Seismic Risk Mitigation for a Portfolio of Reinforced Concrete Frame Buildings through Optimal Allocation of a Limited Budget

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Received 7 February 2018; Accepted 5 August 2018; Published 17 September 2018

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The mitigation of seismic risk for a population of vulnerable civil critical structures (e.g., hospitals, schools, and bridges) is a crucial issue for many governments of earthquake-prone regions. Furthermore, owing to the global economic crisis, limited financial resources make full seismic rehabilitation of entire building stocks challenging. Therefore, a critical decision has to be made on the following key question: what is the most advantageous way of spending the available budget while treating each building in a portfolio differently, by giving it a different level of structural improvement to reduce the overall risk of the portfolio of buildings as much as possible? Herein, a decision-making tool is proposed to address this high-social-impact issue. Starting with a limited amount of information, which is gathered through expeditious surveys on existing buildings, and by involving uncertainties, the overall risk is evaluated from the fragility analysis of each structure. This is conducted via simplified pushover analyses by considering the local seismic hazard. Then, an optimization is performed for each building of the portfolio to select a relevant structural intervention from four alternatives (no intervention, partial retrofit, full retrofit, and demolition and reconstruction), based on both the overall risk reduction and the amount of financial resources. Procedures for quick estimation of fragility curves and installation costs are also discussed as part of the proposed approach. Finally, a practical application is presented with reference to a simulated case study consisting of 46 reinforced concrete school buildings located in Campania, Italy.

1. Introduction

Over the past decades, considerable attention has been given to study the risk of earthquake occurrence, how earthquakes could affect a region when they strike, and how to protect existing structures against them. High regional seismicity and the existing large number of vulnerable structures have made government and policymakers pay more attention to the issue of risk management of urban areas, particularly of critical/strategic buildings and infrastructures. Among the regional-type buildings, schools have always been identified by governments as buildings that must be kept as safe as possible, as they host the innocent future generation of society. Moreover, they play a major role in earthquake risk management, as they can be used as shelter after an earthquake event. Recent studies have shown how the actual seismic risk of Italian schools [1] should be urgently reduced to ensure an adequate safety level for the occupants. In order to mitigate the territorial risk involving a large number of structures, regional policymakers have to make difficult decisions regarding budget allocation. In the past, the authors have dealt with decision-making procedures for the mitigation of the seismic risk of individual buildings [2, 3]. The aim of this study is to investigate the risk management of
a population of reinforced concrete (RC) buildings. It focuses on the case of school buildings, which are taken as a reference; however, it could be applicable to other cases (e.g., hospitals).

In the last decade, risk studies have been a fascinating subject for researchers, particularly in seismic territorial zones. For example, in a risk-based decision-making study, a risk-quantiﬁcation approach was investigated for a group of wood-frame houses in Canada, and potential drawbacks in using simple risk metrics were discussed [4]. In a regional probabilistic risk assessment, the average annual loss was defined as the main parameter for the prioritization criteria for the case of school buildings, and the cumulative average annual loss curves were proposed for the deﬁnition of regional plans [5]. Concerning community resilience planning, a methodology was presented for building a portfolio analysis, which associated the performance of individual buildings with the overall performance of a building portfolio by introducing the concept of a building portfolio fragility function [6]. In another study, a multilayered methodology to assess the vulnerability of a large number of Italian school buildings was implemented [7]. First, the peak ground acceleration (PGA) deﬁcit by means of the difference between the current PGA and design PGA was calculated. Second, the vulnerability index and risk rating based on hazard and building age were assessed. Then, using the assumed failure mechanism and equivalent linearization of the structural response, a simpliﬁed mechanics-based structural methodology was employed in order to calculate the capacity ratio. The idea of a PGA deﬁcit was applied in another similar study [8] to reduce the number of buildings at each phase so that only the most risky buildings are considered in the detailed analysis. A unique seismic risk rate of the whole population of buildings was obtained with integration of the data of the annual probability of an event and collapse. Moreover, the PGA deﬁcit was used in another risk study of public buildings in Basilicata, Italy [9], to prioritize those buildings that need retrofitting according to the time required and annual available fund. Buildings were prioritized by their vulnerability and exposure rates to identify which of them require retrofitting on a limited fund. Cost models were used to estimate the cost of retrofitting options. Each has a different target in terms of capacity-demand ratio. Time-risk curves were used to compare the effectiveness of various strategies based on different cost models. The available fund was assigned to those buildings that could be rehabilitated in one year based on the sum of retrofitting cost. Then, in the following years, the remaining fund was distributed among the rest of the nonretrofitted buildings. Finally, the time (years) required to complete the structural upgrading of all the buildings was obtained by prioritizing the plan and available annual fund.

It has always been challenging to make decisions when many particulars have to be considered. Moreover, decision-making becomes more crucial when a limited amount of resources are available. To allocate a budget fairly, a versatile tool is required to optimize the selection strategy of determining alternatives that satisfy the ﬁnancial constraint. The tool should help in deciding the allocation of the public budget to reduce the seismic risk of regional buildings.

Most of the previous studies on structural safety of large building stocks have been conducted to investigate the vulnerability of buildings; few have investigated the issue of retrofitting risky structures. Hence, owing to the limited studies on this issue, and the fact that a simple but practical tool might help clients to allocate their available budget to reduce the global risk in a given region, we were motivated to conduct this study, which focuses on the definition of a decision-making process and is divided into the following sections:

(1) Assessment of the current overall seismic risk for the building stock
(2) Deﬁnition of a discrete number of alternative interventions for structural improvement of each building
(3) Evaluation of the cost and beneﬁt (reduction of risk) related to each alternative
(4) Deciding on the optimal intervention for each structure to reduce as much as possible the overall risk while staying within the available budget

To implement the abovementioned steps, the adoption of approximate procedures for quick estimation of fragility curves (pre- and postintervention) and installation costs is mandatory, because given the number of structures that needs to be dealt with, it is hardly possible to make these estimations with more rigorous tools (i.e., those generally adopted when a single building is the object of interest). These issues are discussed herein as crucial parts of the proposed framework. The proposed procedure is ﬁnally applied to a simulated case study, which consists of 46 RC school buildings ideally scattered throughout Campania, Italy.

2. Assessment of the Current Overall Seismic Risk for the Building Stock

Seismic risk assessment and loss estimation are essential ﬁrst steps in studying the structural safety of large stocks. Risk is technically deﬁned as the combination of earthquake hazards, seismic vulnerability, and exposure, which, in other words, is the relationship between loss severity and frequency. The challenging part of risk analysis for existing structures is related to incomplete information on architectural and structural details. In such cases, accounting for uncertainties is deﬁnitely mandatory and includes, in addition to information on the earthquake action as for new constructions, information related to the building layout, geometry, structural and nonstructural size, and detailing. This requires the selection of a ﬁnite number of random variables (RVs) that are able to represent, with their realizations, each structure of the set.

In the subsequent sections, by referring to RC frame buildings, the following are presented:
(i) A possible selection of variables to be adopted for sampling of buildings.
(ii) A practical way of dealing with seismic hazards.
(iii) A method of assessing seismic vulnerability of each structure with approximate, quick tools.
(iv) A method of combining vulnerability and hazard information to obtain a frequency of failure for each building. A reliability index is used to quantitatively express how far a given structure is from being “safe” according to the threshold values for structural reliability defined by standards.
(v) An approach to deal with exposure.
(vi) A method of calculating the seismic risk of each building, expressed in terms of economic losses, and then that of the entire stock.

