Research Article

Pounding between Adjacent Frame Structures under Earthquake Excitation Based on Transfer Matrix Method of Multibody Systems

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In this paper, the case of two adjacent frame structures is studied by establishing a mechanical model based on the transfer matrix method of multibody system (MS-TMM). The transfer matrices of the related elements and total transfer equation are deduced, combining with the Hertz-damp mode. The pounding process of two adjacent frame structures is calculated by compiling the relevant MATLAB program during severe ground motions. The results of the study indicate that the maximum error of the peak pounding forces and the peak displacements at the top of the frame structure obtained by the MS-TMM and ANSYS are 6.22% and 9.86%, respectively. Comparing the calculation time by ANSYS and MS-TMM, it shows that the computation efficiency increases obviously by using the MS-TMM. The pounding mainly occurs at the top of the short structure; meanwhile, multiple pounding at the same time may occur when the separation gap is small. The parametric investigation has led to the conclusion that the pounding force, the number of poundings, the moment of pounding, and the structural displacement are sensitive to the change of the seismic peak acceleration and the separation gap size.

1. Introduction

Pounding will occur when the relative displacement of adjacent buildings is greater than the width of their separation gaps under the excitation of earthquake; it will directly affect the failure mode and the degree of damage of the structure. During the 1985 Mexico City earthquake, about 40% of damaged structures were subjected to pounding and about 15% of buildings collapsed due to collision [1]. In the 1989 Loma Prima earthquake, there were about 200 impact events in San Francisco, Oakland, Santa Cruz, and Watsonville involving more than 500 buildings [2]. In the 1995 Hanshin earthquake, the 2008 Wenchuan earthquake, and the 2010 Yushu earthquake, a considerable part of the damage caused by structural pounding was discovered [3–5]. In the past, many countries did not specify the setting of the separation gap that results in the distance between adjacent buildings being very close to or even zero in many buildings, which may lead buildings collide with each other under the excitation of earthquakes. At present, most countries have established regulations on separation gaps, but the pounding may occur in the event of a rare earthquake [6]. Therefore, the pounding of structures during the earthquake has attracted more and more attention from earthquake-resistant workers, and they have carried out many related studies [7].

Nowadays, there are two main methods for structural pounding research: the classical contact method and the contact force-based method. The classical method is based on the momentum conservation of the system and the Newtonian velocity recovery coefficient [8]. Papadrakakis et al. [9], Desroches and Muthukumar [10], and Mahmoud et al. [11] analyzed the structural pounding problem based on the classical method; the application of this method is greatly limited because it cannot reflect the impact factors such as impact force and impact deformation and is not easy to combine with the existing software. An analytical technique based on the contact force-based method is developed, where the contact element is activated when the structures come into contact. The contact element uses an equivalent spring element and an equivalent damper element to simulate the interaction and energy dissipation during the collision which is placed in the event of a pounding and is withdrawn when disengaged. Scholars have conducted extensive researches on the contact force-based method, such
as the linear spring pounding model which uses only one spring element to simulate structural pounding [12]. In order to reflect the nonlinear process of pounding, the Hertz model was used to simulate structural pounding responses [13, 14]. In order to further accurately simulate the structural collision response, the Kelvin model [15] and the Hertz-damp model [16, 17], which can consider the energy consumption of pounding, have been proposed and applied.

A number of researchers have studied the problem of pounding for adjacent structural under earthquake: Efraidmiadou et al. [18, 19] performed seismic pounding response analysis of the different layer height structures. Naderpour et al. [20] studied the case of pounding between two adjacent buildings by the application of single degree-of-freedom structural model. Ghandil and Aldaikh [21] developed a series of SSSI models to accurately study the problem of SSSI-included pounding of two adjacent buildings. Karayannis and Naoum [22] investigated the influence of two adjacent structures with different stories, different layouts, and initial distance on torsion under earthquake. Furthermore, some more recent numerical analyses have been carried out to study the influence of different parameters in pounding of buildings [23, 24].

The multibody system is a system in which a number of rigid and flexible bodies are connected in some way. The current various multibody system dynamics methods have the following common features: it is necessary to establish the global dynamics equations of the system; the global dynamics equation of the complex system involves high-order matrices and makes the computational workload large. Rui et al. established the transfer matrix method of multibody system (MS-TMM) by combining transfer matrix method with modern calculation method in 1993 [25], which has the advantages as follows: without the system global dynamics equations, high programming, low order of system matrix, and high computational efficiency [26]. The MS-TMM was mainly divided into the transfer matrix method for linear multibody systems [27] and discrete time transfer matrix method [28]. The discrete time transfer matrix method is suitable for linear time-varying, nonlinear, large-motion, and general multibody systems. In the field of civil engineering, some applications have been carried out on the application of the MS-TMM. For example, Ding et al. applied this method to the vibration analysis of building structures and the dynamic response analysis of structures under earthquake action. They have studied new single-story frame bent structures [29], portal frame structures [30], frame structures [31], reinforced concrete shear wall structures [32], etc. The results show that the MS-TMM calculation results are similar to the finite element calculation results, and the calculation efficiency is significantly improved.

Many scholars have studied the pounding problem of adjacent structures under earthquakes. However, most of them use finite element software for simulation calculation, such as ANSYS, ABAQUUS, LUSAS, and DRAIN-2DX, which has a large computational workload and is time-consuming. Therefore, it is an important research direction to seek a highly efficient calculation method. In this paper, the MS-TMM is introduced into the study of pounding between two adjacent frame structures under the excitation of earthquake. First, the appropriate pounding model is selected. Next, the mechanical model is established based on MS-TMM. Then, the transfer matrices of elements and the total transfer equation of the structure are derived. Finally, the corresponding MATLAB program is compiled to analyze the influence of the separation gap size and the peak seismic acceleration on pounding process of two unequal adjacent frame structures during severe ground motions. Meanwhile, the results calculated by MS-TMM and ANSYS are compared further.

**2. Pounding Model Selection**

Based on the contact force-based method, scholars have conducted extensive research on structural pounding problems; the structural pounding analysis model is shown in Figure 1, where \( m_1 \) and \( m_2 \) represent the masses of the two colliding bodies, \( u_1 \) and \( u_2 \) are the corresponding displacements, \( v_{10} \) and \( v_{20} \) are the speeds of the initial contact moments of the two colliding bodies, respectively, \( v_1 \) and \( v_2 \) are the corresponding speeds of the colliding bodies at separation moment, \( k \) represents the stiffness coefficient of the contact unit, \( c \) is the damping coefficient, and \( g_p \) is the initial gap.

When different pounding models are used to simulate the structural pounding process, the force-deformation relationship expressions of the contact elements are as follows.

1. **Linear elastic model** [12]:
   \[
   F_c = \begin{cases} 
   k \delta & \delta > 0 \\
   0 & \delta \leq 0
   \end{cases}, \quad \delta = u_1 - u_2 - g_p, \quad (1)
   \]
   where \( F_c \) is the pounding impact force and \( \delta \) is the relative deformation of two colliding bodies during contact.

2. **Hertz model** [13, 14]:
   \[
   F_c = \begin{cases} 
   k \delta^{1.5} & \delta > 0 \\
   0 & \delta \leq 0
   \end{cases}, \quad \delta = u_1 - u_2 - g_p \quad (2)
   \]

3. **Kelvin model** [15]:
   \[
   F_c = \begin{cases} 
   k \delta + c \dot{\delta} & \delta > 0 \\
   0 & \delta \leq 0
   \end{cases}, \quad \dot{\delta} = u_1 - u_2,
   \]
   \[
   c = 2\zeta \sqrt{k \left( \frac{m_1 m_2}{m_1 + m_2} \right)}, \quad (3)
   \]
   \[
   \zeta = \frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}}, \quad e = \frac{v_1 - v_2}{v_{10} - v_{20}}.
   \]
where $\dot{\delta}$ is the relative deformation speed of the collision body during the pounding, $\zeta$ is the corresponding damping ratio, $\dot{u}_1$ and $\dot{u}_2$ are the derivatives of $m_1$ and $m_2$ for time $t$, and $e$ is the Newtonian speed recovery coefficient before and after pounding.

(4) Hertz-damp model [16, 17]:

$$F_e = \begin{cases} \kappa \delta^{1.5} + c \delta & \delta > 0 \\ 0 & \delta \leq 0 \end{cases},$$

$$c = \lambda \delta^{1.5},$$

$$\lambda = \frac{3k(1-e^2)}{4(\nu_{10} - \nu_{20})}.$$

where $\lambda$ is the hysteresis damping coefficient, $k$ is the contact stiffness, reinforced concrete is usually taken as $2.0 \times 10^6 \text{kN/m}^{3/2}$, and $e$ represents the energy recovery coefficient, which is usually taken as 0.65 in the typical concrete structure.

