Research Article

Strength of Flanged and Plain Cruciform Members

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There are two different types of cruciform members used in practice. Flanged cruciform sections are typically fabricated from two hot-rolled WT sections welded to the web of a standard hot-rolled I section, whereas plain cruciform sections are typically fabricated from two symmetric rectangular plates welded in the form of a cross. Cruciform members that are subjected to combined compression and bending are typically limited by torsional buckling unlike conventional compression members (such as W-shapes) that are typically limited by flexural (Euler) buckling about their local weak axis of bending. Detailed guidance on the analysis of flanged and plain cruciform members is scarce in literature. Hence, this paper presents numerical studies on the strength capacities of both flanged and plain cruciform members that are subjected to combined compression and bending effects. Analysis results show the ability of flanged and plain cruciform to resist lateral-torsional buckling over longer unbraced lengths, allowing development of efficient plastic resistance.

1. Introduction

Flanged cruciform members are used in high-load applications as compression members. Flanged cruciform sections are typically fabricated from two hot-rolled WT sections welded to the web of a standard hot-rolled I section as shown in Figure 1. Typical applications include column and lateral bracing members for orthogonal moment frames with large bending moments about both local axes [1] and for high-load applications including housing of mechanical and power equipment [2].

On the contrary, plain cruciform sections, as shown in Figure 2, are typically fabricated from two symmetric rectangular plates welded in the form of a cross. Plain cruciform members are used in lower load applications, such as for transmission line tower legs using the built-up construction with steel angles.

While conventional compression members including W-shapes are typically limited by flexural (Euler) buckling about the local weak axis of bending, cruciform members in combined compression and bending are typically limited by torsional buckling. The American Institute for Steel

Construction (AISC) code [3] does not explicitly address cruciform members. Therefore, sound engineering judgement and practical assumptions are required to perform strength capacity checks for compression, bending, and combined compression and bending of cruciform members. This paper presents numerical studies on the strength capacities of both flanged and plain cruciform members that are subjected to combined compression and bending effects.

2. Literature Review

In this section, literature review on the use of both plain and flanged cruciform members is presented particularly highlighting torsional buckling resistance. However, we note that peer-reviewed research on the behavior of flanged cruciform members is scarce compared to plain cruciform members.

The fundamental work of Timoshenko and Gere [4] forms the basis of the evaluation of torsional buckling resistance for singly, doubly, and unsymmetrical members. Chajes [5] provided a comprehensive formulation of torsional buckling equations and added valuable insights into

FIGURE 1: Typical flanged cruciform.

FIGURE 2: Typical plain cruciform section.

how design codes modify theoretical equations for use in practice. Chajes [5] concluded that, for doubly symmetric open sections, such as flanged and plain cruciforms, the torsional buckling mode is the common mode of failure.

Svensson and Plum [6] provided a theoretical basis for increasing torsional buckling strength of flanged cruciform members by using intermediate stiffener plates welded between adjacent flanges of W and WT sections. The intermediate stiffeners serve as to increase the warping resistance of the member, which is one component of the torsional buckling resistance. However, no experimental investigation into the use of intermediate warping stiffener plates has been identified to qualify or calibrate this theoretical work.

Smith [2] considered torsional buckling of four equal-leg angles used for composite plain cruciform sections. His research evaluated the elastic critical loads for flexural, torsional, and plate buckling and considers inelastic buckling with respect to residual stress distribution in hot-rolled members. The results indicated that the AISC code specifications [3] were not conservative for small to medium slenderness ratios in four angle cruciform members. At the

time, the AISC code checks are provided only for local plate buckling and not for torsional buckling, which has since been refined in later years to include torsional buckling checks. Smith [2] also noted that there is a direct relationship between torsional buckling and local plate buckling validating the fundamental work by Bleich [7].

King [8] prepared a technical brief describing the unique design considerations for plain cruciform and flanged cruciform members as they relate to Eurocode provisions for steel construction. The technical brief provides a basic primer for code checks for flexural and torsional buckling resistance of doubly symmetric flexural cruciform members, as well as suggestions for typical flanged cruciform stiffener details. He also suggested the use of intermediate gusset plates to improve flexural buckling capacity of flanged cruciform members. The incorporation of the intermediate gusset details serves to increase the stiffness of the web subject to primary flexural buckling. The suggested spacing of gussets was taken such that the minor axis slenderness ratio does not exceed that of the full-flanged cruciform member. Neither theoretical nor experimental research into the use of intermediate gusset plates for improvement of flexural buckling resistance of flanged cruciform members has been identified.

