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Research Article

Two-Dimensional Electroosmotic Consolidation Theory of Nonlinear Soil Voltage Distribution Characteristics

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Soil voltage is generally assumed to show a linear relationship with distance from the cathode according to the established electroosmotic consolidation equation. However, this assumption is inconsistent with experimental results. To more reasonably reflect the soil consolidation process during electroosmosis treatment, it is necessary to consider the influence of the actual soil voltage distribution trend when establishing the electroosmotic consolidation equation. Electroosmosis results show that soil voltage exhibits nonlinear distribution characteristics against distance from the cathode. The change trend of soil voltage can be well reflected by cubic polynomial fitting. Then, the anodic electrode was taken as the research object, and a two-dimensional horizontal plane model of electroosmosis was established because it represents practical electroosmosis applications more closely than some other models. Based on this established model, the dissipation equation describing the excess pore water pressure and the soil consolidation equation were derived for the electroosmosis treatment process. The derivation process considered both linear and nonlinear soil voltage distributions, wherein the anode was closed and the cathode was open. Finally, the analytical solution was analyzed and validated with model test cases in terms of the excess pore water pressure and average moisture content of the soil. The trend observed in the measured excess pore water pressure was more consistent with that of the theoretical results calculated assuming a nonlinear soil voltage distribution than that obtained using a linear distribution. In addition, the measured values of the average moisture content in the soil were closer to the values calculated under a nonlinear distribution of soil voltage than to those calculated under a linear distribution. These results further show that the established consolidation equation is reasonable when a nonlinear distribution of soil voltage is considered. The proposed consolidation equation can thus improve the application of electroosmotic methods in the future.

1. Introduction

With rapid economic development and an increase in the population, the land required for infrastructure is increasing in various industries. Soft clay foundations with low permeability, high moisture content, high compressibility, and low strength have been increasingly exploited to alleviate the shortage of usable land [1–3]. For example, in recent years, an increasing number of reclamation and dredging projects have been carried out in China [4, 5]. In such locations, the soft clay foundation must be strengthened before construction to prevent excessive settling and to meet the bearing capacity requirements for the foundation.

Surcharge preloading and vacuum preloading methods have been widely used for reinforcing soft clay foundations for decades. These methods have achieved good results but have also shown many problems. The vacuum preloading method provides limited improvement in the bearing capacity of a foundation. The consolidation rate in the vacuum preloading method is limited by the hydraulic permeability coefficient of soft clay; therefore, soil with high clay content requires prolonged treatment when using this preloading method. The treatment of deep soil has achieved limited success, which makes it difficult to acquire the desired enhancement. Thus, secondary treatment is often needed before construction [6, 7]. Surcharge preloading requires a large amount of materials and higher transportation and construction costs than some other methods [8]. In addition, excessive loading may increase the shear stress, which may lead to shear failure of the foundation [9]. Considering the shortcomings of the abovementioned reinforcement methods, new alternatives must be urgently explored. Examples adopted in recent years have included the electroosmotic method, which is relatively new and has proven to be effective in the reinforcement of soft clay foundations.

When the electroosmotic method is used for treating soft clay foundations, the shear stress does not increase. Furthermore, this method is not limited by the hydraulic permeability coefficient of soft clay particles [10]. Therefore, electroosmosis offers great potential for application in the reinforcement of soft clay foundations. Since its first introduction in 1949, many researchers have undertaken experimental studies on the electroosmotic method [11–14]. However, theoretical studies on the electroosmosis method have not been carried out to the same extent, which has somewhat restricted the development of the technique.

In 1968, Esrig proposed a one-dimensional electroosmotic consolidation theory under the assumption that the flow caused by the voltage and head differences can be superimposed. This theory is used to explain the phenomena of excess pore water pressure increase and dissipation in the electroosmosis process [15]. In 1976, based on Esrig's one-dimensional electroosmotic consolidation theory, Wan proposed a one-dimensional electroosmotic consolidation theory considering external loads. The electrode conversion technology has been shown to benefit the electroosmosis process [16]. In 2004, Su proposed an analytical solution to a two-dimensional consolidation theory using the method of block processing. The excess pore water pressure produced can be positive or negative during the electroosmosis process [17]. In 2013, employing the Lagrangian coordinate system, Wang established a theoretical one-dimensional nonlinear large deformation consolidation equation with an excess pore water pressure variable based on Esrig's one-dimensional consolidation theory [18]. In 2014, Wang extended the general equation of the unidirectional seepage consolidation process to the field of electroosmosis by considering the interaction of flow and current under linear surcharge and the dissipation of excess pore water pressure, consequently establishing a one-dimensional electroosmotic consolidation equation of soft clay under a linear surcharge [19]. Moreover, the analytical solutions of excess pore water pressure and average consolidation degree were given under two situations ((a) cathode open and anode closed and (b) both cathode and anode open). In 2017, Zhou established a one-dimensional electroosmotic consolidation model considering the variation in saturation. A numerical solution of the electroosmotic consolidation equation considering the variation in saturation was obtained based on that model and an example, which was used to analyze the variations in pore water pressure, pore gas pressure, saturation, and settlement with time during the process of electroosmotic consolidation [20].

