

## Research Article

# Experimental Analysis and Failure Criterion of Plain Concrete Subjected to Biaxial Loading under Fixed Lateral Loading

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By using a rock true triaxial apparatus hydraulic servo machine, biaxial loading experiments including biaxial compression-compression and biaxial compression-tension with fixed lateral loading on plain concretes were conducted and the stress-strain curves of plain concrete under various stress ratios were obtained. After determining the peak principal stress, the damage modes of plain concrete under various stress ratios were analyzed and the law of strength in the principal stress direction was studied as well. The experimental findings show that, under the fixed lateral loading, the failure modes of plain concrete under biaxial compression-compression and biaxial compression-tension are very similar to those under the equal proportional loading, but with higher amplitude of variation. In this paper, Kupfer's classical failure criterion was applied to verify the experimental data and the predicted biaxial loading on plain concrete under fixed lateral loading and was regarded as relatively conservative. Meantime, based on Kupfer's failure criterion and octahedral stress space, two different failure criteria had been proposed and verified. The results show that the proposed failure criteria have good applicability. The failure mechanism under fixed lateral loading was discussed and compared with that under the equal proportional loading method. This research is meaningful to plain concrete engineering application and calculation.

## 1. Introduction

Due to the unique advantage, concrete is widely used in engineering projects, such as building construction, bridge structure, dam foundation pit, and unclear prevention shell structure. In these mentioned structures, concretes are normally exposed to the complicated multiaxial compression status, for instance, biaxial compression, biaxial tension and compression, triaxial tension and compression, and triaxial compression [1–3]. In terms of this reason, accurate multiaxial compression analysis is conducive to reduce the waste of concrete and to improve the satisfaction of safety requirement in actual projects.

So far, there are plenty of researches on multiaxial compression on concrete [4–9]. Such as, in Kupfer's research [10], the proportional pressure loading method was applied to study the concrete with different strengths under biaxial compressive loading and the corresponding failure criterion

was initiated. An experiment on the concrete prism specimen under lateral triaxial compression, carried out by Takeda et al. [11, 12], showed that the confining pressure and strain rate in the axial direction had effects on concrete strength and its corresponding strain. By doing pressure loading experiment on concrete with high strength under high values of strain rate, the dynamic triaxial compressive strength of concrete and deformation was studied by Gran. The mechanics of concrete under biaxial compression and triaxial compression with proportional loading was studied by Guo and a simplified failure criterion was proposed as well [1]. Song et al. [13–16] analyzed biaxial tension and compression, triaxial tension and compression, and triaxial compression on concrete by using the proportional loading method and lateral loading method. In addition, multiaxial compression experiment on other kinds of concrete was carried out meantime, for example, lightweight aggregate concrete, steel fiber reinforced concrete, and roller compacted concrete. Based on

these experiments, a failure criterion for different concrete and constitutive relation was proposed. However, there is little available research on the property of concrete under multiaxial compression by fixed lateral loading yet. Regarding to this reason, a proper failure criterion under the lateral loading method to predict concrete strength performance is meaningful.

In this paper, six kinds of fixed lateral stress ratios were designed. By using the rock true triaxial apparatus hydraulic servo machine, biaxial loading experiments including biaxial compression-compression and biaxial compression-tension on plain concretes with three different strengths were conducted. The failure mode and the stress-strain curve for plain concrete under biaxial loading by various lateral stress ratios were obtained. Based on different lateral stress ratios and their peak principal stress, the peak compressive stress and peak tensile stress by different lateral pressures were analyzed. In accordance with Kupfer's biaxial failure criterion, the experimental data were verified. Meanwhile, based on Kupfer's biaxial failure criterion equation and octahedral stress space, the failure criterion for plain concrete under fixed lateral stress was proposed and verified.

## 2. Experiment Introduction

**2.1. Specimen Design.** In this research, plain concretes with the strength level of C30, C40, and C50 were required. C40 and C50 plain concretes were made of ordinary Portland cement (P.O 42.5), and C30 plain concrete was made of ordinary Portland cement (P.O 32.5). Besides, in all these concretes, fine aggregate (river sand with a maximum particle size of 5 mm), water from the same area, gravel with maximum diameter of 16 mm were allowed, and there was no any other additive ingredient in C30 and C40 at all. In accordance with the Regulation of Common Concrete Mix Design (JGJ55-2011), the proportion of plain concrete with different strengths was calculated and is shown in Table 1.

Plain concrete with different strengths was stirred mechanically and poured into 100 mm-sized cube mould. Then, cubes were placed on the vibration table for several minutes to improve the dense of the concrete specimen. After 24 hours, the moulds were removed, and the samples were maintained at a constant temperature and humidity laboratory for another 28 days ( $20^{\circ}\text{C} \pm 3^{\circ}\text{C}$  temperature, 95% humidity). Later on, the biaxial loading experiment under various lateral stress ratios was conducted.

**2.2. Experimental Device and Proposal.** The biaxial loading test on plain concrete with different strengths was conducted by the rock true triaxial apparatus hydraulic servo machine which has the hydraulic servo system and loading actuating head in three dimensions. Besides, on each loading actuating head, load sensor and displacement sensor were attached to ensure the required accuracy by the research. The setup of equipment is shown in Figure 1.

This paper mainly focused on the biaxial loading test of plain concrete, in which the biaxial compression-compression and biaxial compression-tension were carried out. The biaxial

compression-compression loading is shown in Figure 2(a). In this case, the lateral compression was loaded along the  $X$  direction. Once the predefined value was reached, the compression along the  $Z$  direction was loaded until the specimen failure. In the biaxial compression-tensional experiment, shown in Figure 2(b), the lateral compression was loaded in the  $Z$  direction firstly; after the lateral compression reached to the predetermined confining pressure, the splitting tensile load was applied along the  $X$  direction. Therefore, the force in the  $Y$  direction can be treated as the tensile stress, and the biaxial compression-tension loading mode can be done through this method.

In the biaxial loading experiment on plain concrete with different strengths, lateral compression loading schema can be determined by the following method. Firstly, the actual strength  $f_c$  of different concretes was determined by doing uniaxial compressive experiment. Based on the determined actual strength, the value of lateral compression was decided as  $0f_c$ ,  $0.1f_c$ ,  $0.2f_c$ ,  $0.3f_c$ ,  $0.4f_c$ , and  $0.5f_c$ . For each kind of specimen, at least three tests were carried out simultaneously.

According to related researches, it is very important to reduce the friction on loading surface during multiaxial compressive research. Otherwise, the obtained loading value may have significant difference with the actual loading compression. Due to this reason, a common method that uses three layers of polyethylene film coated with machinery grease was applied to reduce the influence of friction. The fixed lateral loading method was determined for this experiment. As for biaxial compression-compression experiment of plain concrete, firstly, start loading along the  $X$  direction with loading rate  $0.05\text{ MPa/s}$  till the specified confining stress. Then along  $Z$  direction, repeat preloading with values 0 to 10 KN for three times to eliminate the gap between test specimen and equipment. After that, initiate  $Z$  directional loading with the displacement of  $0.8\text{ mm/min}$  and collect experimental data until the specimen was damaged. According to the concrete biaxial compression-tensional test, the confining load is first applied in the  $Z$  direction. When the confining pressure reaches the predefined pressure, the splitting load is applied in the direction of  $X$ . In other words, it can be seen as the tensile stress is applied in the direction of  $Y$ . The loading rate is the same as that situation under biaxial compression-compression.