2.1. Sampling of Buildings: Deterministic and Random Variables. A set of variables has to be selected to identify each structure and to account for uncertainties. Owing to the type of information, the variables are divided into two groups: those related to the building geometry or materials and those related to the structural detailing and materials (Table 1). In order to reduce computational effort, the parameters herein referred to as geometry variables are considered as deterministic data that have to be obtained for the building stock by survey or from available technical documents. Information on the second group of data—materials and details—is generally obtained through a combination of destructive and nondestructive tests on structural members. The latter are hardly realizable for each of the several buildings of the set. Therefore, they are assumed to be RVs, whose probability distribution is defined according to the available data in design documents, related references, or expert judgement.

Several possible realizations of each building have to be considered in the process, as a combination of deterministic variables and RVs. Each of those has a given probability to actually reflect the real case that is related to the joint probability of a specific combination of realizations for the involved RVs. According to the relevant literature, the selection of design points could be performed either randomly by sampling methods such as Monte Carlo or based on predetermined points on the distribution function of variable. The full factorial design method is adopted herein for the design of experiments (DoE), and three points are chosen for each RV. The three points correspond to the mean value of that RV and to the mean value plus and minus the standard deviation. It is worth noting that a dense DoE allows capturing of the variability of the RV to reduce the approximate fitted results [10]. On the contrary, it may exponentially increase the analytical effort. The choice herein assumed is considered a suitable trade-off, leading to \(3^{11} = 177,147\) realizations of the same building, where 11 is the total number of RVs proposed to be adopted (Table 1) for RC frame structures.

### Table 1: Deterministic data and random variables assumed for sampling of buildings.

<table>
<thead>
<tr>
<th>Geometry(^1) (deterministic data)</th>
<th>Structural details and materials (random variables)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Column section width, (b_1)</td>
<td>(i) Number of storeys, (n_s)</td>
</tr>
<tr>
<td>(ii) Column section height, (h_c)</td>
<td>(ii) Total height of the building, (h)</td>
</tr>
<tr>
<td>(iii) Stirrup diameter in the column section, (d_{sa})</td>
<td>(iii) Effective width of the frame(^2), (w)</td>
</tr>
<tr>
<td>(iv) Number of frame bays(^3), (n_b)</td>
<td>(iv) Number of frame bays(^3), (n_b)</td>
</tr>
<tr>
<td>(v) Bay length of the frame(^4), (l)</td>
<td>(v) Number of column longitudinal rebars, (n_c)</td>
</tr>
<tr>
<td>(vi) Diameter of column longitudinal rebars, (d_{cl})</td>
<td>(vi) Diameter of column longitudinal rebars, (d_{cl})</td>
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<tr>
<td>(vii) Beam section height, (h_b)</td>
<td>(vii) Beam section height, (h_b)</td>
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<tr>
<td>(viii) Number of beam longitudinal rebars, (n_{bb})</td>
<td>(viii) Number of beam longitudinal rebars, (n_{bb})</td>
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<tr>
<td>(ix) Diameter of beam longitudinal rebars, (d_{bl})</td>
<td>(ix) Diameter of beam longitudinal rebars, (d_{bl})</td>
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<tr>
<td>(x) Concrete compressive strength, (f_{cd})</td>
<td>(x) Concrete compressive strength, (f_{cd})</td>
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<tr>
<td>(xi) Yield strength of the steel rebar, (f_{yd})</td>
<td>(xi) Yield strength of the steel rebar, (f_{yd})</td>
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</tbody>
</table>

\(^1\)Variables \(w\), \(n_s\), and \(l\) have to be determined for each of the two main horizontal orthogonal directions of the structure. \(^2\)The mean distance between consecutive parallel 2D frames in the considered direction. \(^3\)The number of bays for the generic 2D frame in the considered direction. \(^4\)The mean length of the bays for the considered frame.

2.2. Seismic Hazard. In order to assess the seismic risk of a given structure, first, the annual probability of exceeding different intensity levels of an earthquake at the site has to be determined. This type of information is obtained from site hazard analysis. Here, the recent technical document of the National Council of Italian Research [11] is assumed for this and the other steps required to define the structural reliability of buildings. The hazard is defined by the median annual frequency of occurrence of an earthquake (Equation (1)) amplified by a factor to include the epistemic uncertainty. Therefore, the mean annual frequency of occurrence is calculated as Equation (2):

\[
\lambda(s_j) = \frac{1}{f_{R,j}},
\]

\[
s_j = f(T_{R,j}),
\]

\[
\lambda(s_j) = \lambda(s_j) \times \exp \left( \frac{\beta_j}{\beta_{H}} \right),
\]

where \(\lambda(s_j)\) and \(\lambda(s_j)\) are the median and mean annual frequency of occurrence, respectively; \(s_j\) and \(T_{R,j}\), respectively, are the ground motion intensity and the return period of an earthquake with the \(j^{th}\) intensity level \((j = 1, 2, \ldots, n)\), where \(n\) is the number of earthquake scenarios assumed for the reliability analysis; and \(\beta_j\) is the amplification factor that considers the response spectrum of the 16th and 84th percentiles of each seismicity scenario according to the different return periods of the site. In this study, this factor is assumed to be equal to 0.3 (suggested by Masi et al. [9] when no specific information is available). It is also assumed that the number of scenarios \(n\) is 9. The values
of $T_{R,1}$, $T_{R,2}$, \ldots, $T_{R,9}$ are assumed to be, respectively, equal to 30, 50, 72, 101, 140, 201, 475, 975, and 2475 years according to the Italian codes [11, 12].

2.3. Seismic Vulnerability. Among tools useful for seismic vulnerability analysis of existing structures, static nonlinear pushover (PO) analysis is considered one of the more suitable analyses to examine seismic capacity. As this study deals with a large number of buildings, widely used software based on FEM structural models is not easily applicable; thus, it could not provide an effective solution. Hence, a simplified approach is followed to calculate the capacity of each sample building inasmuch as they are highly recommended in such cases. Therefore, among the available methods in the literature, the simplified PO-based earthquake loss assessment method [13] has been chosen for its ease of use and effectiveness demonstrated by the authors through software numerical analysis [14]. The methodology is validated against the results of more sophisticated nonlinear dynamic analyses [15]. As expected for PO analysis, the output results for such an approximate method are the ultimate base shear $V_b$, yield top displacement $\Delta y$, and ultimate top displacement $\Delta u$. The shear capacity is calculated as the minimum value of the shear capacity of the column, the shear capacity corresponding to the flexural capacity of the column, and the shear capacity corresponding to the flexural capacity of the beams supported by columns. The displacement capacities $\Delta y$ and $\Delta u$ are defined as the functions of chord rotation of columns and building height corresponding to the elastic and postelastic displacement, respectively. The displacement at yielding, $\Delta y$, is calculated for the light damage state, whereas $\Delta u$ can be associated with either the life safety (LS) or collapse prevention limit states. In this study, the LS limit state is chosen to represent the ultimate damage state of buildings. According to the method in [16], the chord rotation capacity for the LS limit state is limited to three-fourths of the ultimate rotation capacity. A simplified PO curve has to be determined for each of the several realizations of each building belonging to the set. Here, only the LS limit state is considered as the object of the seismic vulnerability assessment. Actually, in the general case, more performance objectives should be involved. These aspects will be addressed in future improvements on this work. The capacity spectrum method [17] is then suggested as a practical tool to compare the demand and capacity of each structure, evaluate the probability of failure of each building, and finally draw its fragility curve at the LS limit state. With reference to the $i^{th}$ building, the process is as follows:

1. For the $m^{th}$ realization among the $n = 3^{11}$ realizations of that building and with reference to the $j^{th}$ one of the 9 return periods $T_{R,j}$, the final result of the capacity/demand comparison is recorded in a binary mode, i.e., associating an index $I_{SL}$ with 0 when the capacity ($C_j$) is greater than or equal to the demand ($D_j$) and with 1 in the case of failure. (2) The probability of exceeding the LS limit state at the $j^{th}$ intensity level of earthquake motion for this building is calculated by counting the number of system failures ($I_{SL,m} = 1$) weighted by the probability of occurrence of each of the $n$ realizations of RVs.

Therefore, the probability of failure $P_{FSL}^{pre-int}$ of the $i^{th}$ building in the preintervention case when the damage state, DS, due to the $j^{th}$ level of seismic magnitude $s_j$ exceeds the objective DS is evaluated by the following equation, where JPDF$_m$ is the value of the joint probability density function for the $m^{th}$ realization of the building under examination, i.e., the probability that it corresponds to the building in its normal state:

$$P_{FSL}^{pre-int}(ds > DS | s_j) = \sum_{m=1}^{n} \text{JPDF}_m \times (I_{SL})_m$$

Here, JPDF is the joint probability density function of $n$ number of realizations made for a set of RV’s ($\sum \text{JPDF} = 1.0$), and $I_{SL}$ takes into account system failures and is assumed to be equal to 1 when the structure fails and 0 when it does not. That is, for each realization of an RV, the probability of failure in that case would be the product of the sum of damage probability, which indicates the joint probability of all considered RVs and damage indicators whether it occurs ($I_{SL} = 1$) or not ($I_{SL} = 0$).

2.4. Exposure. The third component of risk analysis is known as exposure that can be summarized as the amount of expected social and economic losses due to earthquakes. Globally, exposure may incorporate the quality and quantity of risk measures that could be affected by the seismicity of the site, directly or indirectly. Hence, it quantifies the potential loss to the risk assets, which translate the probability into seismic risk. Depending on the objective limit states of the project, loss could incorporate human life, i.e., the lives of occupants that may be lost when a full collapse occurs. When structures are in a state of almost collapsing (e.g., immediate occupancy or LS limit states according to the study in [18]), in the worst case, only injury is considered, and the expected loss is only considered for the structural and nonstructural components. Exposure in this study is assumed to be related to the overall floor area of the given building, since the latter can be seen as an indirect—certainly simplified—measure of “exposed” value when an economic value for a unit of area is considered.

2.5. Current Seismic Risk. As the loss fraction has been associated with the displacement capacity value at a significant state, when it is considered to be 0.75 of the collapse state, the economic loss at the LS limit state is calculated as 75% of the collapse state. This means that if collapse occurs, the owner has to pay a reimbursement cost equal to rebuilding a new building. However, when a building suffers damage at a lower level, the damage cost has to be assumed as equal to 75% of the reconstruction cost. The physical damage cost corresponding to the LS limit state is calculated by...
where RPLV$_i$ refers to the replacement value of the $i^{th}$ building, which is the amount that should be paid to rebuild a new building, and it is usually defined in per square metres. According to previous regional studies in Italy, the replacement value is assumed to be equal to 1500 €/m$^2$ [19].

After obtaining the hazard data of the site under consideration, vulnerability function of damage distribution, and estimated loss due to probable damages to the buildings, one should calculate the risk through a loss estimation process. To determine how much the reimbursement of loss would be, the seismic risk for the $i^{th}$ building at a given objective damage limit state would be calculated by

$$SR^i_{\text{pre-int}} = E[L_i | DS = LS] \cdot \sum_{j=1}^{9} \Delta \lambda^j(s_j) \cdot \text{PF}^i_{\text{pre-int}}(s_j),$$

(5)

where $\Delta \lambda^j(s_j)$ is the mean annual frequency of earthquake occurrence accounting for the $j^{th}$ number of $s$ ground motion intensity for a site where the $i^{th}$ building is located, $\text{PF}^i_{\text{pre-int}}(s_j)$ is the probability of exceeding the LS limit state when the $i^{th}$ building is subjected to the $s$-times motion intensity, and $E[L_i | DS = LS]$ is the expected loss that the $i^{th}$ building suffers after exceeding the LS limit state.

Then, the overall seismic risk is obtained as

$$SR_{\text{pre-int}} = \sum_{i=1}^{n} SR^i_{\text{pre-int}},$$

(6)

The idea of risk assessment was obtained from the probabilistic framework conducted by the PEER Centre to estimate damage and monetary losses incurred during earthquakes [20–22].

### 2.6. Annual Frequency of Failure

The annual frequency of exceeding a given damage limit state induced by earthquakes with different return periods is the convolution of the fragility function at each ground motion intensity and the counterpart annual frequency of earthquake occurrence.

With the hazard data and fragility for each building, the mean annual frequency of exceeding a given damage limit state could be calculated. Referring to Section 2.6 in [11], the mean annual frequency of exceeding the damage limit state is calculated for 9 points representing different return periods of the earthquake scenario as follows:

$$\lambda^i_{\text{pre-int}} = \int_0^{\infty} \text{PF}^i_{\text{pre-int}}(s_j) \cdot \left| \frac{d \Delta \lambda^j(s_j)}{ds} \right| \cdot ds$$

$$\equiv \sum_{j=1}^{n=9} \text{PF}^i_{\text{pre-int}}(s_j) \cdot |\Delta \lambda^j(s_j)|,$$

(7)

where $\text{PF}^i_{\text{pre-int}}(s_j)$ is the probability of exceeding the LS limit state conditioned to the ground motion intensity $s$ where the $i^{th}$ building is located and $\lambda$ is the mean annual frequency of exceeding the $j^{th}$ intensity level of ground motion intensity of the site under consideration. To attain a consistent evaluation of the annual frequency of exceeding the damage state, the seismic hazard should be defined as the mean annual frequency of being at the $s$ intensity level. Hence, the gradient of the hazard curve is used to show that the annual frequency of being at a given $s$ intensity level is the difference between the frequency of the $s$ and $s + 1$ motion intensities ($\Delta \lambda$).

### 2.7. Vulnerability Index

To decide how to intervene in order to upgrade the building system structurally, it is essential to first examine whether a building is vulnerable or not. To this end, the mean annual frequency of exceeding the damage limit is compared with the allowable reliability level [11] defined as the vulnerability index:

$$\chi^j_{\text{pre-int}} = \frac{\lambda^i_{\text{pre-int}}}{\lambda^j_{\text{pre-int}}},$$

(8)

where $\chi^j_{\text{pre-int}}$ is the vulnerability index of the $i^{th}$ building, $\lambda^i_{\text{pre-int}}$ is the maximum allowable frequency of failure, and $\lambda^j_{\text{pre-int}}$ is the mean annual frequency of the $i^{th}$ building, which is calculated using Equation (7). The reference document [11] suggests different values for the maximum allowable frequency of failure according to the classes of use and limit states. Owing to the fact that buildings are categorized as Class III and analyzed with respect to the LS limit state, the mean annual frequency of 0.0032 is the maximum allowable tolerable risk for school buildings in this study.

