Research Article

Random Vibration Analysis of Coupled Three-Dimensional Maglev Vehicle-Bridge System

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Abstract

This paper presents a framework for the linear random vibration analysis of the coupled three-dimensional (3D) maglev vehicle-bridge system. Except for assembling the equation of motion of vehicle only via the principle of virtual work, the fully computerized approach is further expanded to assemble the governing equation of fluctuating current via the equilibrium relation. A state-space equation couples the equation of motion of the vehicle and the governing equation of fluctuating current. The equation of motion of a real three-span space continuous girder bridge is established by using finite element methods. A separated iteration method based on the precise integration method and the Newmark method is introduced to solve the state-space equation for the maglev vehicle and the equation of motion for the bridge. Moreover, a new scheme to application of the pseudoexcitation method (PEM) in random vibration analysis is proposed to maximize the computational efficiency of the random vibration analysis of the maglev vehicle-bridge system. Finally, the numerical simulation demonstrates that the proposed framework can efficiently obtain the mean value, root mean square (RMS), standard deviation (SD), and power spectral density (PSD) of dynamic response for the coupled 3D maglev vehicle-bridge system.

1. Introduction

The guideway of the maglev vehicle mostly adopts continuous elevated girder bridges to save land and reduce foundation settlement [1]. The reason of the interaction between maglev vehicle and bridge is that flexibility of the bridge causes the change of the air gap between the suspension bogie and the bridge, so the control voltage and control current must correspondingly change with the air gap to provide adequate electromagnetic force supporting the moving maglev vehicle. As the construction deviation and rough track profile for the guideway surface, the stochastic factors exist in the guideway deformation, which leads to random vibration of the maglev vehicle-bridge system. As a result, conducting research efforts on random vibration analysis of the maglev vehicle-bridge system should be a very meaningful work.

In the past decades, many investigators have conducted a large amount of research on the interaction between maglev vehicle and bridge. Popp and Schiehlen previously established a 2-degrees-of-freedom (2-DOFs) maglev vehicle-guideway model [2]. Assuming the car body as a rigid supported by different number of magnets (or bogies) with linear spring-dashpot systems, Cai and Chen conducted a comprehensive study on vertical dynamic responses of the maglev vehicle traveling on a single-span or double-span beam [3–5]. Yau investigated the resonance phenomenon and vibration control of the neuro-PI controller for a series of simplified 2-DOFs maglev vehicles moving on a flexible guideway [6]. He also studied the dynamic behaviors of the simple maglev vehicle-guideway system subjected to environmental excitations [7, 8]. Based on the urban transit maglev (UTM) in Korea, Min et al. simulated the EMS-type vehicle as a detailed 25-DOFs model with a rigid cabin supported by four suspension bogies to investigate the spatial vibration characteristics of the maglev vehicle traveling along the simple supported beam [9]. Xu et al. presented a framework for the dynamic analysis of high-
speed maglev trains running on elastic transitional viaducts [10]. Deng et al. used a simplified levitation force model from the experiment to couple a 6-DOFs HTS maglev vehicle with a double-span continuous girder bridge [11, 12]. These literatures above treated the guideway irregularity as a certain input, and the response results cannot reflect the random properties of the vehicle-guideway system. Then, Zhao and Zhai developed a 10-DOFs model of the maglev vehicle traveling along three types of guideways to observe the vertical random response and ride quality [13]. Shi et al. studied the dynamic response of the high-speed EMS maglev vehicle-guideway system with random irregularity [14]. Certainly, these two literatures indicate that the reaction characteristics of the maglev vehicle-guideway system to different frequencies can be obtained by using the discrete fast Fourier transform from dynamic responses. However, to date, relatively little research attention seems to have provided completely random vibration analysis of the maglev vehicle-guideway system, especially concerning vertical and lateral directions.

In this study, the proposed framework is that the random vibration model of the maglev vehicle-bridge system is decomposed into two independent parts: (i) the vehicle model with the state-space equation and (ii) the bridge model with the equation of motion. These two parts are coupled through linearized electromagnetic forces. In the vehicle model, except for assembling the equation of motion of vehicle only via the principle of virtual work, the fully computerized approach is further expanded to assemble the governing equation of fluctuating current via the equilibrium relation. In the bridge model, the equation of motion of a three-span continuous girder bridge is established using finite element methods. Based on the precise integration method and the Newmark method, a separated iteration method is introduced to solve the state-space equation for the maglev vehicle and the equation of motion of the bridge. To maximize the computational efficiency of the random vibration analysis of the maglev vehicle-bridge system, a new scheme to application of the pseudoexcitation method (PEM) in random vibration analysis is presented. Numerical simulation will indicate that the proposed framework is appropriate and efficient for the linear random vibration analysis of the 3D maglev vehicle-bridge system.

2. State-Space Equation of the 3D Maglev Vehicle

The fully computerized approach to assemble the matrices of equation of motion of the coupled vehicle-bridge system is presented by Guo and Xu [15]. In this approach, the single equation of motion of the bridge is expanded to the overall equation of motion of the coupled vehicle-bridge system, and the additional elements generated from the vehicle are automatically assembled into the corresponding position in the overall equation of motion based on the principle of virtual work. However, except for the general equations of motion from the maglev vehicle and the bridge, the coupled maglev vehicle-bridge system contains electromagnetic forces and control systems. In this study, the fully computerized approach will be further used to assemble the coefficient matrices of the electromagnetic forces. Moreover, unlike the common way of the fully computerized approach to assemble the mass matrix, damping matrix and spring matrix via the total virtual work done by the inertia forces, damping forces and spring forces, and the coefficient matrices of the governing equation of fluctuating current will also be automatically assembled row by row via the equilibrium relation. Here, the fully computerized approach will be further expanded to assemble coefficient matrices of any type of equations.