The linear spring model cannot simulate the energy dissipation and the changes in local stiffness with compression. The Kelvin model also cannot represent changes in the compression stiffness of the pounding; however, when pounding is from the maximum compression to the uncompressed regression, the Kelvin model’s simulated pounding force appears as a tensile force for a period of time before the movement is about to come out of contact, which is inconsistent with the facts. The Hertz model can simulate changes in compression stiffness, but does not represent energy dissipation during pounding. The Hertz-damp model improves this shortcoming of the Hertz model; therefore, this paper selects the Hertz-damp model.

3. Mechanical Model of a Frame Structure

In this paper, a frame structure is used as a multibody system. Since the planar arrangement of the frame structure is generally regular and symmetrical, it is simplified into a planar structure for calculation and analysis. Model simplification concerns three parts: the first part is the simplification of the column, based on the discretization idea, the column is equivalent to several concentrated masses connected by vertical elastic beams; the second part is the beam-column joint, which is equivalent to a plane hinge unit; and the third part is the connection of the plate and the beam, which is simplified as a transverse elastic beam connected to the concentrated mass. The self-weight and external load of the structure are applied to the concentrated mass, ignoring the axial deformation of the elastic beam, only considering the lateral deformation. In summary, the mechanical model of the frame structure is shown in Figure 2.

4. Derivation of Element Transfer Matrix

In this paper, the discrete time transfer matrix method is used to analyze the pounding response of two structures under the excitation of earthquake. For time-varying systems, there is no linear relationship between state vectors; so, we need to introduce a linearization idea to linearize the state vector in order to transfer the force and displacement between points. When the time step is small to a certain extent, the relationship between physical parameters can be approximated, seen as linear in the physical process corresponding to each time step. In this paper, the linearization of acceleration and velocity is performed by the Newmark-$\beta$ method [33]; the relationship between velocity, acceleration, and displacement is as follows:

$$\begin{align*}
\dot{x}_{t+\Delta t} &= Ax_{t+\Delta t} + B, \\
\ddot{x}_{t+\Delta t} &= Cx_{t+\Delta t} + D,
\end{align*}$$

where

$$A = \frac{1}{\beta \Delta t^2},$$

$$B = A[-x(t_{i-1}) - x'(t_{i-1})\Delta t - (0.5 - \beta)x''(t_{i-1})\Delta t^2],$$

$$C = \frac{\gamma}{\beta \Delta t^2},$$

$$D = x'(t_{i-1}) - (1 - \gamma)x''(t_{i-1})\Delta t + \gamma B \Delta t.$$

When calculating the pounding response of two frame structures under earthquake, it is necessary to consider the seismic excitation and pounding force; therefore, the concept of extending the transfer matrix is introduced here to distinguish the transfer matrix based on the linear multibody system transfer matrix method and the
The state vector of elements in physical coordinates is defined as
\[ z = [x, y, \theta_z, m_z, q_x, q_y]^T, \tag{7} \]
where \( x \) is the displacement in \( x \) direction, \( y \) is the displacement in \( y \) direction, \( \theta_z \) is the angular displacement around \( z \), \( m_z \) is the internal moment, \( q_x \) is the internal force in \( x \) direction, and \( q_y \) is the internal force in \( y \) direction. The external force is \( f_{x,c} = -m \ddot{x}_I - c \dot{x}_I - k x_I \).

4.1. Concentrated Mass at One Input End and One Output End. The concentrated mass of mass \( m \) is shown in Figure 3(a), the input end is \( I \) and the output end is \( O \). According to its stress balance and deformation relationship, we have
\[
\begin{align*}
x_O &= x_I, \\
y_O &= y_I, \\
\theta_{z,O} &= \theta_{z,I}, \\
m_{z,O} &= m_{z,I}, \\
q_y,O &= q_y,I, \\
q_{x,O} &= q_{x,I} + f_{x,c} - m \ddot{x}_I - c \dot{x}_I - k x_I. \tag{8}
\end{align*}
\]
The transfer equation is \( z_O = U z_I \); combining equations (8) and (5), we can get the transfer matrix as follows:
\[
U = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-(mA + cC + k) & 0 & 0 & 0 & 1 & 0 & -mB_{x,I} - cD_{x,I} + f_{x,c} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}. \tag{9}
\]

4.2. Concentrated Mass at One Input End and Two Output Ends. The concentrated mass of mass \( m \) is shown in Figure 3(b), the input end is \( I \) and the output ends are \( O_1 \) and \( O_2 \). According to its stress balance and deformation relationship, we have
\[
\begin{align*}
x_{O_1} &= x_{O_2} = x_I, \\
y_{O_1} &= y_{O_2} = y_I, \\
\theta_{z,O_1} &= \theta_{z,O_2} = \theta_{z,I}, \\
m_{z,O_1} + m_{z,O_2} &= m_{z,I}, \\
q_{y,O_1} + q_{y,O_2} &= q_{y,I}, \\
q_{x,O_1} + q_{x,O_2} &= q_{x,I} + f_{x,c} - m \ddot{x}_I - c \dot{x}_I - k x_I. \tag{10}
\end{align*}
\]
The transfer equation is \( U_O z_O = U_I z_I = U_O z_{O_1} U_I z_{O_2} \); combining equations (10) and (5), we can get the transfer matrix as follows:
where

\[
\mathbf{U}_1 = \begin{bmatrix} 0_{3\times3} & \mathbf{I}_3 & \mathbf{O}_{3\times1} \end{bmatrix},
\]
\[
\mathbf{U}_2 = \begin{bmatrix} - (mA + cC + k) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
\mathbf{U}_3 = \begin{bmatrix} \mathbf{I}_3 & \mathbf{O}_{3\times4} \end{bmatrix},
\]
\[
\mathbf{U}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -mB_{x1} + cD_{x1} + f x_c \\ 0 & 0 & 1 \end{bmatrix},
\]
\[
z^0 = [0, 0, 0, 0, 0, 0]^T.
\]

According to the transfer equation, the state vector of the output can be obtained as follows:

\[
z_{O1} = E_1 \mathbf{U}_{O}^{-1} \mathbf{U}_1 (E_1 z_{I1} + E_1 z_{I1}),
\]
\[
= E_1 \mathbf{U}_{O}^{-1} \mathbf{U}_1 \mathbf{E}_1 \mathbf{U}_1 \mathbf{z}_{z1},
\]
\[
= \mathbf{U}^{11} \mathbf{z}_{z1} + \mathbf{U}^{12} \mathbf{z}_{z1},
\]
\[
z_{O2} = E_2 \mathbf{U}_{O}^{-1} \mathbf{U}_1 (E_2 z_{I1} + E_2 z_{I1}),
\]
\[
= E_2 \mathbf{U}_{O}^{-1} \mathbf{U}_1 \mathbf{E}_2 \mathbf{U}_1 \mathbf{z}_{z1},
\]
\[
= \mathbf{U}^{21} \mathbf{z}_{z1} + \mathbf{U}^{22} \mathbf{z}_{z1},
\]

where

\[
E_1 = \begin{bmatrix} \mathbf{I}_7 & \mathbf{O}_{7\times7} \end{bmatrix},
\]
\[
E_2 = \begin{bmatrix} \mathbf{O}_{7\times7} & \mathbf{I}_7 \end{bmatrix},
\]
\[
E_3 = \begin{bmatrix} \mathbf{I}_{7\times7} \end{bmatrix},
\]
\[
E_4 = \begin{bmatrix} \mathbf{O}_{7\times7} \end{bmatrix}.
\]

4.3. Concentrated Mass at Two Input Ends and Two Output Ends. The concentrated mass of mass \( m \) is shown in Figure 3(c), the input ends are \( I_1 \) and \( I_2 \) and the output ends are \( O_1 \) and \( O_2 \). According to its stress balance and deformation relationship, we have

\[
\begin{aligned}
x_{O1} &= x_{O2} = x_{I1} = x_{I2}, \\
y_{O1} &= y_{O2} = y_{I1} = y_{I2}, \\
\theta_{z,O1} &= \theta_{z,O2} = \theta_{z,I1} = \theta_{z,I2}, \\
m_{z,O1} + m_{z,O2} &= m_{z,I1} + m_{z,I2}, \\
q_{y,O1} + q_{y,O2} &= q_{y,I1} + q_{y,I2}, \\
q_{z,O1} + q_{z,O2} &= q_{z,I1} + q_{z,I2} + f x_c - m \ddot{x}_I + \ddot{c} \dot{x}_I - k x_I.
\end{aligned}
\]

