Research Article

Using the Schwarz Alternating Method to Identify Critical Water-Resistant Thickness between Tunnel and Concealed Cavity

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This paper aims to estimate the stability of the water-resistant strata between the tunnel and the small-medium-sized concealed cavity filled with high-pressurized water or other fillings at optional position around tunnel through solving the double-hole problem. The analytical method to identify the critical water-resistant thickness is proposed based on the Schwarz alternating method and Griffith strength criterion, and the program to calculate the critical thickness was prepared according to this method using mathematical software. Parametric study of the critical thickness indicates that the critical water-resistant thickness will increase with the buried depth of the tunnel and cavity size; the lateral pressure coefficient has more complicated influence on the critical thickness, which is affected by cavity position; when the cavity is filled with sand or gravel, the critical water-resistant thickness will decrease with the increase of the filling pressure; and when the cavity is filled with the high-pressurized water, the critical thickness will decrease as the water pressure initially and increase afterwards. The analytical result of the critical thickness is consistent with that obtained by numerical simulation using the user-defined program based on FLAC^{3D}, demonstrating the rationality and feasibility of the proposed method in this study.

1. Introduction

Construction activities of a tunnel in the karst area require engineering measures to prevent water inrush due to exposed cavities with rich water and high pressure. Karst cavities that are not exposed directly in the tunnel excavation, but that are nonetheless impacted by tunnel construction, are referred to as concealed cavities [1]. If the water-resistant thickness between concealed cavity filled with pressurized water and tunnel is not enough, the water-resistant strata suddenly collapses under construction disturbance, and water inrush occurs without any advanced prevention measures. The process of karst water inrush is sudden, instantaneous, and difficult to predict and causes economical loss and serious casualties [2–4]. In km 5 + 608

of Koohrang tunnel with a 23.409 km length located at the center of the High Zagros Mountains in Iran, a huge amount of water rushed into the tunnel. The water discharge began to exceed 1200 l/s after a few hours. Water raised as much as 2.5 m and flooded about 18 km of the tunnel, causing the equipment and machinery in the tunnel to be seriously deformed and/or damaged [5]. In the Maluqing tunnel of Yichang-Wanzhou Railway in China, water inrush and mud gushing occurred in the DK255+978 on 21 January 2006, with the value of water inrush of approximately $7.2 \times 10^5 \text{ m}^3$ and that of mud of about $7.0 \times 10^4 \text{ m}^3$. The water inrush flooded the 3152 m parallel heading and 2508 m main tunnel in the vicinity of exit and caused the damages of a large number of equipment and machinery with economic losses over RMB 10 million [6].

The study on the stability of water-resistant strata and its thickness is of great significance for preventing water inrush in the karst tunnel. Researchers have done many useful works about this serious problem on karst tunnel construction. Zhao Mingjie et al. studied the effects of karst cavities above the tunnel on the displacement and stress of surrounding rock using numerical simulation [7]. Song Zhanping deduced a formula of minimal safe thickness of the top and bottom water-resistant strata and analyzed the instability mechanisms of water-resistant strata during tunnel construction [8]. Tang Junhua et al. established a mechanical model for water inrush of karst collapse pillar below the floor based on the thick plate theory of elastic mechanics [9]. A model is developed to predict the critical thickness of the water-bearing rock between the karst cavity and the tunnel utilizing the plastic theory and the mechanical balance of intermediate strata [10]. A model was established to calculate the minimum thickness of rock pillar by use of the criteria of shear strength and tensile strength [11]. Hui Zhe et al. discussed the structural mechanics model of two types of faults and water inrush in the karst tunnel and then obtained the corresponding formula of the key strata's thickness [12]. Peng Jun et al. focused on the water-inrush hazards in deep karst tunnels and recommended the suitable formula for evaluating the optimal impermeable rock thickness [13]. Li Liping applied the catastrophe theory and the rock beam structure theory to produce the minimal against-inrush thickness equation for different types of rock pillars [14]. Based on the physical simulation experiment, actual engineering cases, and cusp catastrophe theory, the criteria of water inrush in the karst tunnel affected by the concealed cavity were established and verified with the experimental data [15].

Although plenty of research results have been achieved on stability of the water-resistant strata and its thickness in the past years, many problems still exist to accurately identify the water-resistant thickness. Further research must be completed to address issues related to (1) the reasonableness of water-resistant strata between the tunnel and small-medium-sized concealed cavity being simplified into the elastic beam and plate model; (2) the influence of fillings in the karst cavity on the water-resistant strata stability; and (3) examining the stability of strata between the concealed cavity at irregular positions (except directly above, positively lateral to, and directly below the tunnel) and the tunnel. In response to the above issues, the analytical method for analyzing stability of the water-resistant strata in this study was proposed according to the Schwarz alternating method and Griffith strength criteria. Then, the parametric study of the critical water-resistant thickness is conducted based on this method. Finally, considering the Dazhiping tunnel on Yichang-Wanzhou Railway in China, the reliability and rationality of this method is verified through comparison of the critical thickness calculated analytically, the practical thickness, and that obtained by numerical simulation using FLAC^{3D}.

2. Schwarz Alternating Method

The karst cavity is divided into the "open-field" mechanical model, which ignores the corrosion mechanical effect, and the mechanical model of the tunnel, which takes into account the corrosion effect. Song Zhanping studied the stress distribution rule in surrounding rock mass in two models and observed that the differences of the two models are slighter and may be ignored in engineering application when the thickness of the water-resistant strata is more than 2 m [8].

In this study, the concealed cavity around the tunnel is analyzed via the mechanical model of the tunnel. This model consists of the following properties: the cavity existed before tunnel excavation, the corrosion effect on the formation of the cavity is similar to tunnel excavation, and the stress concentration in the surrounding rock mass around the cavity is obvious. Therefore, the tunnel and the small-medium-sized concealed cavity (smaller than 15 m or two times of tunnel span) around it can be simplified as a double-hole problem in the plane strain state. The instability mechanism of the water-resistant strata is then studied by solving the double-hole problem, and then the critical water-resistant thickness between the tunnel and the concealed cavity is identified under the condition that prereinforcement measures are not adopted.