However, the abovementioned electroosmotic consolidation theories all assume that the soil voltage is linearly distributed in the process of derivation. This assumption does not agree with the experimental results obtained in recent years [21, 22]. Therefore, to more reasonably describe soil consolidation during electroosmosis, the actual distribution of soil voltage must be considered when establishing the electroosmosis consolidation equation to improve the theory of electroosmosis consolidation.

For this reason, laboratory electroosmotic tests of typical soft clay from Taizhou used in reclamation were first carried out, and then the distribution of soil voltage was analyzed and discussed. The anodic electrode was studied as the research object, and a two-dimensional horizontal model was established, which produced results that were closer to the observations made in practical engineering applications than the results given by some other models. Based on this model, a two-dimensional electroosmotic consolidation equation was deduced considering the actual distribution of soil voltage. Finally, the rationality of the equation was analyzed and verified by model test cases. The influence of the change in soil voltage on the electroosmosis process was analyzed by comparing the results of the proposed equation with those produced under the assumption that the soil voltage is distributed linearly.

2. Materials and Methods

2.1. Experimental Materials. The horizontal electroosmosis test model tank used in the experiment included a sample tank and a collection tank (Figure 1).

The sample tank used to load the soil sample had internal dimensions of $250\,\mathrm{mm} \times 130\,\mathrm{mm} \times 200\,\mathrm{mm}$ $(L \times H \times W)$. The internal dimensions of the collection tank were $250\,\mathrm{mm} \times 70\,\mathrm{mm} \times 200\,\mathrm{mm}$ $(L \times H \times W)$. A small hole was made at the bottom of the collection tank to collect the water discharged during the electroosmosis process. The electroosmotic experiment was environmentally closed throughout the process; so, the influences of water evaporation on the experimental process are negligible.

The other major test devices used included a DC power supply (IT6863A) and a multimeter. The maximum output voltage of the power supply was 72 V, the maximum output current was 3 A, and the maximum output power was 108 W. A multimeter was used to measure the soil voltage during the electroosmosis test.

The soil samples used in the tests were obtained from a new land area site formed by reclamation of a coastal industrial zone in Taizhou, Zhejiang Province. The physical and mechanical properties of the soil samples are listed in Table 1.

When an electrokinetic geosynthetic (EKG) electrode is used as both a cathode and an anode, it can not only provide filter and drainage channels but also accelerate pore pressure dissipation and soil consolidation. Therefore, a tabular EKG

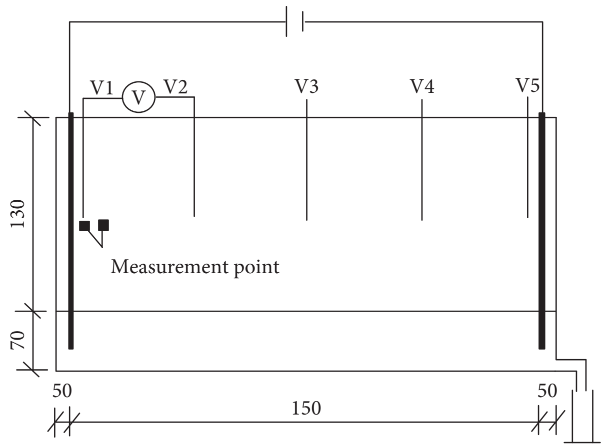

Figure 1: Schematic diagram of the model test device (units: mm).

electrode was selected as the test electrode for use in the tests (Figure 2). Its graphical representation is shown in Figure 3.

