Research Article

Sectional Analysis Procedure for Reinforced Concrete Members Subjected to Pure Torsion

Jongkwon Choi and Seong-Cheol Lee

1 Department of Civil, Architectural and Environmental Engineering, The University of Texas at Austin, Austin, TX 78712, USA
2 Department of Civil Engineering, Kyungpook National University, Daegu, Republic of Korea

Correspondence should be addressed to Seong-Cheol Lee; seonglee@knu.ac.kr

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A sectional analysis procedure for reinforced concrete members subjected to pure torsion is presented in this paper. On the development of the analysis procedure, the reinforced concrete section is modeled with reinforced concrete elements subjected to biaxial stress states, on the basis of the thin-walled tube analogy. Each reinforced concrete element is analyzed with the modified compression field theory (MCFT) to take into account for compression softening and tension stiffening effects in cracked reinforced concrete. Considering analysis results of reinforced concrete elements, equilibrium, and compatibility on the section are checked. For verification of the developed analysis procedure, analytical predictions were compared with test results of 16 reinforced concrete beams subjected to a pure torsional load which are available in the literature. Comparison between predicted and experimentally obtained torque-twist responses showed that the proposed procedure is capable of capturing the ultimate torsional capacity as well as the angle of twist within a reasonable range.

1. Introduction

Due to the continuing advances on the design of concrete structures, the efficient and economical design of the concrete structures were available as such the effect of the torsion to the structural performance has gained renowned interest especially under severe to extreme loading conditions such as earthquake, impact, and tornado. As shown in Figure 1, the torsional loadings on the structures can be divided into the equilibrium torsion and the compatibility torsion [1–3]. As a result of force redistribution, the torsional demand of the members subjected to the compatibility torsion permitted to be reduced to the cracking torque [4]. However, the ultimate torsional capacity is of interest for the typically equilibrium torsional design. Therefore, it is important to accurately predict the torsional capacity of a given structural component.

There are two theories which are used to explain the torsional behavior of reinforced concrete members. The first theory is a skew bending theory developed by Lessig [5] and further developed by Hsu [6–8]. The torsional provision in ACI codes provisions from 1971 to 1989 adopted this theory. This theory is based on the assumption of a skew failure surface (Figure 2). It is assumed that the failure mode involves bending on a skew surface as a result of spiral cracks on three faces of a rectangular beam; the fourth face behaves as a compression zone. The shear and torsion are resisted by three sources: axial contribution of the torsional reinforcement, shear-compression force of concrete, and dowel action of the longitudinal reinforcing bars.

The second theory is a thin-walled tube/space-truss model, depicted in Figure 3, which is developed on the basis of mechanics of materials. Notations in the figure are \( T \) for torque, \( \theta \) for diagonal compressive strut angle, \( A_0 \) for the gross area enclosed by the shear flow path around the perimeter of the tube, \( q \) for shear flow, \( V_i \) for the shear forces acting in the individual sides of the space truss, \( D_i \) for the diagonal compression component of \( V_i \), and \( N_i \) for the axial tension force required in the longitudinal reinforcement, and \( x_0 \) and \( y_0 \) for space truss dimensions. The model in
Figure 3 is adopted in ACI code torsional provisions since 1995. This model is considerably simpler to understand and easily applicable in the torsional design with producing equally accurate results as compared to the skew bending theory. Based on previous test data on both solid and hollow beams, the concrete in the core of the member has little effect on the torsional strength as such its contribution is assumed to be negligible. Therefore, in the thin-walled tube/space-truss model, both solid and hollow members are considered as tubular beams [3].

As shown in Figure 3(a), the reinforced concrete beam can be idealized as a thin-walled tube. It is assumed that torsion is resisted by shear flow $q$ around the perimeter of the member. After torsional cracking, the tubular member is idealized as a space truss consisting of closed stirrups, longitudinal reinforcement, and compression diagonals, as shown in Figure 3(b). It is assumed that the compression diagonals are located between cracks. The crack angle varies between $30^\circ$ and $60^\circ$ depending on numerous factors, and it is generally taken as $45^\circ$ for nonprestressed concrete and $30^\circ$ for prestressed concrete for the design [4].