2.1. Principles of Schwarz Alternating Method. The Schwarz alternating method simplifies the multiconnected regions into a series of simply connected regions when the problem of multiconnected regions is solved [16]. The basic principle can be explained by the common case of the double-hole model in Figure 1. z_1 is the coordinate in the $x_1O_1y_1$ coordinate system, and it is also the coordinate in the global coordinate system. As shown in Figure 1, z_2 is the coordinate in the $x_2O_2y_2$ coordinate system; c is a relative position vector of the two parallel holes; and q , λq , and τ are the uniformly distributed initial stress components and their directions are opposite to the direction regulated in the elastic mechanics. The mutual position relationship of the two holes is arbitrary in order to represent the condition in which the cavity is located at optional position around the tunnel.

The basic procedure to solve the double-hole problem by the Schwarz alternating method is as follows. (1) The stress solution for an infinite region after no. 1 hole excavation under the initial stress condition can be obtained by the Cauchy Integral method. No. 1 hole excavation causes redundant surface traction on the boundary of no. 2 hole, which is not excavated at this time. The redundant surface traction can be obtained by the above stress solution. To balance the redundant surface traction, the reverse surface traction is applied at the edge of no. 2 hole. (2) Another single-hole problem is solved loaded by the reverse surface traction, when no. 2 hole is excavated in the infinite region. This stress solution also creates a nonzero surface traction on the boundary of no. 1 hole. The surface traction is also

FIGURE 1: Mechanical model of double holes at optional position used in the Schwarz alternating method.

redundant and should be obtained again through a new stress solution. (3) The third single-hole problem that should be solved applies corresponding reverse surface traction at the edge of no. 1 hole. The resulting redundant surface traction on the boundary of no. 2 hole can then be calculated. The iterative process is continued until the redundant surface traction on the two holes' boundaries equals zero. The final stress solution is the linear superposition of the stresses for all the single-hole problems during the iterative calculation.

In this paper, one cycle of iteration is defined as the process of solving every single-hole problem in succession. The loading condition of the reverse surface traction induced by the former single hole problem is applied along the hole boundary, except for the first stress solution after no. 1 hole excavation under the initial stress condition.

2.2. Assumptions of the Mechanical Model. The tunnel and small-medium-sized concealed cavity around it can be simplified as an infinite elastic plane containing two holes under the initial uniform in situ stress of q and λq (pressure is positive here). As illustrated in Figure 2, the holes can be at any location. Filling pressure or water pressure refers to the uniform normal pressure along the boundary of the cavity. Assumptions in the iterative solution process based on the Schwarz alternating method are as follows: (1) Statistics show that the cross-section shape of the small-medium-sized concealed cavity is (similar) round or (similar) ellipse in most cases [17, 18]. For this reason, the cross section of the cavity is assumed to be round by utilizing the equivalent circle method. (2) Generally, the complex stress function obtained through two cycles of iteration can satisfy the engineering demand [19]. (3) The difference of the filling pressure or karst water pressure in the cavity is neglected, and the cavity is considered to be a circle with filling or water pressure uniformly.

FIGURE 2: Position relationship of tunnel and concealed karst cavity and their simplified mechanical model.

3. Complex Stress Function and the Critical Water-Resistant Thickness

Assume $\varphi_1(z_1)$ and $\psi_1(z_1)$ are the complex stress functions in $x_1O_1y_1$ (Figure 1). The two complex stress functions of the $x_2O_2y_2$ coordinate system are expressed as $\varphi_2(z_2)$ and $\psi_2(z_2)$. Before and after the coordinate translation, the conversion relationship between the complex stress functions is described as follows [20]:

$$\varphi_2(z_2) = \varphi_1(z_2 + c) \quad (1)$$

$$\psi_2(z_2) = \psi_1(z_2 + c) + \bar{c}\varphi_1'(z_2 + c),$$

$$\varphi_1(z_1) = \varphi_2(z_1 - c) \quad (2)$$

$$\psi_1(z_1) = \psi_2(z_1 - c) - \bar{c}\varphi_2'(z_1 - c).$$

To distinguish the complex stress functions computed in two iterations, the complex functions computed in two iterations in the $x_1O_1y_1$ coordinate system are denoted by $\varphi_{11}(z_1)$, $\psi_{11}(z_1)$, $\varphi_{12}(z_1)$, $\psi_{12}(z_1)$, $\varphi_{13}(z_1)$, and $\psi_{13}(z_1)$. Also, $\varphi_{21}(z_2)$, $\psi_{21}(z_2)$, $\varphi_{22}(z_2)$, $\psi_{22}(z_2)$, $\varphi_{23}(z_2)$, and $\psi_{23}(z_2)$ are the complex stress functions in the $x_2O_2y_2$ coordinate system computed in two iterations. When computing the complex stress functions for hole 1 under the initial stress or the boundary surface force, the second subscript of the complex stress functions is an odd number, but it is an even number for hole 2.