The shape of the tabular EKG is similar to that of the plastic drainage plate used in the vacuum preloading project. The tabular EKG consists of three parts: substrate, wire, and filter membrane. The substrate is made of a conductive plastic. Multiple grooves are imprinted on the surface of the substrate for drainage during the electroosmosis process. Two copper wires are embedded in the substrate to connect to the DC power supply.

2.2. Experimental Methods. The test included five experiments. A new tabular EKG was used as the test electrode material. Five applied voltages were tested (15 V, 22.5 V, 26.25 V, 30 V, and 33.75 V). The test duration was set at 45 h based on previous test results. Specific test parameters are given in Table 2.

The specific test steps were as follows. A suitable amount of dried undisturbed soil was crushed. The required quantity of water was determined based on an assumed initial moisture content of 60% and poured into the soil. An electric mixer was used to evenly mix the soil for the remolded soil samples. The moisture content of the remolded soil samples was measured after the samples were allowed to rest for 24 h. A geotextile was wrapped on the outside of the tabular EKG. Then, the EKG was placed in the corresponding position of the sample tank as the cathode or anode. The test soil samples were loaded in layers. A measurement cylinder was placed under a small hole in the collection tank to collect the water discharged by electroosmosis. Voltage probes were inserted at distances of 37.5 mm, 75 mm, and 112.5 mm from the cathode and near the electrodes to measure the soil voltage. The inserted depth of each voltage probe was 65 mm. The wires, power supply, and electrodes were connected. The power supply was adjusted to the voltage set in the test. The circuit was closed. The current, soil voltage, and discharged water were recorded every 30 min. The test was stopped after 45 h. Finally, the DC power supply was disconnected, and the test device was removed.

3. Distribution of Soil Voltage

Based on the above experiments, five groups of soil voltage distribution data were obtained successively in the process of electroosmosis treatment under different voltages. The experimental data were normalized and are plotted in Figure 4.

Unified fitting of the obtained data shows that the actual soil voltage distribution exhibits a nonlinear response. The soil voltage distribution was found to be consistent with the test results reported by Estabragh et al. [22–24]. There are two main reasons for the nonlinear distribution of the actual soil voltage during the electroosmosis process [25–27]. On the one hand, this nonlinear distribution is caused by the interface resistance on the contact surface between the soil and electrodes. During the electroosmotic test, the water in the soil flows from the anode to the cathode and ultimately discharges. The gap between the soil and electrode increases accordingly. As a result, the interface resistance on the contact surface between the soil and electrode increases; therefore, the actual soil voltage presents a nonlinear distribution. On the other hand, this nonlinear distribution is caused by the complicated electrochemical reaction during the electroosmotic test. During the test, it is clear that the color of the geomembrane on the EKG plate electrode in the anode turns from white to yellow, which indicates that a complicated electrochemical reaction occurs. The complicated chemical reaction between positive and negative ions can also have remarkable effects on the soil voltage distribution.

Cubic polynomials were used to fit the test data under different output voltages by observing the voltage response trends. The relationships between the measured soil voltages and the normalized distance from the cathode under different output voltages were obtained and can be expressed as follows:

$$\begin{aligned} 15\,\mathrm{V}: U' &= 19.69x' - 50.89x'^2 + 46.1x'^3, \\ 22.5\,\mathrm{V}: U' &= 29.43x' - 77.47x'^2 + 70.26x'^3, \\ 26.25\,\mathrm{V}: U' &= 33.04x' - 87.66x'^2 + 80.59x'^3, \\ 30\,\mathrm{V}: U' &= 36.01x' - 93.1x'^2 + 86.93x'^3, \\ 33.75\,\mathrm{V}: U' &= 43.38x' - 116x'^2 + 106.2x'^3, \end{aligned} \tag{1}$$

where U' is the measured soil voltage and x' is the normalized distance from the cathode ($0 < x' < 1$). The fitted R^2 values under output voltages of 15 V, 22.5 V, 26.25 V, 30 V, and 33.75 V were 0.9954, 0.9818, 0.9872, 0.9968, and 0.9969, respectively. The fitted cubic polynomial curves accurately reflect the actual distribution of the soil voltage.

Therefore, it can be assumed that the expression of the soil voltage distribution under any output voltage is as follows:

$$U' = A_1 x' + A_2 x'^2 + A_3 x'^3, \tag{2}$$

where A_1, A_2, and A_3 are coefficients related to the output voltage.