## 3. Experimental Result and Analysis

**3.1. Damage Mode.** When the  $Z$  direction of the plain concrete specimen suffered from compression, due to the effect of Poisson's ratio, tensile strain was generated from both  $X$  and  $Y$  directions. Once the value reached the maximum tensile strain of plain concrete, two to three main cracks, perpendicular to the loading surface, occurred.

In the biaxial compression-compression failure mode study on plain concrete with different strengths, the failure modes generated from the lateral pressure of  $0.1f_c$ ,  $0.3f_c$ , and  $0.5f_c$  in the  $X$  direction are shown in Figures 3(a)–3(c). When the lateral stress ratio were the same, the damage modes of plain concrete were the same as well which means the failure

TABLE 1: Contents of plain concrete (unit: kg/m<sup>3</sup>).

Grade	Cement	Water	Fine aggregate	Coarse aggregate	Fly ash	Mineral powder	Water reducer
C30	461	175	512	1252	—	—	—
C40	473	175	790	1187	—	—	—
C50	398	175	1280	1606	111	151	7

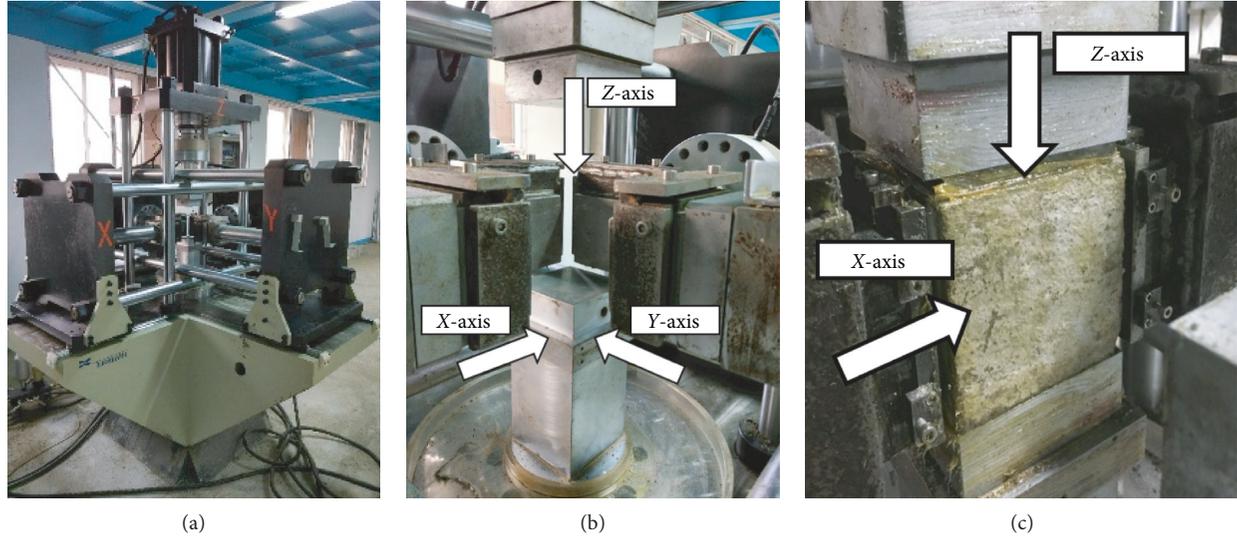


FIGURE 1: Rock fracture seepage flow testing machine. (a) True triaxial loading device. (b) Schematic diagram of the loading device. (c) Schematic diagram of the specimens' loading.

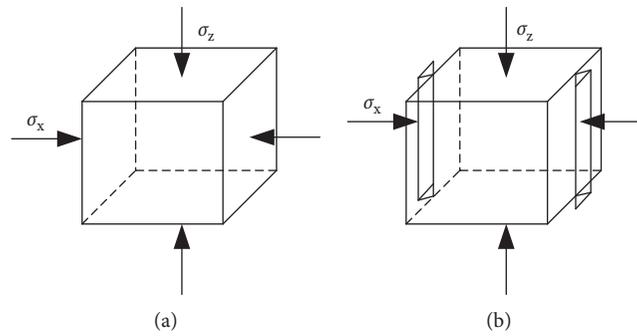


FIGURE 2: Biaxial loading schematic. (a) Biaxial compression-compression. (b) Biaxial compression-tension.

mode of plain concrete has no relation with its strength. When the lateral compression along the X direction was relatively small, in correspondence to, the generated compressive strain was relatively small. Meanwhile, if the vertical load increased in the Z direction, the tensile strain along the X direction and the Y direction was generated. Once the generated tensile strain was larger than the compressive strain in the X direction, the failure mode occurred in this point which was similar with that under uniaxial compression. On the contrary, if the value of compressive strain was greater than that of tensile strain in the X direction, cracks appeared in the Y direction then.

In this study, the lateral pressure in the Z direction is set to  $0.1f_c$ ,  $0.3f_c$ , and  $0.5f_c$ , respectively, to analyze the failure

mode of each test. According to the biaxial compression-tension failure modes of plain concrete in Figures 3(d)–3(f), the biaxial compression-tension failure modes of three different strength plain concretes can be recognized as the tensile failure mode and are almost same. In addition, the failure form has no relation with the value of lateral compression. The biaxial compression-tension failure mechanism of plain concrete is that, under the action of lateral compressive stress in direction Z, the tensile strain is formed by Poisson's ratio effect in both X and Y directions. And meantime, the splitting tensile load applied in direction X generates the tensile stress in the Y direction. When the tensile strain formed in the direction Y and the total value of Poisson's ratio effect along direction Y reach the ultimate