The transfer equation is \( \mathbf{U}_O z_O = \mathbf{U}_I z_I = \mathbf{U}_O \begin{bmatrix} z_{O1} \\ z_{O2} \end{bmatrix} = \mathbf{U}_I \begin{bmatrix} z_{I1} \\ z_{I2} \end{bmatrix} \); combining equations (15) and (5), we can get the transfer matrix as follows:
According to the transfer equation, the state vector of the output can be obtained as follows:

\[
x_{\Omega} = E_{1} U_{1}^{0} I_{1} \left( E_{2} z_{l_{1}} + E_{4} z_{l_{1}} \right),
\]

\[
= E_{1} U_{1}^{0} I_{1} E_{3} z_{l_{1}} + E_{1} U_{1}^{0} I_{2} E_{3} z_{l_{1}},
\]

\[
= U^{11} z_{l_{1}} + U^{12} z_{l_{1}},
\]

\[
x_{\Omega_{2}} = E_{2} U_{1}^{0} I_{1} \left( E_{2} z_{l_{1}} + E_{4} z_{l_{1}} \right),
\]

\[
= E_{2} U_{1}^{0} I_{1} E_{3} z_{l_{1}} + E_{2} U_{1}^{0} I_{2} E_{3} z_{l_{1}},
\]

\[
= U^{21} z_{l_{1}} + U^{22} z_{l_{1}}.
\]

The \( U_{1}, U_{2}, U_{3}, U_{4}, E_{2}, E_{3}, \) and \( E_{4} \) are the same as in equations (12) and (14).

### 4.4. Concentrated Mass at Two Input Ends and One Output End

The concentrated mass of mass \( m \) is shown in Figure 3(d), the input ends are \( I_{1} \) and \( I_{2} \) and the output end is \( O \). According to its stress balance and deformation relationship, we have

\[
\begin{align*}
x_{\Omega} &= x_{1_{l_{1}}} = x_{1_{l_{1}}}, \\
y_{\Omega} &= y_{1_{l_{1}}} = y_{1_{l_{1}}}, \\
\theta_{z_{\Omega}} &= \theta_{z_{1_{l_{1}}}} = \theta_{z_{1_{l_{1}}}}, \\
m_{z_{\Omega}} &= m_{z_{1_{l_{1}}} + m_{z_{1_{l_{1}}}}}, \\
q_{y_{\Omega}} &= q_{y_{1_{l_{1}}} + q_{y_{1_{l_{1}}}}}, \\
q_{x_{\Omega}} &= q_{x_{1_{l_{1}}} + q_{x_{1_{l_{1}}}} + f_{xx} - m_{x_{1_{l_{1}}} - k_{x_{1_{l_{1}}}}}}.
\end{align*}
\]

The transfer equation is \( z_{\Omega} = U z_{l_{1}} = U \left[ \begin{array}{c} z_{l_{1}} \\
\end{array} \right] = U \left( E_{3} z_{l_{1}} + E_{4} z_{l_{1}} \right) \); combining equations (18) and (5), we can get the transfer matrix as follows:

\[
U = \begin{bmatrix}
I_{3} & O_{3 \times 11} \\
U_{1} & U_{1} \\
O_{1 \times 6} & I_{1} & O_{1 \times 7} \\
U_{3} & -U_{3} \\
O_{3 \times 7} & U_{3} \\
O_{1 \times 6} & I_{1} & O_{1 \times 7} \\
\end{bmatrix}
\]

### 4.5. Massless Elastic Beam

The longitudinal elastic beam with a length of \( l \) and a flexural rigidity of \( k \) is shown in Figure 3(e), the input end is \( I \) and the output end is \( O \). According to its stress balance and deformation relationship, we have

\[
\begin{align*}
q_{x_{O}} &= q_{x_{l_{1}}} , \\
q_{y_{O}} &= q_{y_{l_{1}}} , \\
-m_{z_{O}} + m_{z_{O}} q_{x_{O}} l = 0 , \\
x_{O} &= x_{1_{l_{1}}} + \theta_{z_{1_{l_{1}}}} , \\
y_{O} &= y_{1_{l_{1}}} , \\
\theta_{z_{O}} &= \theta_{z_{1_{l_{1}}}} + m_{z_{1_{l_{1}}} + m_{x_{1_{l_{1}}}}} .
\end{align*}
\]

The transfer equation is \( z_{\Omega} = U z_{l_{1}} \); combining equation (20), we can get the transfer matrix as follows:

\[
U = \begin{bmatrix}
1 & 0 & l & l^{2} & l^{3} & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & l \\
0 & 0 & 1 & l & l^{2} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

Similarly, the transfer matrix of transverse massless beams is

\[
U = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & l \\
0 & 1 & l & l^{2} & l^{3} & 0 & 0 \\
0 & 0 & 1 & l & l^{2} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\]
4.6. Planar Elastic Hinge. A plane elastic joint with angular stiffness of $k$, lateral spring stiffness of $k_x$, and longitudinal spring stiffness of $k_y$ is shown in Figure 3(f), and the transfer equation is

$$z_0 = Uz_1,$$  \hspace{1cm} (23)

where

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & -\frac{1}{k_x} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{1}{k_y} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ O_{4\times3} & I_4 \end{bmatrix}. \hspace{1cm} (24)$$

5. Derivation of the Total Transfer Equation of Frame Structure

The transmission direction in this paper is shown in Figure 4. The bottom ends of the frame column $a_{01}, a_{02}, \ldots, a_{0(n-1)}$ are the input ends, and $a_{00}$ is the output end. The mechanical element model number description is shown in the figure: the bottom ends of the frame columns are numbered as $a_{01}, a_{02}, \ldots, a_{0m}$ from left to right; the mass points from the left to the right of the first beam-column joint are numbered as $a_{11}, a_{12}, \ldots, a_{1n}$; the mass points from the left to the right of the second beam-column joint are numbered as $a_{21}, a_{22}, \ldots, a_{2m}$, and so on; the mass point of the beam-column joint of the $n$th row of the $m$th layer is numbered as $A_{nm}$; the first layer of beams is numbered as $L_{11}, L_{21}, \ldots, L_{(n-1)m}$ from left to right, and so on; the number of the $(n-1)$th beam of the $m$th layer is $L_{(n-1)m}$. Take the number $L_{(n-1)m}$ beam and number $A_{nm}$ column as examples to illustrate the internal numbering of the beam and the inside of the column, as shown in Figure 5.

The state vector that defines the elements in physical coordinates is

$$z = [x, y, \theta_x, m_x, q_x, q_y, 1]^T. \hspace{1cm} (25)$$

Taking frame column $A_{01}$ and beam $L_{(n-1)m}$ as examples, the transfer matrices are derived as follows:

$$U_{A_{01}} = U_{A_{01}}U_{L_{(n-1)m}} \cdots U_{A_{n1}}U_{L_{(n-1)m}}^1,$$
$$U_{L_{(n-1)m}} = U_{L_{(n-1)m}}U_{L_{(n-1)m}} \cdots U_{L_{(n-1)m}}U_{L_{(n-1)m}},$$

For the side cross-frame column $A_{11}$ input from the $a_{01}$ and passed up, the transfer equation can be obtained as follows:

$$Z_{a_{11}} = U_{a_{11}}Z_{a_{01}}A_{11} = U_{a_{11}}U_{a_{12}}A_{11} \cdots U_{a_{12}}U_{a_{11}}Z_{a_{01}}A_{11}. \hspace{1cm} (27)$$

For $a_{11}$, the input is $Z_{a_{11}}A_{11}$ and the outputs are $Z_{a_{11}}A_{12}$ and $Z_{a_{11}}L_{1(m-1)}$, and the transfer matrices of the concentrated mass output according to one end input are

$$Z_{a_{11}}A_{11} = U_{a_{11}}^1Z_{a_{11}}A_{11} = E_{11}U_{A_{11}}^1E_1 = U_{A_{11}}U_{A_{11}}Z_{a_{11}}A_{11}^1,$$
$$Z_{a_{11}}A_{12} = U_{a_{11}}^2Z_{a_{11}}A_{12} = E_{21}U_{A_{11}}^1E_1 = U_{A_{11}}U_{A_{11}}Z_{a_{11}}A_{12}^1.$$  \hspace{1cm} (28)