### 2.8. Decision Alternatives

In general, when dealing with mitigation of seismic risk for a group of several buildings, the first decision that has to be made with reference to the single structure is whether the intervention of upgrade should be undertaken or not. If the answer is yes, the question of how to approach the intervention needs to be answered. In this study, decision measures are made for all the possible conditions according to the value of the vulnerability index, which is obtained by the vulnerability assessment of all the buildings. Actually, these decision alternatives are made for those buildings that are vulnerable ($\chi < 1$). In other words, those buildings with $\chi > 1$ do not require any intervention, as they satisfy the criterion of maximum reliability [11].

However, in this study, the buildings with indices $\chi > 0.8$ are determined to be not vulnerable. This choice has been made owing to some reasons. To design a new building, uncertainties and reduction factors are used to determine the building’s capacity. Buildings in reality have a high strength capability; therefore, only buildings with a vulnerability index close to 1, rather than those with the index less than 1, are vulnerable to unwanted damages (the ideal case is when the vulnerability index is less than unity, but this value is not practical). This rule allows the decision-making process to focus more on those buildings with a higher rate of vulnerability; i.e., when for a large number of buildings limited financial resources are available, it is more reasonable to intervene in those buildings that are more vulnerable and which endanger more the occupants’ lives. In this type of
decision approach, more risky buildings are prioritized, and from a decision-making point of view, any remaining budget will be applied to buildings with indices $0.8 \leq \chi < 1$.

When a building or a group of buildings is determined as risky, its performance has to be enhanced. However, there are several ways to retrofit a building and reduce risk. Depending on the rate of vulnerability of a building, it can be either demolished or retrofitted if necessary. For buildings with vulnerability indices $\chi < 0.8$, four possible decision solutions have been assumed for the risk management program:

(i) Demolition and reconstruction  
(ii) No intervention  
(iii) Partial retrofit  
(iv) Full retrofit

Buildings might be demolished or rebuilt if they are considered to be extremely risky. In such a case, it is supposed that a new building would be designed and constructed according to the new building code so that the minimum requirements of the seismic design criteria are satisfied.

By definition, “no intervention” means that a building is left as it is; no retrofitting cost is required. When dealing with a large number of buildings and when the retrofitting budget is less than that required to upgrade all of them, it is inevitable to leave some of those without making any intervention. This choice has been provided to make the sharing of a limited budget for all $N$ buildings possible; otherwise, it would be difficult to decide over a large number of buildings.

Any possible retrofitting option that makes the capacity of a building at least equal to the demand would satisfy the condition of “full retrofitting.” The goal of full retrofitting is to improve the structural performance so that the maximum allowable frequency of failure is at least equal to the annual exceedance of the damage state ($\chi = 1.0$).

Alternatively, a lower-level target design could be accepted so that less amount of the retrofitting budget goes to any building in the portfolio. “Partial retrofitting” is chosen for those buildings with a loss ratio less than 0.7 upgraded to the level that their target capacity is equal to 70% of the demand. The goal of partial retrofitting is to improve the structural performance so that the maximum allowable frequency of failure is at least equal to 70% of the annual exceedance of the damage state ($\chi = 0.7$).

These strategies have been chosen at any of the stage of decision zones to specify how many decisions could be made according to the vulnerability index of any building in the portfolio:

- **Case 1** ($\chi_i \leq 0.2$): (1) Demolition and reconstruction  
- **Case 2** ($0.2 \leq \chi_i \leq 0.7$): (1) No intervention, (2) partial retrofitting, and (3) full retrofitting  
- **Case 3** ($0.7 \leq \chi_i < 0.8$): (1) No intervention and (2) full retrofitting  
- **Case 4** ($\chi_i \geq 0.8$): (1) No intervention

The uncertainty in the risk evaluation could compensate for a small lack of capacity of the system. In this regard, buildings with a $\lambda_{SL}/\lambda_i$ ratio that is less than 0.2 are marked for demolition, and those with a ratio greater than 0.8 do not need any structural intervention. Therefore, the budget would be allocated to those buildings with risk levels between 0.2 and 0.8.

### 3. Cost and Benefit of Investment

Benefit is defined as the profit gained because of the amount of money spent to reduce the seismic risk. In other words, the benefit of an intervention is mainly defined as the decrease in the amount of expected loss once additional structural elements are added to strengthen a structure against the design level of seismic motion. The total cost and total benefit gained from the total possible intervention strategies are given as $3^N \times 2^{N'}$, where $N'$ is the total number of buildings with a risk ratio between 0.2 and 0.7 and $N''$ is the number of buildings with a risk ratio between 0.7 and 0.8.

#### 3.1. Evaluation of Costs.
Among the four decision options, when “no intervention” is the case, evidently, no cost has to be paid, while for the case of “demolition/reconstruction,” the cost is equal to that required to rebuild a new building with the same area of construction. For the two other mitigation options (i.e., partial and full retrofitting), the cost is estimated as a part of reconstruction cost. It has been devised that because retrofitting cost is a function of vulnerability, owing to the fact that the more vulnerable a building is, the larger the amount of money that needs to be paid to repair and fortify it against demanded loads, the cost could be estimated by a bilinear function of the vulnerability rate. To estimate the cost of retrofitting, two cost models are adopted (Figure 1) so that the objective cost (assumed to be a partial cost of the replacement value of building) would be obtained as a function of the vulnerability index.

It should be mentioned that a linear function, which is considered as an approximation, relates the rate of vulnerability to what should be paid in repairing the structural damages. Nevertheless, the experience acquired [9] in executing the strengthening program on public buildings after the 2009 L’Aquila earthquake validated the linear trend assumed to estimate the cost of retrofitting.

In this study, buildings are classified by whether they were constructed before or after 1972, which was the year that the seismic design philosophy of designed buildings was changed [23]. Consequently, it is possible that recent buildings with updated structural codes will have enough structural reliability to resist earthquake loads. Actually, this reference year has been adopted from the study in [9]. It must be noted that the adopted function is the initial step in estimating the cost of retrofitting when no resilient references exist, or no studies have been conducted for large numbers of buildings. Indeed, comprehensive data of different typologies of retrofitting projects according to their rates of vulnerability have to be collected to derive a more reasonable function of cost estimation. As a function of the
year of construction and reliability index of each building, the total cost of retrofitting $N$ buildings is obtained as

$$C_{\text{post-int}} = \sum_{i=1}^{n} C_i^{\text{post-int}},$$

where $C_{\text{post-int}} = 0$, in the case of no intervention; $C_{\text{post-int}} = C_{\text{post-partial}}$, in the case of partial retrofitting; $C_{\text{post-int}} = C_{\text{post-full}}$, in the case of full retrofitting; and $C_{\text{post-int}} = \text{RPLV}$, in the case of demolition/reconstruction.