Although the crack angles presented in the code reasonably represent the actual direction of torsion-induced cracking, the direction of principle compressive stress after cracking (i.e., the angle of effective compression diagonal) differentiates from the principle compressive direction prior to cracking. It is because the direction of principle compressive stress after cracking is dependent on the longitudinal and transverse reinforcement ratios [10]. Therefore, the angle of the compression diagonal of a beam with unbalanced reinforcement ratios between torsional and longitudinal directions may not follow the code recommended angle as such the torsional capacity could be underestimated or overestimated.

In an effort to accurately predict the torsional capacity of reinforced concrete members, a sectional analysis procedure considering concrete softening behavior is proposed. In this analysis procedure, the angle of compression diagonal is calculated on the basis of the current stress status of orthotropic reinforced concrete elements which are taking into account for concrete and reinforcing bars. Furthermore, the concrete compression softening and tension stiffening are also considered for defining the material properties of reinforced concrete elements. Based on this procedure, a sectional analysis program for reinforced concrete beams subjected to pure torsion is developed and presented in this paper.

2. Development of Sectional Analysis Program

With the rapid development of computer programs, the high-fidelity finite element models are available and capable of simulating large-scale reinforced concrete structures. However, the analytical results obtained from the high-fidelity modeling approaches typically depend on the user-defined material properties and require enormous computational cost. Therefore, a simplified analysis procedure using advanced material behavior models is still a favorable choice to analyze the behavior of reinforced concrete because of its less computational efforts yet holds reasonable accuracy of the predictions. The section analytical procedure proposed in this paper aims toward predicting the rational ultimate torsional capacity of the beam under a pure torsional load. Furthermore, the fundamentals of simplified
sectional analysis procedure are basically stemming from the thin-walled tube analogy as such it could provide a better understanding of the torsional mechanisms for the users.

2.1. Fundamental Assumptions and Formulation. The proposed sectional analysis procedure is derived on the basis of the thin-walled tube. In an effort to simplify the analysis procedure, several assumptions that are typically known to be true for the theory of torsion for a shaft with a uniform circular cross section have been adopted. Those assumptions are as follows: (a) the twist along the axis of a reinforced concrete member is uniformly distributed throughout the member; (b) the plane section normal to the longitudinal axis of reinforced concrete member before twist remains plane after the twist; and (c) the shear strain due to the twist is linearly distributed in the radial direction (Figure 4).

The shear strains can be obtained by the following equation:

\[ \gamma = \frac{d\phi}{dx} \cdot \rho, \]  

where \( \gamma \) is an angle of twist which is assumed to be constant over the unit length \( dx \), \( d\phi \) is an angle of rotation with respect to the centroid of the section, and \( \rho \) is the radial distance measured from the centroid of the section.

As mentioned above, the sectional analysis procedure proposed was developed on the basis of the thin-walled tube analogy. In the general design practice, the cover concrete contribution is typically ignored after the cracking of the concrete as such the space truss analogy is used to consider the spalling of the concrete [4]. However, in the proposed analysis procedure, softening responses of concrete due to biaxial loading conditions were employed after the cracking of the reinforced concrete beam since even cracked concrete may have contribution on the tensile behavior due to bond mechanism between rebar and concrete, which is called as the tension stiffening effect. In addition, the spalling of the concrete typically occurs at or beyond the ultimate torque [6, 11, 12], so even cover concrete contributes the torsional behavior.

Figure 5(a) shows a rectangular element taken from one side of an idealized tubular beam. It is assumed that each side of the rectangular element aligned with the longitudinal and transverse direction of the applied torque. To avoid the confusion of the coordinate system, the direction of the torsional shear flow within the section is denoted as \( t \), and the direction where the torque is applied.

---

**Figure 3:** (a) Thin-walled tube analogy and (b) space truss analogy [3, 4].
is denoted as \( l \). Based on this definition, the \( t \)-direction and \( l \)-direction are always perpendicular to each other for any rectangular element taken from any side of the idealized tubular beam. When a beam is subjected to a pure torsion, normal stresses of the rectangular unit element in transverse directions can be assumed as zero because the shear strains owing to pure torsion only induces shear stresses (Figure 5(a)). Because of no constraint in the out-of-plane direction of the rectangular unit element, the out-of-plane (i.e., radial) stresses are also assumed zero. On the other hand, the stress component aligned with longitudinal direction of the beam may not be zero due to the second assumption of this analysis procedure: (b) the plain section normal to the longitudinal axis of reinforced concrete member before twist remains plane after the twist. In the finite element formulation, therefore, the section can be modeled as consisting of plane stress elements where normal stress in the transverse direction is zero. In addition, the equilibrium in the longitudinal axis direction should be satisfied through the entire section. If there is no axial force applied, the axial force evaluated through the integration of the longitudinal stress along the entire section should be zero.

