2 and 3, but due to the design restriction in pressure plate method which means that a pressure over 1200 kPa cannot be exerted and maintained in the experiment, points exceeding 1200 kPa in SWCC are obtained by fitting with VG model [14]. To be precise, the datum acquired from the test is injected into Van-Genuchten model in ORIGIN to obtain soil-water characteristic curve in a wider range of suction. The fitting equation is

$$\theta = (\theta_s - \theta_r)\left[\frac{1}{1 + a^n (u_a - u_w)^n}\right]^m + \theta_r, \quad (22)$$

where θ is the volumetric water content; θ_s is the saturated volumetric water content; θ_r is the residual volumetric water content; and a, m, and n are independent coefficients (satisfy the condition $m = 1 - 1/\mathrm{n}$ as much as possible). The fitting results from (22) for the soil samples are shown in Table 4.

Table 2: Main physical indexes of coarse-grained soils.

Soil samples	$\gamma_{d(max)}$	Particle size (mm) and content (%)							
		≥5 mm	5~2 mm	2~0.5 mm	0.5~0.25 mm	0.2~0.075 mm	0.07~0.01 mm	0.0~0.005 mm	≤0.005 mm
SS	2.08	0%	0.3%	1.2%	26.5%	41.8%	27.8%	2.1%	0.3%
FS	1.91	0%	1.6%	8.7%	34.4%	39.8%	13.8%	1.5%	0.2%
MS	1.84	1.9%	7.6%	17.5%	48.1%	19.9%	4.6%	0.3%	0.1%
CS	1.78	2.1%	11.6%	48.3%	32.7%	2.1%	1.2%	0.1%	0.1%

Table 3: The test results of the steady rising height of capillary water.

Sample	Compaction degree	Maximum height (cm)
CL	0.9	331
	0.85	317
	0.8	301
CH	0.9	355
	0.85	334
	0.8	315
ML	0.9	293
	0.85	267
	0.8	239
CHE	0.9	391
	0.85	358
	0.8	339
SS	0.9	194
	0.85	181
	0.8	159
FS	0.85	140
	0.8	121
MS	0.85	91
	0.8	81
CS	0.85	73

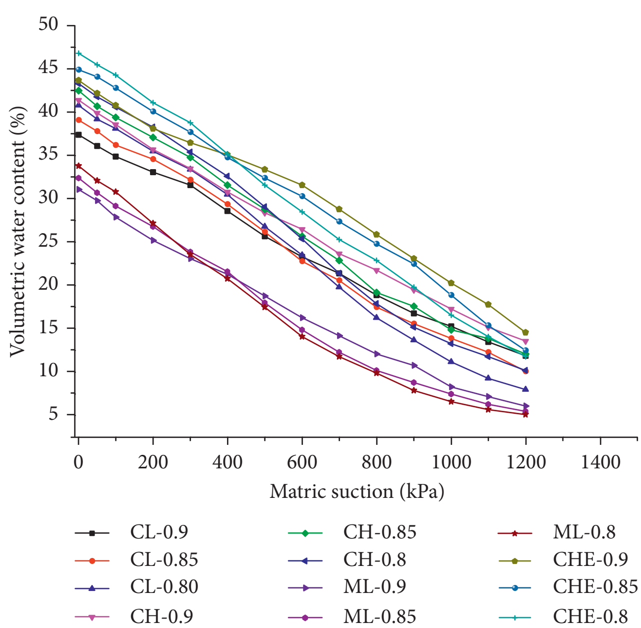

Figure 2: Soil-water characteristic curve test result for fine-grained soils.