2.1. Equation of Motion of the Maglev Vehicle. As shown in Figure 1, a detailed 3D model of the EMS-type maglev vehicle with the middle-low speed consists of a cabin body and 10 levitation bogies [16]. Each bogie has 4 electromagnets and there are 40 electromagnets in all. Secondary suspensions connecting the cabin with bogies are simulated by using spring-dashpot systems. Interaction ways between left and right bogies are also simplified as spring-dashpot systems. The linking modes between secondary suspensions and the cabin have fixed sliding table and lateral sliding table. The fixed sliding table is a mechanical device of the hinge joint that can transmit translational loads, while the lateral sliding stable only transmits vertical load. The main parameters of the maglev vehicle are listed in Table 1.

The cabin and bogies in the vehicle are assumed to be rigid bodies. Each rigid body has 5-DOFs in which two translational and three rotation displacements at the geometry center of the rigid body only except for the running direction. Total DOFs of all rigid bodies in the maglev vehicle is 55. Thus, the displacement vector of the maglev vehicle is written as follows:

\[ X_m = \begin{bmatrix} y_c, z_c, \Phi_c, \varphi_c, \Theta_c, y_{b1}, z_{b1}, \Phi_{b1}, \varphi_{b1}, \Theta_{b1}, \ldots, y_{bpq}, z_{bpq}, \Phi_{bpq}, \varphi_{bpq}, \Theta_{bpq} \end{bmatrix}^T, \]  

(1)

where \( y, z, \Phi, \varphi, \) and \( \Theta \) are the vertical displacement, lateral displacement, rolling rotation, pitching rotation, and yawing rotation, respectively, the subscripts \( c \) and \( b \) represent the cabin body and levitation bogie, the value of the subscript \( p \) is selected as one for left bogies or two for right bogies, and the subscript \( q \) (\( q = 1, 2, 3, 4, 5 \)) represents the \( q \)th bogie. For instance, \( \Phi_{b24} \) is the rotation by \( x \)-axis at the 4th bogie on the right. The distribution of loads for the cabin and the first bogie is shown in Figure 2, in which the spring-damping force represents the spring force plus the damping force; \( F_{expb}, F_{expb}, F_{expb} \), and \( F_{cxpl} \) are, respectively, the longitudinal, vertical, and lateral spring-damping forces between the cabin and bogies, \( F_{byg} \) and \( F_{byg} \) are, respectively, the vertical and lateral spring-damping forces connecting the left bogie with the right left bogie, and \( \Delta F_{typb} \) and \( F_{exp} \) are, respectively, the vertical and lateral electromagnetic forces,
where the subscript \( e \) represents the electromagnet. Here, we know that \( 1 \leq l \leq 4 \), \( 1 \leq k \leq 10 \), \( 1 \leq g \leq 20 \), and \( 1 \leq r \leq 20 \).

The equation of motion of the maglev vehicle is usually described by

\[
M_m \ddot{X}_m + C_m \dot{X}_m + K_m X_m = \Delta F_m, \tag{2}
\]

in which \( M_m \), \( C_m \), and \( K_m \) are the global mass matrix, damping matrix, and stiffness matrix of the vehicle, respectively, and \( \Delta F_m \) is the force vector. The total virtual work done by the 55 inertial forces, 56 damping forces, and 56 spring forces in the maglev vehicle can help to assemble the mass matrix, damping matrix, and stiffness matrix in equation (2) by using the fully computerized approach [15].

2.2. Expression of Fluctuating Electromagnetic Force. The calculation of the nonlinear electromagnetic force mainly relates to two variables of current and air gap, of which expression is [17]

\[
F_{ey} = \frac{\mu_0 A_0 N_0^2 i^2}{4h}, \tag{3}
\]

where \( \mu_0 \) is the vacuum permeability, \( A_0 \) is the effective areas of the magnetic poles, \( N_0 \) is the number of turns of the windings, \( h \) is the vertical air gap between the electromagnet on the bogie and the guideway in the vertical direction, \( i \) is the control current in the circuit, and \( F_{ey} \) is the vertical electromagnetic force. By applying the Taylor series expansion at the balanced point for equation (3) and ignoring
the effect of the high order, the nonlinear electromagnetic force can be described by [17]  
\[ F_{e\phi} = G_0 + k_i \Delta i - k_h \Delta h, \]  
(4)  
with \( k_i = \mu_0 A_0 N^2 \phi_0^2/2l_0^2 \) and \( k_h = \mu_0 A_0 N^2 \phi_0^2/2h_0^3 \), where \( G_0 \) is the equivalent gravity of the maglev vehicle, \( i_0 \) and \( h_0 \) are desired values of control current and air gap when the maglev vehicle maintains static at the balanced point, \( \Delta i \) is the fluctuating control current, \( \Delta h \) is the fluctuating air gap in the vertical direction, and \( k_i \) and \( k_h \) are the equivalent stiffness.

In the middle-low speed maglev vehicle, the lateral guide force does not have the independent control system. The lateral guide force \( F_{e\phi pu} \) can be simplified as the following expression [9]:  
\[ F_{e\phi pu} = -k_{ez} \Delta y_{e\phi pu}, \]  
(5)  
with \( k_{ez} = 2G_0/\pi d_0 \), \( k_{ez} \) is the equivalent stiffness, \( \Delta y_{e\phi pu} \) is the relative displacement between the bogie and guideway in the lateral direction, and \( d_0 \) is the width of the electromagnet.