The relationship of the stresses \( \{\sigma\} \) and strains \( \{\varepsilon\} \) in an element can be expressed with the stiffness matrix of the element \( [D] \), as follows:

\[
\{\sigma\} = [D]\{\varepsilon\},
\]  

(2)

where \( \{\sigma\} = [\sigma_l \quad \sigma_t \quad \tau_{lt}]^T \), \( \{\varepsilon\} = [\varepsilon_l \quad \varepsilon_t \quad \gamma_{lt}]^T \), and

\[
[D] = \begin{bmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & D_{33}
\end{bmatrix}.
\]  

(3)

For the local stress condition of a beam subjected to pure torsion as described in Figure 4, the transverse component of stress \( \sigma_t \) is assumed to be negligible (\( \sigma_t = 0 \)). Hence, the transverse strain \( \varepsilon_t \) can be computed for given values of \( \varepsilon_l \) and \( \gamma_{lt} \), using the following equation:

\[
\{\varepsilon_t\} = -[D_{22}]^{-1} \begin{bmatrix} D_{12} & D_{23} \end{bmatrix} \begin{bmatrix} \varepsilon_l \\ \gamma_{lt} \end{bmatrix}.
\]  

(4)
Consequently, the normal stress in the longitudinal direction and the shear stress can be calculated as follows:

\[
\begin{bmatrix}
\sigma_l \\
\tau_{lt}
\end{bmatrix} = \begin{bmatrix}
D_{11} & D_{13} \\
D_{31} & D_{33}
\end{bmatrix} \begin{bmatrix}
\varepsilon_l \\
\gamma_{lt}
\end{bmatrix} + \begin{bmatrix}
D_{12} \\
D_{32}
\end{bmatrix} \left\{ \varepsilon_l \right\}.
\]

(5)

To satisfy the axial force equilibrium, the sum of the force \( P \) across the beam section should be equal to the applied axial load:

\[ P = \int_A \sigma_l dA. \]

(6)

The torque applied at given state of twist is calculated as follows:

\[ T = \int_A \tau_{lt} dA, \]

(7)

where \( \tau_{lt} \) is torsional shear stresses and \( r \) is the distance of torsional shear flow \( \tau_{lt} \) measured from the centroid of the section.

### 3. Constitutive Modeling

As shown in the previous section, the element within the sectional analysis program is subjected to biaxial stress condition as such concrete constitutive relationships considering the biaxial stress condition should be taken into account. Therefore, the analytical procedure of the modified compression field theory (MCFT) is employed in this sectional analysis procedure [13]. On development of a sectional analysis procedure considering the biaxial stress effect, compression softening and tension stiffening models were considered within the program. The following sections present constitutive models for concrete and embedded rebar. Specifically, for the concrete, the compressive and tensile behaviors of concrete considering the compression softening and tension stiffening effects and the reinforcement behavior are presented. Based on the constitutive models, the stiffness matrix formulation for reinforced concrete membrane elements is presented as well.

#### 3.1. Compressive Behavior of Concrete

The concrete compressive base curve used in this program is Thorenfeldt model [14]. This model well estimates the response of the reinforced concrete member with low- to high-concrete strength and was modified by Vecchio and Collins [15]. The principal compressive stresses within a reinforced concrete member subjected to a biaxial stress condition are softened due to the transverse tensile strains (refer to Figure 5). To account for the compression softening, the compressive base curve (i.e., uniaxial compressive stress-strain responses) is simply multiplied by a softening parameter \( \beta_d \) (equations (8) and (9)). The compression softening model used in the sectional analysis program is the original formulation of MCFT [16]:

\[
f_{c2} = \beta_d \cdot f_{c2}^{\text{base}},
\]

(8)

\[
\beta_d = \frac{1}{0.85 - 0.27(\varepsilon_1/\varepsilon_2)}
\]

(9)

where \( \varepsilon_1 \) and \( \varepsilon_2 \) represent the average principal tensile and compressive strains in the concrete, respectively.