3.1. Stress Function in Surrounding Rock Mass. The Cauchy Integral method is used to solve the mechanical problems of the simply connected region because this paper uses the equivalent method for the cross section of the concealed cavity and tunnel in the Schwarz alternating method. Based on the conversion equations (1) and (2) of the complex stress function in the two coordinate systems, the complex stress functions computed after two iterations are as follows:

$$\varphi_{11}(z_1) = \frac{(1+\lambda)q}{4}z_1 + \frac{(\lambda-1)qr_1^2}{2z_1}$$

$$\psi_{11}(z_1) = \frac{(1-\lambda)q}{2}z_1 + \frac{(\lambda-1)qr_1^4}{2z_1^3} + \frac{pr_1^2}{z_1} - \frac{(1+\lambda)qr_1^2}{2z_1}, \quad (3)$$

$$\begin{aligned}\varphi_{21}(z_2) &= \frac{(1+\lambda)q}{4}(z_2+c) + \frac{(\lambda-1)qr_1^2}{2(z_2+c)} \\ \psi_{21}(z_2) &= \frac{(1-\lambda)q}{2}(z_2+c) + \frac{(\lambda-1)qr_1^4}{2(z_2+c)^3} + \frac{pr_1^2}{z_2+c} - \frac{(1+\lambda)qr_1^2}{2(z_2+c)} + \bar{c} \cdot \frac{(1+\lambda)q}{4} - \bar{c} \cdot \frac{(\lambda-1)qr_1^2}{2(z_2+c)^2},\end{aligned}\quad (4)$$

$$\begin{aligned}\varphi_{22}(z_2) &= \frac{(1-\lambda)qr_1^2}{2\bar{c}^2} \left[\frac{(r_2^2/\bar{c})^3}{(z_2+(r_2^2/\bar{c}))^2} - \frac{3(r_2^2/\bar{c})^2}{z_2+(r_2^2/\bar{c})} \right] + \frac{(\lambda-1)qr_2^2}{2z_2} + \frac{(\lambda+1)qr_1^2}{(2r_2^2/z_2)+2\bar{c}} \\ &\quad + \frac{(1-\lambda)qr_1^4}{2((r_2^2/z_2)+\bar{c})^3} + \frac{(\lambda-1)qcr_1^2}{2((r_2^2/z_2)+\bar{c})^2} + \frac{pr_1^2r_2^2}{\bar{c}(\bar{c}z_2+r_2^2)}\end{aligned}\quad (5)$$

$$\begin{aligned}\psi_{22}(z_2) &= -\frac{(1+\lambda)qr_2^2}{2z_2} + \frac{(1-\lambda)qr_1^2}{(2r_2^2/z_2)+2\bar{c}} + \frac{(\lambda-1)qr_1^2r_2^2}{2z_2c^2} + \frac{pr_1^2r_2^2}{(\bar{c}z_2+r_2^2)^2}, \\ \varphi_{12}(z_1) &= \frac{(1-\lambda)qr_1^2}{2\bar{c}^2} \left[\frac{(r_2^2/\bar{c})^3}{(z_1-c+(r_2^2/\bar{c}))^2} - \frac{3(r_2^2/\bar{c})^2}{z_1-c+(r_2^2/\bar{c})} \right] + \frac{(\lambda-1)qr_2^2}{2(z_1-c)} + \frac{(\lambda+1)qr_1^2}{(2r_2^2/(z_1-c))+2\bar{c}} \\ &\quad + \frac{(1-\lambda)qr_1^4}{2((r_2^2/(z_1-c))+\bar{c})^3} + \frac{(\lambda-1)qcr_1^2}{2((r_2^2/(z_1-c))+\bar{c})^2} + \frac{pr_1^2r_2^2}{\bar{c}(\bar{c}z_1-c\bar{c}+r_2^2)}\end{aligned}\quad (6)$$

$$\begin{aligned}\psi_{12}(z_1) &= -\frac{(1+\lambda)qr_2^2}{2(z_1-c)} + \frac{(1-\lambda)qr_1^2}{(2r_2^2/(z_1-c))+2\bar{c}} + \frac{(\lambda-1)qr_1^2r_2^2}{2(z_1-c)c^2} + \frac{pr_1^2r_2^2}{(\bar{c}z_1-c\bar{c}+r_2^2)^2} - \frac{r_2^2}{z_1-c}\varphi'_{12}(z_1) - \bar{c}\varphi'_{12}(z_1), \\ \psi_{13}(z_1) &= \frac{(1-\lambda)qr_1^2}{2c^2} \left[\frac{3(r_2^2/c)^2}{((r_1^2/z_1)-\bar{c}+(r_2^2/c))^2} - \frac{3((r_2^2/c))^2}{((r_1^2/z_1)-\bar{c}+(r_2^2/c))^2} \right] + \frac{(1-\lambda)qr_2^2}{(2r_1^2/z_1)-2\bar{c}} - \frac{(1+\lambda)qr_1^2}{(2r_2^2/(r_1^2/z_1))-\bar{c}+2c} \\ &\quad \cdot \frac{(1-\lambda)qr_1^4}{2((2r_2^2/(r_1^2/z_1)-\bar{c})+c)^3} + \frac{(1-\lambda)q\bar{c}r_1^2}{2((2r_2^2/(r_1^2/z_1)-\bar{c})+c)^2} - \frac{r_1^2}{z_1}\varphi'_{12}(0) - \frac{r_1^2}{z_1}\varphi'_{13}(z_1),\end{aligned}\quad (7)$$

where q , λ , r_1 , r_2 , and p , respectively, represent vertical in situ stress, lateral pressure coefficient, equivalent radius of the karst cavity, equivalent radius of the tunnel, and filling pressure or karst water pressure in the cavity. As the complex stress is little affected by $\varphi_{13}(z_1)$ and $\psi_{13}(z_1)$, the process to deduce $\varphi_{13}(z_1)$ and $\psi_{13}(z_1)$ ignores the items concerned with water pressure or filling pressure p . For $\varphi_{13}(z_1)$, refer to the appendix.