Table 1: Basic physical and mechanical properties of the soil samples.

Specific gravity	Liquid limit (%)	Plastic limit (%)	Plasticity index	Particle composition (%)		
				Sand	Silt	Clay
2.62	39	21	18	0	79	21

Figure 2: Tabular EKG.

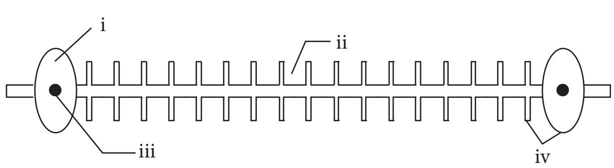

Figure 3: Structure of the tabular EKG. (i) Covering layer, (ii) drainage groove, (iii) copper wire, and (iv) conductive plastics.

Furthermore, the output voltage and coefficients (A_1, A_2, and A_3) were fitted. The fitting results are shown in Figure 5, which shows strong linear relationships between the output voltage of the power supply and the coefficients (A_1, A_2, and A_3).

The relevant expressions can be obtained as follows:

$$A_1 = 1.2202U + 1.3158, \tag{3}$$

$$A_2 = -3.296U - 1.305, \tag{4}$$

$$A_3 = 3.0931U - 0.5492, \tag{5}$$

where U is the output voltage of the DC power supply. The fitted R^2 values for A_1, A_2, and A_3 are 0.9820, 0.9652, and 0.9387, respectively.

4. Establishment of Electroosmotic Consolidation Equation

4.1. Calculation Diagram. In view of the rectangular arrangement of electrodes in practical engineering applications, the electroosmotic two-dimensional consolidation theory proposed by Su was used to establish a horizontal two-dimensional plane model of the electroosmosis method, similar to the method used in practical contexts [17], as shown in Figure 6.

The anodic electrode was considered as the research object. This model provides a reference for the subsequent derivation of the dissipation equation of excess pore water pressure and soil consolidation equation considering the case where the anode is closed and the cathode is open.

4.2. Essential Assumptions. To establish the horizontal two-dimensional consolidation equation of the electroosmosis method, the following assumptions need to be made. (1) The seepage of the soil was considered in only the horizontal direction. The upper and lower boundaries were closed, and the water flow caused by the voltage difference and head difference could be superimposed. (2) The soil was homogeneous, saturated, and isotropic, and the compression of the soil was completely due to skeletal deformation caused by a decrease in the pore volume. The compression of soil particles and water could be ignored, and the volume of discharged pore water was equal to the volume of vertical compaction. (3) The water flow caused by differences in the ion concentration and temperature could be ignored. (4) The hydraulic and electric permeability coefficients of the soil did not change with time.

4.3. Equation Establishment. According to essential assumption (1), in the process of electroosmosis, the water flow in the soil was a coupled flow driven by hydraulic and voltage gradients; thus, the following expression can be obtained:

$$q_{\mathrm{h}} = k_{\mathrm{h}} i_{\mathrm{h}} + k_{\mathrm{e}} i_{\mathrm{e}}, \tag{6}$$

where q_{h} is the water flow, k_{h} is the hydraulic permeability coefficient of the soil, k_{e} is the intrinsic electric permeability coefficient of the soil, i_{h} is the hydraulic gradient, and i_{e} is the voltage gradient.

For two-dimensional electroosmotic consolidation, the following equation can be obtained according to the electroosmotic coupled flow equation (6):

$$q_{\mathrm{h}x} = \frac{k_{\mathrm{h}x}}{\gamma_w}\frac{\partial u}{\partial x} + k_{\mathrm{e}x}\frac{\partial \varphi}{\partial x}, \tag{7}$$

$$q_{\mathrm{h}y} = \frac{k_{\mathrm{h}y}}{\gamma_w}\frac{\partial u}{\partial y} + k_{\mathrm{e}y}\frac{\partial \varphi}{\partial y}. \tag{8}$$

The total seepage in the soil can be expressed as follows:

$$q_{\mathrm{h}} = q_{\mathrm{h}x} + q_{\mathrm{h}y}. \tag{9}$$

According to the consolidation theory proposed by Terzaghi, the following equation can be obtained:

$$\operatorname{div}(q_{\mathrm{h}}) = m_{\mathrm{v}}\frac{\partial u}{\partial t}. \tag{10}$$

Table 2: Summary of the test parameters.