3.2. Seismic Risk Reduction. The benefit of the intervention is mainly defined as the decrease in the amount of seismic risk once structural elements are added to strengthen existing vulnerable buildings. Referring to the three parts of seismic risk evaluation, when doing retrofitting, the seismic hazard does not change, as the building is not moved to another location with less seismicity. Furthermore, it is assumed that the building occupancy before and after retrofitting remains unchanged, which means that the number of occupants and the total area of buildings do not change. However, the structural rehabilitation (e.g., partial retrofitting or full retrofitting) enhances the performance of the building and reduces the expected loss. The increase in safety level and reliability of the structural system reduces the total seismic risk in terms of both structural/nonstructural damage and risk of occupants’ lives.

To obtain the benefits of retrofitting, one needs to analyze the structural system to measure the response of the upgrades, as they have to comply with the criteria stipulated by the seismic code. The difficulty lies in determining how to analyze an existing building when there are several buildings. In this risk management study, structural analysis is not performed using common sophisticated structural software; rather, a solution which is based on a simplistic idea of global risk estimation is utilized. Owing to their processing times and computational effort, the common analytical provisions would not be practical options for constructing fragility curves for all the buildings. Hence, a simplified method has been proposed by the authors to estimate the post-intervention fragility curves. Clearly, the typical analytical methods can derive the fragility curves more precisely; however, it is believed that this estimation, as it is established in certain design criteria of the adopted seismic code, could provide a reasonable estimate of the fragility curves of buildings when retrofitting strategies are applied to existing structural systems.

3.3. Postintervention Fragility Curves. It is assumed that the two fragility curves of the partial and full retrofitting programs could be made using a suitable scale factor. That is, at each of the nine points of the fragility function, a coefficient relating to the building characteristics and intensity level of the earthquake marks the new points of fragility after the intervention.

To determine the scale factor required to draw the new fragility curve of the upgraded building, it is assumed that the scale factor has a linear function of the earthquake return period. This means that, for rarer events, less reduction in the probability of meeting the damage limit state is expected by implementing retrofitting. This assumption is based on the fact that, for more frequent earthquakes, the probability of failure is usually small, and the trend increases significantly for higher intensity levels.

The linear function of the objective scale factor is made up of two points A and B (Figure 2), and two satisfactory limits have to be satisfied. First, for each building, the starting point (point A) is the scale factor of the first return period which, according to the Italian seismic design code, is 30 years. Second, owing to the nature of the cumulative distribution function as it is used to fit the fragility curve, the probability of failure should converge to 100% at a specific return period of the earthquake attained for an objective damage limit state. Hence, a given intensity level has to be fixed, and the probability of failure at this intensity level and larger levels is limited to 100%; i.e., at this intensity level,
exceeding the design damage limit state certainly occurs (point B). To fix this point, the return period at which the building suffers the near collapse limit state of performance is adopted. Indeed, in this study, as the significant damage limit state is chosen to retrofit the buildings, such an assumption could be rational.

In other words, by retrofitting the structure, the expected damages are reduced at the objective limit state although higher levels of damages are still probable for the rarer events. Thus, the return period of point B (Figure 2) is calculated for the collapse prevention limit state. According to the study in [12], the return period of an earthquake, which has a 5% probability of exceeding the 75-year lifetime of the building, is 1462 years (class of use is assumed as Class III).

Once the scale factor at the return period of 30 years for the ith building is known from the linear function (Figure 2), other scale factors for the rest of the return periods could be calculated under the condition that the convolution of fragility and hazard functions has not exceeded the value of 0.0032 (the maximum allowable frequency of failure according to the study in [11]) for full retrofitting and 0.0032/0.7 = 0.0045 for partial retrofitting. In this case, 70% of the global strength criteria of the seismic design is acceptable. The nine scale factors are then multiplied by each of the nine fragility points’ preintervention to obtain the fragility point postintervention for both partial and full retrofitting.

The scale factor is then used to determine the new coordinates of the fragility functions. The probability of exceeding the LS limit state when any of the two intervention strategies is adopted for the ith building is then calculated:

$$P_{i}^{\text{postpartial}} = SF_{i}^{\text{partial}} \times PF_{i}^{\text{pre-int}}$$

$$P_{i}^{\text{postfull}} = SF_{i}^{\text{full}} \times PF_{i}^{\text{pre-int}}.$$  \hspace{1cm} (10)

Knowing the probability of exceeding a specific damage state for pre- and postintervention cases, three fragility curves could be generated as shown in Figure 3.

3.4. Postintervention Seismic Risk. Of the three components of seismic risk formulation, only the fragility function changes by adopting any of the determined retrofitting strategies (partial or full). The seismic risks of the ith building when any intervention plan is made are calculated by

$$SR_{i}^{\text{postpartial}} = E\left[L^{i} | DS = LS \right] \cdot \sum_{j=1}^{9} \Delta \lambda_{i} \cdot PF_{i}^{\text{postpartial}}(s_{j})$$

$$= E[L]^{i}_{\text{postpartial}},$$

$$SR_{i}^{\text{postfull}} = E\left[L^{i} | DS = LS \right] \cdot \sum_{j=1}^{9} \Delta \lambda_{i} \cdot PF_{i}^{\text{postfull}}(s_{j})$$

$$= E[L]^{i}_{\text{postfull}}.$$  \hspace{1cm} (11)

where $PF_{i}^{\text{postpartial}}$ and $PF_{i}^{\text{postfull}}$ are the probabilities of exceeding the LS limit state of the ith building when a decision is made to repair the building partially or fully. Then, the total postintervention seismic risk is obtained as

$$SR_{\text{post-int}} = \sum_{j=1}^{n} SR_{i}^{j}_{\text{post-int}},$$  \hspace{1cm} (12)

where $SR_{i}^{j}_{\text{post-int}} = SR_{i}^{j}_{\text{post-int}}$, in the case of no intervention; $SR_{i}^{j}_{\text{post-int}} = SR_{i}^{j}_{\text{postpartial}}$, in the case of partial retrofitting; and $SR_{i}^{j}_{\text{post-int}} = SR_{i}^{j}_{\text{postfull}}$, in the case of full retrofitting and demolition/reconstruction.

4. Decision of the Optimal Intervention

The optimal decision would be the one that minimizes the expected cost while maximizing the benefit yield. The cost-risk relation in the portfolio risk management project has a downward trend, but it may remain unchanged in special cases where the maximum possible benefit of a project is fulfilled and more investment does not change the global objective of the decision solution. By knowing the different possible costs of retrofitting based on determined levels of

Figure 2: Scale factor for the postintervention fragility curve of all the return periods (TR in years).

Figure 3: Pre- and postfragility curves: (a) status quo, (b) partial retrofitting, and (c) full retrofitting.
upgrading and the conjugated reduced risk values due to each level of performance improvement, one could plot the total cost of retrofitting against the quantity of total post-intervention seismic risk. For some of the buildings, more than one possible decision choice should be considered (0.2 < χ < 0.8); thus, various possible combinations of cost and risk could be summed. A summation of each combination of choices provides the decision-makers with a plot of cost-risk, which they can use to trace the investment and associated risk reduction trends to find the best option among all the buildings in the portfolio (Figure 4).

When no intervention is implemented, all the buildings are the status quo. In other words, the total seismic risk of all the buildings in the portfolio is equal to the sum of the seismic risk when no intervention is made in the buildings (i.e., initial total risk). By commencing the investment (i.e., retrofitting), the total seismic risk would be reduced. Clearly, increasing the total cost reduces the total seismic risk.