In the same way, we continue to pass up and sort out the following equation:

$$Z_{a_{11}}L_{1m} = U_{a_{11}}U_{A_{1m}}A_{12}^1 \cdots U_{A_{1m}}U_{A_{11}}Z_{a_{11}}, \hspace{1cm} (29)$$

Equation (29) can be organized into a matrix form:

$$\begin{bmatrix} Z_{a_{11}}L_{11} \\ Z_{a_{11}}L_{12} \\ \vdots \\ Z_{a_{11}}L_{1(m-1)} \\ Z_{a_{11}}L_{1m} \end{bmatrix} = \begin{bmatrix} U_{a_{11}}^1U_{A_{11}} \\ U_{a_{12}}U_{A_{11}}^1U_{A_{11}} \\ \vdots \\ U_{A_{1m}}U_{A_{1m}}^1U_{A_{11}} \\ U_{A_{1m}}U_{A_{1m}}^1U_{A_{1m}} \end{bmatrix} \begin{bmatrix} U_{a_{11}} \ U_{a_{12}}U_{A_{11}} \ \vdots \ U_{A_{1m}} \ U_{A_{1m}} \end{bmatrix} = Z_{a_{11}}A_{11} = U_{A_{11}}Z_{a_{11}}A_{11}^1, \hspace{1cm} (30)$$

where $Z_{a_{11}}L_{11}, Z_{a_{11}}L_{12}, \ldots, Z_{a_{11}}L_{1(m-1)}, Z_{a_{11}}L_{1m}$ are state vectors at the left end of beams $L_{11}, L_{12}, \ldots, L_{(m-1),1}, L_{1m}$ and the transfer matrix of each beam of the first span is represented by $U_{L_{11}}, U_{L_{12}}, \ldots, U_{L_{(m-1),1}}, U_{L_{1m}}$; according to its transfer matrix and transfer equation, the state vector at the right end of each beam is
\begin{align*}
\begin{bmatrix}
Z_{a21} & L_{11} \\
Z_{a22} & L_{12} \\
\vdots & \vdots \\
Z_{a2(n-1)} & L_{1(n-1)} \\
Z_{a2n} & L_{1n}
\end{bmatrix} &= \begin{bmatrix}
U_{L_{11}} \\
U_{L_{12}} \\
\vdots \\
U_{L_{1(n-1)}} \\
U_{L_{1n}}
\end{bmatrix}
\begin{bmatrix}
\mathbf{U}_{a11} \\
\mathbf{U}_{a12} \\
\vdots \\
\mathbf{U}_{a1(n-1)} \\
\mathbf{U}_{a1n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a21} \\
\mathbf{U}_{a22} \\
\vdots \\
\mathbf{U}_{a2(n-1)} \\
\mathbf{U}_{a2n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a11} \\
\mathbf{U}_{a12} \\
\vdots \\
\mathbf{U}_{a1(n-1)} \\
\mathbf{U}_{a1n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a31} \\
\mathbf{U}_{a32} \\
\vdots \\
\mathbf{U}_{a3(n-1)} \\
\mathbf{U}_{a3n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a21} \\
\mathbf{U}_{a22} \\
\vdots \\
\mathbf{U}_{a2(n-1)} \\
\mathbf{U}_{a2n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a41} \\
\mathbf{U}_{a42} \\
\vdots \\
\mathbf{U}_{a4(n-1)} \\
\mathbf{U}_{a4n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a31} \\
\mathbf{U}_{a32} \\
\vdots \\
\mathbf{U}_{a3(n-1)} \\
\mathbf{U}_{a3n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a51} \\
\mathbf{U}_{a52} \\
\vdots \\
\mathbf{U}_{a5(n-1)} \\
\mathbf{U}_{a5n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a41} \\
\mathbf{U}_{a42} \\
\vdots \\
\mathbf{U}_{a4(n-1)} \\
\mathbf{U}_{a4n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a61} \\
\mathbf{U}_{a62} \\
\vdots \\
\mathbf{U}_{a6(n-1)} \\
\mathbf{U}_{a6n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a51} \\
\mathbf{U}_{a52} \\
\vdots \\
\mathbf{U}_{a5(n-1)} \\
\mathbf{U}_{a5n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a71} \\
\mathbf{U}_{a72} \\
\vdots \\
\mathbf{U}_{a7(n-1)} \\
\mathbf{U}_{a7n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a61} \\
\mathbf{U}_{a62} \\
\vdots \\
\mathbf{U}_{a6(n-1)} \\
\mathbf{U}_{a6n}
\end{bmatrix} \\
\begin{bmatrix}
\mathbf{U}_{a81} \\
\mathbf{U}_{a82} \\
\vdots \\
\mathbf{U}_{a8(n-1)} \\
\mathbf{U}_{a8n}
\end{bmatrix} &= \begin{bmatrix}
\mathbf{U}_{a71} \\
\mathbf{U}_{a72} \\
\vdots \\
\mathbf{U}_{a7(n-1)} \\
\mathbf{U}_{a7n}
\end{bmatrix}
\end{align*}
\( U_{L1} \) can be defined as follows:

\[
U_{L1} = \begin{bmatrix}
U_{L11} & U_{L12} & \cdots & U_{L1(m-1)} & U_{L1m} \\
\end{bmatrix}.
\] (32)

For the intermediate frame column \( a_2 \), pass up from input \( a_{02} \), the transfer equation of \( a_{21} \) is

\[
Z_{a_{21}A_{21}} = U_{A_{21}} Z_{a_{02}A_{21}}.
\] (33)

Element \( a_{21} \) is a concentrated mass at one input end and two output ends, the input ends are \( Z_{a_{21}A_{21}} \) and \( Z_{a_{21}L_{11}} \), the output ends are \( Z_{a_{21}L_{21}} \) and \( Z_{a_{21}A_{22}} \), so we have

\[
Z_{a_{21}L_{21}} = U_{a_{21}} Z_{a_{21}L_{11}} + U_{a_{21}} Z_{a_{21}A_{22}},
\]

\[
Z_{a_{21}A_{22}} = U_{a_{21}} Z_{a_{21}L_{11}} + U_{a_{21}} Z_{a_{21}A_{22}}.
\] (34)

In the same way, we continue to pass up and sort out

\[
Z_{a_{2m}L_{2m}} = U_{a_{2m}} Z_{a_{2m}L_{1m}} + U_{a_{2m}} Z_{a_{2m}A_{2m}} Z_{a_{2(m-1)}L_{2(m-1)}} + \cdots + U_{a_{2m}} Z_{a_{2m}A_{2m}} Z_{a_{2(m-2)}L_{2(m-2)}} + \cdots + U_{a_{2m}} Z_{a_{2m}A_{2m}} Z_{a_{21}L_{11}}
\]

\[
+ U_{a_{2m}} Z_{a_{2m}A_{2m}} Z_{a_{2(m-1)}A_{22}} \cdots U_{a_{2m}} Z_{a_{2m}A_{2m}} Z_{a_{20}A_{21}}.
\] (35)

which is organized into a matrix form

\[
\begin{bmatrix}
Z_{a_{21}L_{21}} \\
Z_{a_{21}L_{22}} \\
\vdots \\
Z_{a_{2(m-1)}L_{2(m-1)}} \\
Z_{a_{2m}L_{2m}}
\end{bmatrix} = \begin{bmatrix}
U_{a_{21}} & U_{a_{22}} & U_{a_{21}} & \cdots & 0 & 0 \\
U_{a_{22}} & U_{a_{22}} & U_{a_{22}} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
U_{a_{2(m-1)}} & U_{a_{2(m-1)}} & U_{a_{2(m-1)}} & \cdots & U_{a_{2(m-1)}} & 0 \\
U_{a_{2m}} & U_{a_{2m}} & U_{a_{2m}} & \cdots & U_{a_{2m}} & U_{a_{2m}}
\end{bmatrix}.
\] (36)
$U_{A2}^1$ and $U_{A2}^2$ can be defined as follows:

$$U_{A2}^1 = \begin{bmatrix}
U_{a_{21}}^{11} & 0 & \cdots & 0 & 0 \\
U_{a_{22}}^{12} & U_{A_{21}} & U_{a_{22}}^{21} & U_{a_{22}}^{11} & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
U_{a_{2m}}^{12} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

$$U_{A2}^2 = \begin{bmatrix}
U_{a_{21}}^{12} & U_{A_{21}} & U_{a_{22}}^{22} & U_{a_{21}}^{11} & \cdots & 0 & 0 \\
U_{a_{22}}^{12} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
U_{a_{2m}}^{12} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