Trahair [9] considered strength design of the classic plain cruciform section. Similar to Smith [2], Trahair evaluated the elastic critical loads for flexural, torsional, and plate buckling and considers inelastic buckling with respect to residual stress distribution in hot-rolled members. His interest was identifying differences in design code checks for plain cruciform sections across the world. He stated that AISC and Eurocode codes consider torsional buckling in addition to local and flexural buckling when compared to the Australian code provisions which ignore torsional buckling. Trahair [9] showed that design of the plain cruciform section for local buckling typically meets the requirements of torsional buckling resistance. Furthermore, it was shown that the AISC and Eurocode approach to torsional buckling, which is limited to the elastic torsional buckling resistance, results in reduced load capacity for plain cruciform sections with low stiffness. Hence, the AISC and Eurocode approaches were found to be overly conservative with respect to code checks for torsional buckling resistance of a plain cruciform section, and furthermore the Australian code check for local buckling was deemed to be sufficient to guard against torsional buckling resistance of the plain cruciform section when postbuckling and inelastic buckling resistance are considered. More recently, Nasrabadi et al. [1] and Kiani et al. [10] included panel zone design detailing for seismic applications of flanged cruciform members using finite element analysis.

3. Guidelines for Strength Design

A clear guideline is needed for determining capacity of cruciform members. Basic steel design code checks for typical building applications in the United States are according to AISC [3]. The design of cruciform members is not explicitly addressed in the AISC specification; however, AISC code checks for members with geometry similar to

flanged and plain cruciform members (built-up, doubly symmetric) provide a minimum basis for code checks. Based on this, the nominal flexural buckling resistance of cruciform members, P_{nf} , is determined by

$$P_{nf} = F_{crf} A_g, \quad (1)$$

where A_g is the gross area of the section and F_{crf} is the limiting flexural buckling stress given by (2), depending on the level of the slenderness ratio, KL/r .

$$F_{crf} = [0.658^{F_y/F_e}] F_y \quad \text{when} \quad \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}, \quad (2)$$

$$F_{crf} = 0.877 F_e \quad \text{when} \quad \frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}},$$

where E is the modulus of elasticity of steel, F_y is the yield strength of steel, K is the effective length factor, L is the laterally unbraced length of the member, r is the radius of gyration of the member, and F_e is the elastic buckling strength given by (3). The elastic buckling strength can be determined from the classical Euler buckling equation given below:

$$F_e = \frac{\pi^2 E}{[KL/r]^2}. \quad (3)$$

The nominal torsional buckling resistance of cruciform members, P_{nt} , is determined from

$$P_{nt} = F_{crt} A_g. \quad (4)$$

The critical torsional buckling strength, F_{crt} , for a doubly symmetric member is limited to the elastic limit and is given by

$$F_{crt} = \left(\frac{\pi^2 E C_w}{[K_z L]^2} + GJ \right) \frac{1}{I_x + I_y}, \quad (5)$$

where G is the shear modulus of steel, C_w is the warping constant, J is the torsional constant, and I_x and I_y are moment of inertia about the principal axes. It is to be noted that the Eurocode provisions [8] for critical torsional buckling stress of a doubly symmetric member is limited to the elastic limit, p_e , as shown in

$$p_e = \left(\frac{n^2 \pi^2 E H}{L^2} + GJ \right) \frac{1}{I_o}, \quad (6)$$

where n is the number of half-sine waves along the outstands of the member. For members restrained at both ends, this should be taken as 1.0. On comparison, when substituting $I_o = I_x + I_y$ and $K = 1/n$, it is clear that (5) from the AISC provisions takes similar form as (6) from the Eurocode provisions.

The controlling compressive resistance of the cruciform member is then taken as the minimum of the flexural buckling resistance and the torsional buckling resistance. The composite cruciform moment of inertias, I_x and I_y , and similarly the elastic and plastic section moduli should be based on the composite shape for the flanged cruciform (one

W section and two WT sections). The composite flanged cruciform warping constant, C_w , is the sum of the C_w values for each "I" shape (taken as that for the selected W shape plus that of an equivalent W shape composed of the two selected WT shapes). The composite flanged cruciform torsional constant, J , is the sum of the individual shapes (one W section and two WT sections).