3.2. The Critical Water-Resistant Thickness. After two iterations are completed, the complex stress function of surrounding rock mass around the tunnel is [20]

$$\begin{aligned}\varphi_1(z_1) &= \varphi_{11}(z_1) + \varphi_{12}(z_1) + \varphi_{13}(z_1) \\ \psi_1(z_1) &= \psi_{11}(z_1) + \psi_{12}(z_1) + \psi_{13}(z_1).\end{aligned}\quad (8)$$

Therefore, the stress of surrounding rock mass can be obtained via the following equations:

$$\begin{aligned}\sigma_r + \sigma_\theta &= 4\text{Re}(\varphi'_1(z_1)) \\ \sigma_\theta - \sigma_r + 2i\tau_{r\theta} &= 2[z_1\varphi''_1(z_1) + \psi'_1(z_1)]e^{2i\theta},\end{aligned}\quad (9)$$

where σ_r , σ_θ , and $\tau_{r\theta}$ represent radial stress, circular stress, and shear stress in surrounding rock mass, respectively.

The relationships of each stress component between the rectangular and polar coordinate system are described as follows [21]:

$$\begin{aligned}\sigma_1 &= \frac{\sigma_r + \sigma_\theta}{2} + \sqrt{\left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2 + \tau_{r\theta}^2} \\ \sigma_3 &= \frac{\sigma_r + \sigma_\theta}{2} - \sqrt{\left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2 + \tau_{r\theta}^2} \\ \sigma_1 - \sigma_3 &= 2\sqrt{\left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2 + \tau_{r\theta}^2}.\end{aligned}\quad (10)$$

Through (8)–(10), we can obtain the stress state of the water-resistant strata by means of the Schwarz alternating method, but for identifying the critical water-resistant thickness to prevent water inrush, we need to judge stability of the water-resistant strata under different c using the specific strength criterion. The Griffith strength theory is

suitable for the brittle karst limestone with significant difference between compressive and tensile strength [22]. The Griffith strength theory is described as follows:

$$\begin{aligned} (\sigma_1 - \sigma_3)^2 - 8\sigma_t(\sigma_1 + \sigma_3) &= 0 & \sigma_1 + 3\sigma_3 > 0 \\ \sigma_3 = -\sigma_t & & \sigma_1 + 3\sigma_3 < 0, \end{aligned} \quad (11)$$

where σ_1 , σ_3 , and σ_t represent the major principal stress, the minor principal stress, and tensile strength, respectively. The stability criterion for the water-resistant strata based on (11) can be expressed as follows [23]:

$$\begin{aligned} k &= \frac{(\sigma_1 - \sigma_3)^2}{8\sigma_t(\sigma_1 + \sigma_3)} & \sigma_1 + 3\sigma_3 > 0 \\ k &= \frac{\sigma_3}{-\sigma_t} & \sigma_1 + 3\sigma_3 < 0. \end{aligned} \quad (12)$$

Using the above equations, we can calculate the safe factor k of each point in the water-resistant strata between the tunnel and concealed cavity. If $k \geq 1$, it indicates that the water-resistant strata will be destroyed at this point. If all points in the strata between the filled cavity and tunnel are under the condition that the k value is no less than 1, the water-resistant strata is under an unstable state [23] and that karst disasters such as water inrush and mud gushing may occur during tunnel construction.

It is complicated to manually identify the critical water-resistant thickness using the above deductive process of solving the stress state in the water-resistant strata. However the complex stress function is explicit after a certain number of iterations, and it is easy to automatically implement the solving process via programming on mathematical software. The specific flow chart to identify the critical water-resistant thickness with mathematical software is shown in Figure 3.

4. Parametric Study

4.1. Influence of Karst Cavity Position. From (3)–(7), the parameter c in the complex stress function indicates the relative position of the karst cavity and tunnel. When the concealed cavity is located directly above or below the tunnel, c is a pure imaginary number. When the concealed cavity is positively lateral to the tunnel, c is a real number. When the cavity is located in a position other than the above three regular positions, c is generally a complex number. Writing c as a form of the module and argument allows for the easy searching of c in the iterations. It is easy to process the cavity at the irregular position around the tunnel because one of the basis of the Schwarz alternating method is a complex variable function. The Schwarz alternating method considering the influence of the filling pressure or water pressure in the cavity solves the question that the water-resistant strata between the tunnel and the small-medium-sized concealed cavity being wholly simplified into the elastic beam and plate model is unreasonable.

The Schwarz alternating method does not consider the influence of gravity in computing the stress state of the

FIGURE 3: Flow chart to identify critical water-resistant thickness.

water-resistant strata. The mechanical model is under a two-dimensional stress state, and the equivalent method is used to process the cavity and tunnel section. Therefore, the model is under the left and right, up and down symmetric condition. This paper discusses only the case in which the cavity is located in the upper-right side of the tunnel. In the computation, we assume that the vertical in situ stress $q = \gamma H$, the average unit weight is 25 kN/m^3 [24], and the equivalent radius r_2 of tunnel cross section remains unchanged. Based on the typical cross-section size of single-track tunnels on the Yu-huai railway and Yi-Wan railway which were constructed in karstic terrain, $r_2 = 4.25 \text{ m}$. When the influence of cavity size is not discussed, r_1 is 3 m ; the tensile strength of water-resistant strata is 1 MPa [24]. The filling pressure or water pressure p is 0.5 MPa . The analysis results are shown in Figures 4–6.

Figures 4–6 show that the critical water-resistant thickness of strata between the tunnel and concealed cavity at optional position will increase as the buried depth of tunnel increases. When the lateral pressure coefficient is 1.2 , the critical water-resistant thickness will increase as the inclination of the connection line between the cavity center and tunnel increases. When the lateral pressure coefficient is less than 1 , the critical water-resistant thickness will decrease as the inclination increases. When the buried depth (buried depth of tunnel is 300 m) and the lateral pressure coefficient ($\lambda = 1.2$) keep unchangeable, the critical water-resistant thickness will increase with the increase of the karst cavity size r_1 .

FIGURE 4: Relationship between the critical water-resistant thickness and buried depth of tunnel.

FIGURE 5: Relationship between the critical water-resistant thickness and lateral pressure coefficient.

FIGURE 6: Relationship between the critical water-resistant thickness and size of the karst cavity.