#### 3.2. Tensile Behavior of Concrete

In a cracked reinforced concrete member, concrete has some contribution to carry the tensile stresses due to the bond mechanism between the concrete and embedded reinforcing bars; this is called the tension stiffening effect [13, 16–18]. For the better prediction of the torsion-dominant behavior of reinforced concrete structures, the tension stiffening effect should be considered. To account for the tension stiffening effect, a model proposed by Vecchio and Collins [16] is employed in this program:

\[
f_{c1} = \frac{f_{c1}^e}{1 + \sqrt{200\varepsilon_1}} \leq f_{c1, \text{max}},
\]

(10)

where \( f_{c1}^e \) is the principal average concrete tensile stress, \( f_{c1} \) is the concrete cracking strength, \( \varepsilon_{c1} \) is the principal average concrete tensile strain, and \( f_{c1, \text{max}} \) is the maximum principal average concrete tensile stress considering local yielding of rebar at a crack.

#### 3.3. Behavior of Reinforcement

Generally, it is reasonable to assume a bilinear response for the reinforcement, as shown in Figure 6.

#### 3.4. Stiffness Matrix Formulation

The stiffness matrix formulation can be fundamentally developed based on the cracked reinforced concrete treated as an orthotropic material as such the material properties are evaluated in the principal stress and strain directions considering cracks [19].

As shown in Figures 7 and 8, secant moduli pertaining to the concrete material stiffness in the principal stress directions are calculated as follows:

\[
E_{c1} = \frac{f_{c1}^e}{\varepsilon_{c1}},
\]

(11)

\[
E_{c2} = \frac{f_{c2}^e}{\varepsilon_{c2}}.
\]

(12)

The secant shear modulus can be calculated using the following equation:

\[
G_{c21} = \frac{E_{c1} \cdot E_{c2}}{E_{c1} + E_{c2}}.
\]

(13)

The orthotropic concrete material stiffness matrix \([D_c]^t\) in the principle directions is assembled as follows:

\[
[D_c]^t = \begin{bmatrix}
E_{c1} & 0 & 0 \\
0 & E_{c2} & 0 \\
0 & 0 & E_{c3}
\end{bmatrix}.
\]

(14)
OY hen, the concrete material stiffness matrix is transformed to the \( l, t \)-coordinate system using an appropriate transformation matrix \( [T_c] \):

\[
[D_c] = [T_c]^T [D_c] [T_c].
\] (15)

Using the same approach used to evaluate the local concrete material matrix \( [D_c] \), stiffness contributions from both longitudinal and transverse reinforcement layers are developed from their respective secant moduli. On the basis of the assumption that reinforcing bars only carry stresses in the local axial direction and does not carry the shearing stresses (dowel action), the contribution from the \( i \)th layer of reinforcement is defined as follows:

\[
[D_s]_i = \begin{bmatrix}
\rho_i E_{si} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\] (16)

where \( \rho_i \) is the \( i \)th layer reinforcement ratio. The secant moduli for reinforcement are calculated as follows:

\[
E_{si} = \frac{f_{si}}{\varepsilon_{si}},
\] (17)

The reinforcement secant stiffness matrix is transformed to the \( l, t \)-coordinate system using an appropriate transformation matrix \( [T_s] \):

\[
[D_s]_i = [T_s]^T [D_s]_i [T_s].
\] (18)

The transformation matrix for concrete and reinforcement in equations (15) and (18), respectively, can be evaluated from the following equation:

\[
[T] = \begin{bmatrix}
\cos^2 \varphi & \sin^2 \varphi & \cos \varphi \sin \varphi \\
\sin^2 \varphi & \cos^2 \varphi & -\cos \varphi \sin \varphi \\
-2 \cos \varphi \sin \varphi & 2 \cos \varphi \sin \varphi & (\cos^2 \varphi - \sin^2 \varphi)
\end{bmatrix},
\] (19)

where \( \varphi = \theta_c \) which is the inclination angle of the concrete principal tensile stress from the \( l \)-axis and \( \varphi = \theta_{si} \) which is the inclination angle of the \( i \)th layer reinforcement from the \( l \)-axis.