Test number	Applied voltage (V)	Test duration (h)	Initial moisture content (%)
T1	15	45	61.58
T2	22.5	45	62.18
T3	26.25	45	61.73
T4	30	45	61.57
T5	33.75	45	62.05

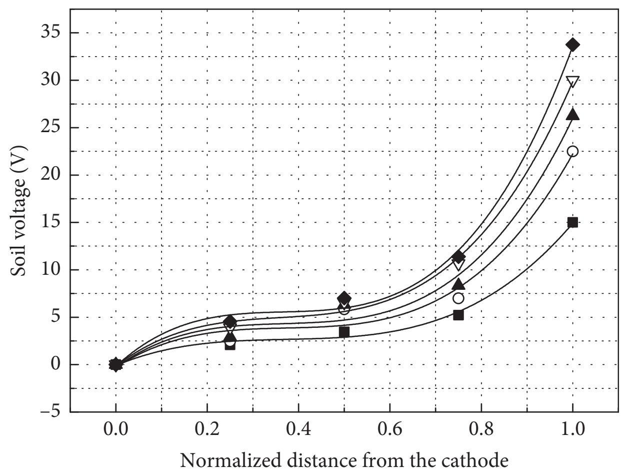

Figure 4: Fitted soil voltage distribution curves at different output voltages.

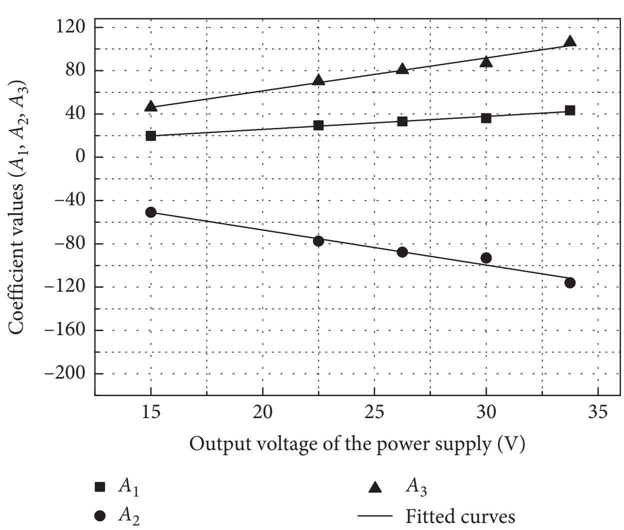

Figure 5: Fitted curves for the output voltage of the power supply and the coefficient values.

Furthermore, equation (10) can be expressed in the following form:

$$\frac{\partial q_{hx}}{\partial x}+\frac{\partial q_{hy}}{\partial y}=m_v\frac{\partial u}{\partial t}. \tag{11}$$

In addition, according to assumptions (2) and (4), the following equations can be obtained:

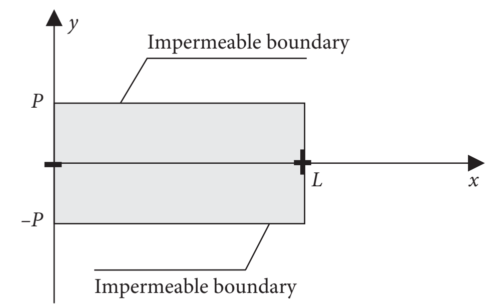

Figure 6: Electroosmosis method calculation diagram for planar two-dimensional strain.

$$\begin{aligned} k_{hx}&=k_{hy}=k_h,\\ k_{ex}&=k_{ey}=k_e. \end{aligned} \tag{12}$$

By combining equations (7)–(11), the following equation can be obtained:

$$\frac{k_h}{\gamma_w}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+k_e\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}\right)=m_v\frac{\partial u}{\partial t}. \tag{13}$$

Equation (13) can be further converted into the following form:

$$\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+k_e\frac{\gamma_w}{k_h}\times\left(\frac{\partial^2 \varphi}{\partial x^2}+\frac{\partial^2 \varphi}{\partial y^2}\right)=m_v\frac{\gamma_w}{k_h}\frac{\partial u}{\partial t}. \tag{14}$$

Assuming that the soil voltage is linearly distributed, the following expression is obtained:

$$\varphi(x,y,t)=\frac{x}{L}\times U. \tag{15}$$

In addition, according to the soil voltage distribution law mentioned above, namely equation (2), it can be assumed that the soil voltage expression with a nonlinear distribution between the cathode and anode is as follows:

$$\varphi(x,y,t)=A_1\frac{x}{L}+A_2\frac{x^2}{L^2}+A_3\frac{x^3}{L^3}. \tag{16}$$

In equations (15) and (16), U is the output voltage of the DC power supply, x is the distance from the cathode, and L is the distance between the anode and the cathode.