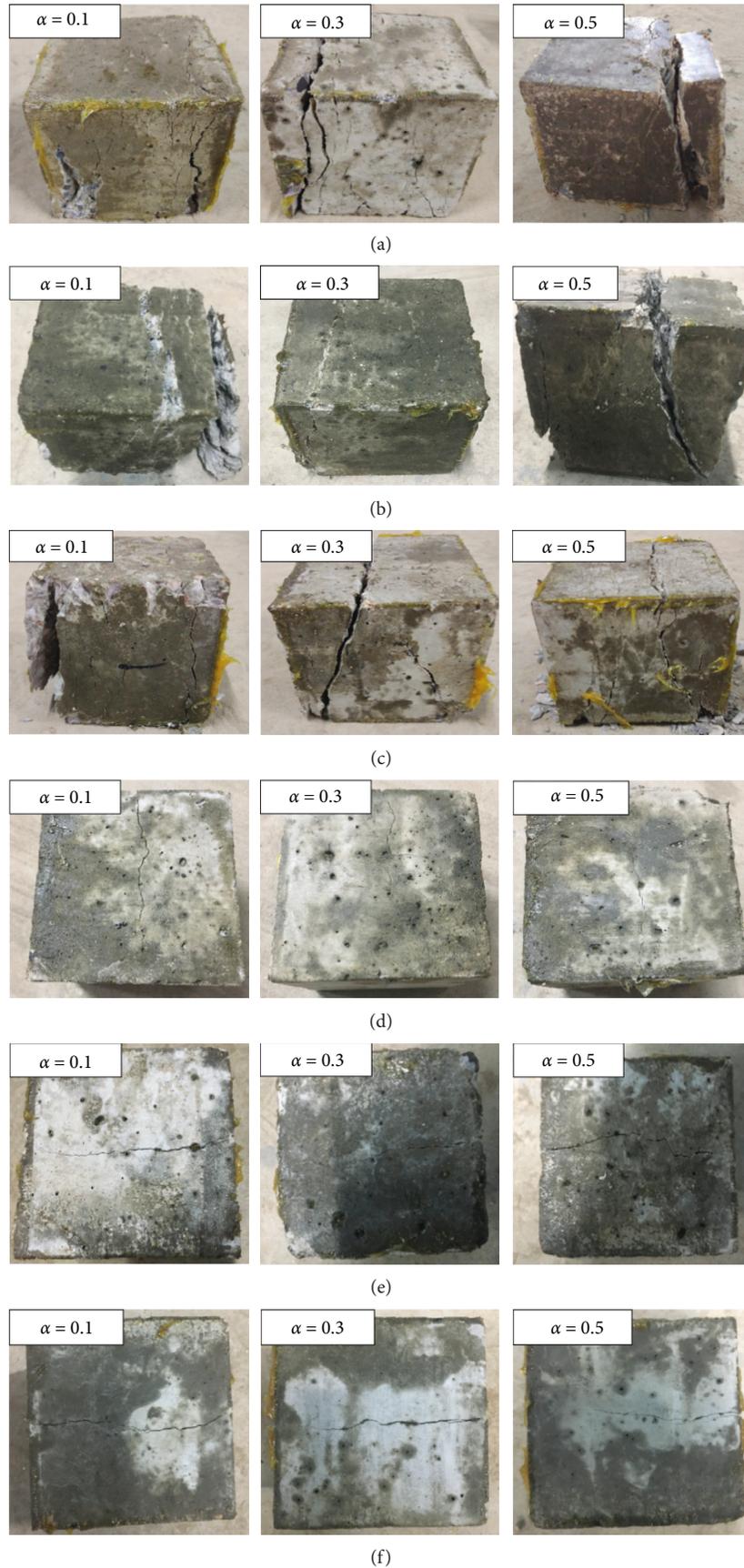


FIGURE 3: Biaxial loading failure mode. Biaxial compression-compression: (a) C30; (b) C40; (c) C50. Biaxial compression-tension: (d) C30; (e) C40; (f) C50.

tensile strain of the plain concrete, then the splitting failure mode appears in plain concrete which has no relation with the concrete strength.

Based on the above analysis of failure modes, the failure modes of concrete under different loading conditions are obtained which are shown in Figure 4. Figure 4(a) illustrates the failure mode of concrete under biaxial compression-compression when the uniaxial compression and lateral force are small, while Figure 4(b) illustrates the failure mode of concrete under biaxial compression-compression when the lateral force of concrete is high, and Figure 4(c) illustrates the failure mode of concrete under uniaxial tension and biaxial tension-compression of concrete.

**3.2. Stress-Strain Curve.** According to the designed loading method, biaxial loading experiment on plain concrete with different strengths by using the rock true triaxial apparatus hydraulic servo machine was carried out. The experimental stress was derived from the ratio of loading pressure and loading area which is obtained from the loading sensor. Meantime, the experimental strain was derived from the ratio of loading deformation and specimen height. In turn, the biaxial loading stress-strain curves for concrete with different strengths under different stress ratios were presented as shown in Figure 5.

Regarding to the biaxial compression-compression stress-strain curves of plain concrete presented in Figure 5, it has been found that the uniaxial compression curve had similar shape with the biaxial compression curve. Besides, it also can be found that, in the curves, there was a linear increasing trend firstly, followed by nonlinear increasing shape. Once the peak value of stress is reached, the ratio between stress and strain dropped immediately until the specimen failure occurred. In addition, based on the figure, the peak value of stress under biaxial compression was significantly higher than that under uniaxial compression. Correspondingly, the peak strain value under biaxial compression was also larger over the value under uniaxial compression. This trend can be applied to three different strengths of plain concrete.

Figure 6 shows the biaxial compression-tensile stress-strain curves of plain concrete with three different strengths and that the developing trend of uniaxial splitting stress-strain curve is the same as that of the lateral pressure stress-strain curve. The stress-strain curve develops from the elastic stage, and when stress reaches the peak stress, the curve shows direct descent. According to the preliminary analysis of the biaxial compression-tensile stress-strain curves of three different strength concretes, the splitting stress gradually decreases with the increase of lateral compressive stress.

**3.3. Eigenvalue of Stress-Strain Curve.** With the purpose to study the influence on plain concrete caused by lateral pressure, according to the relation between biaxial loading stress-strain curves presented in Figures 5 and 6, the principal compressive stress and the principal tensile stress of plain concretes with different strength are listed

in Table 2. By doing numerical analysis on principal stress of different concretes under various lateral stress ratios, the biaxial loading strength of plain concrete had been studied.

**3.4. Peak Stress.** In this paper, biaxial loading experiment with plain concretes C30, C40, and C50 was conducted. According to Table 2, their uniaxial compression values were 21.14 MPa, 27.61 MPa, and 45.45 MPa, respectively. The biaxial compression-compression peak stresses of plain concrete under various lateral stress ratios and envelopes were shown in Figure 7(a).

According to Figure 7(a), when plain concrete C30 was under biaxial compression, its peak stress value was in the range of 21.14 and 30.23 MPa which increased by 27% to 43% compared with that under uniaxial compression. The peak stress value for C40 was changed to the range between 27.61 and 47.08 MPa which increased by 39% to 71%. Similarly, compared with uniaxial compression, the peak stress value for C50 under biaxial compression increased from 10% to 34% with the value in the range of 45.45–60.99 MPa. Meanwhile, when  $\sigma_x/f_c$  was in the range from 0 to 0.2, peak stress  $\sigma_z$  increased along with the lateral stress ratio increasing. If the value of  $\sigma_x/f_c$  was between 0.2 and 0.5, the change of peak stress  $\sigma_z$  became steady with lateral the stress ratio increasing. This change had no relation with plain concrete strength. On the contrary, this trend of biaxial compression peak stress and lateral stress ratio had been found in the experiments conducted with proportional loading. When  $\sigma_x/f_c$  was in the range from 0 to 0.2, peak stress  $\sigma_z$  increased along with the increase in the lateral stress ratio. If the value of  $\sigma_x/f_c$  was between 0.2 and 0.7, the change of peak stress  $\sigma_z$  became steady with lateral stress ratio increasing.