**Table 1: Main parameters of the maglev vehicle.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cabin body mass ((m_c))</td>
<td>kg</td>
<td>23292</td>
</tr>
<tr>
<td>Bogie mass ((m_b))</td>
<td>kg</td>
<td>846</td>
</tr>
<tr>
<td>Roll inertia of cabin body ((J_{oc}))</td>
<td>kg\cdot m(^2)</td>
<td>66800</td>
</tr>
<tr>
<td>Pitch inertia of cabin body ((J_{op}))</td>
<td>kg\cdot m(^2)</td>
<td>210000</td>
</tr>
<tr>
<td>Yaw inertia of cabin body ((J_{oz}))</td>
<td>kg\cdot m(^2)</td>
<td>193000</td>
</tr>
<tr>
<td>Roll inertia of bogie ((J_{ob}))</td>
<td>kg\cdot m(^2)</td>
<td>963</td>
</tr>
<tr>
<td>Pitch inertia of bogie ((J_{op}))</td>
<td>kg\cdot m(^2)</td>
<td>1800</td>
</tr>
<tr>
<td>Yaw inertia of bogie ((J_{oz}))</td>
<td>kg\cdot m(^2)</td>
<td>268</td>
</tr>
<tr>
<td>Longitudinal stiffness of secondary suspension ((k_{oc}))</td>
<td>N/m</td>
<td>900000</td>
</tr>
<tr>
<td>Vertical stiffness of secondary suspension ((k_{oy}))</td>
<td>N/m</td>
<td>140000</td>
</tr>
<tr>
<td>Lateral stiffness of secondary suspension ((k_{oz}))</td>
<td>N/m</td>
<td>6000</td>
</tr>
<tr>
<td>Longitudinal damping of secondary suspension ((c_{oc}))</td>
<td>N\cdot s/m</td>
<td>50000</td>
</tr>
<tr>
<td>Vertical damping of secondary suspension ((c_{oy}))</td>
<td>N\cdot s/m</td>
<td>70000</td>
</tr>
<tr>
<td>Lateral damping of secondary suspension ((c_{oz}))</td>
<td>N\cdot s/m</td>
<td>4000</td>
</tr>
<tr>
<td>Stiffness of spring dampers connecting the left and right bogies ((k_{bp}, k_{hp}))</td>
<td>N/m</td>
<td>(4 \times 10^4, 4300)</td>
</tr>
<tr>
<td>Damping of spring dampers connecting the left and right bogies ((c_{bp}, c_{hp}))</td>
<td>N\cdot s/m</td>
<td>(1 \times 10^4, 2100)</td>
</tr>
<tr>
<td>Resistance ((R_0))</td>
<td>(\Omega)</td>
<td>1.6</td>
</tr>
<tr>
<td>Air gap at balanced point ((h_0))</td>
<td>mm</td>
<td>10</td>
</tr>
<tr>
<td>Pole area ((A_0))</td>
<td>(\text{m}^2)</td>
<td>0.383</td>
</tr>
<tr>
<td>Coil turn ((N_0))</td>
<td></td>
<td>660</td>
</tr>
<tr>
<td>Vacuum permeability ((\mu_0))</td>
<td>(\text{H/m})</td>
<td>(4\pi \times 10^{-7})</td>
</tr>
<tr>
<td>Width of electromagnet ((d_0))</td>
<td>(\text{m})</td>
<td>0.164</td>
</tr>
<tr>
<td>Length of cabin</td>
<td>(\text{m})</td>
<td>16</td>
</tr>
<tr>
<td>Width of cabin</td>
<td>(\text{m})</td>
<td>2.62</td>
</tr>
<tr>
<td>Height of cabin</td>
<td>(\text{m})</td>
<td>2.9</td>
</tr>
</tbody>
</table>
For the independent vehicle model, the influence of the guideway on the vehicle is considered as the external input. From equations (4) and (5), the calculation of external forces acting on the maglev vehicle is related to the following parameters: fluctuating currents, vertical and lateral displacements of bogies, and vertical and lateral displacements of the guideway. Hence, the force vector in equation (2) can be described in the following matrix-vector form:

\[
\Delta F_m = A_m X_m + B_m \Delta I_m + F_m U + Q_m V, \tag{6}
\]

\[
\Delta I_m = \begin{bmatrix} \Delta i_1, \Delta i_2, \Delta i_3, \ldots, \Delta i_{n-1}, \Delta i_n \end{bmatrix}^T, \tag{7a}
\]

\[
U = \begin{bmatrix} u_{11}, u_{12}, u_{13}, \ldots, u_{pr} \end{bmatrix}^T, \tag{7b}
\]

\[
V = \begin{bmatrix} v_{11}, v_{12}, v_{13}, \ldots, v_{pr} \end{bmatrix}^T, \tag{7c}
\]

where \(\Delta I_m\) is the vector of fluctuating current, and \(U\) and \(V\) are the guideway displacement vectors in vertical and lateral directions (including bridge displacement and guideway irregularity), and \(\Delta i_n\) is the \(n\)th control current \((1 \leq n \leq 40)\). \(A_m, B_m, F_m,\) and \(Q_m\) are, respectively, the \(55 \times 55, 55 \times 20, 55 \times 40,\) and \(55 \times 40\) coefficient matrices of the fluctuating electromagnetic force. It has been pointed out that one bogie has four electromagnets as shown in Figure 1(c). The uniformly distributed forces between electromagnets on the bogies and ferromagnetic rails on the parallel track girders are assumed as concentrated forces. This assumption meets the requirement of computational accuracy, in which the quantity of forces between one bogie and the guideway is at least four [2].