By introducing a variable ξ, the following expression can be obtained:

$$\xi(x,y,t)=u(x,y,t)+\frac{k_e\gamma_w}{k_h}\varphi(x,y,t). \tag{17}$$

By substituting ξ into equation (14) and combining equations (15) and (16), the governing equation of the calculation model takes the following form irrespective of whether the soil voltage is linearly or nonlinearly distributed:

$$\frac{\partial \xi}{\partial t} = C_{\mathrm{h}}\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right), \tag{18}$$

where $C_{\mathrm{h}} = k_{\mathrm{h}}/m_{\mathrm{v}}\gamma_w$ is the horizontal consolidation coefficient of the soil.

4.4. Equation Solution

(i) Boundary conditions:

Because the cathode is open, $x = 0$, $y = 0$, $u = 0$, and $\varphi = 0$; therefore, the following expression can be obtained:

$$\xi(0, 0, t) = 0. \tag{19}$$

Because the anode is closed, $\xi_x + \xi_y = 0$; therefore, the following expression can be obtained:

$$\xi_x(L, 0, t) + \xi_y(L, 0, t) = 0. \tag{20}$$

Because the boundaries are impermeable, $y = \pm P$; therefore, the following expression can be obtained:

$$\xi_y(x, \pm P, t) = 0. \tag{21}$$

(ii) Initial conditions:

$$\xi(x, y, 0) = u(x, y, 0) + \frac{k_{\mathrm{e}}\gamma_w}{k_{\mathrm{h}}}\varphi(x, y, 0). \tag{22}$$

4.4.1. Equation of Linear Distribution of Soil Voltage. Equations (15), (18)–(22) were simultaneously used to solve the dissipation equation for excess pore water pressure considering a linearly distributed soil voltage. The characteristic function system corresponding to the definite solution problem is as follows:

$$\left\{\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P}\right\}. \tag{23}$$

Thus, the formal solution of the governing equation can be assumed to take the following form:

$$\xi(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P} \times \exp\left(-(\lambda+\mu)\times C_{\mathrm{h}}t\right). \tag{24}$$

According to the boundary conditions and initial conditions, C_{mn} can be obtained.

$$C_{mn} = \frac{16k_{\mathrm{e}}\gamma_w(-1)^{m-1} \times U}{k_{\mathrm{h}}(2m-1)^2(2n-1)\pi^3}. \tag{25}$$

By combining equations (17) and (24), the following equation can be obtained:

$$u(x, y, t) = -\frac{k_{\mathrm{e}}\gamma_w}{k_{\mathrm{h}}}\varphi(x, y, t) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P} \times \exp\left(-(\lambda+\mu)\times C_{\mathrm{h}}t\right). \tag{26}$$

4.4.2. Equation for Nonlinear Distribution of Soil Voltage. Equations (16), (18)–(22) were simultaneously used to solve the dissipation equation for excess pore water pressure considering a nonlinearly distributed soil voltage. The characteristic function system corresponding to the definite solution problem is as follows:

$$\left\{\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P}\right\}. \tag{27}$$

Similarly, the formal solution of its definite solution is as follows:

$$\xi(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_{mn}\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P} \times \exp\left(-(\lambda+\mu)\times C_{\mathrm{h}}t\right), \tag{28}$$

$$\begin{aligned} D_{mn} = &\frac{16k_{\mathrm{e}}\gamma_w(-1)^{m-1} \times (2A_2 + A_1 + 3A_3)}{k_{\mathrm{h}}(2m-1)^2(2n-1)\pi^3} \\ &- \frac{64k_{\mathrm{e}}\gamma_w A_2}{k_{\mathrm{h}}(2m-1)^3(2n-1)\pi^4} \\ &- \frac{384k_{\mathrm{e}}\gamma_w(-1)^{m-1}A_3}{k_{\mathrm{h}}(2m-1)^4(2n-1)\pi^5}. \end{aligned} \tag{29}$$