Changing trend of tensile stress of plain concrete affected by lateral compressive stress under biaxial compression-tension is shown in Figure 7(b). In this paper, the uniaxial tensile strengths of concretes C30, C40, and C50 were 1.93 MPa, 2.55 MPa, and 2.67 MPa, respectively. When the lateral stress ratio was in the range of 0.1 to 0.5, the tensile stress of all three plain concretes gradually decreased. It can be noticed that the tensile stress on C30 plain concrete was reduced to 0.83 MPa, the peak decreased by 56.99%, and the average stress was 11.40% lower than 0.1 tensile stress; the tensile stress on C40 plain concrete was reduced to 1.23 MPa, the peak decreased by 51.76%, and the average stress was 10.35% lower than 0.1 tensile stress; the tensile stress on C50 plain concrete was reduced to 0.96 MPa, the peak decreased by 64.04%, and the average stress was 12.81% lower than 0.1 tensile stress. This law of the change is basically the same as the biaxial compression-tension experiment of plain concrete under the equal proportion loading.

## 4. Failure Criterion

**4.1. Kupfer's Biaxial Failure Criterion of Concrete.** In the study of the biaxial failure criterion of concrete, the most common failure criterion on principal stress space was established by Kupfer, regarding to experimental data and

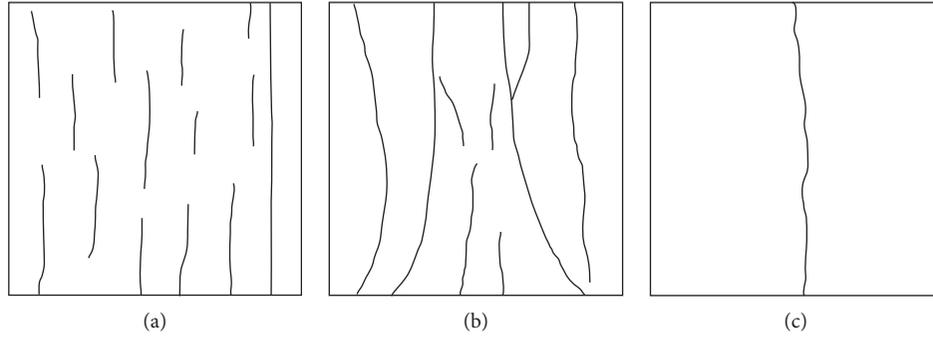


FIGURE 4: Failure modes of concrete under different loading modes. (a) Failure mode 1. (b) Failure mode 2. (c) Failure mode 3.

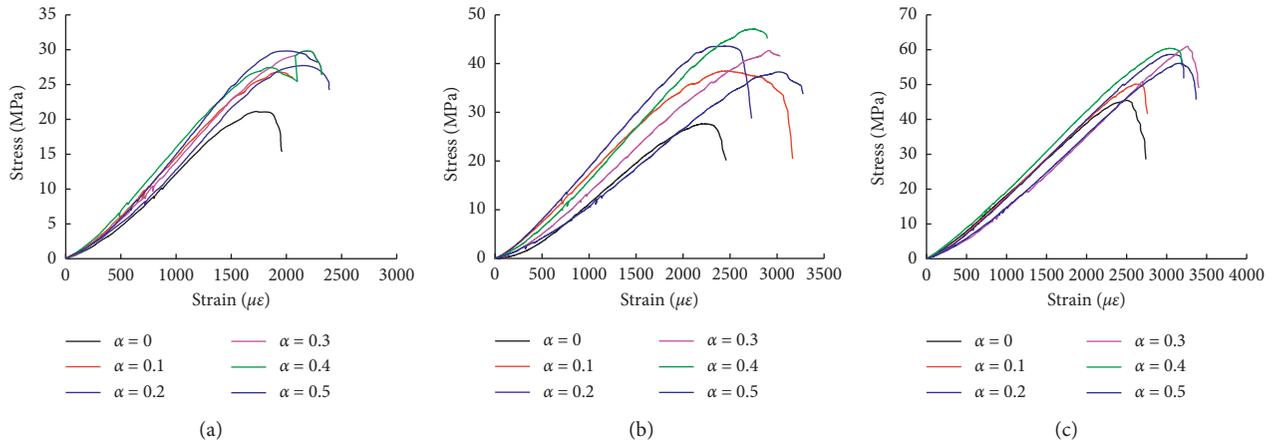


FIGURE 5: Biaxial compression-compression stress-strain curves of plain concrete. Ordinary concrete: (a) C30; (b) C40; (c) C50.

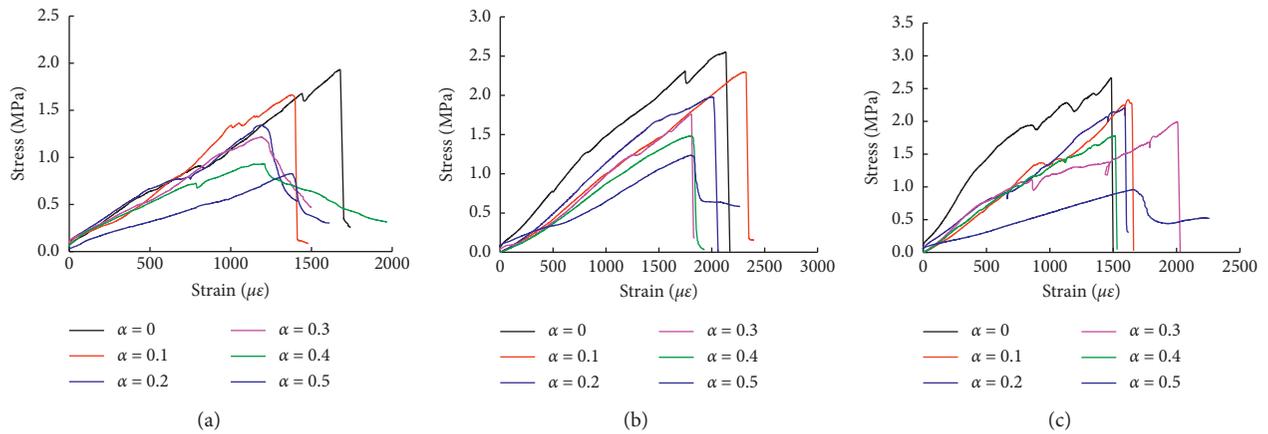


FIGURE 6: Biaxial compression-tension stress-strain curves of plain concrete. Ordinary concrete: (a) C30; (b) C40; (c) C50.

other researches. The equation of the failure envelope curve under biaxial compression is as follows:

$$\left(\frac{\sigma_Z}{f_c} + \frac{\sigma_X}{f_c}\right)^2 - \frac{\sigma_Z}{f_c} - 3.65 \frac{\sigma_X}{f_c} = 0. \quad (1)$$

By calculating equation (1), the ratio of stress was described as follows:

$$\begin{cases} \sigma_Z \leq \sigma_{Zc} = \frac{1 + 3.65\alpha}{(1 + \alpha)^2} f_c, \\ \sigma_Z \leq \sigma_{Zc} = \alpha \sigma_{Xc}, \end{cases} \quad (2)$$

$$\alpha = \frac{\sigma_Z}{\sigma_X}, \quad 0 \leq \alpha \leq 1.0,$$

TABLE 2: The eigenvalues of biaxial loading stress-strain curves of plain concrete with different strengths.