The lateral pressure coefficient has complex influence on the critical water-resistant thickness. When $\lambda = 1$, the mechanical model is under the axisymmetric state both of the geometry and loading. At this time, the critical water-resistant thickness is identical at different positions. When the lateral pressure coefficient approaches 1, the difference between the critical water-resistant thickness at different positions becomes small. When $\lambda < 1$, the critical water-resistant thickness will reduce with the increase of the lateral pressure coefficient. The differences are significant at different inclinations. The critical water-resistant thickness curve will become flat with the increase of the inclination. When the inclination is relatively large, the critical water-resistant thickness will first increase or first decrease and then increase with the growth of the lateral pressure coefficient. When $\lambda > 1$, the critical water-resistant thickness will increase with the increase of the lateral pressure coefficient. The critical water-resistant thickness is different for different inclinations. The critical water-resistant thickness curve will become flat with the decrease of the inclination. The critical water-resistant thickness will decrease with the increase of the lateral pressure coefficient for the smaller inclination.

4.2. Influence of Water Pressure or Filling Pressure. The concealed cavity around the tunnel is often filled with the high-pressurized water or sand and gravel. Therefore, it is necessary to analyze the influence of the water pressure or filling pressure in the karst cavity on the stability of the water-resistant strata. The effect of water pressure or filling pressure is illustrated by the example of the concealed cavity directly above the tunnel in this paper. The calculated parameters are the same as them in Section 4.1, except for the water pressure or filling pressure. The results are shown in Figure 7.

As seen in Figure 7, the critical water-resistant thickness will first decrease and then increase with the increase of the water pressure or filling pressure in the cavity. This is because the Schwarz alternating method is only an elastic solution for multihole problem based on continuous medium theory and cannot demonstrate the influences of hydraulic fracturing and fracture flow. In the process of mathematical solution, p is equivalent to the support force of tunnel lining. So, as p is comparatively small, the growth of p can reinforce the stability of the water-resistant strata. When the water pressure in the karst cavity is 0.2 MPa, 0.5 MPa, 0.8 MPa, and 2.0 MPa, respectively, the safe factor k to assess the stability of the water-resistant strata is presented in Figure 8. As shown in Figure 8, with the increase of p , the curve of k moves left on the whole, and the range of the unstable state in the water-resistant strata is gradually narrowed. That is to say, the critical thickness of strata to prevent water inrush and mud gushing is getting small. When p is relatively high, the unfavorable effects will occur, specifically the case shown in the second half of the curve in Figure 7. When the cavity is filled with sand and gravel, its pressure will not be higher than the in situ stress, and only the

FIGURE 7: Relationship between the critical water-resistant thickness and p based on the Schwarz alternating method and Griffith strength criterion.

first half of the curve is displayed. When the cavity is filled fully with the high-pressurized water, it is difficult to obtain the trend shown in the second half part of the curve based on the Schwarz alternate method and Griffith strength criteria in Figure 7 because generally the hydraulic pressure in the karst cavity difficultly reaches 15 MPa according to the in situ measured results of the karst tunnel in China [25, 26], even though the water pressure in the cavity may be higher than the in situ stress.

Xi has analyzed the influence of water pressure on the critical water-resistant thickness by UDEC and found that the relationship between the critical water-resistant thickness and water pressure is the same as that in Figure 7 [27], but the water pressure of turning point is much less than the corresponding water pressure in Figure 7. Additionally based on some published references [28–29], the hydraulic fracturing does not need so high water pressure. According to the above analysis, the schematic relationship between the critical water-resistant thickness and water pressure or filling pressure is plotted in Figure 9. It can be observed from this figure that the critical water-resistant thickness will decrease as the water pressure initially and increase afterwards. Though the trend of curve shown in Figures 7 and 9 is similar, the meaning of water pressure corresponding to the turning point is different in two figures. As shown in Figure 7, the water pressure to the turning point is the cutoff point from the favorable influence on the stability of water-resistant strata to the unfavorable factor based on the Schwarz alternating method and Griffith strength criterion, but the water pressure is the critical pressure for the hydraulic fracturing in Figure 9. As seen in Figure 9, the curve is divided into the failure stage by Griffith strength criteria and the hydraulic fracturing stage, when the cavity is filled with water. The water-resistant thickness will be on the decline with the increase of the sand or gravel pressure,

FIGURE 8: Influence of p on k of the water-resistant strata.

FIGURE 9: Schematic relationship between the critical thickness and water pressure or filling pressure in the cavity.

and the whole is the failure stage by Griffith strength criteria in Figure 9.

5. Validation and Application

DK137+540–DK137++800 of the Dazhiping tunnel is situated in the west wing of the Yangchang River anticline. The solution and gash breccia, limestone, and dolomite strata of Jialingjiang formation in the lower Triassic are uncovered. The rock masses are broken, and the grade of rock masses is III based on *Code for Design on Tunnel of Railway of China*. This section is in the horizontal circulation zone of karst water, and karst structure with high-pressurized water is in good development. The results investigated by the electrical conductivity imaging

FIGURE 10: Cavity size and position relationship of tunnel and cavity.

FIGURE 11: Relationship of the principal stress and the water-resistant thickness at the middle point of water-resistant strata.

system (EH-4) indicate that the section DK137+704–DK137+786 belongs to the karst low-resistance abnormal body. Based on the findings of the geophysical prospecting method and drilling probing, there is a karst cavity filled by water with high pressure in this section, which can easily cause water inrush disasters. The longitudinal section (along the tunnel axis) of this cavity is the size of a soccer field, and the longitudinal span is about 13.5–15.8 m. The cavity with the overburden thickness of 110 m is located directly above the tunnel vault. The hydraulic pressure in this cavity is about 0.73–0.89 MPa measured by in situ testing. The cavity size and the position relationship between the karst cavity and tunnel are shown in Figure 10.

