Research Article

Study on Flexural Bearing Capacity Calculation Method of Enclosure Pile with Partial Excision in Deep Foundation Pit

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During deep foundation pit construction, the structural clearance intrusion, which is caused by the complex formation conditions and the inefficient drilling equipment, is usually detected due to the vertical deviation of piles. To meet construction requirements, pile parts intruding into the structural clearance are supposed to be excised. However, the sectional flexural strength of the pile is bound to decrease with partial excision, which would reduce the bearing capacity of the enclosing structure during construction. In this paper, a theoretical derivation of the normal sectional flexural strength of the partially excised circular pile is proposed. The derivation adopts the assumption of the plane section and steel ring equivalence and can be solved by the bisection method. Furthermore, the calculation method is applied to the pile evaluation of a practical engineering; also, the method is verified by the numerical method. The application results show that the excision of rebar and pile’s sectional area will cause a rapid linear decline in the sectional flexural strength. After excising 18 cm radial thickness of the circular pile (ϕ800 mm) and 6 longitudinal rebars, the sectional flexural strength of the pile decreases to 58% from the origin, which cannot meet the support requirement. The analysis indicates that pile reinforcements must be carried out to maintain the construction safety.

1. Introduction

With the development of underground space engineering, large amounts of foundation pits have appeared in China [1–4]. For security construction, supporting structure system of the retaining wall with internal supports is necessary during pit excavation [5–9]. At present, the circular bored pile has been commonly applied to the construction of retaining wall due to its low cost and small disturbance [10–15]. However, in the challenging field conditions such as the strata of the expansive soil mass [16], loosen rock mass [17], sand and pebble [18], and ultrathick layers [19], or the utilization of the drilling equipment with low precision [20], the vertical direction deviations of piles [21] and concrete crack [22] may occur frequently due to the lateral displacement of the ground. The vertical direction deviations will cause serious engineering problems, for instance, the intrusion of the structural clearance and the variation of the bearing system. Thus, the intruding part of the pile is supposed to be excised [23]. With the large-scale excision of the pile, longitudinal rebars inside the pile may even be cut off, which will lead to a reduction of the bearing capacity [24]. In severe cases, it may even cause the overall instability and overturning of the foundation pit retaining structure [25]. To ensure the reliability of supporting system with excised piles, the bearing capacity evaluation of the partially excised pile is supposed to be carried out.

Currently, the calculation method of the normal sectional flexural strength of the complete circular pile has been clearly defined [26], but with no specific calculation method for the pile body with partial excision. Hence, based on the practical engineering, this paper performs a calculation
method of the normal sectional flexural strength for the partially excised pile. The formulas can provide references for the pit safety assessment and appropriate reinforcement measures.

2. Engineering Background

The background engineering is a tunnel located on a constructing passenger-dedicated line in China. The total length of the tunnel is 1200 m with the start and termination mileage of DK33+310 to DK34+510. Among them, the sections of DK33+310~DK33+730 and DK34+160~DK34+510 are constructed via the open-cut method with rectangular foundation pits. The strata around the engineering are mainly composed of artificial accumulation layer clay (0~9 m), Quaternary Holocene alluvial silty clay (9~13 m), silty soil (13~15 m), and quaternary upper Pleistocene alluvial sand soil (below 15 m), as shown in Figure 1. The mechanical parameters of each layer are shown in Table 1.

The supporting structure system of foundation pits includes the retaining bored piles and the internal support structure. The design parameters of bored piles are Φ800 mm@1.1~1.3 m with a length of 29 m, and each of them is reinforced by the annular longitudinal rebars of 22ϕ25 mm. For the internal support structure, it is composed of Φ609 mm steel pipe supports, 3~4 channels are, respectively, set inside the foundation pit according to the excavation depth. The geological profile and supporting structure system are shown in Figure 1.