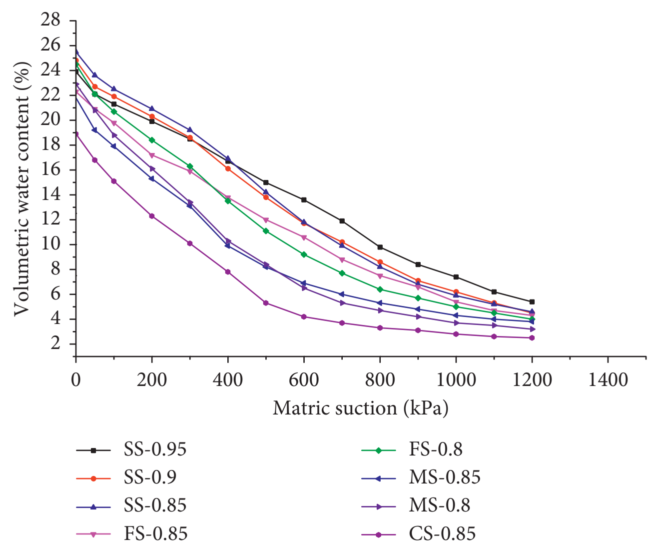

Figure 3: Soil-water characteristic curve test result for coarse-grained soils.

When selecting suction calculation range, the value range should be considered to represent the entity of soil as well as the measurement range of suction. After many attempts, we propose that fine-grained soils matric suction range be controlled at 200 kPa~10000 kPa and that the range of coarse-grained soils be controlled at 50 kPa~4000 kPa. (The external values of the test range are calculated by fitting the results of VG model.)

According to the theory introduced above, soil-water characteristic curve is used to calculate the radius r_0. Calculation results of soil sample of CL with compaction degree 0.9 are listed in Table 5 as an example while the numerical integration calculation steps are as follows:

(1) Get the SWCC of the soil and select the data of 200 kPa~20000 kPa (the data outside the range of test are obtained by fitting with VG model).

(2) Calculate RH (relative humidity) of matrix suction according to (13).

(3) Translate mass water content into per unit mass of soil-water filled pore volume (volumetric water content).

(4) Use (12) to calculate the Kelvin radius r_k^i under the suction of different substrates with which the contact angle α is 0 (soil in dehydration process).

Table 4: Fitting results of SWCC parameters for soil samples.

Samples	rd	θ_s (%)	θ_r (%)	a	n	m
CL	0.9	38.2163	1.7842	0.0012	1.7798	0.6021
	0.85	39.9741	2.0259	0.0013	1.9266	0.5988
	0.8	40.5377	1.9624	0.0014	2.3215	0.5002
CH	0.9	41.3376	1.0621	0.0012	1.6198	0.5549
	0.85	42.2937	1.2063	0.0013	1.8688	0.4889
	0.8	43.2986	1.4079	0.0014	2.1398	0.4766
ML	0.9	30.6177	0.8823	0.0016	1.8423	0.4501
	0.85	31.7787	0.7213	0.0018	1.9932	0.4821
	0.8	33.1944	0.5056	0.0020	1.9161	0.4533
CHE	0.9	43.6997	2.2268	0.0010	2.0968	0.5001
	0.85	44.4501	0.9385	0.0012	2.0152	0.4896
	0.8	46.7998	1.3176	0.0013	1.9668	0.5102
SS	0.9	23.2396	0.7603	0.0014	1.9339	0.5983
	0.85	23.9630	0.5382	0.0017	1.9117	0.6021
	0.8	24.8004	0.6886	0.0017	2.0129	0.5891
FS	0.85	21.7180	0.2819	0.0018	1.6740	0.6213
	0.8	23.9116	0.5884	0.0022	1.6523	0.6011
MS	0.85	21.4904	0.3096	0.0026	1.3913	0.6542
	0.8	22.2219	0.2342	0.0027	1.5630	0.7311
CS	0.85	18.5616	0.1443	0.0033	1.5067	0.7001

Table 5: Equivalent radius calculated form of soil sample of CL with compaction degree 0.9.