The total virtual work of fluctuating electromagnetic forces and guide forces acting on the maglev vehicle can be computed by the following expressions:

\[
\delta W_{cyp} = \sum_{p=1}^{20} \sum_{r=2}^{20} \Delta F_{cyp} \delta \Delta h_{cyp} + \sum_{p=1}^{20} \sum_{r=2}^{20} \Delta F_{cyp} \delta \Delta h_{cyp}
\]

\[
= \sum_{p=1}^{20} \sum_{r=2}^{20} \left( k_p \Delta i_n - k_p \Delta h_{cyp} \right) \delta \Delta h_{cyp} + \sum_{p=1}^{20} \sum_{r=2}^{20} \left( k_p \Delta i_n - k_p \Delta h_{cyp} \right) \delta \Delta h_{cyp}, \tag{8a}
\]

\[
\delta W_{cpr} = \sum_{p=1}^{20} F_{cpr} \delta \Delta h_{cpr} = \sum_{p=1}^{20} \left( -k_c \Delta z_{cpr} \delta \Delta h_{cpr} \right), \tag{8b}
\]

where \(\delta W_{cyp}\) is the total virtual work of vertical electromagnetic forces, \(\delta W_{cpr}\) is the total virtual work of guide forces, \(F_{cyp}\) is the vertically fluctuating electromagnetic force, and \(\Delta h_{cyp}\) is the vertically relative displacement. After equations (8a) and (8b) are expanded, they will produce many terms and each term falls into one of the four categories: \(\delta y_p A_p y_p\), \(\delta y_p A_g f_p\), \(\delta y_p A_\eta u_p\), and \(\delta y_p A_\eta v_p\), where \(1 \leq \eta \leq 55, 1 \leq \gamma \leq 55, 1 \leq \mu \leq 20,\) and \(1 \leq \chi \leq 40.\) Here, \(y_p\) is the \(\eta\)th element of \(X_m\), \(\delta y_p\) is the \(\chi\)th element of \(X_m, u_p\) is the \(\mu\)th element of \(\Delta I_m\), and \(u_p\) and \(u_p\) are the \(\chi\)th elements of \(U\) and \(V\), respectively. All these coefficients \(A_A, A_b, A_c,\) and \(A_d\) are added to the corresponding positions of the coefficient matrices \(A_m, B_m, F_m,\) and \(Q_m\) under the help of the guide vectors, respectively. For instance, the coefficient \(A_A\) will be added in the position of the matrix \(A_m\) at the \(y\)th row with the \(n\)th column.

2.3 Governing Equation of Fluctuating Current. The EMS-type maglev vehicle cannot remain stable without the control law [18]. The control law is very important in the ride quality of the maglev vehicle traveling along the guideway. The fluctuation of current in equation (4) mainly depends on control voltage. The relationship between current and voltage can be described as [19]

\[
\Delta i_n = \frac{L}{h_0} \Delta h_{en} - \frac{R}{L} \Delta i_n + \frac{1}{L} \Delta U_{en}, \tag{9}
\]

\[
L = \frac{h_0 N^2 A_0}{2\eta_0}, \tag{10}
\]

where \(L_0\) is the inductance of the magnet winding, \(\Delta h_{en}\) is the first-order derivative of the vertical air gap related to the \(n\)th control current, and \(\Delta U_{en}\) is the \(n\)th control voltage. In order to maintain the maglev vehicle moving over the guideway stably, the linear negative feedback control is used to get the reliable voltage. The control voltage expression is obtained by

\[
\Delta U_{en} = -k_v \Delta h_{en} - k_d \Delta h_{en}, \tag{11}
\]

where \(k_v, k_d,\) and \(k_d\) are the feedback gain coefficients of acceleration, velocity, and displacement, respectively, and \(\Delta h_{en}\) and \(\Delta h_{en}\) are the vertical acceleration and air gap related to the \(n\)th control current, respectively. For instance, \(\Delta h_{en}\) is equal to \((\Delta h_{cyp} + \Delta h_{cyp})/2).\) Here, the important function of the feedback control law is to compensate deviation against the target value. For the maglev vehicle-vehicle system in this paper, the control voltage is adjusted by using the feedback control law to ensure that the maglev vehicle fluctuates around the equilibrium point. If the air gap between the bogie and the guideway is less than 10 mm, the larger control voltage from the feedback control law produces the larger control current to obtain sufficient electromagnetic force. Thus, the bogie can move toward the equilibrium point. In practice, each variable of motion can be measured by using sensors on the bogies.

Hence, we know that the first-order derivative of control current concerns with the vertical displacement, velocity, and acceleration, and the control current of the vehicle, as well as the displacement and velocity of the guideway from equations (9) and (11). The general expression of the governing equation of fluctuating current can be formulated as the following matrix form:

\[
\Delta I_m = D_m \Delta X_m + J_m \Delta X_m + O_m X_m + G_m \Delta I_m + H_m U + S_m \Delta U, \tag{12}
\]

where \(D_m, J_m, O_m, G_m, H_m,\) and \(S_m\) are the \(20 \times 55, 20 \times 55,\)

\(20 \times 55, 20 \times 20, 20 \times 40,\) and \(20 \times 40\) coefficient matrices of the governing equation of the fluctuating current,
respectively. Substituting equation (11) into equation (9), the governing equation of the \( n \)th fluctuating current can be described by
\[
\Delta i_n = -\frac{k_1}{L_0} \Delta y_n + \left( \frac{i_0}{h_0} - \frac{k_2}{L_0} \right) \Delta h_m - \frac{k_3}{L_0} \Delta h_m - \frac{R_0}{L_0} \Delta i_n. \tag{13}
\]

Expanding the expression above, we know that each item should belong to one of following six categories: \( A_{1j} \Delta y_n, A_{1j} \Delta y_n, A_{1j} \Delta y_n, A_{1j} \Delta h_m, A_{1j} \Delta h_m, \) and \( A_{1j} \Delta h_m. \) Finally, all these coefficients from six types of items are added to the corresponding positions in the \( n \)th row of \( D_m, J_m, O_m, G_m, H_m, \) and \( S_m, \) respectively.