3. Clearance Intrusion

According to the actual lofting survey, retaining piles at the section DK33+630~DK33+710 have significantly intruded into the main structure clearance; the maximum intruding thickness even reaches 18 cm. To satisfy the space requirements of the main structure, the intruding part is supposed to be excised. Then 6 main stressed rebars would be cut off at the excision part, as shown in Figure 2. The excision is bound to cause the flexural strength reduction of the supporting structure. The instability risk of the whole supporting system may further increase especially on the demolition process of the internal support. Therefore, it is necessary to evaluate the bearing capacity of the excised pile for the safety of the subsequent construction process.

4. Calculation of Flexural Strength of Retaining Piles

4.1. Normal Section Flexural Strength of Complete Piles.

In the current designing, friction retaining bored piles is usually regarded as flexural members. According to the Chinese code for concrete structure design (GB 50010-2010) [26], the normal circular section bearing capacity of reinforced concrete flexural members (Figure 3) with longitudinal rebars (no less than 6) uniformly distributed along the periphery can be calculated as follows:

\[
M = \alpha_1 f_c A_0 r \left(2 \sin^3 \frac{\pi}{3} \alpha + A_0 \sin \frac{\pi}{3} \alpha + \frac{\pi}{2} \alpha_1 f_c A_0 \left(1 - \frac{\sin \frac{2 \pi}{3} \alpha}{2 \pi \alpha} \right) + (\alpha - \alpha_1) f_c A_0 = 0, \right)
\]

where \(A_0\) is the sectional area of the complete pile; \(A_0\) is the sectional area of all the longitudinal rebars of the complete pile; \(r\) is the radius of the circular pile; \(r_0\) is the circumference radius of the longitudinal rebar centroid; \(f_c\) is the compressive strength of concrete; \(f_s\) is the tensile strength of longitudinal rebars; \(\alpha\) is the ratio of the central angle (rad) to \(2\pi\) in the concrete compressive zone; \(\alpha_1\) is the concrete strength coefficient (when concrete grade \(\leq C50\), \(\alpha_1 = 1.0\); for \(C80\) concrete, \(\alpha_1 = 0.94\); for \(C50~C80\) concrete, \(\alpha_1\) can be determined by linear interpolation); and \(\alpha_i\) is the sectional area ratio of longitudinal tensile rebars to all the longitudinal rebars.

Equation (1) is a transcendental equation that needs to be solved by the iterative method. The bisection algorithm [27]
The second steel supports

Excision length
≈ 6m

The max
excision thick
≈ 18cm

Structure
clearance
Broken zone

Retaining bored piles

Figure 2: Clearance intrusion and excision of pile body.

Figure 3: Circular section with uniformly circumferentially distributed rebars.

and trial algorithm [28] can be applied. In this paper, the bisection algorithm is selected, its basic idea is firstly to solve the equilibrium equation of axial force in equation (1) and obtain \( \alpha \) and the corresponding \( a_i \), and then by substituting \( \alpha \) and \( a_i \) into the equilibrium equation, the ultimate normal section bearing capacity of the circular pile can be obtained [29]. The specific calculation steps above are listed as follows:

1. By translating the equilibrium equation of axial force in equation (1) into \( F(\alpha) = 0 \), the characteristic function \( F(\alpha) \) is defined as follows:

\[
F(\alpha) = \frac{f_c A_0}{f_c A_0} \frac{\alpha (1 - \sin \frac{2 \pi a}{2\pi a})}{1.25 - 3\alpha}.
\]

2. After determining the design parameters of the pile and feasible root range, the root (\( \alpha \)) of equation \( F(\alpha) = 0 \) is obtained via the bisection algorithm. Correspondingly, \( a_i \) can be obtained through the substitution of \( \alpha \).

3. By substituting \( \alpha \) and \( a_i \) into equation (1), the normal section flexural strength of the complete circular pile is given as follows:

\[
M_u = \frac{2}{3} A_0 f_c A_0 \frac{\sin^3 \frac{\pi a}{\pi}}{\pi} + A_0 f_c r_s \frac{\sin \pi a + \sin \pi a_i}{\pi}.
\]

4.2. Normal Section Flexural Strength of Piles with Partial Excision. For the partially excised pile, no design code can be referred currently. Thus, the corresponding calculation method is proposed in this paper, with the situations that the radial broken thickness of the pile is \( d \) and the number of residual longitudinal rebars is \( n \), as shown in Figure 4(c).