$u_a - u_w$ (kPa)	W(g/g)	RH (%)	V_p (cm^3/g)	r_k (Å)	t (Å)	r_p (Å)	Δv_p (cm^3/g)	(r_p)avg (Å)	$\sum (V_p)$ (cm^3/g)
200	0.176	99.85	0.176	7200	41.827	7241.8	—	—	—
300	0.168	99.78	0.168	4800	36.539	4836.5	0.008	6039.2	0.008
400	0.152	99.71	0.152	3600	33.198	3633.2	0.016	4234.9	0.024
500	0.137	99.64	0.137	2880	30.818	2910.8	0.015	3272.0	0.039
600	0.124	99.57	0.124	2400	29.001	2429.0	0.013	2669.9	0.052
700	0.114	99.49	0.114	2057	27.549	2084.7	0.010	2256.8	0.062
800	0.101	99.42	0.101	1800	26.349	1826.3	0.013	1955.5	0.076
900	0.089	99.35	0.089	1600	25.335	1625.3	0.011	1725.8	0.087
1000	0.081	99.28	0.081	1440	24.461	1464.5	0.008	1544.9	0.095
1100	0.072	99.20	0.072	1309	23.696	1332.8	0.010	1398.6	0.104
1200	0.063	99.13	0.063	1200	23.018	1223.0	0.009	1277.9	0.113
2000	0.054	98.56	0.054	720	19.414	739.4	0.009	981.2	0.122
4000	0.048	97.14	0.048	360	15.409	375.4	0.006	557.4	0.128
6000	0.036	95.74	0.036	240	13.461	253.5	0.012	314.4	0.140
10000	0.025	93.00	0.025	144	11.354	155.4	0.011	204.4	0.150
20000	0.018	86.48	0.018	72	9.011	81.011	0.007	118.2	0.158

(5) Calculate the thickness of the water film t^i under the suction of different substrates by (16).

(6) Calculate actual pore radius r_p^i by results (4) and (5) applying (15).

(7) Calculate the reduction of pore volume water Δv_p in per unit mass of soil when the matrix suction is changed form one substrate to another. (Here, Δv_p is the pore volume corresponding to the water that is discharged.)

(8) Calculate the average pore radius (r_p)avg in the process of evacuation of pore water by averaging the result of (6) for per step. (Here (r_p)avg is the average pore radius corresponding to the pore volume v_p.)

(9) Add the pore volume to calculate the cumulative pore volume $\sum (V_p)$ of per unit mass, and draw relationship diagram between the pore volume Δv_p and the average pore radius (r_p)avg.

(10) Calculate the r_0 by (18).

The average pore radius r_0 of soil sample is calculated by the method above. After determining the radius r_0, the retardation coefficient β of different soil samples will be obtained. And the results of coefficient β are shown in Table 6.

As clearly observed from the above chart, the coefficient of the fine-grained soils was about 21 cm^{-2} while the range barely fluctuates with value relatively fixed. But for coarse-grained soils, β changes from 23 cm^{-2} to 26 cm^{-2}. One main reason we argue for coarse-grained soils is that the proportion of large pore size increased significantly, contributing greatly to a maximum capillary rise. As a result, for

Table 6: Calculation result of coefficient β for soil samples.

Sample	Compaction degree	Maximum height (cm)	Capillary radius (Å)	Coefficient β
CL	0.9	331	2128	21.3
	0.85	317	2212	21.4
	0.8	301	2336	21.3
CH	0.9	355	2014	21.0
	0.85	334	2107	21.3
	0.8	314	2234	21.4
ML	0.9	293	2359	21.7
	0.85	267	2650	21.2
	0.8	239	2881	21.8
CHE	0.9	391	1811	21.2
	0.85	358	1989	21.1
	0.8	339	2090	21.2
SS	0.9	194	3305	23.4
	0.85	181	3399	24.4
	0.8	159	3841	24.6
FS	0.85	140	4499	23.8
	0.8	121	4950	25.0
MS	0.85	91	6374	25.9
	0.8	81	7116	26.0
CS	0.85	73	7854	26.2

Table 7: Test and calculation results of maximum capillary rise height for 44 kinds of soils.