AISC [3] provides the derivation of torsional buckling equations from elastic buckling loads using classic theory of elasticity methods, but with modifications for plastic buckling. However, for doubly symmetric sections such as the case in flanged cruciform members, the torsional buckling stress is limited to the elastic buckling stress. Inelastic postbuckling strength is usually ignored because of difficulty in quantifying its value. The commentary also claims that torsional buckling is an uncommon mode of failure in doubly symmetric compression members without slender elements, except in cases where torsional unbraced lengths are significantly greater than the weak-axis unbraced lengths and in the case of doubly symmetric members. However, Trahair [9] provides a compelling case for evaluating local buckling in plain cruciform members with or without slender elements. Hence, we suggest that it is prudent to check local buckling in flanged cruciform members as well.

The evaluation of cruciform members under combined flexural and axial loading can be considered using interaction equations. This requires development of pure bending strength of the cruciform member. Because no reference to plain or flanged cruciform members is given in code provisions, caution must be exercised by the designer to select appropriate and conservative assumptions for the prediction of bending strength for cruciform members before using interaction curves. The combined compression and flexure interaction equations of AISC applicable to flanged cruciform members take the forms shown in (7) based on the ratio of the magnitude of axial load to axial resistance.

$$\frac{P_r}{P_c} + \frac{8}{9} \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad \text{when} \quad \frac{P_r}{P_e} \geq 0.20, \quad (7)$$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}} \right) \leq 1.0 \quad \text{when} \quad \frac{P_r}{P_e} \leq 0.20,$$

where P_r is the required axial strength using LRFD load combinations, P_c is the available factored axial strength, M_r is the required flexural strength using LRFD load combinations, and M_c is the factored flexural strength.

4. Numerical Evaluation

A set of analysis programs has been developed in MATLAB to determine strength of cruciform members. The first program performs capacity checks for flanged cruciform members using W and WT shapes. The second program performs capacity checks for plain cruciform members. The programs perform checks based upon a full set of geometric and load inputs. However, the programs do not utilize optimization algorithms.

4.1. Independent Assessment of Flanged Cruciform Sections. The flanged cruciform program performs strength checks for any given flanged cruciform member. Program inputs to define the section are simplified to allow the user to enter the desired “W” and “WT” sections. For example, the user may input “W16×100” and “WT8×50” as the proposed sections, and the program will automatically build and analyze a flanged cruciform member comprised of one W16×100 and two WT8×50 sections. The program also automatically retrieves the required geometric parameters from the AISC [3] Shapes Database without the need to get individual section properties. Additional inputs to the program include unbraced lengths, effective length factors, material properties, and design loads (axial load and biaxial flexure).

The output of the program is a combined axial and flexural interaction ratio (less than 1.0 is satisfactory), and a text statement indicating whether the input parameters satisfy AISC code checks. Additionally, the output includes evaluation of the biaxial symmetry of the proposed flanged cruciform section (a tolerance of $\pm 10\%$ is the default tolerance). If the symmetry about either axis differs more than the tolerance, a caution message is provided. The program does not perform local buckling checks.

Due to lack of literature on flanged cruciform sections, validation of the program has been relatively difficult. We resorted to cross-validation by using the work of Svensson and Plum [6], which included calculations for the Euler buckling loads about both axes and the torsional buckling load. Our flanged cruciform program, modified with direct inputs of the composite section parameters to match the example inputs, yields the same Euler and torsional buckling loads reported by Svensson and Plum [6].

The flanged cruciform program was then used to demonstrate the capacity of these members in applications with high axial load with combined biaxial flexure making it well suited for use in orthogonal moment frames. In addition, the program was used to perform a parametric study of strength capacities for three proposed members: W16×100 and two WT8×50 sections; W10×50 and two WT8×25 sections; and W10×26 and two WT8×13 sections. Figure 3 shows the ratio of the torsional buckling resistance to flexural buckling resistance of each of the three sections at various unbraced lengths. In the case of these three cases, the torsional buckling resistance is typically greater than that of the flexural buckling resistance at low slenderness. However, with increasing the flanged cruciform member slenderness, the torsional buckling mode becomes critical. We note this trend is contrary to the behavior of plain cruciform sections, wherein low to moderate slenderness promotes the importance of torsional buckling.