In this study, the portion of the budget allocated to retrofitting the portfolio of buildings is applied to all N buildings. This is noteworthy because the optimum allocation could be achieved by spending the budget only on those buildings considered to be higher in terms of risk rate. This is the case when the budget is very tight (Case 1). Alternatively, when the available budget is larger than the maximum required cost, then all the buildings would be upgraded fully (Case 3). The budget falls within the minimum and maximum total cost when all N buildings are under consideration; the decision tool should optimize the proper selection (Case 2).

Actually, the decision tool should be able to determine the best possible solution according to the available resources and project constraints. The goal is to find a point that minimizes the objective function and at the same time satisfies the constraints. The optimum solution is one that obtains the maximum total seismic risk reduction after intervention in which the total cost does not exceed the budget:

\[
\min SR_{\text{post-int}} \quad \text{subject to } C_{\text{post-int}} \leq \text{available budget},
\]

where \( SR_{\text{post-int}} \) is the postintervention risk and \( C_{\text{post-int}} \) is the retrofitting cost of the \( i^{\text{th}} \) building.

5. Application of the Procedure to a Simulated Case Study

To validate the feasibility of the presented decision procedure with the aim of managing the risk for a portfolio of RC school buildings in Italy, a numerical study is carried out. In this regard, 46 ideal buildings are chosen; each point in Figure 5 represents a school building in the Campania region of Southern Italy. The decision problem is programmed to manage all the required parts of the analysis mentioned in the prior sections [24].

To prepare this first explorative application of the proposed procedure, some assumptions are made to reduce the complexity, but not the validity, of the process: 2D frames are analyzed instead of 3D frames, the RC moment-resisting frame is considered, masonry infill influence is neglected, and only the LS limit state is considered.

The buildings have the same structural system, while their characteristics (i.e., geometry and structural features) are different. The seismicity of the buildings is unique for each case depending on their location. Each building has a specific geometry (width, length, and height) although its structural parameters (i.e., material strength and structural detailing) are determined by the appropriate probability distributions of the two classes of buildings (pre-1972 and post-1972). In fact, geometry-based variables are used to form the frames of the case studies, while a structural-based variable is used to make a vulnerability analysis of each generated sample, probabilistically. From the possible realization of the RVs (Table 1), to obtain the current seismic risk of each building, 118,098 (310 × 2) analyses are conducted at each of the nine levels of hazard seismicity.

5.1. Building Population Generation. To generate sample buildings, the Latin hypercube sampling method is used to randomly select predefined values from the set of deterministic variables assumed in Table 2.

The statistics of the parameters chosen to specify the geometry of the frames are shown by the histograms in Figure 6. The distribution of variables demonstrates the attractiveness of the random selection, as the distribution of all the data in the assumed interval is almost uniform and all the values within the interval are covered as well.

It is worth mentioning that the values used to study the practicality of the proposed method are assumed by the authors; however, these values are basically collected by conducting a survey of existing buildings. Furthermore, the data in Table 2 (Section 2) are used to make a structural analysis of the 2D frames in the preliminary risk assessment. All the buildings are supposed to have type “B” soil according to the local seismic code classification [10].

5.2. Seismic Hazard. The annual frequency of exceeding any of the intensity levels \( a_j \) is obtained by

\[
\lambda(a_j) = \frac{1}{T_{R,j}}, \quad j = 1, 2, \ldots, 9,
\]

where \( T_{R,j} \) is the return period of an earthquake associated with each \( j^{\text{th}} \) ground motion intensity using the Italian seismic hazard database [25]. According to the reference seismic code [12], the seismic hazard has to be calculated for nine return periods to consider both seismic hazard and demand. The annual frequency (Equation (14)) is the median hazard curve that has to be transferred to the mean curve, which illustrates the mean annual frequency of occurrence of an event. To do so, an amplification factor is introduced [11] to change the median to mean annual frequency of exceeding the intensity of the earthquake scenario (Section 2.2).
As an example, with respect to the analytical data, one building (no. 39) among the 46 studied buildings is chosen to better describe the results. In Figure 7, the mean annual frequency of exceeding the ground motion intensity levels for the site where the selected building is located is shown.

According to the reference guideline of this research [11], which limits the seismic risk of buildings to the minimum reliability of exceeding the damage state, the annual frequency of surpassing the given intensity levels is used to calculate the seismic hazard of the buildings.

5.3. Seismic Vulnerability in the Current State. The vulnerability of a building is assessed by appraising whether the capacity of the building exceeds its demand. To make the structural analysis, the following values of introduced RVs are extracted from relevant studies in the literature, or if a relevant source is not available, they are assumed by expert judgement as discrete distributions.

A building sampling is introduced by combining the structural parameters (Table 3), which is performed by selecting three values of the mean and mean ± standard deviation of each probability distribution of an RV when it is distributed continuously. For discrete distributions, the predefined values have been determined as given above. In addition, other assumptions have to be made in frame modelling, such that the interstorey height and bay length are uniform, column/beam cross sections are equal in size at different storeys, and storey masses are equal for all floors. The values of the required design parameters, such as concrete and steel rebar strain, gravitational loads, and elastic modulus of materials, are taken from typical values of structural design.

5.3.1. Evaluation of Probability of Failure. Adopting the simplified method (Section 2.3 [13]), the capacity of each building is obtained as the displacement capacity at the LS limit state. Eventually, the displacement capacities of the buildings are obtained by determining which of the three failure conditions is predominant in each deterministic analysis of sample frames.

In this study, the seismic demand is determined by comparing the inelastic response spectrum (demand curves [12]) with the inelastic behaviour of the equivalent single-degree-of-freedom (SDOF) system of realized frames (capacity curves). The elastic response spectra for the site where buildings are located are transformed into inelastic spectra. The transformation is performed based on the application of the capacity spectrum method [17].
The vectors of capacity and demand are obtained, and the probability of failure is obtained by comparing these vectors, stochastically. The capacity spectrum method is used to determine whether the capacity surpasses the demand by superimposing the capacity curves onto the inelastic response spectra.

To calculate the probability of failure, the demand displacement values are compared with the associated capacity displacement values (Figure 8). The probability of failure at each intensity of ground motion is calculated by the multiplication of failure counter \((I_{SL})\) with the associated joint probability density function of the RVs used to analyze the system at each realization. Then, for each building, the nine values of probability of exceeding the LS limit state with regard to the seismicity of the zone associated with the nine return periods of earthquake are obtained. Figure 9 illustrates the fragility curve of the nominated building fitted by the normal cumulative distribution function.

The vectors of capacity and demand are obtained, and the probability of failure is obtained by comparing these vectors, stochastically. The capacity spectrum method is used to determine whether the capacity surpasses the demand by superimposing the capacity curves onto the inelastic response spectra.