Stress ratio $\alpha$	Strength grade					
	C30		C40		C50	
	Compressive stress	Tensile stress	Compressive stress	Tensile stress	Compressive stress	Tensile stress
0	21.14	1.93	27.61	2.55	45.45	2.67
0.1	26.78	1.66	38.45	2.28	50.13	2.33
0.2	27.73	1.34	43.56	1.99	58.65	2.20
0.3	29.82	1.22	42.67	1.76	60.99	1.99
0.4	27.15	0.93	47.08	1.48	60.37	1.78
0.5	30.23	0.83	38.29	1.23	56.08	0.96

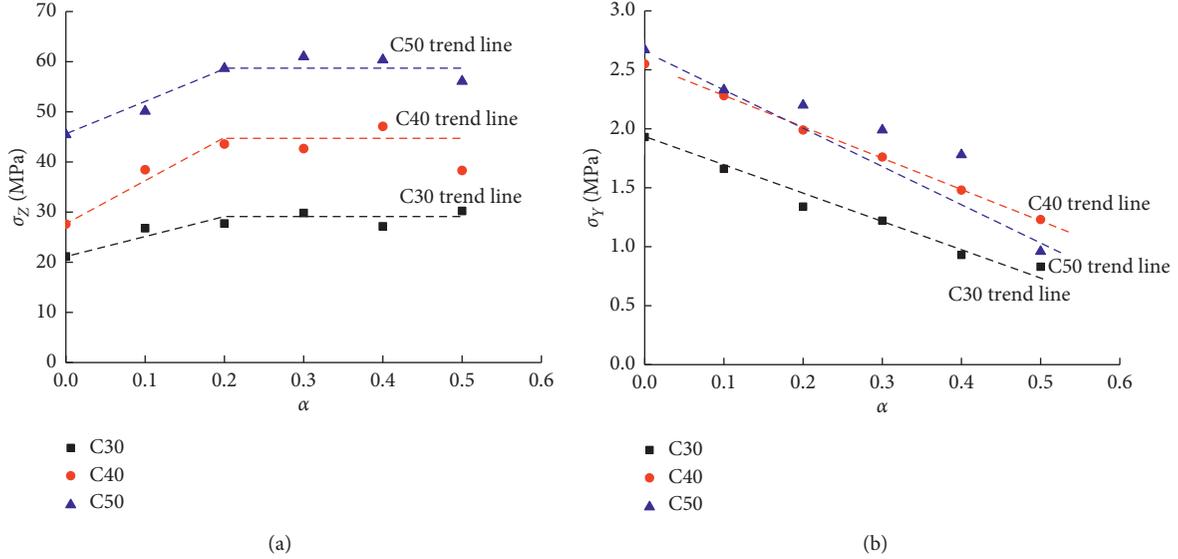


FIGURE 7: Biaxial loading peak stress of plain concrete. (a) Biaxial compression-compression. (b) Biaxial compression-tension.

where  $\sigma_Z$  and  $\sigma_X$  represent the ultimate strengths in Z and X directions when concrete was under biaxial compression, respectively.  $f_c$  stands for the ultimate strength of concrete under uniaxial compression, and  $\alpha$  stands for the ratio of stress.

In the study of the failure criterion of concrete under the biaxial tension-compression, an equation of the failure criterion proposed by Kupfer is described as

$$\frac{\sigma_Y}{f_t} + 0.8 \frac{\sigma_Z}{f_c} = 1, \quad (3)$$

where  $\sigma_Z$  means the lateral compressive stress of concrete;  $\sigma_Y$  stands for the tensile stress of concrete; and  $f_c$  and  $f_t$  are the uniaxial compressive strength and uniaxial tensile strength of concrete, respectively.

The relationship between experimental data on plain concrete under biaxial loading achieved in this paper and numerical data derived from Kupfer's classical biaxial failure criterion is presented in Figure 8.

The failure criterion proposed by Kupfer is relatively conservative compared with that in the experiment in this paper, which is obviously lower than the principal stress variation coefficient obtained in this paper. The reason is that the Kupfer biaxial loading failure criterion is based on the equal proportion loading method and the biaxial loading method used in this paper is the fixed lateral loading mode. But with the lateral stress ratio, the variation trend of the

principal stress is basically the same and has nothing to do with the loading way.

Based on the biaxial loading failure criterion of concrete proposed by Kupfer, a new failure criterion under lateral loading pressure is shown as follows:

$$\left( \frac{\sigma_Z}{f_c} + \frac{\sigma_X}{f_c} \right)^2 + a \frac{\sigma_Z}{f_c} + b \frac{\sigma_X}{f_c} = 0, \quad (4)$$

$$\frac{\sigma_Y}{f_t} + c \frac{\sigma_Z}{f_c} = 1. \quad (5)$$

By doing the Matlab calculation with experimental data, the biaxial loading failure criterion under fixed lateral loading was derived as shown in equations (6) and (7), and its relationship is presented in Figure 9:

$$\left( \frac{\sigma_Z}{f_c} + \frac{\sigma_X}{f_c} \right)^2 - 1.02 \frac{\sigma_Z}{f_c} - 4.20 \frac{\sigma_X}{f_c} = 0, \quad (6)$$

$$\frac{\sigma_Y}{f_t} + 1.10893 \frac{\sigma_Z}{f_c} = 1. \quad (7)$$

According to the change in the peak stress increasing coefficient with the lateral stress ratio and Kupfer's biaxial compression-compression failure criterion, the failure criterion of two-dimensional curve was described as follows:

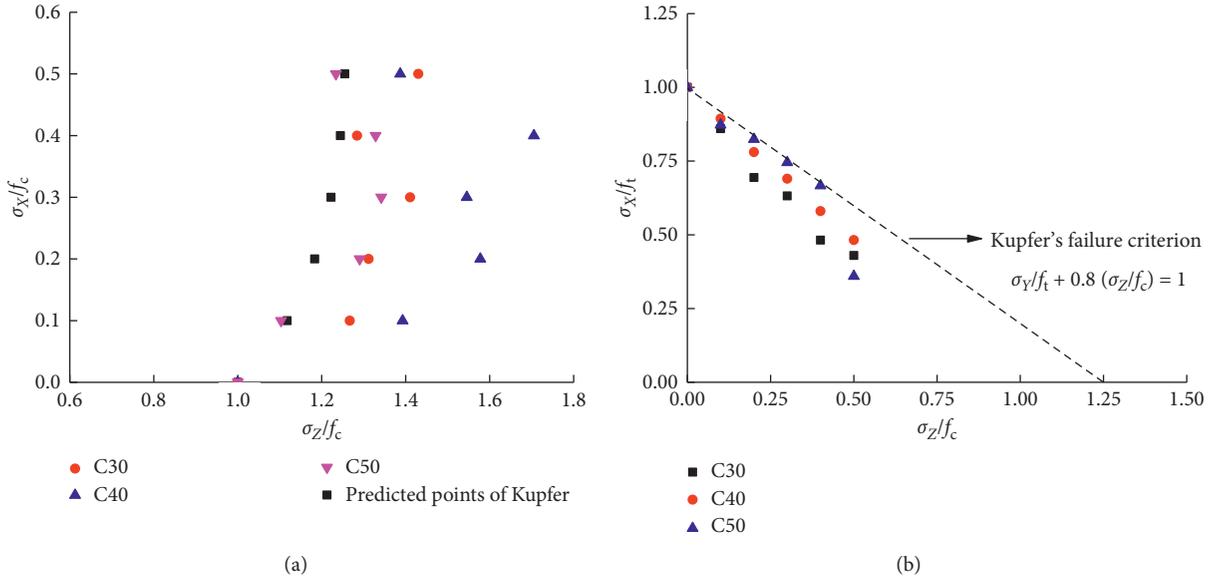


FIGURE 8: Comparison of Kupfer's failure criterion and experimental data. (a) Biaxial compression-compression. (b) Biaxial compression-tension.

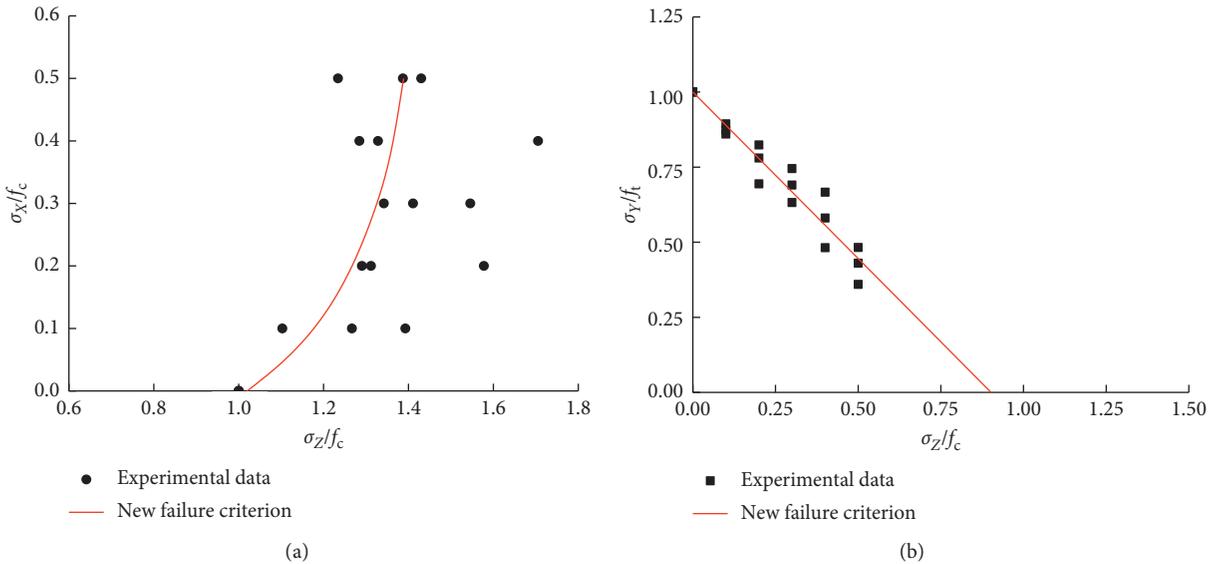


FIGURE 9: New failure criterion based on Kupfer's failure criterion. (a) Biaxial compression-compression. (b) Biaxial compression-tension.

$$\frac{\sigma_Z}{f_c} = d + e \frac{\sigma_X}{f_c} + f \left( \frac{\sigma_X}{f_c} \right)^2, \quad (8)$$

where  $\sigma_Z$  and  $\sigma_X$  represent the ultimate strengths in the Z and X directions when concrete was under biaxial compression, respectively.  $f_c$  stands for the ultimate strength of concrete under uniaxial compression. Symbols  $d$ ,  $e$ , and  $f$  stand for the involved coefficient.

In accordance with the experimental data and equation (9), the biaxial compression-compression failure criterion of the two-dimensional curve is described in Figure 10.

Based on the above finding, it had been found that the biaxial compression-compression damage mode and the

behavior of change for plain concrete were consistent. According to one variable quadratic equation failure criterion, the biaxial compression-compression failure criterion was derived as

$$\frac{\sigma_Z}{f_c} = 1.01034 + 2.67559 \frac{\sigma_X}{f_c} - 4.01067 \left( \frac{\sigma_X}{f_c} \right)^2. \quad (9)$$

**4.2. Octahedral Stress Space.** In the study of the true three-axis test of concrete, the independent loads on three directions are  $\sigma_X$ ,  $\sigma_Y$ , and  $\sigma_Z$ , respectively. In the three-dimensional coordinate system, the angles between water

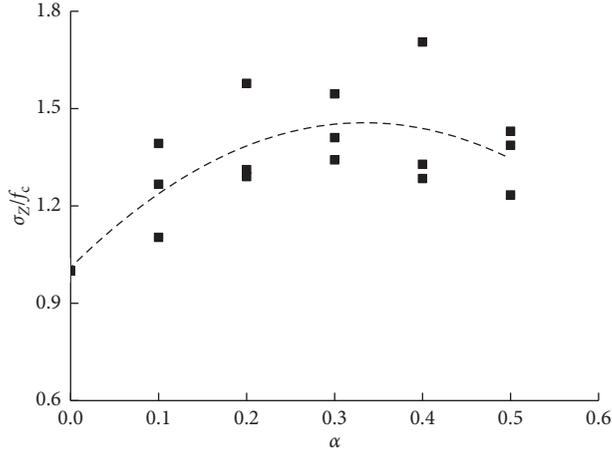


FIGURE 10: Biaxial compression-compression failure criterion of the two-dimensional curve.

pressure axis and all three axes are the same and  $\sigma_X = \sigma_Y = \sigma_Z$ . The plane perpendicular to the hydrostatic pressure axis is the PI plane; the normal stress on this plane is  $\sigma_{oct}$  and the shear stress is  $\tau_{oct}$ ; its expression is shown as follows:

$$\sigma_{oct} = \frac{\sigma_X + \sigma_Y + \sigma_Z}{3}, \quad (10)$$

$$\tau_{oct} = \frac{1}{3} \sqrt{(\sigma_Z - \sigma_X)^2 + (\sigma_X - \sigma_Y)^2 + (\sigma_Y - \sigma_Z)^2}. \quad (11)$$