4.2.1. Basic Assumptions. For the simplification of the mechanical calculation model, several assumptions are introduced: (a) retaining piles are regarded as the flexural members, and the stress deformation of piles meets the plane-section assumption; (b) the influence of the concrete tensile stress on flexural strength is not considered; (c) the tensile rebar is assumed as the ideal elastoplastic material; (d) the relationship between the equivalent rectangular height \( x_c \) and the actual height of the compressive zone \( x_s \) is that when the concrete strength grade is less than C50, \( x = 0.8x_c \); when the concrete strength grade reaches C80, \( x = 0.74x_c \); others can be determined by linear interpolation; and (e) the ultimate compressive strain of concrete \( \varepsilon_u = 0.0033 \).

4.2.2. Flexural Strength Derivation. The uniformly distributed rebars are regarded as the equivalent steel ring, as shown in Figures 4(a) and 4(b). Thus, the thickness of the equivalent steel ring \( t_s \) is given by

\[
t_s = \frac{A_0}{2\pi r_s}.
\]

According to the above basic assumptions, the normal section flexural strength calculation model of piles with partial excision can be established, as shown in Figure 5. Then, the distance of the neutral axis to the section centroid axis \( x_h \) is equal to \( (r-x) \); correspondingly, the half of the central angle to the neutral axis \( \theta_h \) is calculated as

\[
\cos \theta_h = \frac{x_h}{r} = \frac{r - x}{r} = \frac{r - 0.8x_c}{r} = \frac{r - 0.8\xi (2r - d)}{r}.
\]

Then,

\[
\theta_h = \arccos \left[ 1 - \frac{0.8\xi (2r - d)}{r} \right],
\]

where \( \xi \) is the height coefficient of the compressive zone:

\[
\xi = \frac{x_s}{2r - d}.
\]

Furthermore, based on the plane-section assumption, the distance of the position that the steel ring reaching compressive yield strength to the centroid axis \( x_s' \) can be determined by the following equation:

\[
\frac{\varepsilon_u}{\xi (2r - d)} \frac{f'c x_s'}{x_s}.
\]
By rearranging equation (8), the following equation is obtained:

\[ x'_{s} = \frac{\xi(2r-d)f'_s}{\varepsilon_u} + x_0 = \frac{\xi(2r-d)f'_s}{\varepsilon_u} + \frac{r - \xi(2r-d)}{\varepsilon_u} \]

where \( x_0 \) is the distance of the actual neutral axis to the section centroid axis, which is calculated by \( x_0 = r - \xi(2r-d) \); \( f'_s \) is the compressive strength of the longitudinal rebars, and \( E_s \) is the elastic modulus of longitudinal rebars; \( \theta'_s \) is the half of the central angle to the compressive yield position of the rebar.

In combination with equations (9) and (10), the corresponding half of the central angle \( \theta'_s \) is written as

\[ \theta'_s = \arccos \left[ \frac{\xi(2r-d)f'_s}{r_s\varepsilon_u} + \frac{r - \xi(2r-d)}{r_s\varepsilon_u} \right] \leq \pi. \]

Similarly, the distance of the position that the steel ring reaching tensile yield strength to the centroid axis \( x_s \) is determined by

\[ \frac{\varepsilon_u}{\xi(2r-d)} = \frac{f_s/E_s}{x_s + x_0} \]

where \( f_s \) is the tensile strength of the longitudinal rebar. \( x_s \) and \( \theta_s \) satisfy the following equations in the same way:

\[ x_s = \left[ \frac{\xi(2r-d)f_s}{E_s\varepsilon_u} + r - \xi(2r-d) \right] \geq -r_s, \]

\[ \cos \theta_s = \frac{x_s}{r_s} \]