4.2. Independent Assessment of Plain Cruciform Sections. The plain cruciform program performs strength checks for any proposed plain cruciform member. The plain cruciform section is defined by inputs for the desired total section depth along each axis and the desired thickness of the cruciform legs on each axis. Additional inputs to the program include

FIGURE 3: Torsional to flexural buckling ratio for select flanged cruciform members (1 ft = 0.305 m).

unbraced lengths, effective length factors, material properties, and design loads (axial load and biaxial flexure). The plain cruciform program was indirectly calibrated by reproducing the elastic flexural and torsional buckling from Trahair [9]. The plain cruciform section used for validation has a leg thickness of 10 mm and variable leg width to leg thickness ratios of 10, 20, and 30. A modified slenderness ratio, as shown in (8), is implemented for comparison purposes.

$$\lambda_{oy} = \sqrt{\frac{N_y}{N_{oy}}} \quad (8)$$

where N_y is the squash load and N_{oy} is the Euler buckling load. If N_{oz} represents the elastic torsional buckling load, plot depicting the ratios N_{oz}/N_y and N_{oy}/N_y versus the modified slenderness for the varying values of b/t provides an insight into the cruciform behavior. The plain cruciform program was used to produce the result shown in Figure 4.

Our results are consistent for flexural buckling when compared to results from available literature [9, 10]. For low modified slenderness, our results for torsional buckling of plain cruciform sections diverge with results reported in available literature [9, 11]. However, the plain cruciform program uses a warping coefficient, C_w , equal to zero by default. This is because torsional buckling resistance is minor for practical designs of plain cruciform sections when the members have moderate to high modified slenderness [12]. One recommendation is to incorporate a warping stiffness of rectangular sections using the classical torsional rigidity given by

FIGURE 4: Torsional to flexural buckling loads for select plain cruciform members.

$$C_w = \frac{b^3 T^3}{9}. \quad (9)$$

The plain cruciform program was modified to account for this warping stiffness ($C_w \neq 0$), with results plotted in addition to results from the program default ($C_w = 0$). When $C_w = 0$ is used, the plain cruciform program yields values close to what is reported in literature [9] with the exception of very low modified slenderness values of less than 0.2. On the other hand, when the program is modified to add C_w ($C_w \neq 0$), it predicts higher torsional buckling resistance for low to moderate modified slenderness values. Results converge as the modified slenderness approaches 0.8. The results indicate that for very low modified slenderness (very short members), sections with higher leg slenderness, given the same leg thickness, have a higher torsional buckling resistance. Although this result is inconsistent with findings by Trahair [9], we believe that for very low slenderness members, the warping component of torsional buckling resistance becomes highly dominant over St. Venant torsional resistance. Since the warping coefficient is a function of b^3 , given an equal modified slenderness, the section with the more slender leg (larger b) will produce a higher warping torsional resistance.

The plain cruciform program was used to demonstrate the significant limitation of plain cruciform sections when subjected to flexural loading. Relative to axial resistance of a given plain cruciform section, the section develops little flexural resistance about either axis. As such, plain cruciform sections should be limited to purely axial applications such

FIGURE 5: Axial resistance for comparative plain and flanged cruciform members (1 ft = 0.305 m and 1 kip = 4.45 kN).

as space trusses or struts. Furthermore, the program demonstrates the susceptibility of plain cruciform sections with low to moderate slenderness to fail under torsional buckling at loads significantly less than that of flexural buckling. This is consistent with the conclusion presented by Smith [2] for built-up plain cruciform members.

4.3. Comparison of Flanged and Plain Cruciform Sections. A comparative study was conducted to investigate the comparative behavior of typical flanged and plain cruciform members. In the study, three flanged cruciform members, W16×100 + 2 WT8×50, W10×50 + 2 WT8×25, and W10×26 + 2 WT8×13, were compared to their plain cruciform counterpart sections (16" (40.6 cm) depth × 0.585" (1.5 cm) leg thickness; 16" (40.6 cm) depth × 0.38" (1 cm) leg thickness; and 16" depth (40.6 cm) × 0.25" (0.65 cm) leg thickness). The selected material yield strength for all members was 36 ksi (248 MPa). The results for axial and flexural resistance at different unbraced lengths ranging from 10 feet (3.33 m) to 40 feet (13.3 m) are shown in Figures 5–8.