The reinforced concrete stiffness matrix is assembled by adding the reinforcement scant stiffness matrix to the concrete secant stiffness matrix:

\[
[D] = [D_c] + \sum_i [D_s]_i.
\] (20)

### 4. Analysis Algorithm

The simplified sectional analysis procedure is summarized in Figure 9. The sectional analysis is performed using the angle of the twist-controlled procedure. Therefore, the first step of the sectional analysis procedure is to set an initial angle of twist \( \phi \). Based on the assumption of the linear distribution of shear strain from the center of the section, the shear deformation \( \gamma_{lt} \) along the section can be calculated. The initial value of the longitudinal strain \( \varepsilon_l \) for each sectional element is assumed at the first iteration step. Then, the longitudinal strains can be iteratively calculated until the longitudinal force equilibrium of the model considering constitutive models is satisfied. Once the longitudinal strains are obtained, the axial force can be calculated. The iteration should be conducted until the axial force equilibrium is satisfied with the applied axial force. Once the equilibrium is satisfied, torsion for the given angle of twist can be calculated from the shear stresses along the reinforced concrete elements.
5. Program Verification and Discussion

To access the adequacy of the sectional analysis procedure, the test data available in the open literature pertaining to the rectangular section reinforced concrete beams under pure torsional loading condition have been used for verification.

Fang and Shiau [10] reported the results from a series of reinforced concrete beam tested under a pure torsional loading condition. The purpose of their research is to investigate the adequacy of the design provisions of torsion when it comes to the high-strength concrete. The major variables are the concrete strength and reinforcement ratios in both longitudinal and transverse directions. The sixteen reinforced concrete beams were tested to evaluate the differences in torsional behavior of strength and deformation between high- and normal-strength concrete (HSC and NSC hereafter). The overall dimension of the beam was $350 \times 500 \text{ mm}$ with $20 \text{ mm}$ clear concrete cover, and it remained the same for all beam specimens. The entire length of the beam was $3100 \text{ mm}$ that the middle portion of $1600 \text{ mm}$ was the main test region. Each end portion of beams was heavily reinforced to ensure the failure of the middle test region. Uniaxial concrete compressive strength ranging from $68.4$ to $78.5 \text{ MPa}$ was defined as a high-strength concrete, and the concrete strength ranging from $33.5$ to $35.5 \text{ MPa}$ was defined as a normal-strength concrete.

To cover the various torsional design practices, the total reinforcement ratios of both longitudinal and transverse reinforcement were varying from $1.2$ to $4.0\%$. The respective concrete strength, reinforcement ratios, and typical reinforcement layout within the beam specimens selected for each beam specimen are summarized in Table 1 and Figure 10 respectively.

To generate the input data for sectional analysis, the reinforced concrete sections are idealized as an equivalent thin-wall rectangular tube with a wall thickness $t$, as shown in Figure 11. Each side element of the thin-wall rectangular tube is assumed as a sectional element. The wall thickness $t$ of an equivalent thin-wall tube is typically taken as $0.75A_{cp}/P_{cp}$ per ACI 318–14 [4], where $A_{cp}$ is the area enclosed by the outside perimeter of the concrete cross section and $P_{cp}$ is the outside perimeter of the concrete cross section. The tributary length of each side element is defined as respective length minus the equivalent wall thickness. The reinforcement ratio of each beam specimen used in the analysis inputs was recalculated based on the equivalent element dimensions and reinforcement distribution within the section. Specifically, it was assumed that the corner reinforcing bars which are located at the intersection of adjacent side elements equally contributed to both elements as such the half of the reinforcing bar area were included for each side element to calculate reinforcement ratio.

The analytically computed torque to angle of twist responses along with the actual test responses are presented from Figures 12–15. The structural test responses for each beam were obtained from Fang and Shiau [10]. In general, it can be seen that the analytical predictions show good agreement with the test results for the prepeak torsional behavior of RC beams under pure torsion. Some difference on the torque corresponding to the initial crack was found since concrete cracking strength ($f_{cr}$) was simply assumed to be $0.33\sqrt{f'_c}$ in equation (12), which is defined in ACI 224.2R-92 [20]. The design equations referencing the compressive strength typically generate conservatively estimations for other concrete material properties as such the cracking loads are typically underestimated. The sectional analysis program does not well capture the postpeak responses. Specifically, in the analysis for the high-strength concrete, a sudden drop on the torque was observed after the maximum torque. Since the torsional design is on the basis of the ultimate strength, the postpeak behavior was not of interest for developing this analysis program. However, if performance-based behavior is of interest, an appropriate postpeak behavioral model for concrete should be incorporated within the program.