Based on the octahedral stress space, the failure criterion equation of concrete under the fixed lateral loading mode is proposed. In order to simplify the failure criterion for the tensile and pressure meridian damage of recycled concrete with different substitution rates, which is the five-parameter failure criterion of Song model [13], a two-parameter-based equation and a three-parameter-based equation were proposed to simplify the criterion as follows:

$$\frac{\tau_{oct}}{f_c} = j + k \frac{\sigma_{oct}}{f_c}, \quad (12)$$

$$\frac{\tau_{oct}}{f_c} = g + h \frac{\sigma_{oct}}{f_c} + i \left( \frac{\sigma_{oct}}{f_c} \right)^2. \quad (13)$$

Through the mathematical regression analysis of formulae (12) and (13) with the experimental data obtained in the biaxial stress test, new formulae (14) and (15) was derived then. Furthermore, based on the octahedral stress space, the curve of concrete under biaxial stress with the fixed lateral loading method is described in Figure 11:

$$\frac{\tau_{oct}}{f_c} = -0.08629 - 0.91883 \frac{\sigma_{oct}}{f_c}, \quad (14)$$

$$\frac{\tau_{oct}}{f_c} = -0.05808 - 1.34663 \frac{\sigma_{oct}}{f_c} + 0.56325 \left( \frac{\sigma_{oct}}{f_c} \right)^2. \quad (15)$$

According to the analysis of Figure 11, it has been found that the failure criterion of plain concrete under biaxial stress with the fixed lateral loading method can be explained more adequately by the two-parameter-based equation and three-

parameter-based equation. Especially, the failure criterion described by the three-parameter-based equation is more accurate than that of the equation with two parameters.

## 5. Verification of New Failure Criteria

Based on the biaxial compression-compression and biaxial tension-compression experimental data of three different strength grades of concrete and the failure criteria of Kupfer concrete under biaxial compression-compression and biaxial tension-compression, the failure criterion equations of concrete under biaxial compression-compression and biaxial tension-compression under constant lateral loading are proposed, as shown in formulae (6) and (7). According to the failure law of concrete biaxial compression-compression, the failure criterion equation of concrete biaxial compression-compression quadratic equation is proposed, as shown in formula (9). At the same time, according to the octahedron stress space, the failure criterion equation of concrete under biaxial loading is proposed, as shown in formulae (14) and (15). In order to verify the applicability of the above failure criteria proposed in this paper, comparative analysis is conducted between the experimental data in the literature [17–23] and the failure criteria proposed, as shown in Figure 12.

According to the experimental data of the relevant literature, the failure criteria proposed in this paper are validated. As shown in Figure 12, due to the randomness and discreteness of concrete, the experimental data of different literatures are relatively discrete. However, the application of the failure criterion proposed in this paper to validate the literature data has better applicability and can better describe the failure law of concrete. Among them, the biaxial tension-compression failure criterion which is based on the Kupfer failure criterion and the octahedron stress space failure criterion has the best applicability.

## 6. Discussion

Under the working condition of biaxial compression-compression, the principal compressive stress of plain concrete is influenced by the lateral compressive stress, which is obviously higher than the uniaxial compressive strength. This difference should be caused by the lateral pressure stress restraint which inhibits the expansion of the microcracks in the concrete and enhances the occlusion of the aggregate. In terms of this reason, the compressive strength is greater than the uniaxial compressive strength.

With the working condition of biaxial compression-tension, the lateral compressive stress on the plain concrete in the Z direction is affected by Poisson's ratio. Also, the tensile strain is generated in both X and Y directions. With the increase of the lateral compressive stress, the tensile strain increases gradually. At this time, the tensile stress is applied in the direction Y. The critical value of the ultimate tensile strain is gradually reduced with the increase in the lateral compressive stress. Hence, the principle tensile stress of concrete gradually decreases with the increase of lateral compressive stress.

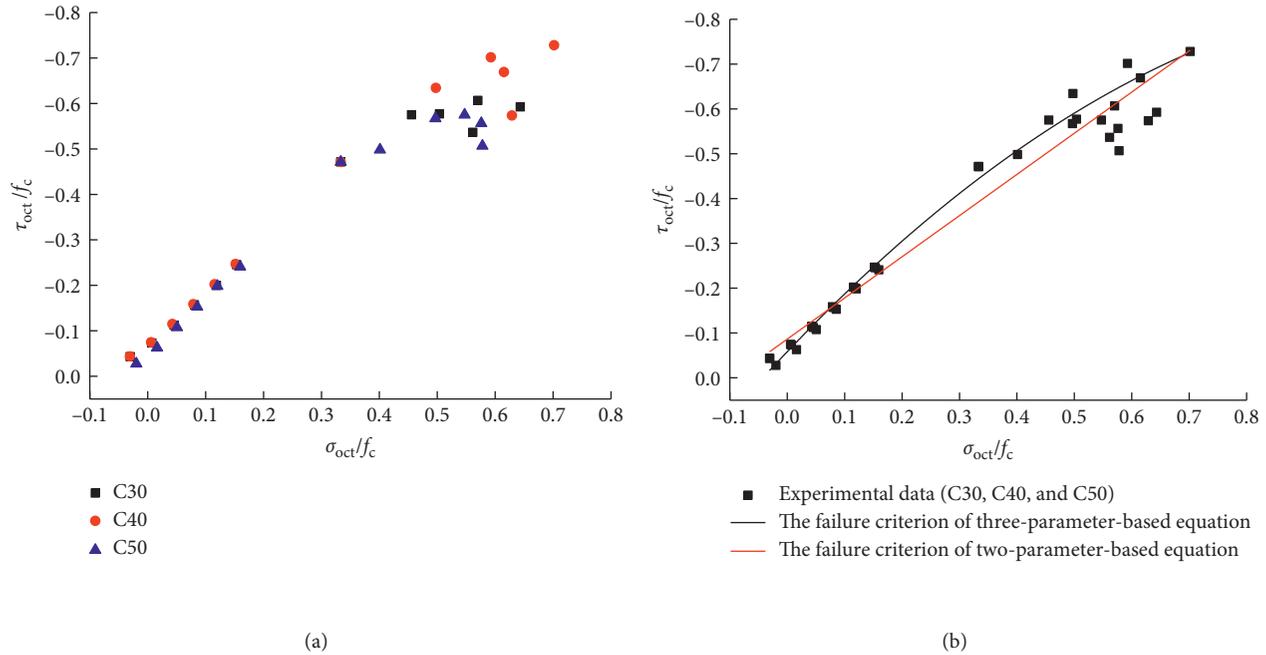


FIGURE 11: The new failure criterion based on octahedral stress space. (a) Experimental data based on octahedral stress space. (b) Two different kinds of failure criterion.

Regarding to the equal proportion loading method, the failure criterion for concrete under biaxial stress proposed by Kupfer is more conservative to predict the principal compressive stress and the principal tensile stress of plain concrete under the fixed lateral loading pattern. When applying the proportional loading method for plain concrete under biaxial compression-compression, the lateral compressive stress and the principal pressure stress are loaded simultaneously; the lateral compressive stress is changed from 0 MPa to the predefined confining pressure. Therefore, the principal pressure stress is also increased. In the fixed lateral loading mode, when the lateral compressive stress reaches the preset value, the principal compressive stress begins to be loaded until the specimen is destroyed. Apparently, the principal compressive stress is more affected by the lateral compressive stress under the fixed lateral loading mode; that is, the increase of principal pressure stress is higher than that of equal proportion loading. In the same principle, the effect of lateral compressive stress on the principal tensile stress under the fixed lateral loading is higher than that of equal proportions loading mode, and the decrease range of principal tensile stress with increase of the lateral compressive stress is higher.