Results confirm the exceptional flexural performance of flanged cruciform members relative to plain cruciform members at all slenderness levels. The analysis also confirms that flanged cruciform members are well suited for applications under large axial compression and biaxial flexure. A similar finding for FRP can be found in [13].

Conversely, the analysis shows that plain cruciform sections are very economical under pure axial applications. Results also demonstrate that the high biaxial flexural strength of flanged and plain cruciform members enables

FIGURE 6: Normalized axial resistance for comparative plain and flanged cruciform members (1 ft = 0.305 m and 1 kip/in² = 6.895 MPa).

FIGURE 7: Flexural resistance for comparative plain and flanged cruciform members (1 ft = 0.305 m and 1 kip = 4.45 kN).

maintenance of a stable flexural resistance across a large range of unbraced length. This is due to the ability of flanged and plain cruciform to resist lateral-torsional buckling, allowing development of efficient plastic resistance, over

FIGURE 8: Normalized flexural resistance for comparative plain and flanged cruciform members (1 ft = 0.305 m and 1 kip/in² = 6.895 MPa).

longer unbraced lengths when compared to conventional shapes. As the limitation of this study, strength check for connections and local concentrated end bearings are not included here.

5. Conclusions

This paper presents numerical studies on the strength capacities of both flanged and plain cruciform members that are subjected to combined compression and bending effects. Two numerical programs were developed, and they were used to conduct a comparative study of the behavior of three flanged cruciform to their plain cruciform counterpart sections. Analysis results show the ability of flanged and plain cruciform to resist lateral-torsional buckling over longer unbraced lengths, allowing development of efficient plastic resistance. It is to be noted that required checks for stability, service, and fatigue should be considered in addition to the member strength analysis presented in this research. To enhance the findings of this paper, further comparative studies using finite element analysis are recommended.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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References

- [1] M. Nasrabadi, S. Torabian, and S. Mirghaderi, "Panel zone modelling of flanged cruciform columns: an analytical and numerical approach," *Engineering Structures*, vol. 49, pp. 491–507, 2012.
- [2] E. Smith, "Buckling of four equal leg angle cruciform columns," *Journal of Structural Engineering*, vol. 109, no. 2, pp. 439–450, 1983.
- [3] AISC 360, *Specification for Structural Steel Buildings*, American Institute of Steel Construction, Chicago, IL, USA, 2016.
- [4] S. P. Timoshenko and J. M. Gere, *Theory of Elastic Stability*, McGraw-Hill, New York, NY, USA, 2nd edition, 1961.
- [5] A. Chajes, *Principles of Structural Stability Theory*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1974.
- [6] S. A. Svensson and C. M. Plum, "Stiffener effects on torsional buckling of columns," *Journal of Structural Engineering*, vol. 100, no. 3, pp. 758–772, 1983.
- [7] F. Bleich, *Buckling Strength of Metal Structures*, McGraw-Hill, New York, NY, USA, 1952.
- [8] C. King, *Design of Cruciform Sections using BS 5950-1:2000*, The British Constructional Steelwork Association Ltd, London, UK, 2006.
- [9] N. Trahair, "Strength design of cruciform steel columns," *Engineering Structures*, vol. 35, pp. 307–313, 2011.
- [10] B. Kiani, S. Torabian, and S. Mirghaderi, "Local seismic stability of flanged cruciform sections (FCSs)," *Engineering Structures*, vol. 96, pp. 126–138, 2014.
- [11] G. Urgessa and T. Arciszewski, "Blast response comparison of multiple steel frame connections," *Finite Elements in Analysis and Design*, vol. 47, no. 7, pp. 668–675, 2011.
- [12] P. B. Dinis, D. Camotim, and N. Silvestre, "On the local and global buckling behaviour of angle, T-section and cruciform thin-walled members," *Thin-Walled Structures*, vol. 48, no. 10–11, pp. 786–797, 2010.
- [13] G. Urgessa, S. Horton, M. M. Reda Taha, and A. Maji, "Significance of Stress-Block Parameters on the Moment Capacity of Sections Under-Reinforced with FRP", ACI Special Publication 230, pp. 1531–1550, American Concrete Institute, Farmington Hills, MI, USA, 2005.