By combining equations (13) and (14), the half of the central angle to the tensile yield position of the rebar \( \theta_s \) is obtained:

\[ \theta_s = \arccos \left[ -\frac{\xi(2r-d)f_s}{r_s\varepsilon_u} + \frac{r - \xi(2r-d)}{r_s\varepsilon_u} \right] . \]
Therefore, the stress at each point of the steel ring can be obtained by combining equations (11) and (15):

\[
\sigma_{\theta\theta} = \begin{cases} 
-f_{\theta}^s, & 0 < \theta \leq \theta_s', \\
\frac{r_s \cos \theta - x_0}{r_s \cos \theta_s - x_0} f_{\theta}^s, & \theta_s' < \theta \leq \theta_s, \\
f_{\theta}^s, & \theta_s < \theta \leq \varphi,
\end{cases}
\]  

(16)

where \(\varphi\) is the half of the central angle to the excised position of the pile, which is calculated by \(\varphi = \pi - \arccos[(r - d)/r]\); the tensile stress is positive.

Furthermore, by setting the sectional area of all the longitudinal rebars of the excised pile \(A_s\), the moment of steel ring stress to the neutral axis with pile excised \(M_s\) can be obtained:

\[
M_s = 2 \int_{0}^{\varphi} \left| x - x_0 \right| \sigma_{\theta\theta} dA_s,
\]

(17)

where \(dA_s = t_s r_s d\theta \) and \(x = r_s \cos \theta\).

That is,

\[
M_s = 2 \int_{0}^{\theta_s'} (x - x_0) f_{\theta}^s \frac{A_s}{2\pi} d\theta + 2 \int_{\theta_s'}^{\theta_s} (x_0 - r_s \cos \theta) f_{\theta}^s \frac{A_s}{2\pi} d\theta
\]

\[
\times \left( \frac{r_s \cos \theta - x_0}{r_s \cos \theta_s - x_0} \right) \times f_{\theta}^s \frac{A_s}{2\pi} d\theta + 2 \int_{\theta_s}^{\varphi} (x_0 - r_s \cos \theta) f_{\theta}^s \frac{A_s}{2\pi} d\theta
\]

\[
= f_s A_s \frac{2\pi}{\pi} \left\{ r_s \left( \sin \theta_s' + \sin \theta_s - \sin \varphi \right) + x_0 \left( \theta_s' - \theta_s \right) \\
- \left( \frac{4 \sin^2 \theta_s'}{3(2\theta_s' - \sin 2\theta_s')} r \right) \frac{2r_s \left( \sin \theta_s - \sin \theta_s' \right) x_0 - x_0^2 (\theta_s' - \theta_s)}{r_s \cos \theta_s - x_0} \right\}.
\]

(18)

Subsequently, according to Figure 5, the distance of the resultant force of concrete in the compressive zone to the sectional centroid axis \(z_h\) is expressed as

\[
z_h = \frac{4 \sin^3 \theta_s'}{3(2\theta_s' - \sin 2\theta_s')} r.
\]

(19)

The equivalent stress map of the concrete in the compressive zone corresponding to the area of bow compressive zone \(A_c\) is written as

\[
A_c = (\theta_s' - \sin \theta_s \cos \theta_s) r^2.
\]

(20)

From the above two equations, the moment of the resultant force in the concrete compressive zone to the neutral axis can be acquired:

\[
M_c = \alpha_1 f_c A_c (z_h - x_0) = \alpha_1 f_c (\theta_s' - \sin \theta_s \cos \theta_s) r^2.
\]

(21)

From equations (18) and (21), the normal section flexural strength of the pile body with partial excision is given by

\[
M_{pu} = M_s + M_c.
\]

(22)

4.2.3. Internal Force of Section and Its Equilibrium Equation.

From the computational scheme of Figure 5, the resultant compressive force of the concrete in the compressive zone \(D_c\) is given by

\[
D_c = \alpha_1 f_c A_c = \alpha_1 f_c (\theta_s' - \sin \theta_s \cos \theta_s) r^2.
\]