Table 2 summarizes the ultimate torque and the angle of twist obtained from both the structural tests and the sectional analyses. To compare the analysis results and test results, the torque and angle of twist obtained from analysis were divided by the respective experimental test results. As can be seen in the analysis-to-test ratio presented in Table 2, the ultimate torques predicted by the analysis showed good agreement with the test results. The average analysis-to-test ratio for the ultimate torques is $0.99$ with $8\%$ coefficient of variation (CoV).

Table 3 shows the prediction comparison between various models and the analysis results obtained from this study. As compared in the table, the proposed analysis procedure can well predict the torsional capacity of beams with various concrete strengths and reinforcement ratios.

Meanwhile, the angle of twist corresponding to the maximum torque was little underestimated by the analysis; the average analysis-to-test ratio is $0.78$ with $13\%$ coefficient of variation. Considering difficulties on ductility of
structural behavior of RC beams under pure torsion, it can be seen that even the predictions on the angle of twist corresponding to the maximum torque are within the acceptable range.

The analysis-to-test results in Table 2 are tabulated in Figure 16. As mentioned above, the ultimate torques obtained from both sectional analysis procedure and structural testing have a near-linear relationship. Meanwhile, the angle of twist
corresponding to the maximum torque obtained from the sectional analysis procedure somewhat underestimated the test results, but still within the acceptable range.

6. Conclusions

In this paper, a sectional analysis procedure based on the thin-walled tube analogy has been proposed to predict the torsional behavior of reinforced concrete beams subjected to pure torsion. For the development of the analysis procedure, the reinforced concrete section was idealized to consist of reinforced concrete elements subjected to biaxial stress states. On the analysis of each reinforced concrete element, the modified compression field theory was employed so that the constitutive relations in cracked reinforced concrete could be rigorously considered in the analysis. With the analysis results for reinforced concrete elements, compatibility and equilibrium on the section were also checked. Consequently, the developed analytical procedure predicts the torque-twist angle response of reinforced concrete beams under pure torsion, including the ultimate torque and the corresponding twist angle.

For the verification of the developed analysis procedure, 16 reinforced concrete beams under pure torsion were analyzed. The comparison of the analytical predictions and the test results showed good agreement on the ultimate torques; the average analysis-to-test ratio for the ultimate torques was 0.99 with 8% coefficient of variation. The angle
Figure 13: Analysis and test results comparison for beam 2: (a) H-06-12, (b) H-12-12, (c) N-06-12, and (d) N-12-12.

Figure 14: Analysis and test results comparison for beam 3: (a) H-07-16, (b) H-12-16, (c) N-07-16, and (d) N-12-16.
Figure 15: Analysis and test results comparison for beam 4: (a) H-20-20 and (b) N-20-20.

Table 2: Ultimate torque and the corresponding angle of twist: test versus sectional analysis results.

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<th>Specimen ID</th>
<th>Reinforcement layout</th>
<th>Test</th>
<th>Analysis</th>
<th>Analysis/test</th>
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<td>H-06-06</td>
<td></td>
<td>T_{test} (kN·m)</td>
<td>ϕ_{test} (rad/m)</td>
<td>T_{analysis} (kN·m)</td>
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<td></td>
<td>126.7</td>
<td>0.033</td>
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<td>0.038</td>
<td>140.2</td>
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<td>155.3</td>
<td>0.038</td>
<td>145.4</td>
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<td>0.040</td>
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<tr>
<td>CoV</td>
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Table 3: Comparison of calculated and experimental ultimate torques (reproduced from [10]).

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<td>1.02</td>
</tr>
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<td>0.92</td>
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<td>1.06</td>
</tr>
<tr>
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<td>Beam 2</td>
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<td>0.74</td>
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of twist corresponding to the maximum torque was also predicted within acceptable range; the average analysis-to-test ratio was 0.78 with 13% coefficient of variation.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare no conflicts of interest regarding the publication of this paper.

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**References**


[20] ACI Committee 224, 224.2R-92: *Cracking of Concrete Members in Direct Tension (Reapproved 2004),* American Concrete Institute, Farmington Hills, MI, USA, 1992.