Test number	Soil sample	Void ratio e	d_{10} (cm)	Coefficient β (cm^{-2})	h_a (cm)	Capillary radius (Å)	Measured height (cm)	Results by formula (e/er)/cm	Results by (1) (e/er)/cm	Results by (2) (e/er)/cm	Results by (3) (e/er)/cm
1	CL	0.89	0.001	21	178	2253	309	317 (8/2.5%)	530 (221/72%)	281 (−28/9.1%)	312 (3/1.0%)
2	CL	0.92	0.001	21	202	2158	325	331 (6/1.8%)	530 (205/63%)	271 (−54/17%)	336 (11/3.4%)
3	CL	1.05	0.0008	21	194	2070	324	345 (21/6.5%)	552 (228/70%)	297 (−27/8.3%)	328 (4/1.2%)
4	CL	0.88	0.0007	21	184	2268	319	315 (−4/1.3%)	565 (246/77%)	404 (85/27%)	318 (−1/0.3%)
5	CL	0.88	0.0009	21	192	2001	341	357 (16/4.7%)	540 (199/58%)	317 (−24/7.0%)	326 (−15/4.4%)
6	CL	0.81	0.001	21	199	2171	322	329 (7/2.2%)	530 (208/65%)	309 (−13/4.0%)	333 (11/3.4%)
7	CL	0.98	0.0011	21	195	2089	318	340 (22/6.9%)	520 (202/64%)	231 (−87/27%)	329 (11/3.5%)
8	CL	1.01	0.0007	21	180	2035	344	351 (7/2.0%)	565 (221/64%)	350 (6/1.7%)	314 (−30/8.7%)
9	CH	0.96	0.0008	21	214	2312	317	309 (−8/2.5%)	552 (235/74%)	327 (10/3.2%)	348 (31/9.8%)
10	CH	0.84	0.0006	21	231	2158	328	331 (3/0.9%)	580 (252/77%)	494 (166/51%)	365 (37/11%)
11	CH	0.73	0.0007	21	209	2165	343	330 (−13/3.8%)	565 (222/65%)	492 (149/43%)	343 (0/0%)
12	CH	0.96	0.0006	21	218	2232	329	320 (−9/2.7%)	580 (251/76%)	433 (104/32%)	352 (23/7.0%)
13	CH	1.01	0.0007	21	199	2158	315	331 (16/5.1%)	565 (250/79%)	355 (40/13%)	333 (18/5.7%)
14	CH	0.89	0.0009	21	233	2191	347	326 (−21/6.1%)	540 (193/56%)	310 (−37/11%)	367 (20/5.8%)
15	ML	0.74	0.0018	21	145	2421	288	295 (7/2.4%)	472 (184/64%)	186 (−102/35%)	279 (−9/3.1%)
16	ML	0.90	0.0025	21	165	2524	276	283 (7/2.5%)	439 (163/59%)	110 (−166/60%)	299 (23/8.3%)

Table 7: Continued.

Test number	Soil sample	Void ratio e	d_{10} (cm)	Coefficient β (cm^{-2})	h_a (cm)	Capillary radius (Å)	Measured height (cm)	Results by formula (e/er)/cm	Results by (1) (e/er)/cm	Results by (2) (e/er)/cm	Results by (3) (e/er)/cm
17	ML	0.86	0.0029	21	137	2655	259	269 (10/3.9%)	424 (165/64%)	100 (−159/61%)	271 (12/4.6%)
18	ML	0.78	0.0031	21	131	2716	247	263 (16/6.5%)	418 (171/69%)	104 (−143/58%)	265 (18/7.3%)
19	ML	0.74	0.0043	21	158	2636	260	271 (11/4.2%)	385 (125/48%)	78 (−182/70%)	292 (32/12%)
20	ML	0.82	0.0019	21	122	2758	248	259 (11/4.4%)	466 (218/88%)	161 (−87/35%)	256 (8/3.2%)
22	MH	0.72	0.003	21	101	2892	236	247 (11/4.7%)	421 (185/78%)	116 (−120/51%)	235 (−1/0.4%)
23	MH	0.78	0.0022	21	95	2869	257	249 (−8/3.1%)	452 (195/76%)	146 (−111/43%)	229 (−28/11%)
24	MH	0.69	0.0037	21	88	3161	221	226 (5/2.3%)	400 (179/81%)	97 (−124/56%)	222 (1/0.5%)
25	MH	0.81	0.0042	21	94	2737	259	261 (2/0.8%)	388 (129/50%)	73 (−186/72%)	228 (−31/12%)
26	MH	0.72	0.0035	21	88	3079	240	232 (−8/3.3%)	406 (166/69%)	99 (−141/59%)	222 (−18/7.5%)
27	SM	0.62	0.009	25	78	4167	199	180 (−19/9.5%)	312 (113/57%)	40 (−159/72%)	211 (12/6.0%)
29	SM	0.60	0.0085	25	83	4121	204	182 (−22/10.8%)	318 (114/56%)	49 (−155/59%)	216 (12/5.9%)
30	SM	0.62	0.0081	25	80	4190	185	179 (−6/3.2%)	323 (138/75%)	50 (−135/80%)	213 (28/15%)
34	SS	0.42	0.019	25	58	4098	197	183 (−14/7.1%)	238 (41/21%)	31 (−166/84%)	191 (−6/3.0%)
35	SS	0.53	0.012	25	71	4167	184	180 (−4/2.2%)	284 (100/54%)	39 (−145/79%)	204 (20/10%)
36	SS	0.46	0.021	25	61	4491	178	167 (−11/6.2%)	228 (50/28%)	26 (−152/85%)	194 (16/9.0%)
37	SS	0.43	0.015	25	58	4190	180	179 (−1/0.6%)	262 (82/46%)	39 (−141/78%)	191 (11/6.1%)
38	FS	0.52	0.031	25	43	5474	139	137 (−2/1.4%)	190 (51/37%)	16 (−123/88%)	176 (37/27%)
39	FS	0.42	0.052	25	41	6356	110	118 (8/7.3%)	139 (29/26%)	11 (−99/90%)	174 (64/58%)
40	FS	0.49	0.047	25	39	6466	118	116 (−2/1.4%)	149 (31/26%)	11 (−107/91%)	172 (54/46%)
41	FS	0.42	0.055	25	42	7353	98	102 (4/4.1%)	133 (35/36%)	11 (−87/89%)	175 (77/79%)
42	MS	0.38	0.092	25	34	7732	86	97 (11/12.8%)	82 (−4/5%)	7 (−79/92%)	167 (81/94%)
43	MS	0.33	0.1	25	30	8152	83	92 (9/10.8%)	74 (−9/11%)	8 (−75/90%)	163 (80/96%)
44	MS	0.41	0.0864	25	29	8523	79	88 (9/11.4%)	88 (9/11%)	7 (−72/91%)	162 (83/105%)