Although the maglev vehicle is a complicated multirigid-body dynamic system with 55-DOFs and 20 control currents, the coefficient matrices for the equation of motion of the vehicle and the governing equation of fluctuating current in the computer program can be assembled conveniently by using the fully computerized approach. Apart from these familiar parameters such as dynamic properties, positions of all the rigid bodies and springs and damping devices, and control parameters of electromagnetic forces, the required input data about the vehicle to the computer program also include the guide matrices, which help the generating elements get into corresponding positions of the coefficient matrices. For different types of the maglev vehicle, the fully computerized approach to assemble the matrix can quickly form the equation of motion and the governing equation of fluctuating current after the required input data.

### 2.4. State-Space Equation of the Maglev Vehicle

The equation of motion of the vehicle is a second-order differential equation, while the governing equation of fluctuating current is a first-order differential equation. The coupled relationships exist between these two equations. In this study, a state-space equation of the vehicle is derived to solve these two equations. The 55-DOFs vehicle has 130 state variables including the 55 displacements and 55 velocities of the vehicle and 20 fluctuating currents, which can be expressed as
\[
X = [X_m, X_m, \Delta I_m]^T. \tag{14}
\]

The dynamic behaviors of the 3D maglev vehicle system containing the maglev vehicle model, control electromagnetic forces, and control currents are coupled and described in the linear state form as follows:

\[
\dot{X} = AX + BU + CU + DV, \tag{15}
\]

\[
A = \begin{bmatrix}
0 & 1 \\
-M_m^{-1}C_m & -M_m^{-1}B_m \\
O_m - D_mM_m^{-1}K_m + D_mM_m^{-1}A_m & J_m - D_mM_m^{-1}C_m \end{bmatrix}, \tag{16}
\]

\[
B = \begin{bmatrix}
0 \\
M_m^{-1}F_m \\
H_m + D_mM_m^{-1}F_m
\end{bmatrix}, \tag{17}
\]

\[
C = \begin{bmatrix}
0 \\
0 \\
S_m
\end{bmatrix}, \tag{18}
\]

\[
D = \begin{bmatrix}
0 \\
M_m^{-1}Q_m \\
D_mM_m^{-1}Q_m
\end{bmatrix}. \tag{19}
\]

### 3. Equation of Motion of the Three-Span Space Continuous Girder Bridge

The guideway model takes a real three-span space continuous girder bridge as shown in Figure 3. The continuous girder bridge has a total length of 280 m with the main span measured at 110 m and the two side spans of 85 m. It is also a single-box and single-cell concrete bridge. The midsection of the midspan is illustrated in Figure 4, and the detailed size of the cross section can be seen in Reference [20]. Young’s modulus and Poisson’s ratio of the bridge are 3.55e10 Pa and 0.3, respectively. A 3D dynamic finite element model is established by using 3D Euler–Bernoulli beam elements [21]. In the model, track girders on the bridge are considered, and F-shape rails used to support the vehicle are neglected because of their slight stiffness compared to the continuous girder bridge. Therefore, the equation of motion of the bridge is described as
\[
M_{br} \ddot{Y} + C_{br} \dot{Y} + K_{br} Y = F_{br}, \tag{20}
\]

where \( Y \) is the displacement vector of the bridge, \( M_{br} \) and \( K_{br} \) is the mass and stiffness matrices of the bridge, \( C_{br} \) is the...
Rayleigh damping matrix, and the damping ratio of the bridge is taken as 0.03, and $F_{br}$ is the time-dependent vector from the electromagnetic forces acting on the bridge:

$$F_{br} = N_1 (G_{br} + k_x N_3 X - k_y U) + k_x N_2 (N_2 X - V),$$

(21)

where $N_1$ and $N_2$ are the function matrices to solve the node forces of the bridge from the vertical electromagnetic forces and the lateral guide forces, $N_3$ is the shape function matrix to solve the fluctuating currents, and $N_4$ and $N_5$ are the function matrices to solve the vertical and lateral displacements of the electromagnets. $G_{br}$ is the vector of the equivalent gravity and its value of each element is equal to $G_0$.

4. Pseudoexcitation Method

4.1. Proposed Scheme for the PEM. The nonlinear maglev vehicle-guideway system has been changed into a linear system to conduct random vibration analysis using the PEM. From the expression of electromagnetic force in equation (4), the maglev vehicle-bridge system contains the random input from the guideway irregularity and the deterministic input from the gravity of the maglev vehicle. Here, the response results for the deterministic input are treated as the mean values of stochastic behaviors by adopting time-history analysis [22]. For the random input of the dynamic system, its self-power spectral density $S_{xx}(w)$ needs to be converted into the pseudoexcitation, which can be described as [22]

$$\bar{x}(t, w) = \sqrt{S_{xx}(w)} e^{j\omega t},$$

(22)

where $i = \sqrt{-1}$ and $e^{j\omega t}$ is a series of harmonic waves at given discrete frequencies. It should be emphasized that $w$ is the spatial angular frequency of guideway irregularity and $v$ is the traveling speed of the maglev vehicle. The whole computation of the random vibration using the PEM is carried out in the complex field. Then, by using the response result multiplied by its own conjugate complex, the self-power spectral density of the desired system response is expressed as the following form [23]:

$$\bar{y}(t, w) \cdot \bar{y}^*(t, w) = H(w)\sqrt{S_{xx}(w)} e^{j\omega t} \cdot e^{-j\omega t} \sqrt{S_{xx}(w)} H^*$$

$$\cdot (w) = |H(w)|^2 S_{xx}(w) = S_{yy}(t, w).$$

(23)