(37)

where $Z_{a_{21}L_{21}}, Z_{a_{22}L_{22}}, \ldots, Z_{a_{2(m-1)}L_{2(m-1)}}, Z_{a_{2m}L_{2m}}$ are state vectors at the left end of the beam $L_{21}, L_{22}, \ldots, L_{2(m-1)}, L_{2m}$ and the transfer matrices of the second span beam are represented by $U_{L_{21}}, U_{L_{22}}, U_{L_{23}}, \ldots, U_{L_{2m}}$; according to the transfer matrix and the transfer equation of a span beam, the state vector of the right end of each beam can be obtained as follows:

$$\begin{bmatrix}
Z_{a_{21}L_{21}} \\
Z_{a_{22}L_{22}} \\
\vdots \\
Z_{a_{2(m-1)}L_{2(m-1)}} \\
Z_{a_{2m}L_{2m}}
\end{bmatrix} = U_{L_{21}} \begin{bmatrix}
Z_{L_{21}} \\
Z_{L_{22}} \\
\vdots \\
Z_{L_{2(m-1)}} \\
Z_{L_{2m}}
\end{bmatrix}$$

(38)

The transfer of the state vector of the intermediate frame is the same as that of the frame column $a_2$, so it can be derived in the same way. The transfer equation of the frame column $a_j$ is

$$\begin{bmatrix}
Z_{a_{j1}L_{j1}} \\
Z_{a_{j2}L_{j2}} \\
\vdots \\
Z_{a_{j(m-1)}L_{j(m-1)}} \\
Z_{a_{jm}L_{jm}}
\end{bmatrix} = \begin{bmatrix}
U_{a_{j1}}^{11} & U_{A_{j1}} & U_{a_{j2}}^{21} & U_{a_{j1}}^{11} & \cdots & 0 & 0 \\
U_{a_{j2}}^{12} & U_{A_{j1}} & U_{a_{j2}}^{22} & U_{a_{j2}}^{11} & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
U_{a_{jm}}^{12} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix} \begin{bmatrix}
Z_{L_{j1}} \\
Z_{L_{j2}} \\
\vdots \\
Z_{L_{j(m-1)}} \\
Z_{L_{jm}}
\end{bmatrix}$$

(39)
$U_{A_j}^1$ and $U_{A_j}^2$ can be defined as follows:

$$U_{A_j}^1 = \begin{bmatrix}
U_{A_j}^{11} & 0 & \cdots & 0 & 0 \\
U_{A_j}^{12} & U_{A_j}^{21} & U_{A_j}^{11} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
U_{A_j}^{12} & U_{A_j}^{21} & U_{A_j}^{12} & \cdots & 0 \\
U_{A_j}^{12} & U_{A_j}^{21} & U_{A_j}^{12} & \cdots & 0
\end{bmatrix}$$

$$U_{A_j}^2 = \begin{bmatrix}
U_{A_j}^{12} & U_{A_j}^{22} & U_{A_j}^{11} & \cdots & U_{A_j}^{21} \\
U_{A_j}^{12} & U_{A_j}^{22} & U_{A_j}^{12} & \cdots & U_{A_j}^{21} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
U_{A_j}^{12} & U_{A_j}^{22} & U_{A_j}^{12} & \cdots & U_{A_j}^{21}
\end{bmatrix}$$

where $Z_{a_1L_1}$, $Z_{a_2L_2}$, $\cdots$, $Z_{a_{(n-1)}L_{(n-1)}}$, $Z_{a_{n}L_{n}}$ are state vectors at the left end of beams $L_1$, $L_2$, $\cdots$, $L_{(n-1)}$, $L_n$, and using $U_{L_1}$, $U_{L_2}$, $\cdots$, $U_{L_{(n-1)}}$, $U_{L_n}$ to represent the transfer matrix of

$$\begin{bmatrix}
Z_{a_1L_1} \\
\vdots \\
Z_{a_{(n-1)}L_{(n-1)}} \\
Z_{a_{n}L_{n}}
\end{bmatrix} = \begin{bmatrix}
U_{L_1} \\
\vdots \\
U_{L_{(n-1)}} \\
U_{L_n}
\end{bmatrix}$$

$$Z_{a_{mn}} = U_{a_{mn}} Z_{a_{mn}L_{(n-1)m}}$$

For the rightmost column $a_{mn}$, $Z_{a_{mn}L_{(n-1)m}}$, $Z_{a_{(n-1)}L_{(n-1)m}}$, $Z_{a_{n}L_{n}m}$, and using $U_{L_1}$, $U_{L_2}$, $\cdots$, $U_{L_{(n-1)}}$, $U_{L_n}$ to represent the transfer matrix of

$$\begin{bmatrix}
Z_{a_1L_1} \\
\vdots \\
Z_{a_{(n-1)}L_{(n-1)}} \\
Z_{a_{n}L_{n}}
\end{bmatrix} = \begin{bmatrix}
U_{L_1} \\
\vdots \\
U_{L_{(n-1)}} \\
U_{L_n}
\end{bmatrix}$$

$$Z_{a_{mn}} = U_{a_{mn}} Z_{a_{mn}L_{(n-1)m}}$$

From the transfer relationship, the following equation can be obtained in turn:

$$Z_{a_{mn}} = U_{a_{mn}} Z_{a_{mn}L_{(n-1)m}}$$

Equation (45) can be organized into a matrix form as follows:

$$Z_{a_{mn}} = U_{a_{mn}} Z_{a_{mn}L_{(n-1)m}}$$

Substituting equation (43) into equation (44),

$$Z_{a_{mn}} = U_{a_{mn}} Z_{a_{mn}L_{(n-1)m}}$$

For $a_{mn}$, the input end is $Z_{a_{mn}A_{ni}}$ and the output end is $Z_{a_{mn}A_{ni}}$, so the transfer equation is expressed as follows:

$$Z_{a_{mn}A_{ni}} = U_{A_{ni}} Z_{a_{mn}A_{ni}}$$

Substituting equation (43) into equation (44),


\[
Z_{d_{A}} = \begin{bmatrix} \mathbf{U}_{A_{1}}^{11} & \mathbf{U}_{A_{2}}^{11} & \cdots & \mathbf{U}_{A_{n}}^{11} \\
\mathbf{U}_{A_{1}}^{12} & \mathbf{U}_{A_{2}}^{12} & \cdots & \mathbf{U}_{A_{n}}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{U}_{A_{1}}^{n} & \mathbf{U}_{A_{2}}^{n} & \cdots & \mathbf{U}_{A_{n}}^{n} \end{bmatrix}
\]

Substituting the above derivation formula into equation (46), the relationship between the output end and the input end is

\[
\begin{align*}
Z_{d_{A}} &= \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{n}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{1}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{1}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{1}} + \cdots \\
&= \begin{bmatrix} Z_{d_{A_{1}}} \\
Z_{d_{A_{2}}} \\
\vdots \\
Z_{d_{A_{n}}} \end{bmatrix} = \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{n}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{1}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{1}} + \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{1}} + \cdots \tag{47}
\end{align*}
\]

Then, the total transfer equation of the frame structure is

\[
\mathbf{U}_{all} Z_{all} = 0, \tag{48}
\]

\[
\mathbf{U}_{all} = \begin{bmatrix} \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{1}}} \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{2}}} \\
\vdots \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{n}}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{A_{1}}^{11} & \mathbf{U}_{A_{2}}^{11} & \cdots & \mathbf{U}_{A_{n}}^{11} \\
\mathbf{U}_{A_{1}}^{12} & \mathbf{U}_{A_{2}}^{12} & \cdots & \mathbf{U}_{A_{n}}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{U}_{A_{1}}^{n} & \mathbf{U}_{A_{2}}^{n} & \cdots & \mathbf{U}_{A_{n}}^{n} \\
\end{bmatrix} = \begin{bmatrix} \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{1}}} \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{2}}} \\
\vdots \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{n}}} \end{bmatrix} + \begin{bmatrix} \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{1}}} \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{2}}} \\
\vdots \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{n}}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{A_{1}}^{11} & \mathbf{U}_{A_{2}}^{11} & \cdots & \mathbf{U}_{A_{n}}^{11} \\
\mathbf{U}_{A_{1}}^{12} & \mathbf{U}_{A_{2}}^{12} & \cdots & \mathbf{U}_{A_{n}}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{U}_{A_{1}}^{n} & \mathbf{U}_{A_{2}}^{n} & \cdots & \mathbf{U}_{A_{n}}^{n} \\
\end{bmatrix} + \begin{bmatrix} \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{A_{1}}} \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{A_{2}}} \\
\vdots \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-2)}} Z_{d_{A_{n}}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{A_{1}}^{11} & \mathbf{U}_{A_{2}}^{11} & \cdots & \mathbf{U}_{A_{n}}^{11} \\
\mathbf{U}_{A_{1}}^{12} & \mathbf{U}_{A_{2}}^{12} & \cdots & \mathbf{U}_{A_{n}}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{U}_{A_{1}}^{n} & \mathbf{U}_{A_{2}}^{n} & \cdots & \mathbf{U}_{A_{n}}^{n} \\
\end{bmatrix} + \cdots \tag{49}
\]