(21) the numerical value of the coefficient β is considered to be 21 cm^{-2} and 25 cm^{-2} for fine-grained soils and coarse-grained soils, respectively.

5. Verification and Discussion

In this study, in order to verify the accuracy of the maximum capillary rise height formula proposed by the author, capillary rise tests are carried out on 44 kinds of soil samples, and thereafter, a comparison is made between the capillary rise estimated from our equation (21) and the estimations from (1)–(3). Here, the coefficient C for (2) takes 5×10^{-5} m^2

The test results and calculation results of the maximum capillary rise height for 44 kinds of soil samples are shown in Table 7.

It can be seen from the table that the estimating error of the maximum height from the formula proposed by the author is always less than 10% and the maximum error is 22 cm (204 cm), indicating its relatively higher accuracy compared with other formulas mentioned in this paper. This also means that the results coincide better with the actual situation and have higher practical value.

Accurately determining the maximum capillary rise height of the soils is a complicated issue. In this paper, we calculated the average pore radius by method of SWCC and then determined the maximum capillary rise height using the pore radius. Obtaining the pore distribution data from the SWCC data is the core of the calculation method. We can easily get the pore distribution curve through the soil-water characteristic curve. However, similar to the particle size curve, we need to find a set of parameters to evaluate this curve as well as calculate the maximum capillary rise height. At first, we tried to use r_{50} as the evaluation parameter but later found when fine-soil samples are at different degrees of compaction, the r_{50} values are very close. According to the definition, as long as the soils have the same matrix suction at the saturation of 50%, the values of r_{50} still remain the same. It is difficult to use a specific pore radius to represent the equivalent capillary radius in the calculation of the maximum capillary rise height of the soil; therefore, we chose the weighted average of the pore radius r_0 as the calculation parameter. Whether there is a better way to evaluate the distribution of soil pore and to analyze the behavior of soils is a project that we still need to explore.

Another important parameter is β which is an empirical one obtained from the test. We use r_0 to distinguish different soil samples, so we hope different soil samples would obtain the same values of β as much as possible. Test results show that β of fine-grained soil is very close and can be directly taken to 21, but β also has the tendency to augment with the increase of average pore radius r_0 and this trend is particularly evident in coarse-grained soils. Therefore, there is an issue about how to pick the value of β in the actual calculation process and we argue that β of fine-grained soils can be fixed to 21, but for coarse-grained soil the value of β is not a settled figure, ranging from 24 to 26. Moreover, the value rises with the increase of the content of the coarse particles. Here the author suggests taking an average of 25.