From equations (22) and (23), the dynamic system has two major variables of time and frequency during computation. The conventional scheme adopted by much more research is to conduct a complete time-history analysis under each one of discrete frequencies [24–28]. The total number of independent computations for this way in the dynamic system reaches $N_w \times N_t$, where $N_w$ and $N_t$ are the numbers of discrete frequency and time, respectively. This leads to numerous repetitive calculations, especially for the required huge time cost to solve inverse matrices at each time step. Thus, the pseudoexcitation of the maglev vehicle-bridge system should be expressed as

$$\bar{X}(t) = \begin{bmatrix} \sqrt{S_{xx}(w_{N_1})} e^{j\omega_{N_1}(\tau - D_{pr})} \\ \sqrt{S_{xx}(w_{N_2})} e^{j\omega_{N_2}(\tau - D_{pr})} \\ \vdots \\ \sqrt{S_{xx}(w_{N_p})} e^{j\omega_{N_p}(\tau - D_{pr})} \end{bmatrix},$$

(24)

where $D_{pr}$ is the longitudinal distance from the first electromagnetic force to the $r$th electromagnetic force. In this
study, a new scheme that all discrete frequencies are computed simultaneously under each time step is proposed to further manifest the advantage of the computational efficiency of the PEM. The proposed scheme can be achieved because there is no interaction relationship between any two discrete frequencies. Thus, any repetitive calculations cannot exist in the computer program, so the computational efficiency of the PEM is maximized by using the proposed idea. The advantage of the proposed scheme will be illustrated in the following numerical simulation. Here, for the maglev vehicle-bridge system, the pseudoexcitation cannot be a vector as shown in equation (24) but a matrix expressed as

\[
\mathbf{X}(t) = \begin{bmatrix}
S_{xx}(w_1) e^{jw_1(\tau - D_{11})} & S_{xx}(w_2) e^{jw_2(\tau - D_{12})} & \cdots & S_{xx}(w_{N_v}) e^{jw_{N_v}(\tau - D_{1N_v})} \\
S_{xx}(w_1) e^{jw_1(\tau - D_{21})} & S_{xx}(w_2) e^{jw_2(\tau - D_{22})} & \cdots & S_{xx}(w_{N_v}) e^{jw_{N_v}(\tau - D_{2N_v})} \\
\vdots & \vdots & \ddots & \vdots \\
S_{xx}(w_1) e^{jw_1(\tau - D_{N_v1})} & S_{xx}(w_2) e^{jw_2(\tau - D_{N_v2})} & \cdots & S_{xx}(w_{N_v}) e^{jw_{N_v}(\tau - D_{N_vN_v})}
\end{bmatrix},
\]  

(25)

where \( \mathbf{R}_{up} \) and \( \mathbf{R}_{vp} \) are the vertical and lateral pseudoexcitations of guideway irregularities. For instance, if \( p = 1 \) indicates that the vertical pseudoexcitation \( \mathbf{R}_{up} \) is on the left. We know that \( \mathbf{R}_v = [\mathbf{R}_{v11}, \mathbf{R}_{v12}]^T \) and \( \mathbf{R}_u = [\mathbf{R}_{u11}, \mathbf{R}_{u12}]^T \).

The vectors of \( \mathbf{U} \) and \( \mathbf{V} \) are the vertical and lateral shape functions of the bridge. The first derivative of \( \mathbf{U} \) in equation (15) is computed by

\[
\mathbf{U} = \nabla N_6 \mathbf{Y} + \mathbf{R}_u + j\omega \mathbf{R}_u.
\]  

(32)

4.2. PSD of Guideway Irregularity. Many researches have shown that the guideway irregularity is one of the main reasons that induce random vibration of the vehicle and guideway [29]. Frequency characteristic of guideway irregularity can be expressed by the PSD function. However, to date the PSD function specialized for the maglev vehicle guideway irregularity has not been measured. As a result, the PSD function proposed by the Federal Railroad Administration is adopted to describe the spatial irregularity characteristics of two parallel F-shape rails, which have four types of track irregularities: elevation irregularity, superelevation irregularity, alignment irregularity, and gauge irregularity [30, 31]. They are expressed in the following forms:

\[
S_1(w) = \frac{A_1 \Omega_1^2}{(w^2 + \Omega_1^2)(w^2 + \Omega_2^2)},
\]

(26)

\[
S_2(w) = \frac{A_1 b_1^2 \Omega_1^2 \omega^2}{(w^2 + \Omega_1^2)(w^2 + \Omega_2^2)(w^2 + \Omega_3^2)},
\]

(27)

\[
S_3(w) = \frac{A_2 \Omega_2^2}{(w^2 + \Omega_1^2)(w^2 + \Omega_3^2)},
\]

(28)

\[
S_4(w) = \frac{A_3 \Omega_3^2 \omega^2}{(w^2 + \Omega_2^2)(w^2 + \Omega_3^2)},
\]