To calculate the probability of failure, the demand displacement values are compared with the associated capacity displacement values (Figure 8). The probability of failure at each intensity of ground motion is calculated by the multiplication of failure counter \((I_{SL})\) with the associated joint probability density function of the RVs used to analyze the system at each realization. Then, it is summed over \(n\) number of realizations (Equation (3)). The failure counter occurs when the demand displacement exceeds the capacity displacement (Section 2.3). For example, the probability of exceeding the significant damage state for the selected building at a given intensity level \((a_g = 0.063 g; T_R = 101 \text{ years})\) of an earthquake is

\[
P_{f,\text{pre-int}} \left( ds > DS = LS \big| s = 0.063 \right) = \sum_{n=1}^{118098} JPDF \times (I_{SL})_m = 0.449,\]

where \(I_{SL}\) is the numerator of zero or one multiplied by the JPDF of all realizations of the determined values of all the RVs.

5.4. Current Seismic Risk. For all 46 buildings, the probability of exceeding the LS limit state for all nine earthquake return periods was obtained. The seismicity data of each building were combined with the calculated vulnerability to obtain the mean annual frequency of exceeding the determined damage limit state. Eventually, seismic risk was quantified by the expected annual monetary loss. The calculated mean annual frequencies were compared with the maximum allowable frequency of exceeding the damage limit [11] to obtain the vulnerability index. As an example, the seismic risk of the nominated building (no. 39) is presented as follows:

**Table 2: Assumed intervals for the adopted parameters of building geometry.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum value</th>
<th>Steps</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of storeys ((n_h))</td>
<td>2</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>Interstorey height of the frame ((h))</td>
<td>3.0 m</td>
<td>0.5 m</td>
<td>4.5 m</td>
</tr>
<tr>
<td>Effective width of the frame ((w))</td>
<td>4.0 m</td>
<td>2.0 m</td>
<td>10 m</td>
</tr>
<tr>
<td>Number of bays in length of the frame ((n_l))</td>
<td>2</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Bay length of the frame ((l))</td>
<td>2.0 m</td>
<td>0.5 m</td>
<td>6.0 m</td>
</tr>
<tr>
<td>Year of construction</td>
<td>1920</td>
<td>1</td>
<td>2013</td>
</tr>
</tbody>
</table>

![Figure 5: Distribution of 46 building samples across the Campania region.](image-url)
5.5. Seismic Risk Mitigation. The first step in decision-making is to check the reliability indices of the buildings. The indices are shown by the histograms plotted in Figure 10, which shows the vulnerability distribution among all buildings. This issue would be important as the decision is being made for part of the decision axis, which is defined on the abscissa of this plot.

In Figure 10, some of those buildings that are safe, as their vulnerability index results in a very large value, are discarded. In this figure, the black lines represent the boundary limit of the decision zones. Once the current
Seismic risk is known for the building stock, the best way to reduce the risk of those exposed buildings should be determined. The decision-making for the available budget to reduce the risk of those exposed buildings should be made on the scale of two factors: cost and seismic risk.

The cost of retrofitting is estimated by linear functions that relate the reliability index to the cost as part of the reconstruction cost of a building (Section 3.1). The reliability indices obtained are used to approximate the required cost for partial and full retrofitting based on whether the building was constructed before or after 1972. It should be mentioned that, for some cases, more than one cost would be required according to different decision alternatives (0.2 < χ < 0.8).

Referring to Section 3.4, the postintervention seismic risk was obtained by recalculating the probability of exceeding the damage state when partial and full retrofitting activities are implemented. To find the reduced probability of damage, scale factors were obtained for all return periods (Section 3.3). In Figure 11, the fragility curves of pre- and postintervention of building no. 39 superimposed by the hazard curve are illustrated. The fragility curves of those buildings that could be partially retrofitted (0.2 ≤ χ ≤ 0.7) or fully retrofitted (0.2 ≤ χ ≤ 0.8) are shown in Figures 12 and 13.

The decision solution is introduced by finding the maximum possible risk reduction by any of the introduced strategies, while the optimum choice has been bound to the

---

**Table 3: Considered random variables in structural analysis.**

<table>
<thead>
<tr>
<th>Uniform discrete distribution (pre-1972)</th>
<th>Uniform discrete distribution (post-1972)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dₜₑ (mm) = 6, 8</td>
<td>dₜₑ (mm) = 8, 10</td>
</tr>
<tr>
<td>Sₜₑ (mm) = 250, 300, 350</td>
<td>Sₜₑ (mm) = 150, 200, 250</td>
</tr>
<tr>
<td>nₑ = 3, 4, 6</td>
<td>nₑ = 6, 8, 10</td>
</tr>
<tr>
<td>dₜ (mm) = 12, 16, 20</td>
<td>dₜ (mm) = 14, 18, 22</td>
</tr>
<tr>
<td>nₑₜ = 3, 4, 6</td>
<td>nₑₜ = 4, 6, 8</td>
</tr>
<tr>
<td>dₑₜ (mm) = 12, 16, 20</td>
<td>dₑₜ (mm) = 14, 18, 22</td>
</tr>
</tbody>
</table>

**Continuous distribution (pre-1972)**

<table>
<thead>
<tr>
<th>fₑ (MPa): μ = 369.7 and σ = 29.57 [26]</th>
<th>fₑ (MPa): μ = 550 and σ = 33 [26]</th>
</tr>
</thead>
<tbody>
<tr>
<td>fₑ (MPa): μ = 25 and σ = 7.75 [26]</td>
<td>fₑ (MPa): μ = 36 and σ = 7.2 [26]</td>
</tr>
<tr>
<td>hₑ (mm): μ = 600 and σ = 96 [27]</td>
<td>hₑ (mm): μ = 480 and σ = 67 [27]</td>
</tr>
</tbody>
</table>

bₑ and hₑ (mm) [27]

if nₑ ≤ 3, μ = 450 and σ = 50
if nₑ = 4, μ = 490 and σ = 147
if nₑ = 5, μ = 650 and σ = 195
if nₑ ≥ 6, μ = 700 and σ = 196

---

**Figure 8:** Inelastic capacity curve and inelastic response demand spectrum for all the realizations of the example building (no. 39) at TR = 101 years and \(a_g = 0.063\) g.

**Figure 9:** Fitted fragility curve for exceeding the LS limit state for the example building (no. 39).

**Figure 10:** Histogram of vulnerability indices of buildings with \(χ < 3\).
budget. The cost of intervention and conjugated seismic risk due to each strategy have to be combined over all $N$ buildings of the portfolio. For buildings where $\chi \leq 0.2$ and $\chi \geq 0.8$, just one choice is determined, while a decision is made when $0.2 < \chi < 0.8$. For the 46 buildings chosen for the case study,

(i) Zone 1: $\chi \leq 0.2$ ($n_1 = 1$ building and $t_1 = 1$ mitigation option)

(ii) Zone 2: $0.2 < \chi < 0.7$ ($n_2 = 13$ buildings and $t_2 = 3$ mitigation options)

(iii) Zone 3: $0.7 < \chi < 0.8$ ($n_3 = 1$ building and $t_3 = 2$ mitigation options)

(iv) Zone 4: $\chi \geq 0.8$ ($n_4 = 31$ buildings and $t_4 = 1$ mitigation option)

Hence, the solution that reduces the seismic risk of all $N = 46$ buildings could be obtained by $Q = 1^1 \times 3^{13} \times 2^1 \times 1^{31} = 3,188,646$. Each combination denotes a unique solution that can be used as intervention in the buildings. The summation of each combination of cost and risk provides the total value that could be spent and gained by taking any of the strategies and mapping them out. Each solution corresponds to a given total postintervention seismic risk and a certain total cost (i.e., the sum of the costs of the individual interventions to be made, according to that specific combination, to each building of the set). It is worth noting that the last two quantities are, respectively, the ordinate and the abscissa of any of the points in Figure 4. One point out of all the plotted points represents the best fit of the project. It is the one that yields the minimum seismic risk, at the same time being close as much as possible to the budget line.