6. Conclusion

After theoretical calculation and large numbers of indoor experiments, the following conclusions can be summarized as follows:

(1) The average pore radius formula is deduced on summary of experiments and statistical theory, and the calculation theory can be used to distinguish the pore distribution characteristics of different compaction degree of soil samples with different particle size distribution. Therefore, it can offer references for similar research and also can be used for estimating the maximum height of capillary. In particular, for fine-grained soils the calculation results for the maximum capillary height are relatively accurate.

(2) Using the soil pore radius to calculate the maximum capillary rise height is an accurate and feasible approach. During the calculation process, the coefficient β of the fine-grained soil was about 21 to 22, while β of the coarse-grained soil was about 24 to 26. The greater the number of coarse particles is, the greater the retardation coefficient β is.

(3) The maximum capillary rise height of the soil is mainly determined by the distribution of the large size pore, and meanwhile, the effect of the small size pore is quite insignificant.

Notations

h_c: Maximum capillary rise height of soils
D_{10}: Particle size d_{10} of soils
d_{10}: Fine-grain diameter of ten percent of total soil mass
e: Void ratio of soils
h_a: Height corresponding to air-entry pressure value of soils
α: Liquid-solid contact angle
r: Radius of capillary channel
ρ_w: Density of water
T_S: Surface tension of water
A: Total capillary gas-liquid area in each cross section
ϕ_A: Number of gas-liquid interface in total capillary gas-liquid area A
$f(r)$: Pore size distribution (PSD) function of soil
β: Path retardation coefficient
S: Matric suction
R: Universal gas constant
T: Thermodynamic temperature
μ_{v0}: Free water saturated vapor pressure in equilibrium state under the condition of temperature T
μ_{v1}: Vapor pressure value of solution at the state of balance
v_m: Partial molar volume of water vapor
u_a: Pore air pressure
u_w: Pore water pressure
RH: Relative humidity
v_p: Pore volume of soil
r_k: Kelvin pore radius
r_p: Actual pore radius
t: Thickness of adsorbed water film
τ: Effective diameter of adsorbed water molecules
N_A: Avogadro constant 6.02×10^{23}/mol
d_{60}: Fine-grain diameter of 60 percent of total soil mass
C_u: Nonuniform coefficient of soil
r_0: Average pore radius
θ: Volumetric water content
θ_S: Saturated volumetric water content
θ_r: Residual volumetric water content
a, m, n: Independent coefficients of VG model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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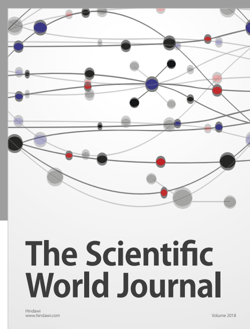

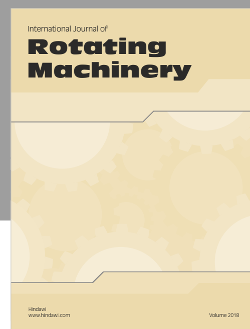

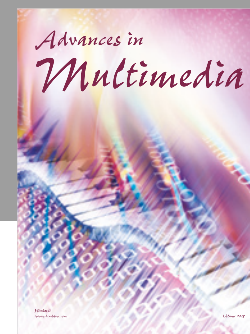

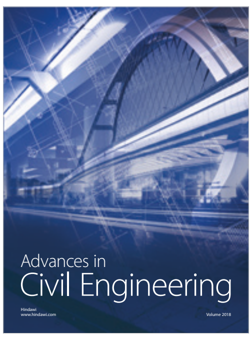

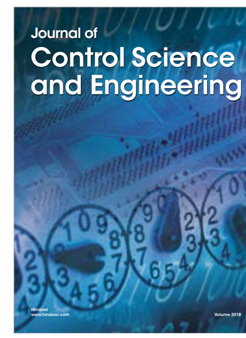

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