(29)

where \( \Omega_1 = 0.8246 \text{ rad/m} \), \( \Omega_2 = 0.0206 \text{ rad/m} \), and \( \Omega_3 = 0.438 \text{ rad/m} \) are the cutoff frequencies, \( A_1 = 1.5 \times 10^{-7} \text{ m}^2/\text{rad/m} \), \( A_2 = 0.6 \times 10^{-7} \text{ m}^2/\text{rad/m} \), and \( A_3 = 14 \times 10^{-8} \text{ m}^2/\text{rad/m} \) are the roughness constants, \( b_1 \) is the half of the distance between two parallel F-shape rails, and the value of \( \omega \) ranges from 0.314 rad/m to 6.28 rad/m. Based on the elevation irregularity, superelevation irregularity, alignment irregularity, and gauge irregularity, the pseudoexcitations of vertical and lateral space guideway irregularities can be described as

\[
\mathbf{R}_{up} = \mathbf{X}_1(t) + (-1)^p b_1 \mathbf{X}_2(t),
\]

(30a)

\[
\mathbf{R}_{vp} = \mathbf{X}_3(t) - \frac{(-1)^p}{2} \mathbf{X}_4(t),
\]

(30b)
where $F_{m,t+\Delta t}$ is the external input of the state-space equation and denoted by
\[ F_{m,t+\Delta t} = B\left(N_{m,t+\Delta t}/Y_{t+\Delta t} + \tilde{R}_{m,t+\Delta t}\right) + C\left(\frac{\partial N_{m,t+\Delta t}/Y_{t+\Delta t}}{\partial x}\right)Y_{t+\Delta t} + N_{s,t+\Delta t}/Y_{t+\Delta t} + i\omega v\tilde{R}_{m,t+\Delta t} + D\left(N_{s,t+\Delta t}/Y_{t+\Delta t} + \tilde{R}_{m,t+\Delta t}\right). \]

Equivalent forces in equation (33b) yield
\[ F_{br,t} = C_{br}\left(\frac{2Y_{t}}{\Delta t} + \dot{Y}_{t}\right) + M_{br}\left(\frac{4Y_{t}}{\Delta t^{2}} + \frac{4\ddot{Y}_{t}}{\Delta t} + \dot{Y}_{t}\right), \]

\[ F_{br,t+\Delta t} = N_{t+\Delta t}(\dot{Y}_{t} + k_{i}N_{t}/X_{t+\Delta t} + k_{h}N_{t}/Y_{t+\Delta t} + k_{l}N_{t}/X_{t+\Delta t} - N_{t+\Delta t}/Y_{t+\Delta t} - N_{t+\Delta t}/\tilde{Y}_{t+\Delta t}}), \]

Figure 5: RMSs of vertical displacements for (a) the cabin and (b) the bridge midpoint when the maglev vehicle with one car moves on the bridge at the speed of 100 km/h.

Figure 6: Mean values of vertical accelerations for (a) the second cabin and (b) the bridge midpoint.
where \( Y_t, \hat{Y}_t, \text{ and } \check{Y}_t \) are the computing results at time \( t \) and their values are \( j^{+1}Y_t, j^{+1}\hat{Y}_t, \text{ and } j^{+1}\check{Y}_t \), respectively. The equivalent stiffness in equation (33b) is given as

\[
K = K_{br} + \frac{2C_{br}}{\Delta t} + \frac{4M_{br}}{\Delta t^2}.
\]  

(37)

During the separated iteration, the initial iterative displacement of the bridge \( Y_{t+\Delta t}^{\text{br}} \) is selected as \( Y_t \), and the first-order derivative of \( j^{+1}Y_{t+\Delta t} \) in equation (34) is described by

\[
j^{+1}\check{Y}_{t+\Delta t} = -2(Y_{t+\Delta t} - Y_t) - \check{Y}_t.
\]  

(38)

After the iteration steps terminate at each time step, the acceleration of the bridge can be calculated as

\[
j^{+1}\ddot{Y}_{t+\Delta t} = M^{-1}\left(j^{+1}F_{br,t+\Delta t} - C_{br,t+\Delta t}\dot{Y}_{t+\Delta t} - K_{br,t+\Delta t}Y_{t+\Delta t}\right).
\]  

(39)

When the bridge displacements between the current iteration step and the previous iteration step almost are the
same, the iterations at the time step will be ended. The convergence condition is

$$\max\left(\max\left(\left|y_{t+\Delta t}^{i+1} - y_{t+\Delta t}^i\right|\right) > 10^{-10}\right).$$ (40)

When the pseudoexcitation of the guideway irregularities $\tilde{R}_{u,t}$ and $\tilde{R}_{v,t}$ in equations (34) and (35) is set as zero vectors, equations (33a) and (33b) can be used to solve the mean values of random responses only considering the influence of the gravity on the maglev vehicle-bridge system by conducting time-history analysis. Moreover, when the deterministic input $G_{br}$ is selected as the zero matrix, equations (33a) and (33b) are also used to solve dynamic responses of the maglev vehicle-bridge system subjected to random input of guideway irregularity by using the PEM. As a result, only the time-history analysis seems to appear in these two types of numerical simulations. The computer program is very convenient for both the time-history analysis and the random vibration analysis, and the required change in the program is very small when either of these two analyses is conducted. It can also maximize the computational efficiency of the PEM in the random vibration analysis of the maglev vehicle-bridge system.