5.1. Identification of the Critical Water-Resistant Thickness by Analytical Solution. As can be seen in Figure 10, the transverse section of this double-track tunnel is 11.8 m in height and 13.10 m in width. We can obtain $r_2 = 6.23$ m based on the equivalent circle method. The average width

of cross section of this karst cavity filled with high-pressurized water is 8.45 m and its height is 6.65 m, so $r_1 = 3.78$ m. Based on the position relationship of the cavity and tunnel, c can be obtained at the different water-resistant thickness. If the average unit weight of overlying strata is 26.5 kN/m^3 , the overburden pressure q is 2.92 MPa, then $\lambda = 1.3$, $\sigma_t = 1.0$ MPa, and the average hydraulic pressure p in the karst cavity is 0.81 MPa. Putting the above parameters into the program designed in accordance with the method in this paper, we can compute the stress state of the water-resistant strata. As an example, the relationship between the stress at the middle point of the water-resistant strata and the water-resistant thickness is shown in Figure 11.

From Figure 11, the major principal stress at the middle point will reduce with the increase of the water-resistant thickness. However, the minor principal stress will increase with the increase of the water-resistant thickness. Based on (12), as the water-resistant thickness increases, the principal stress difference ($\sigma_1 - \sigma_3$) will decrease, and stability of this point will enhance. This

FIGURE 12: The critical water-resistant thickness of the engineering example on Yichang-Wanzhou Railway.

changeable law meets the actual condition, which indicates that the method that uses the Schwarz alternating method and Griffith strength criteria is reasonable and feasible for assessing stability of the water-resistant strata between the small-medium-sized concealed cavity and tunnel.

The distance from the tunnel vault to the cavity bottom is about 3.4 m–4 m, and the average distance is 3.7 m in this project. The safe factor k of the water-resistant strata is shown in Figure 12. The maximum k is 1.752 and the minimal k is 1.08 as seen from Figure 12. The k will gradually reduce with the growth of the water-resistant thickness under the above-listed conditions, as shown in Figure 12. When the water-resistant thickness increases to 4.3 m, the minimum k of this strata reduces to 0.992. Based on the above analysis, the critical water-resistant thickness is regarded as 4.3 m under this condition. Because the actual thickness of water-resistant strata for this section is less than 4.0 m, the water-resistant strata between the cavity and tunnel is under unstable condition, and water inrush seriously threatens tunnel construction when the tunnel goes through this unfavorable geological section characterized by high karstic rocks. The lead pipe-shed support and advanced grouting technology are used in tunnel construction for this section.

5.2. Numerical Investigation of the Critical Water-Resistant Thickness. Identification of the failure zone of water-resistant strata by fast Lagrange analysis of continua in three dimensions (FLAC^{3D}) was further developed through the user-defined program in the FISH environment according to the experimental results of mechanical properties and the failure mechanism of karst limestone under natural and saturated states [30]. Also, a method to analyze the stability of water-resistant strata is established based on FLAC^{3D}. The developed method is used to assess the stability of water-resistant strata in the Dazhiping tunnel as mentioned above, and the results are shown in Figure 13. The mechanical parameters used in

FIGURE 13: Failure zone scope in the water-resistant strata obtained by the user-defined program on FLAC^{3D}. (a) The water-resistant thickness of 3.5 m. (b) The water-resistant thickness of 4.0 m. (c) The water-resistant thickness of 4.5 m. (d) The water-resistant thickness of 5.0 m.

this numerical simulation are the same as those in analytical solution in Section 5.1.

As shown in Figure 13, the failure zone scope in the water-resistant strata is obviously reduced with the thickness increase of water-resistant strata. When the water-resistant thickness increases to 4.5 m, the failure zones just transfix and coalesce, and water inrush will immediately occur. The critical water-resistant thickness obtained by the numerical simulation is 4.5 m, which is pretty close to the value identified by the theoretical method. The comparison of the critical water-resistant thickness obtained by the two methods, respectively, shows that using the Schwarz alternating method and Griffith strength criterion to identify the water-resistant thickness between the tunnel and concealed cavity is reasonable and feasible.

6. Discussion

The small-medium-sized concealed karst cavity filled with high-pressurized water is usually distributed around the tunnel. Tunnel excavation often creates stress concentration, and crack initiates and propagates at the location of stress concentration in the water-resistant strata between the tunnel and concealed cavity subjected to karst water pressure. Therefore, accurately calculating the stresses at the edge of the tunnel or cavity is important to assess the stability of the water-resistant strata. At present, some numerical and experimental methods can be used to study the stress distribution in the water-resistant strata and then analyze its safety, but they cannot quickly or correctly obtain the stresses and displacements at any point around the tunnel and cavity. More importantly, one can clearly know the stress and stability of the water-resistant strata in theory by means of an analytical method. Likewise, it is worth noting that the analytical results can be used for the validation and reference of numerical procedures and tests [31]. As far as the analytical model is concerned, the tunnel and the small-medium-sized concealed cavity can be simplified as two holes embedded in an isotropic and homogeneous infinite plate under the initial uniform in situ stress at infinity and water pressure internally and normally acting on the boundary of one hole to represent the cavity. The stress state of an infinite plate containing two circular holes, in particular the region between double holes, has also been studied by many scholars and in lots of analytical methods, such as the bipolar coordinates method used by Ling [32], the conformal mapping method in complex variable techniques used by Haddon [33], and the Schwarz alternating method by Sokolnikoff [34]. As an analytical method, the Schwarz alternating method reduces a problem of doubly connected regions in an elastic infinite plate to a sequence of problems in a simply connected region [34]. In addition, the Schwarz alternating method can be used as an effective tool to solve the problems of two holes having arbitrary shapes and arrangements [20]. Hence, the Schwarz alternating method is used to study the distribution of stress for the water-resistant strata area in an infinite plate containing two holes (one is the tunnel hole and the other is the cavity hole) and then identify the critical water-resistant thickness based on the strength criterion.