(23)

The resultant of the tensile and compressive stress of the steel ring \(D_s\) is given by

\[
D_s = 2 \int_{0}^{\varphi} \sigma_{\theta\theta} dA_s = 2 \int_{0}^{\theta_s'} f_{\theta}^s \frac{A_s}{2\pi} d\theta
\]

\[
+ 2 \int_{\theta_s'}^{\theta_s} \left( \frac{r_s \cos \theta - x_0}{r_s \cos \theta_s - x_0} \right) f_{\theta}^s \frac{A_s}{2\pi} d\theta + 2 \int_{\theta_s}^{\varphi} \left( \frac{r_s \cos \theta - x_0}{r_s \cos \theta_s - x_0} \right) f_{\theta}^s \frac{A_s}{2\pi} d\theta
\]

\[
= f_s A_s \frac{2\pi}{\pi} \left\{ \varphi - \theta_s' - \theta_s + r_s \left( \sin \theta_s - \sin \theta_s' \right) - x_0 \left( \theta_s - \theta_s' \right) \right\}.
\]

(24)

Let \(C = \varphi - \theta_s' - \theta_s' + \left( (r_s \cos \theta_s - \sin \theta_s') (r_s \cos \theta_s - x_0) \right) - \left( x_0 (\theta_s' - \theta_s') (r_s \cos \theta_s - x_0) \right) \) and based on the cross section internal force equilibrium condition \(D_c = D_s\) in the horizontal direction, the equilibrium equation of axial force is obtained:

\[
f_c (\theta_s' - \sin \theta_s \cos \theta_s) r^2 - A_o f_s C = 0.
\]

(25)

On the basis of the bisection algorithm and the combination of equations (6), (10), (11), (15), and (16), equation (25) can be solved. By substituting \(\xi\), \(\theta_s'\), \(\theta_s\), \(\varphi\), and \(x_0\) into equations (18), (21), and (22), the normal section flexural strength of the pile with partial excision is obtained.

5. Field Application

To further verify the calculation method and present its application, the calculation method is applied to the safety evaluation of the retaining structure in the construction period of the previous engineering.

5.1. Numerical Simulation. For the bearing capacity analysis of the excised pile, the maximum bending moment that piles undertake during construction is supposed to be determined. In this paper, 3D numerical method is applied to the safety analysis and mechanical calculation of the supporting structure, and the calculation model is shown in Figure 6. In the modeling process, soil parameters are shown in Table 1 with the constitutive relationship of Mohr–
Coulomb, and parameters of the retaining pile and internal steel support (steel pipe) of the background engineering are listed in Table 2. The entire construction cycle of the detailed construction state is considered in the simulation as shown in Table 3.

5.2. Internal Force Calculation of Retaining Piles. With the calculation of internal force under each construction state, the bending moment envelope of all the retaining piles in each depth during construction is obtained, as shown in Figure 7. The maximum positive bending moment of the retaining pile is 818.5 kN·m in the whole construction cycle, and the position of the maximum bending moment is just within the excision region of the pile body.

5.3. Safety Evaluation of Pile with Excision

5.3.1. Flexural Strength of the Complete Pile. By setting the root range (α) of equation $F(\alpha) = 0$ as 0~2, the relation curve of the characteristic function $F(\alpha)$ related to $\alpha$ can be obtained from equation (2), as shown in Figure 8. Through the bisection method, the root of equation $F(\alpha) = 0$ is calculated as 0.2973 and 0.4704. Considering the limitation of the maximum and the minimum reinforcement ratio, the reasonable value range of $\alpha$ is within [0.18, 0.36] [27], and the results $\alpha = 0.2973$ and $\alpha = 0.6554$ are finally obtained. Then by substituting the relevant parameters into equation (3), the normal section flexural strength of the complete retaining pile $M_\text{n} = 1128$ kN·m is given. The calculation value is close to the calculation result of 1042 kN·m in the literature [28], which indicates the feasibility of the bisection method and the reliability of the calculation result.