6. Numerical Simulations

6.1. Computational Efficiency of the Proposed Scheme. To verify the computational efficiency and correctness of the proposed scheme, the solution to the RMS of the random vibration response for a maglev vehicle with one car traveling along the continuous girder bridge is conducted by using three methods, i.e., the proposed PEM-based scheme, the conventional PEM-based scheme, and the Monte Carlo (MC) method [34]. Major computational
procedures for the PEM using the proposed scheme or the conventional scheme are illustrated in the previous Sections 4 and 5. The MC method is to statistically analyze response results from substantial random samples. The accuracy of the MC method for random vibration analysis increases with the number of samples. In this study, all calculations are conducted on a personal computer with Intel (R) Core (TM) i7-8700 processors. Figures 5(a) and 5(b) show the time-history RMSs of the vertical displacements for the cabin and the bridge midpoint. The proposed scheme, the conventional scheme, and the MC method with the guideway irregularity samples of 500 consume 1,120 s, 9,487 s, and 15,707 s for computing the RMSs of dynamic responses of the vehicle-bridge system, respectively. Also, the proposed scheme and the conventional scheme have the same computational results, in which the stable RMS curves have higher precision than the fluctuating curves of the MC method with 500 samples. Consequently, the PEM using the proposed scheme for the linear random analysis of the maglev vehicle-bridge system is very efficient compared with the conventional scheme and the MC method.

6.2. Mean Value. Figures 6(a) and 6(b) show the mean values of vertical accelerations for the second cabin and the bridge midpoint. It should be noted that the random vibration analysis of the maglev vehicle in this paper is mainly conducted under the case of the maximum commercial operating speed of 100 km/h. Here, the mean values of lateral accelerations for the vehicle and bridge are not given because their lateral responses are smaller without specific

Figure 10: Time-dependent PSDs of the bridge midpoint acceleration for (a) vertical and (b) lateral directions.
excitations. Major interaction between the maglev vehicle and three-span continuous girder bridge is the vertical motions when the guideway surface is smooth. When $t = 2.39$ s, the second car in the maglev vehicle begins to move on the bridge. The significant response of the second cabin focuses on the case of the vehicle moving on the bridge, especially for the vehicle on the bridge midpoints of side spans and main span, while the effects on the three-span continuous girder bridge mainly concentrate on the main span. The maximum values of vertical accelerations for the second cabin and bridge midpoint are 0.16 m$^2$/s and 0.09 m$^2$/s. Besides, the negative feedback control in the maglev vehicle is a reasonable design because the oscillation in the vertical acceleration curve of the cabin can reduce to zero rapidly after the second car leaves the bridge at $t = 13.05$ s.

6.3. $3\sigma$ Rule. With the assumption that the random process of guideway irregularity is a Gaussian distribution, the $3\sigma$ rule can be used to estimate upper or lower boundaries of response, and its accuracy rate has 99.73%. Figures 7(a) and 7(b) show the estimated boundaries for vertical and lateral accelerations of the cabin, and response results of any arbitrary two irregularity sample samples stay within the scope of the boundaries except for very few points. Thus, the extreme value of response that is of most interest in practical engineering can be picked from its corresponding boundary. When the vehicle moves on the bridge, the maximum vertical and lateral accelerations of the cabin can get to 0.32 m$^2$/s and 0.03 m$^2$/s, respectively. Moreover, the curve of the lateral acceleration of the cabin shown in Figure 7(b) appears to be low fluctuation, and this is a clear demonstration that lateral responses of the maglev vehicle mainly come from the influence of the guideway irregularity. By comparing with accelerations of the second cabin in Figures 6(a) and 7(a), it can also be found that the random irregularity still has great influence on vertical motion of the vehicle. Certainly, response accelerations of the bridge midpoint for any two irregularity sample curves also almost stay within boundaries estimated using the $3\sigma$ rule as shown in Figures 8(a) and 8(b). Maximum vertical and lateral accelerations of the bridge midpoint are 0.11 m$^2$/s and 0.03 m$^2$/s, respectively.

6.4. Time-Dependent PSD of Dynamic Response. Time-dependent PSDs of vertical and lateral accelerations for the second cabin are given in Figures 9(a) and 9(b). When the maglev vehicle begins to move on the ground, PSD of the cabin rapidly enlarge around zero frequency. There are a series of peaks distributed in the frequency range of 0–2.2 rad/m for the PSD of the cabin vertical acceleration (see Figure 9(a)) and then they slowly diminish with the increasing frequency. Here, the maximum peak appears at the frequency 0.16 rad/m. On the contrary, the influence of frequency on the lateral acceleration of the cabin is concentrated in the range of 0–0.52 rad/m.

Displayed in Figures 10(a) and 10(b) are the time-dependent PSDs of bridge midpoint accelerations for vertical and lateral directions. Before the maglev vehicle reaches the bridge, the vertical and lateral PSDs of the bridge midpoint acceleration are zeros. With the increase of moving time of the vehicle on the bridge, the PSD of the vertical acceleration for the bridge midpoint appears some peaks and valleys in the frequency range of 0.2–0.8 rad/m, in which major frequencies concentrate on 0.24 and 0.66 rad/m. For the lateral acceleration of the bridge midpoint, the effect of frequency mainly focuses on 0.38 rad/m (see Figure 10(b)).

7. Conclusions

This study has been presented a framework for the linear random vibration analysis of a coupled 3D maglev vehicle-bridge system. The framework coupled the motion of the maglev vehicle, the control system, the dynamic behavior of the bridge, and the guideway irregularity. The proposed scheme has maximized the advantage of the PEM in the computational efficiency of the random vibration analysis. In addition, any type of matrices in the maglev vehicle-bridge system can be assembled automatically by using the fully computerized approach. It is very useful for solving the dynamic problem of various maglev vehicle-bridge models.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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