The Schwarz alternating method, the earliest known domain decomposition method, was introduced in a seminal paper by Hermann Schwarz in 1870 [35]. It has been proved to be convergent for doubly connected regions in two dimensions by Mikhlin in 1934 [36] and in three dimensions by Soboleff in 1936 [37]. Since the 1960s, Salerno and Ukadgaonker have begun to solve the problem of two circular holes via this method [38, 39]. Zimmerman gives the second order solution for an elastic plate containing two equally sized circular holes by using the Schwarz alternating method [40]. Ukadgaonker and Patil analyzed the stress state of a plate containing two elliptic holes subjected to uniform pressure and tangential

shear stresses on the hole boundaries [41]. Kooi and Verruijt presented an analytical expression for calculating the stress and displacement around two parallel deep tunnels by using the Schwarz alternating method [42]. Zhang and Lu and Zhang et al. conducted a series of studies on two circular or multiple elliptic holes under uniform loads at infinity, using the Muskhelishvili techniques and the Schwarz alternating method [16, 43, 44]. Zhang et al. provided the accurate stress solution for two elliptical holes in the infinite region by using the Schwarz alternating method and Fourier series expansions for the resulting stresses on the tunnel boundary [45]. Chen et al. proposed a simplified analysis solution for stresses and deformation characteristic of tunnel due to the influence of adjacent ellipse karst cave by means of the Schwarz alternating method [46]. An analytical solution for shallow twin tunnels assuming an elastic half space is provided by Fu et al. through this method [47]. Using the Schwarz alternating method, the analytical expressions of component stresses in an unlimited plane with two elliptical caves are obtained by Rao et al. [48]. All the aforementioned works by using the Schwarz alternating method are based on the assumption of elasticity. Wang et al. derived a new analytical solution for the calculation of stresses and displacements using this method for two parallel twin tunnels located at a short distance apart accounting for the effect of rock time-dependent behavior and sequential excavation in visco-elastic rock [49]. To the author's knowledge, only the calculation to find the stress distribution characteristics of adjacent horizontal parallel tunnels in an elastoplastic infinite medium utilizing the complex function theory, the Schwarz alternating method and the D-P yield criterion [50]. In order to solve the contradiction between the existing fact of plastic zone around tunnels after one cycle of iteration in the Schwarz alternating method and the complex function theory of elasticity utilized continuously in successive iteration, the complex stress function, which cannot meet the precision need for two close parallel tunnels in literature [50], is obtained only through one cycle of iteration.

The elastic-plastic solutions for stress and displacement around a single circular or cylindrical hole have received a tremendous amount of scientific attention, and comprehensive research achievements are obtained. The majority of these solutions are based on the linear Mohr-Coulomb (M-C) failure criterion or nonlinear Hoek-Brown (H-B) failure criterion for an elastic-brittle-plastic or elastic-perfectly plastic rock mass [51-56]. The limitation of these solutions achieved by M-C or H-B criterion is due to ignoring the influence of the intermediate stress on stress and displacement distribution around the single circular opening. In fact, the strength of geomaterials (such as soil and rock mass) is often observed to be dependent on the intermediate principal stress, and moreover, this effect varies from case to case and the extent of the effect is related to the material type and the stress state [57, 58]. The closed-form elastoplastic solutions are derived for the stresses, displacement, and the

extent of the plastic zone of an opening in a rock mass by the unified strength theory which can consider the influence of all the three principal stresses [59, 60]. A large-strain elastoplastic analysis is presented for a cylindrical cavity embedded in an infinite medium which employs invariant, nonassociated deformation-type theories for Drucker–Prager (D–P) solids, accounting for arbitrary hardening, with the equivalent stress as the independent variable [61, 62]. These closed-form analytical solutions based on the Drucker–Prager yield criterion and non-associated flow rule have capability to reflect the response of a pressure-sensitive material and can be able to account for the effect of hydrostatic pressure neglected in the solutions based on the unified strength theory. The study for stress distribution of an elastic-plastic medium containing only one opening with simple geometry like circle is relatively easy in the abovementioned literatures. However, when the two openings (especially with non-circular shape) are embedded in rock mass or other medium, the analysis to find the elastoplastic field around holes is more difficult. So far, no exact or highly accurate analytical solutions have been reported in the literatures for an infinite elastoplastic medium containing two circular holes under the most simple load type according to the second paragraph of this section because the presence of the other hole makes the problem mathematically intractable due to elastic iteration process to solve stress distribution due to the incompatibility between the elastic iteration process to solve stress distribution by applying the Schwarz alternating method and response of geomaterials.

Obert and Duvall described brittle of a material such as cast iron and brittle rocks to end by fracture at or after the yield stress [63]. Brittleness is described as a property of the material to shatter with little or no ductility [64]. The ductility of a material is the ability of the material to tolerate a large inelastic deformation with no loss of its load-carrying capacity [65]. In contrast, the brittleness of a material is demonstrated by its decrease in load-carrying capacity as the strain increases with little or no inelastic deformation. Karst limestone rock, in which the tunnel hole and karst cavity hole is embedded, shows the obvious brittleness through analysis of lots of experimental results by Guo et al. and Wang [30, 66]. This is the reason for choosing the Griffith strength criterion suitably applied to the brittle material as the strength criterion to estimate stability of the water-resistant strata and identify the critical water-resistant thickness in this paper, although the Griffith strength criterion is regarded as not meeting the requirements of modern geotechnical analysis well. Most types of rocks may exhibit the different ductile-brittle behaviors, and an attempt will be made in further study to solve the elastoplastic stress field of the elastic-plastic geomaterial-containing tunnel hole and cavity hole by using the Muskhelishvili complex variable function techniques, the Schwarz alternating method with random times of iteration, and D–P criterion. This proposed method is promising to have more accurate results than those of method in literature [50], despite the fact that there exist some deficiencies about this line of thinking.