\[
\mathbf{Z}_{all} = \begin{bmatrix} Z_{d_{A_{1}}} \\
Z_{d_{A_{2}}} \\
\vdots \\
Z_{d_{A_{n}}} \end{bmatrix} = \begin{bmatrix} Z_{d_{A_{1}}} \\
Z_{d_{A_{2}}} \\
\vdots \\
Z_{d_{A_{n}}} \end{bmatrix} + \begin{bmatrix} Z_{d_{A_{1}}} \\
Z_{d_{A_{2}}} \\
\vdots \\
Z_{d_{A_{n}}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{1}}} \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{2}}} \\
\vdots \\
\mathbf{U}_{A_{n}} \mathbf{U}_{L_{(n-1)}} Z_{d_{A_{n}}} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{A_{1}}^{11} & \mathbf{U}_{A_{2}}^{11} & \cdots & \mathbf{U}_{A_{n}}^{11} \\
\mathbf{U}_{A_{1}}^{12} & \mathbf{U}_{A_{2}}^{12} & \cdots & \mathbf{U}_{A_{n}}^{12} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{U}_{A_{1}}^{n} & \mathbf{U}_{A_{2}}^{n} & \cdots & \mathbf{U}_{A_{n}}^{n} \\
\end{bmatrix} + \cdots \tag{50}
\]

### 6. Example Analysis

Two adjacent frame structures are selected as models, one of which is a 5-layer structure (J1) and the other is a 10-layer structure (J2); the height of each layer of the two structures is 3.2 m, the concrete grade is C30, the strength grade of the longitudinal reinforcement of the column and the beam is HRB400, the strength grade of the stirrup is HPB300, and the longitudinal reinforcement of the column and the beam is 800 mm. The structural schematic is shown in Figure 6.

A simplified model of the structure based on the MS-TMM is shown in Figure 7. J1 simplifies each frame column into four concentrated masses and four sections of massless beams; the first span of each beam is simplified into two elastic hinges, three concentrated masses, and three sections of massless elastic beams; the second span of each beam is simplified into two planar elastic hinges, seven concentrated masses, and seven sections of massless elastic beams. J2 simplifies each frame column into four concentrated masses and four sections of massless beams; simplifies each of the first and third span of each beam into two planar elastic hinges, seven concentrated masses, and seven sections of massless elastic beams; simplifies each beam of the second span into two elastic hinges, three concentrated masses, and three sections of massless elastic beams. The specific parameters are shown in Table 1.

When the peak acceleration of seismic is large, the structure may exhibit elastoplastic deformation. Therefore, the stiffness and damping of the structural material should be considered as a function of time, that is, determines the structural restoring force model. Currently used resilience models include bilinear restoring model and trilinear restoring model. The bilinear restoring model is too simple and rough; so, this paper uses the trilinear restoring model, as shown in Figure 8.

The steps for calculating the pounding response of two structures under earthquake based on MS-TMM are shown in Figure 9. In this paper, the calculation is achieved by compiling the relevant MATLAB program.

In this paper, two natural earthquake waves (the El Centro earthquake wave and the Taft earthquake wave)
Figure 6: Structure diagram. (a) Structural plan. (b) Structural elevation.
and one artificial earthquake wave (the Nanjing earthquake wave) are chosen. We adjust the peak acceleration to 35 cm/s², 70 cm/s², and 220 cm/s², respectively, and the pounding response of the structure under different separation gap sizes is calculated, as shown in Table 2. In order to compare and analyze the pounding response of adjacent structures under the excitation of earthquake based on MS-TMM, this paper also uses the element software ANSYS to perform modeling and calculation. In the analysis of ANSYS, because the structure is regular, BEAM161 is selected as the model for both the beam and the column elements, and SHELL63 is selected as the model for the floor elements, the contact type is defined as ASSC, as shown in Figure 10.

7. Pounding Force Analysis

The time history of the pounding force based on MS-TMM and ANSYS is shown in Figures 11 to 14; it can be seen that the results obtained by the two methods are similar. It is found that changing the peak value of seismic acceleration and the width of separation gap will have a great influence on pounding force, the number of pounding, and the moment of pounding. Under the earthquake with same peak acceleration, as the width of the separation gap decreases, the pounding force and the number of poundings increase. When the separation gap width is the same, as the peak acceleration of the earthquake increases, the pounding force and the number of poundings increases. It

<table>
<thead>
<tr>
<th>Structure</th>
<th>Elastic beam length (m)</th>
<th>Concentrated mass (kg)</th>
<th>Column</th>
<th>Planar elastic hinge</th>
<th>First span beam</th>
<th>Second span beam</th>
<th>Third span beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>0.8</td>
<td>1723.8</td>
<td>1.6 \times 10^9</td>
<td>1.6 \times 10^7</td>
<td>6.3 \times 10^9</td>
<td>0.75</td>
<td>2752.8</td>
</tr>
<tr>
<td>J2</td>
<td>0.8</td>
<td>2337.3</td>
<td>1.6 \times 10^9</td>
<td>1.6 \times 10^7</td>
<td>6.3 \times 10^9</td>
<td>0.6</td>
<td>2681.28</td>
</tr>
</tbody>
</table>
can be seen from Figure 11 that when the acceleration peak is 35 cm/s² and the separation gap width is 30 mm, J₁, a₁₅ and J₂, a₃₅ will not pound. As shown in Figure 12, when the peak acceleration is 70 cm/s² and the width of the gap is 60 mm, there will be no pounding between J₁, a₁₅ and J₂, a₃₅. As shown in Figure 13, when the peak acceleration is 220 cm/s² and the width of the gap is 150 mm, there will be no pounding between J₁, a₁₅ and J₂, a₃₅. It can be seen from Figure 14 that when the acceleration peak is 400 cm/s² and when the gap size is 220 mm, J₁, a₁₅ and J₂, a₃₅ will not pound.

According to Table 3, we can see, under the excitation of the El Centro wave with a peak acceleration of 35 cm/m², when the widths of the separation gaps are 10 mm and 20 mm, respectively, the errors of the peak value of the pounding force calculated by the two methods are 6.22% and 4.62%, respectively. Under the excitation of the El Centro wave with a peak acceleration of 220 cm/m², when the widths of the separation gaps are 100 mm and 130 mm, respectively, the errors of the peak value of the pounding force calculated by the two methods are 5.37% and 4.39%, respectively. Under the excitation of the El Centro wave with a peak acceleration of 400 cm/m², when the widths of the separation gap are 200 mm and 210 mm, respectively, the errors of the peak value of the pounding force calculated by the two methods are 6.25% and 5.32%, respectively. Evidently, the results of the two methods are almost the same; however, using ANSYS to calculate the pounding response of two adjacent frame structures under the excitation of earthquake takes about 8 hours, while the calculation by MS-TMM only takes about 20 minutes; its time consumption is only 1/24 of ANSYS, and the calculation speed advantage is very obvious.

Figure 9: The process for calculation of pounding response based on MS-TMM.

Table 2: Seismic wave peak acceleration peak and separation gap size description table.

<table>
<thead>
<tr>
<th>Seismic acceleration peak</th>
<th>35 cm/s²</th>
<th>70 cm/s²</th>
<th>220 cm/s²</th>
<th>400 cm/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation gap (mm)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3: SEISMIC WAVE PEAK ACCELERATION PEAK AND SEPARATION GAP SIZE DESCRIPTION TABLE.

<table>
<thead>
<tr>
<th>Seismic acceleration peak</th>
<th>35 cm/s²</th>
<th>70 cm/s²</th>
<th>220 cm/s²</th>
<th>400 cm/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separation gap (mm)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>
Figure 10: Analyzed ANSYS model of two adjacent frame structures.