## 7. Conclusion

In this paper, biaxial compression-compression experiment and biaxial compression-tension experiments of plain concrete were studied under fixed lateral loading. Six different lateral stress ratios and three different concrete strengths were selected. By analyzing the change law of the principal compressive stress and the principal tensile stress of the plain concrete influenced by the lateral compressive stress, the laws of biaxial mechanical performance and damage under the fixed lateral loading were studied. The main conclusions are as follows:

- (1) With the same lateral stress ratio, the damage modes of concretes with different strengths were same. For biaxial compression-compression of concrete, when the lateral stress ratio was relatively small, its damage mode was similar with uniaxial compression. On the contrary, when the lateral stress ratio was relatively large, cracks appeared in the specimens. For biaxial compression-tension of concrete, the splitting failure mode occurred in concrete and it had no relation with the value of lateral compressive stress.
- (2) Compared with the experimental findings obtained with the equal proportion loading method and the fixed lateral loading method, it is suggested that the change law of plain concrete under biaxial compression-compression influenced by the lateral compressive stress was basically the same. When  $\sigma_x/f_c$  was in the range from 0 to 0.2, peak stress  $\sigma_z$  increased along with the increase in the lateral stress ratio. If the value of  $\sigma_x/f_c$  was between 0.2 and 0.5, the change of peak stress  $\sigma_z$  became steady with the increase in the lateral stress ratio. This change had no relation with concrete strength.
- (3) Under the working condition of biaxial compression-tension of plain concrete, the law of principal tensile stress influenced by lateral compressive stress is basically the same in the equal proportion loading method and fixed lateral loading method, but the effect of lateral pressure stress on the principal tensile stress of plain concrete under fixed lateral loading mode is more obvious.
- (4) After doing numerical analysis, it has been found that the calculated value of Kupfer's biaxial loading failure criterion was relatively conservative compared with

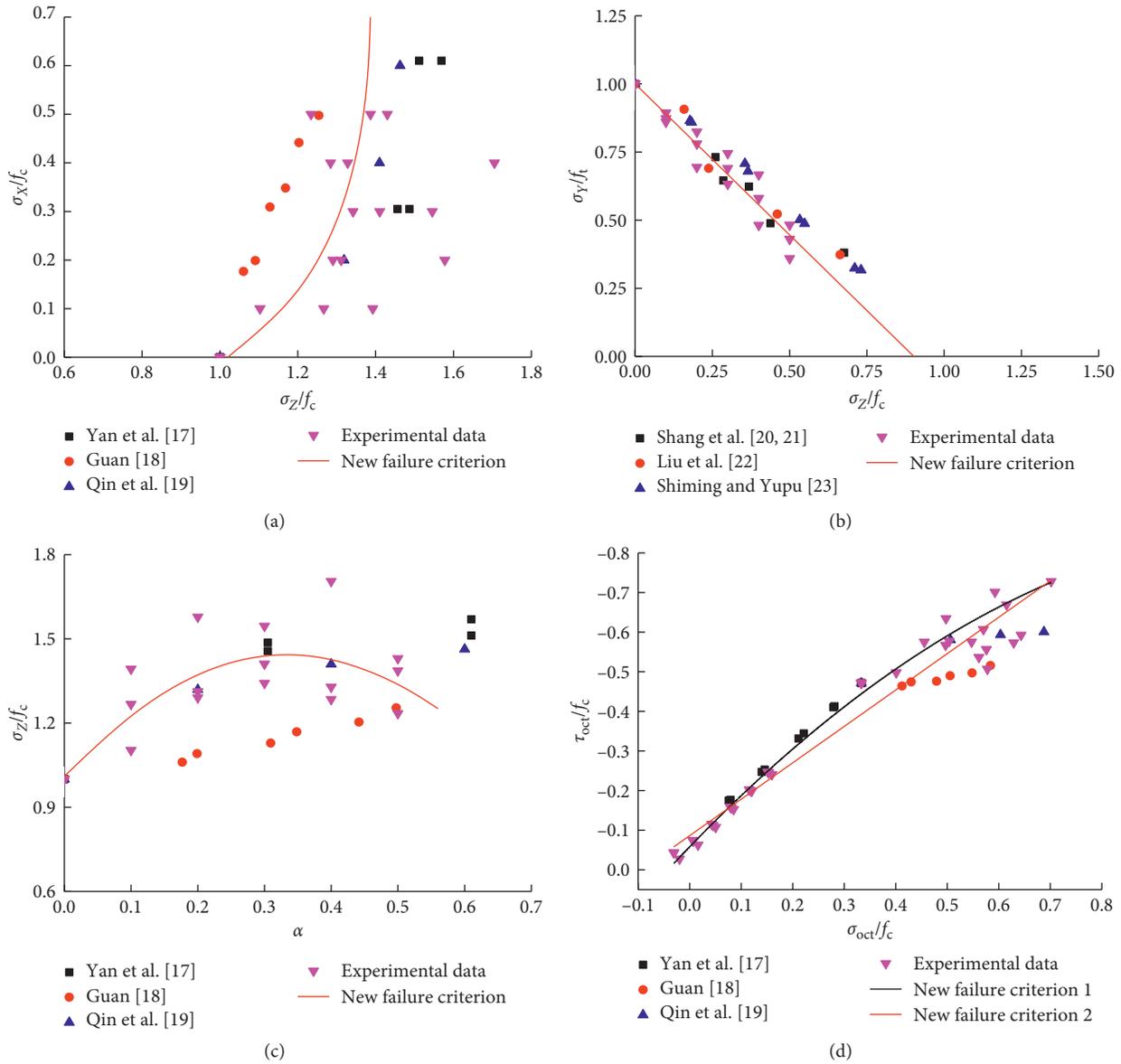


FIGURE 12: Verification of different failure criteria. The failure criterion of (a) biaxial compression-compression (based on Kupfer’s failure criterion); (b) biaxial tension-compression (based on Kupfer’s failure criterion); (c) biaxial compression-compression (based on unary quadratic equation); (d) octahedral stress space.

experimental data collected under lateral stress loading method. However, the change of maximum stress predicted by Kupfer’s biaxial failure criterion and experimental data was consistent. In addition, regarding to Kupfer’s failure criterion and octahedral stress space, the biaxial loading failure criterion equation under lateral pressure was proposed. The failure criteria proposed in this paper are validated by the experimental data in the literature. The results show that the failure criterion proposed in this paper has good applicability.

**Data Availability**

The nature of the data is the experimental data of plain concrete subjected to biaxial loading under fixed lateral

loading. The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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