5.3.2. Flexural Strength of the Partially Excised Pile. For the normal section flexural strength of partially excised piles, with the radial length of the excised pile as $d = 18$ cm and 6 longitudinal tensile rebars cut off (residual rebar number $n = 16$), similarly, the equilibrium equation of axial force (equation (25)) can be solved via the proposed method. Firstly, the characteristic function of the equilibrium equation $F(\xi)$ is introduced as follows:

$$F(\xi) = f_c (\theta_h - \sin \theta_h \cos \theta_h) \times \frac{r^2}{\pi} - \frac{CA_f f_s}{\pi},$$

$$C = \varphi - \theta_s - \theta_s' + \frac{r_s (\sin \theta_s - \sin \theta_s')}{r_s \cos \theta_s - x_0} - \frac{x_0 (\theta_s - \theta_s')}{r_s \cos \theta_s - x_0}$$

(26)

where $\theta_h$, $x_0$, $\theta_s$, and $\theta_s'$ are the functions related to $\xi$ as presented in equations (6), (10), (11), and (15).

In combination with the design parameters in Table 3 and the pile excision conditions, equations $F(\xi) = 0$, (6), (10), (11), and (15) can be solved by the bisection method as well, the relation curve of the characteristic function $F(\xi)$ related to $\xi$ and the root ($\xi = 0.4378$) are given, as shown in Figure 9. Correspondingly, the section bearing parameters of the excised pile solved from the equation system are shown in Table 4. By substituting the section bearing parameters into equations (18), (21), and (22), the bending moments of the excised pile are given: $M_s = 358.82$ kN·m and $M_c = 302.94$ kN·m. Finally, the normal section flexural strength of the excised pile is solved as $M_{pu} = 661.76$ kN·m.

5.3.3. Verification and Analysis. For the verification of the sectional flexural strength calculation method, the failure process of the single pile is simulated by setting a 1.0 m displacement in the excised region. With the increasing of pile’s deformations, the stress redistribution and concrete damages of piles occur. The compressive fracture of concrete ($\varepsilon_u = 0.002$) is regarded as the sign of the ultimate flexural bearing capacity in the numerical model. Thus, the relation curves between the compressive stress of concrete risk points and the failure time step are drawn in the failure process, as shown in Figure 10.

The peak stress reflects the stress state approaching damage of concrete risk points, and also, the corresponding peak time can be obtained from Figure 10. Then, the bending moment of the pile (containing concrete, rebars) in the peak time maps to the sectional flexural strength, which can be
directly extracted from the numerical model. After simulation, the sectional flexural strength of the complete pile is calculated as 1085 kN·m in the numerical model; for the excised pile, it is 585 kN·m. The simulated sectional flexural strength results are close to the values calculated by the derived method (deviation: 3.8% for the complete pile; 11.4% for the excised pile), which verifies the reliability of the method.

Comparing the flexural strength of the complete pile with that of the partially excised pile, the maximum flexural strength of the excised pile is only 58% than that of the complete pile, which is less than the maximum flexural strength of 818.5 kN·m. The calculation and comparison indicate that excised piles cannot meet the safety requirements of the practical engineering and need to be reinforced.

5.4. Discussion. In order to further study the impact of pile partial excision on the normal section flexural strength, the flexural strength ratio of the excised pile to the completed pile ($M_p/M_c$) and the sectional area ratio of the excised pile to the completed pile ($A_p/A_c$) are defined after longitudinal rebars being cut off. Based on the proposed method, the influence of the partial excised region size on the flexural strength of the pile body is analyzed, as shown in Figure 11.
According to the analysis, with the increase of the partial excised region of the pile body, the number of cutting-off rebars increases and the flexural strength of pile body’s normal section decreases rapidly and linearly, which brings potential safety risks to the retaining structure.