Figure 11: Time-history diagram of $f_2$, $a_{35}$-$f_1$, $a_{15}$ pounding force under seismic wave (peak acceleration is 35 cm/s²). (a) 10 mm, (b) 20 mm, and (c) 30 mm.
Select the point of J1, a15 and the point J2, a35 for study. The displacement time-history diagrams are shown in Figures 15 to 22. It can be seen from the comparison that the trends of the displacement time-history diagram obtained by the two methods are almost the same. As illustrated in the figure, the acceleration peak of seismic wave and the separation gap both have an effect on the structural displacement response. The main feature is that the displacement on the pounding side is limited, which makes the displacement reaction produce obvious directional difference, and the difference becomes more significant with the increase of the acceleration peak and the decrease of the separation gap. Statistical comparisons are made between the displacement peaks obtained by the two methods, as shown in Tables 4 and 5.

As shown in Tables 4 and 5, under the excitation of seismic wave with peak acceleration of 35 cm/s², when the separation gaps are 10 mm, 20 mm, and 30 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 6.51%, 4.87%, and 4.50%, and 9.86%, 5.01%, and 5.71%, respectively. Under the excitation of the El Centro wave with an acceleration peak of 70 cm/s², when the separation gaps are 10 mm, 40 mm, and 50 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 9.86%, 5.01%, and 5.71% and 8.22%, 5.57%, and 6.20%, respectively. Under the excitation of the El Centro wave with an acceleration peak of 220 cm/s², when the separation gaps are 100 mm, 130 mm, and 150 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 5.17%, 7.83%, and 4.47% and 5.28%, and 7.20%, 5.75%, respectively.

Figure 12: Time-history diagram of J2, a35-J1, a15 pounding force under seismic wave (peak acceleration is 70 cm/s²). (a) 10 mm, (b) 40 mm, and (c) 60 mm.

8. Displacement Analysis

Select the point of J1, a15 and the point J2, a35 for study. The displacement time-history diagrams are shown in Figures 15 to 22. It can be seen from the comparison that the trends of the displacement time-history diagram obtained by the two methods are almost the same. As illustrated in the figure, the acceleration peak of seismic wave and the separation gap both have an effect on the structural displacement response. The main feature is that the displacement on the pounding side is limited, which makes the displacement reaction produce obvious directional difference, and the difference becomes more significant with the increase of the acceleration peak and the decrease of the separation gap. Statistical comparisons are made between the displacement peaks obtained by the two methods, as shown in Tables 4 and 5.

As shown in Tables 4 and 5, under the excitation of seismic wave with peak acceleration of 35 cm/s², when the separation gaps are 10 mm, 20 mm, and 30 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 6.51%, 4.87%, and 4.50%, and 5.16%, 7.42%, and 5.70%, respectively. Under the excitation of the earthquake with a peak acceleration of 400 cm/s², the J1, a15 peak displacement will decrease slightly with the increase of the separation gap width; for J2, a310, it will decrease slightly with the increase of the separation gap width.

Under the excitation of the El Centro wave with a peak acceleration of 35 cm/s², when the separation gaps are 10 mm, 20 mm, and 30 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 6.51%, 4.87%, and 4.50%, and 5.16%, 7.42%, and 5.70%, respectively. Under the excitation of the El Centro wave with an acceleration peak of 70 cm/s², when the separation gaps are 10 mm, 40 mm, and 50 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 9.86%, 5.01%, and 5.71% and 8.22%, 5.57%, and 6.20%, respectively. Under the excitation of the El Centro wave with an acceleration peak of 220 cm/s², when the separation gaps are 100 mm, 130 mm, and 150 mm, respectively, the peak displacement errors of J1, a15 and J2, a310 calculated by MS-TMM and ANSYS are 5.17%, 7.83%, and 4.47% and 5.28%, and 7.20%, 5.75%, respectively.

Figure 12: Time-history diagram of J2, a35-J1, a15 pounding force under seismic wave (peak acceleration is 70 cm/s²). (a) 10 mm, (b) 40 mm, and (c) 60 mm.
Under the excitation of the El Centro wave with an acceleration peak of 400 cm/s², when the separation gaps are 100 mm, 200 mm, and 210 mm, respectively, the peak displacement errors of \( J_1, a_{15} \) and \( J_2, a_{310} \) calculated by MS-TMM and ANSYS are 5.83%, 6.76%, and 5.16% and 6.24%, 4.15%, and 5.47%, respectively. However, using ANSYS to calculate the pounding response of two adjacent frame structures under the excitation of earthquake takes about 8 hours, while the calculation by MS-TMM only takes about 20 minutes; its time consumption is only 1/24 of ANSYS, and the calculation speed advantage is very obvious.

9. Analysis of Pounding Process Based on MS-TMM

It is found that the influence of the three seismic waves on the pounding response of adjacent buildings is basically the same in the above researches; so, we only study the pounding response of adjacent buildings under the excitation of the El Centro wave in next research. The MS-TMM is used to calculate the pounding response of \( J_1 \) and \( J_2 \) under the excitation of the El Centro wave, and the deformation of the structure is drawn at the moment when the pounding force is greater than 400 KN, as shown in Figures 23 to 26. It can be seen from the figure that the pounding mainly occurs at the apex of the shorter structure. Meanwhile, multiple pounding at the same time may occur when the separation gap is small, which will have a more adverse effect on the structure. Seismic peak acceleration and separation gap size will affect the moment of pounding. The pounding force is small except for a few moments, and the rest is small when the separation gap is small.

10. Analysis of Shear Force Based on MS-TMM

As illustrated in Figures 27 and 28, under the same peak seismic excitation, although the separation gap is different, the shear force of each column of the adjacent two structures is consistent before the first pounding. The shear force histories of \( J_1-A_{13} \) and \( J_2-A_{31} \) at the peak seismic acceleration of 220 cm/s² and 400 cm/s² are quite different from these whose peak seismic acceleration are 35 cm/s² and 70 cm/s², since the structural stiffness is reduced and the period is increased under the seismic excitation of higher peak acceleration; as a result, the smaller frequency in the seismic wave is amplified. In addition, the influence of pounding on the base shear force of the shorter structure is greater than that of the base shear force of the higher structure, this is because the pounding force of the two structures is the same, but the base shear force of the shorter structure is smaller than the base shear of the higher structure.

Figure 13: Time-history diagram of \( J_2, a_{35-J_1, a_{15}} \) pounding force under seismic wave (peak acceleration is 220 cm/s²). (a) 100 mm, (b) 130 mm, and (c) 150 mm.
As illustrated in Figure 29, when the peak value of the seismic acceleration is 35 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 95 KN and 138 KN, respectively; when the peak value of the seismic acceleration is 70 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 181 KN and 298 KN, respectively; when the peak value of the seismic acceleration is 220 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 347 KN and 587 KN, respectively; and when the peak value of the seismic acceleration is 400 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 485 KN and 845 KN, respectively. These indicate that the maximum shear force of the two structural side columns increases significantly by increasing the peak seismic acceleration.

Table 3: Peak pounding force of J2, a_{35-J1}, a_{15} under seismic wave.

<table>
<thead>
<tr>
<th>Seismic peak acceleration</th>
<th>Separation gap</th>
<th>El Centro-MS-TMM (KN)</th>
<th>El Centro-ANSYS (KN)</th>
<th>Error (%)</th>
<th>Nanjing-MS-TMM (KN)</th>
<th>TAFT-MS-TMM (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 cm/s²</td>
<td>10 mm</td>
<td>161</td>
<td>143</td>
<td>5.59</td>
<td>147</td>
<td>169</td>
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<tr>
<td></td>
<td>20 mm</td>
<td>134</td>
<td>129</td>
<td>3.88</td>
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<td>140</td>
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<td>0</td>
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<td>10 mm</td>
<td>239</td>
<td>225</td>
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<td>235</td>
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<tr>
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<td>40 mm</td>
<td>181</td>
<td>173</td>
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<td>189</td>
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<tr>
<td></td>
<td>60 mm</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
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<td>100 mm</td>
<td>412</td>
<td>391</td>
<td>5.37</td>
<td>389</td>
<td>424</td>
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<tr>
<td></td>
<td>130 mm</td>
<td>95</td>
<td>91</td>
<td>4.39</td>
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<td></td>
<td>150 mm</td>
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<tr>
<td>400 cm/s²</td>
<td>100 mm</td>
<td>697</td>
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<td>6.25</td>
<td>637</td>
<td>683</td>
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<td>200 mm</td>
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<td>8.9</td>
<td>5.32</td>
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<td>210 mm</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

As illustrated in Figure 29, when the peak value of the seismic acceleration is 35 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 95 KN and 138 KN, respectively; when the peak value of the seismic acceleration is 70 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 181 KN and 298 KN, respectively; when the peak value of the seismic acceleration is 220 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 347 KN and 587 KN, respectively; and when the peak value of the seismic acceleration is 400 cm/s², the maximum shear forces at the bottom of J1-A11 and J2-A31 are 485 KN and 845 KN, respectively. These indicate that the maximum shear force of the two structural side columns increases significantly by increasing the peak seismic acceleration.
Figure 15: Time-history diagram of $J_1$, $a_{15}$ displacement under seismic wave (peak acceleration is 35 cm/s^2). (a) 10 mm, (b) 20 mm, and (c) 30 mm.