7. Conclusions

The analytical method to identify the critical water-resistant thickness between the tunnel and cavity is established based on the Schwarz alternate method and Griffith strength criterion. Parametric study of the critical water-resistant thickness is conducted. The feasibility of this method is verified by the numerical simulation through engineering application. The following conclusions can be drawn:

- (1) For the small-medium-sized concealed cavity at optional position around the tunnel, a method is proposed to estimate stability of the water-resistant strata between the tunnel and karst cavity filled with water or other fillings and identify the water-resistant thickness using the Schwarz alternate method and Griffith strength criterion in this study. The program to calculate the critical water-resistant thickness was prepared according to this method on mathematical software.
- (2) The critical water-resistant thickness will increase with the increase of the buried depth of tunnel and cavity size. When the lateral pressure coefficient is 1.2, the critical water-resistant thickness will increase as the inclination of the connection line between the cavity center and tunnel increases. When lateral pressure coefficient is less than 1, the critical water-resistant thickness will reduce with the increase of inclination.
- (3) The lateral pressure coefficient has complex influences on the critical water-resistant thickness. When $\lambda = 1$, the critical water-resistant thickness is identical at different positions. When $\lambda < 1$, the critical thickness will reduce with the increase of the lateral pressure coefficient. When $\lambda > 1$, the critical thickness will increase with the increase of the lateral pressure coefficient. The critical water-resistant thickness will decrease with the increase of the lateral pressure coefficient for the smaller inclination.
- (4) When the cavity is filled with sand and gravel, its pressure will not be higher than the in situ stress, and the critical water-resistant thickness will decrease with the increase of the filling pressure; when the cavity is filled fully with the high-pressurized water, the critical thickness will decrease as the water pressure initially and increase afterwards in face, and the water pressure to the turning point is the critical pressure for the hydraulic fracturing.
- (5) To verify the theoretical method established in this study, the analytical result of the critical water-resistant thickness for the Dazhiping tunnel on Yichang-Wanzhou Railway is compared to that obtained by the numerical simulation. The critical thickness determined by a user-defined program based on FLAC^{3D} is very close to that calculated by the analytical method, therefore demonstrating the rationality and feasibility of the proposed method.

Appendix

$$\begin{aligned}
 \varphi_{13}(z_1) = & \frac{(1+\lambda)qr_2^2}{(2r_1^2/z_1) - \bar{c}} + \frac{(\lambda-1)qr_1^2}{(2r_2^2z_1/(r_1^2 - z_1\bar{c})) + c} + \frac{(1-\lambda)qr_1^2r_2^2}{2\bar{c}^2((r_1^2/z_1) - \bar{c})} + \left(\frac{z_1r_2^2}{r_1^2 - z\bar{c}} + c \right) \\
 & \cdot \left\{ \frac{(1-\lambda)qr_1^2}{2c^2} \left[\frac{3((r_2^2/c))^2}{((r_1^2/z_1) - \bar{c} + (r_2^2/c))^2} - \frac{2((r_2^2/c))^3}{((r_1^2/z_1) - \bar{c} + (r_2^2/c))^3} \right] \right. \\
 & + \frac{(1-\lambda)qr_2^2}{2((r_1^2/z_1) - \bar{c})^2} + \frac{(1+\lambda)qr_1^2r_2^2}{2((z_1r_2^2/(r_1^2 - z\bar{c})) + c)^2((r_1^2/z_1) - \bar{c})^2} \\
 & + \left. \frac{3(1-\lambda)qr_1^4r_2^2}{2((z_1r_2^2/(r_1^2 - z\bar{c})) + c)^4((r_1^2/z_1) - \bar{c})^2} + \frac{(\lambda-1)q\bar{c}r_1^2r_2^2}{((z_1r_2^2/(r_1^2 - z\bar{c})) + c)^3((r_1^2/z_1) - \bar{c})^2} \right\} \frac{(3(\lambda-1)qr_1^4r_2^4)/(2c^2(c\bar{c}-r_2^2)^2)}{(A.1)} \\
 & \cdot \left[\frac{((r_1^2c/(c\bar{c}-r_2^2)))^3}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^2} + \frac{3((r_1^2c/(c\bar{c}-r_2^2)))^2}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))} \right] + \frac{(\lambda-1)qr_1^2r_2^6}{c^2(c\bar{c}-r_2^2)^3} \\
 & \cdot \left[\frac{((r_1^2c/(c\bar{c}-r_2^2)))^4}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^3} + \frac{4((r_1^2c/(c\bar{c}-r_2^2)))^3}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^2} + \frac{6((r_1^2c/(c\bar{c}-r_2^2)))}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))} \right] \\
 & + \frac{(\lambda-1)qr_2^2}{2\bar{c}^2} \left[\frac{r_1^6}{\bar{c}(\bar{c}z_1 - r_1^2)^2} + \frac{3r_1^4}{\bar{c}^2z_1 - \bar{c}r_1^2} \right] - \frac{(1+\lambda)qr_1^2r_2^2}{2(c\bar{c}-r_2^2)^2} \left[\frac{((r_1^2c/(c\bar{c}-r_2^2)))^3}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^2} + \frac{3((r_1^2c/(c\bar{c}-r_2^2)))^2}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))} \right] \\
 & + \frac{3(1-\lambda)qr_1^4r_2^2}{2(c\bar{c}-r_2^2)^4} \left[\bar{c}^2z_1 - \frac{z_1^3(r_1^2 - \bar{c}z_1)^2}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^4} \right] + \frac{(1-\lambda)q\bar{c}r_1^2r_2^2}{(c\bar{c}-r_2^2)^3} \left[\frac{\bar{c}z_1^4 - r_1^2z_1^3}{(z_1 - (r_1^2c/(c\bar{c}-r_2^2)))^3} - \bar{c}z_1 \right].
 \end{aligned}$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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