6. Conclusions

(1) Based on the assumption of the steel ring equivalence and plane-section assumption, a theoretical derivation of the normal sectional flexural strength of the partially excised circular pile is proposed with the radial broken thickness $d$ and the residual number of longitudinal rebars $n$. The derived equation system can be solved via the bisection method.

(2) The derived method is applied to the practical evaluation of the retaining pile, and the numerical model verifies the evaluation result. After cutting off 6 longitudinal rebars, the sectional flexural strength of the excised pile reaches 58% of the designing value, which is less than the maximum bending moment calculated by the excavation numerical model. Thus, the flexural strength cannot meet the construction requirement and the retaining structure reinforcement must be carried out.

(3) The influence of the pile excision region size on the flexural strength is discussed. The analyzing result indicates that with the increase of the excised region, the reduction of longitudinal stressed rebars and the pile body section region will lead to a rapid linear decrease in the flexural strength.
decline in the pile’s normal section flexural strength. Pile strength decreases may bring potential danger to the retaining structure.

**Nomenclature**

\( M \): Bending moment of the normal circular section  
\( M_{p0} \): Normal section flexural strength of the complete pile  
\( M_{pp} \): Normal section flexural strength of the excised pile  
\( M_l \): Moment of steel ring stress to the neutral axis with pile excised  
\( M_r \): Moment of the resultant force in the concrete compressive zone to the neutral axis  
\( A_0 \): Sectional area of the complete pile  
\( A_{00} \): Sectional area of all the longitudinal rebars of the complete pile  
\( A_{01} \): Sectional area of all the longitudinal rebars of the excised pile  
\( A_c \): Area of bow compressive zone  
\( D_c \): Resultant compressive force of the concrete in the compressive zone  
\( D_r \): Resultant of the tensile and compressive stress of the steel ring  
\( d \): Radial excision distance of pile  
\( d_o \): Outer diameter of steel pipes  
\( L \): Length of the retaining pile  
\( L_o \): Length of the steel pipe  
\( n \): Number of residual longitudinal rebars  
\( r \): Radius of the circular pile  
\( r_{01} \): Circumference radius of the longitudinal rebar centroid  
\( f_c \): Compressive strength of concrete  
\( f_y \): Tensile strength of longitudinal rebars  
\( f_{py} \): Compressive strength of longitudinal rebars  
\( f_{sy} \): Compressive strength of the steel pipe  
\( a \): Ratio of the central angle (rad) to \( 2\pi \) in the concrete compressive zone  
\( a_0 \): Concrete strength coefficient  
\( a_1 \): Sectional area ratio of longitudinal tensile rebars to all the longitudinal rebars  
\( t_i \): Thickness of the equivalent steel ring  
\( t_{eq} \): Wall thickness of the steel pipe  
\( x \): Equivalent rectangular height  
\( x_{0c} \): Actual height of the compressive zone  
\( x_{00} \): Distance of the actual neutral axis to the section centroid axis  
\( x_{0b} \): Distance of the neutral axis to the section centroid axis  
\( x_{0t} \): Distance of the position that the steel ring reaching compressive yield strength to the centroid axis  
\( x_{0y} \): Distance of the position that the steel ring reaching tensile yield strength to the centroid axis  
\( z_0 \): Distance of the resultant force of concrete in the compressive zone to the sectional centroid axis  
\( z_{0c} \): Height coefficient of the compressive zone  
\( E_c \): Elastic modulus of concrete  
\( E_y \): Elastic modulus of longitudinal rebars  
\( E_y \): Elastic modulus of the steel pipe  
\( \theta_c \): Half of the central angle to the neutral axis  
\( \theta_{py} \): Half of the central angle to the compressive yield position of the rebar  
\( \theta_{sy} \): Half of the central angle to the tensile yield position of the rebar  
\( \varphi \): Half of the central angle to the excised position of the pile  
\( \varepsilon_{cu} \): Ultimate compressive strain of concrete  
\( \sigma_{0y} \): Stress at each point of the steel ring.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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