Figure 16: Continued.
Figure 29 also shows that pounding suppresses the maximum base shear force of the shorter structural side column and enlarges the maximum base shear force of the higher structural side column, since the pounding mainly occurs in the negative movement of the two structures under the excitation of the El Centro wave; meanwhile, the base shear force of the two structural side columns is positive, and the pounding produces a positive force on the shorter structure and a negative force on the higher structure.
Figure 18: Time-history diagram of $J_1$, $a_{12}$ displacement under seismic wave (peak acceleration is 400 cm/s²). (a) 100 mm, (b) 200 mm, and (c) 210 mm.

Figure 19: Continued.
11. Conclusion

In this paper, a mechanical model based on the transfer matrix method of multibody systems (MS-TMM) is established, the transfer matrix of the related elements and overall transfer equation are deduced, combining the Hertz-damp model, and the corresponding MATLAB program is compiled to determine the pounding process.
Figure 21: Time-history diagram of $J_2$, $\alpha_{310}$ displacement under seismic wave (peak acceleration is 220 cm/s$^2$). (a) 100 mm, (b) 130 mm, and (c) 150 mm.

Figure 22: Continued.
of two adjacent frame structures during severe ground motions. The following conclusions are drawn from the results:

The pounding response trends of two adjacent frame structures that under the El Centro wave obtained by the MS-TMM are similar to the responses obtained

---

### Table 4: Peak displacement of $\alpha_{15}$ under the excitation of seismic wave.

<table>
<thead>
<tr>
<th>Seismic peak acceleration</th>
<th>Separation gap</th>
<th>El Centro-MS-TMM (mm)</th>
<th>El Centro-ANSYS (mm)</th>
<th>Error (%)</th>
<th>Nanjing-MS-TMM (mm)</th>
<th>TAFT-MS-TMM (mm)</th>
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</thead>
<tbody>
<tr>
<td>35 cm/s²</td>
<td>10 mm</td>
<td>13.25</td>
<td>12.44</td>
<td>6.51</td>
<td>10.76</td>
<td>12.51</td>
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<td></td>
<td>20 mm</td>
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<td>4.87</td>
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<td>30 mm</td>
<td>17.17</td>
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<td>14.17</td>
<td>18.39</td>
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<td>70 cm/s²</td>
<td>10 mm</td>
<td>26.62</td>
<td>24.23</td>
<td>9.86</td>
<td>24.47</td>
<td>29.36</td>
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<tr>
<td></td>
<td>40 mm</td>
<td>32.29</td>
<td>30.75</td>
<td>5.01</td>
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<td>5.71</td>
<td>33.79</td>
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<td>100 mm</td>
<td>94.35</td>
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<td>5.17</td>
<td>96.61</td>
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<td>7.83</td>
<td>113.54</td>
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<td>150 mm</td>
<td>126.73</td>
<td>121.31</td>
<td>4.47</td>
<td>124.49</td>
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<td>400 cm/s²</td>
<td>100 mm</td>
<td>179.27</td>
<td>169.39</td>
<td>5.83</td>
<td>189.03</td>
<td>170.16</td>
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<td>200 mm</td>
<td>162.60</td>
<td>152.3</td>
<td>6.76</td>
<td>167.81</td>
<td>163.25</td>
</tr>
<tr>
<td></td>
<td>210 mm</td>
<td>163.71</td>
<td>155.67</td>
<td>5.16</td>
<td>167.81</td>
<td>163.25</td>
</tr>
</tbody>
</table>

---

### Table 5: Peak displacement of $\alpha_{310}$ under the excitation of seismic wave.

<table>
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<tr>
<th>Seismic peak acceleration</th>
<th>Separation gap</th>
<th>El Centro-MS-TMM (mm)</th>
<th>El Centro-ANSYS (mm)</th>
<th>Error (%)</th>
<th>Nanjing-MS-TMM (mm)</th>
<th>TAFT-MS-TMM (mm)</th>
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</thead>
<tbody>
<tr>
<td>35 cm/s²</td>
<td>10 mm</td>
<td>37.91</td>
<td>36.05</td>
<td>5.16</td>
<td>36.92</td>
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<td>20 mm</td>
<td>40.37</td>
<td>37.58</td>
<td>4.72</td>
<td>40.67</td>
<td>41.30</td>
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<td>30 mm</td>
<td>40.45</td>
<td>40.00</td>
<td>5.70</td>
<td>41.81</td>
<td>42.02</td>
</tr>
<tr>
<td>70 cm/s²</td>
<td>10 mm</td>
<td>70.94</td>
<td>65.55</td>
<td>8.22</td>
<td>69.84</td>
<td>73.49</td>
</tr>
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<td></td>
<td>40 mm</td>
<td>84.72</td>
<td>80.25</td>
<td>5.57</td>
<td>84.26</td>
<td>87.35</td>
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<td>6.20</td>
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<td>172.22</td>
<td>163.58</td>
<td>5.28</td>
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<td>130 mm</td>
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<td>7.20</td>
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<td>171.28</td>
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<td>151.46</td>
<td>5.75</td>
<td>164.22</td>
<td>168.93</td>
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<td>286.38</td>
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<td>281.14</td>
<td>266.57</td>
<td>5.47</td>
<td>279.57</td>
<td>285.19</td>
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</table>
Figure 23: J₁-a₁, J₂-a₃ deformation diagram of the structure pounding moment under the excitation of El Centro wave (35 m/s²). (a) 10 mm and (b) 20 mm.

Figure 24: J₁-a₁, J₂-a₃ deformation diagram of the structure pounding moment under the excitation of El Centro wave (70 m/s²). (a) 10 mm and (b) 40 mm.
by ANSYS, the biggest error of the peak pounding force is 6.22%, the biggest error of the maximum displacement for the top of the structure with fewer stories and the structure with more stories is 9.86% and 8.22%, respectively, while the calculation time based on the MS-TMM method was approximately 1/24 of that based on the ANSYS method. Therefore, analysis of the pounding between adjacent frame
Figure 27: Base shear force history for $J_{1-A_{11}}$. (a) 35 cm/s$^2$, (b) 70 cm/s$^2$, (c) 220 cm/s$^2$, and (d) 400 cm/s$^2$.

Figure 28: Continued.
structures under earthquake excitation using the MS-TMM not only guarantees calculation accuracy but also has high computational efficiency.

(2) The pounding force and the number of poundings increase with the decrease of the separation gap size when the two adjacent structures under same peak seismic excitation; the pounding force and the number of poundings increase with the increase of peak seismic excitation when the two adjacent structures have same separation gap size.

(3) Pounding of two adjacent structures will occur when the acceleration peak is 35 cm/s$^2$ and the separation gap is not more than 20 mm, the acceleration peak is 70 cm/s$^2$ and the separation gap is not more than 40 mm, the acceleration peak is 220 cm/s$^2$ and the separation gap is not more than 130 mm, the acceleration peak is 400 cm/s$^2$ and the antiseismic joint width is not more than 200 mm. The displacement response of the two structures shows obvious directional differences due to the pounding. The difference becomes more significant with the increase of the acceleration peak and the decrease of the separation gap.

(4) The pounding mainly occurs at the top of the structure with fewer stories, while multiple pounding at the same time may occur when the separation gap is small. Seismic peak ground acceleration and separation gap size will affect the moment of pounding. The pounding force is small except for a few moments, and the rest is small when the separation gap is small. The influence of pounding on the base shear force of the shorter structural side column is greater than the base shear force of the higher structure. Pounding suppresses the maximum base shear force of the shorter structural side column and enlarges the maximum base shear force of the higher structural side column.

**Figure 28:** Base shear force history for J2-A31. (a) 35 cm/s$^2$, (b) 70 cm/s$^2$, (c) 220 cm/s$^2$, and (d) 400 cm/s$^2$.

**Figure 29:** Maximum value of base shear force.
Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments
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References