The range of the cost-risk curve when an intervention is made on $N$ buildings is plotted with the coordinate of cost-risk when no intervention is made (Figure 14). This illustration could show how much seismic risk is decreased by the budget-oriented intervention with respect to the point when no intervention is made (red point). The yellow point indicates how much should be spent to just demolish and reconstruct those buildings with $\chi < 0.2$ and leave the rest without intervention. The blue point represents the optimum allocation of the budget ($€2.5 \times 10^7$), and the green point shows the maximum project cost. According to these two coordinates, the amount of risk reduction by any decision made over the population can be drawn. The main objective is to allocate the budget to the right place so that
the client will know how much should be spent on each building and what would be the decision strategy for intervention (blue circle in Figure 14).

Hence, the final report of the decision-making would be as indicated in Table 4, where the level of the current seismic risk and the strategies of how to intervene structurally in each building are addressed. In this table, the values of the current seismic risk of the nominated buildings are presented to decide how to intervene in those buildings considered vulnerable (i.e., $\chi_{\text{pre-int}} < 0.8$) according to the design objective with a limited available budget. In other words, for $N$ buildings of a portfolio, the cost ($C_{\text{post-int}}$) associated with each decision alternative ($Q$) has to be paid to determine how to intervene in the building ($D$: demolition/reconstruction, $N$: no intervention, $P$: partial retrofitting, and $F$: full retrofitting) to reduce the seismic risk ($SR_{\text{post-int}}$) as much as possible compared with the risk level of building when no decision has been made ($SR_{\text{pre-int}}$).

The results show that the reduction would be more significant for those buildings with higher rates of vulnerability (buildings no. 3, no. 11, no. 17, no. 18, and no. 37 are clearer). For these buildings, an average of 60% of the initial risk is diminished.

Moreover, it would be interesting to observe the captured seismic risk from the optimum allocations of the assigned budget by the risk values when no intervention is made. The two bar charts, which are similar to the figures above, are plotted to determine how much risk is reduced by the allocations of the budget (Figure 15).

6. Conclusions

Italy is located in a high seismic hazard zone, with several buildings that were designed before the national seismic design code was issued. Even the more recent buildings require further assessment to ensure that they comply with the current criteria for structural design. This issue forces responsible urban policymakers to make urgent prevention decisions and actions regarding preevent risk management programs to strengthen the regional buildings. In some cases, the governmental budget is already approved; therefore, a wise and rational decision should be made to allocate funds to the right parties. In Italy, this type of decision has been made arbitrarily and unsystematically, as no solid principle had been established. In this regard, decision-makers face a challenge: there are a large number of vulnerable buildings and there is no plan describing how the available budget should be allocated. The aforementioned missing point in seismic risk management of a building portfolio inspired this study, which attempts to develop a decision-making procedure to determine how to distribute the budget wisely.

The study is built based on a seismic risk analysis of buildings, and its goal is to determine how to appropriately manage the budget to lessen future risks. The seismic risk is calculated in dimensions of monetary loss by combining the hazard, vulnerability, and exposure data. The algorithm for making decisions on how to reduce the seismic risk of the entire portfolio is established by estimating the cost of retrofitting and calculating the benefit gained by such an investment. The reliability index of buildings is used to determine whether buildings are vulnerable or not. A linear function of the repair cost-vulnerability index is taken from the literature of Italian vulnerability studies to obtain the approximate cost of retrofitting.

Owing to the fact that it would be hard to handle the structural analysis with in-use sophisticated engineering software in this type of large-scale problem, a simplified method is proposed to figure out what the reduction would be in terms of probability of failure according to each of the retrofitting options (i.e., partial and full retrofitting). This study does not claim that the simplified method is the unique one but any other methods that could result in quick observation of objective structural response parameters would be used in the future. Therefore, the postintervention seismic risk would be easily evaluated for all the buildings. Four intervention strategies are assumed for each building: demolition/reconstruction, no intervention, partial retrofitting, and full retrofitting. The best intervention choice is defined as the one that meets all the requirements and limits of the clients. From all the combinations of interventions for all the buildings, the total cost and total seismic risk of the postintervention are calculated. The optimum intervention is selected as the one that is less than the budget limit and yields the minimum seismic risk during postintervention. Finally, one is able to know how to intervene in each building and how much should be spent for each building to obtain the perceived minimum seismic risk.

As has been already mentioned, the authors tried to make a simplified but practical method to share the limited budget among the buildings; however, to build the decision
algorithm, some assumptions and limitations were inevitable, such as the following: analyzed buildings are all consistent with the parameters of regular buildings; interstorey height, floor mass and structural sections, and detailing of beams/columns are all equal at all floors; RVs are assumed to be stochastically independent; and structural analysis is performed for a 2D frame, which represents a 3D building. Rather than adopting a simplified method, different methods could be attempted to conduct the structural analysis. The development of new simplified methods that include more structural details will provide 3D analyses of buildings. The adopted simplified PO analysis uses the equivalent SDOF system rather than the multiple-degree-of-freedom system. According to the objective buildings, just one damage limit state has been studied; however, more damage limit states could be investigated. The effect of masonry infills is neglected in evaluation of the lateral capacity of the frame. The cost of retrofitting is calculated using a linear function of the cost-risk rate presented in a previous similar study.

### Table 4: Optimum budget allocations required to make the best possible structural intervention decision.

<table>
<thead>
<tr>
<th>Building</th>
<th>λ</th>
<th>$\chi_{\text{pre-int}}$</th>
<th>Decision zone</th>
<th>$\text{SR}_{\text{pre-int}}$ (k€)</th>
<th>$C_{\text{Post-int}} = 1648005$ (k€)</th>
<th>$\text{SR}_{\text{Post-int}}$ (k€)</th>
<th>$C_{\text{Post-int}} = 1648005$ (k€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00092</td>
<td>&gt;3</td>
<td>4</td>
<td>2.3</td>
<td>N</td>
<td>0</td>
<td>2.2</td>
</tr>
<tr>
<td>2</td>
<td>0.00189</td>
<td>1.689</td>
<td>4</td>
<td>2.5</td>
<td>N</td>
<td>0</td>
<td>2.5</td>
</tr>
<tr>
<td>3</td>
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| Total    | 356.1 |                      | 24,959.5     | 207.0                        |

"D": demolition and reconstruction; "N": no intervention; "P": partial retrofitting; "F" full retrofitting.
A comprehensive cost analysis could provide a national resource for similar studies. Certainly, the above deficiencies will be addressed in the future work. All in all, this manuscript has to be intended as the first preliminary development of a framework that needs further improvement before leading to significant and applicable results.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The research activity has been supported by the University of Naples “Parthenope” with a 2017 grant within the call “Support for Individual Research for the 2015–17 Period” issued by Rectoral Decree No. 793/2017. The above-mentioned support is gratefully acknowledged. Finally, the authors sincerely thank Professor Iunio Iervolino, University of Naples Federico II (Italy), for his expert support and valuable suggestions.

References