By combining equations (17) and (28), the following equation can be obtained:

$$u(x, y, t) = -\frac{k_{\mathrm{e}}\gamma_w}{k_{\mathrm{h}}}\varphi(x, y, t) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{mn}\sin\frac{(2m-1)\pi x}{2L} \times \sin\frac{(2n-1)\pi y}{2P} \times \exp\left(-(\lambda+\mu)\times C_{\mathrm{h}}t\right). \tag{30}$$

In equations (26) and (30), the variables λ and μ are defined as follows:

$$\begin{aligned} \lambda &= \left(\frac{(2m-1)\pi}{2L}\right)^2, \\ \mu &= \left(\frac{(2n-1)\pi}{2P}\right)^2, \end{aligned} \tag{31}$$

where $m = 1, 2, 3, \ldots$ and $n = 1, 2, 3, \ldots$.

Furthermore, the calculated excess pore water pressure at any time could be obtained according to the corresponding excess pore water pressure equation. Then, the expression for the radial consolidation degree at a point at any depth between the anode and the cathode in the process of electroosmosis is determined as follows:

$$U = 1 - \frac{u(x, y, t)}{u(x, y, 0)}. \quad (32)$$

5. Example Analysis

To verify the applicability of the two-dimensional electro-osmotic consolidation equation deduced above, which considers the distribution of the actual soil voltage, this equation was analyzed and discussed in terms of the excess pore water pressure distribution and the average moisture content of the soil.

5.1. Analysis of Excess Pore Water Pressure. The excess pore water pressure changes were monitored in real time at preinstalled monitoring points during the laboratory electroosmosis tests. To verify the suitability of the consolidation equation, the measured values were compared with the theoretical values for the two kinds of soil voltage distributions.

The material, device, and procedure of the test were essentially the same as those mentioned above. The only difference was that micropore water pressure sensors are embedded at distances of 0.015 m, 0.03 m, 0.045 m, 0.06 m, 0.075 m, 0.09 m, 0.105 m, 0.12 m, and 0.135 m from the cathode and near the electrodes. The microsensors are shown in Figure 7.

Figure 7: Micropore water pressure sensors.

The embedded depth of each microsensor was 65 mm. Because the microsensor was small, its interference with the seepage field could be neglected. The wire head was fed through the model slot during the test. The excess pore water pressure data were acquired using a reading instrument.

In addition, the applied voltage was 20 V. The correlation coefficients were calculated by combining equations (3)–(5), and the results showed that $A_1 = 25.7198$, $A_2 = -67.225$, and $A_3 = 61.328$. In addition, the following soil parameters were obtained through the preliminary tests: $k_e = 5 \times 10^9\ \mathrm{m^2 \cdot (s \cdot V)^{-1}}$, $k_h = 2 \times 10^{-8}\ \mathrm{m \cdot s^{-1}}$, $c_h = 8 \times 10^{-8}\ \mathrm{m^2 \cdot s^{-1}}$, and $L = 0.15\ \mathrm{m}$. By introducing these parameters into equations (26) and (30), the values calculated for the excess pore water pressure under linearly and nonlinearly distributed soil voltages could be obtained, respectively. The measured and calculated values are plotted in Figure 8.

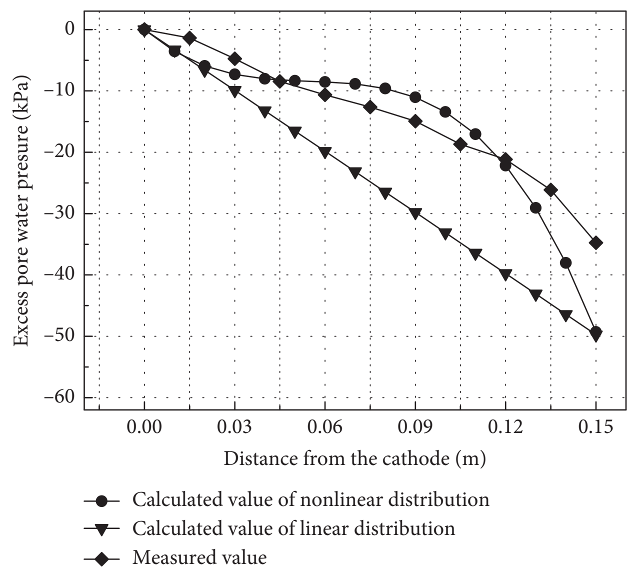

Figure 8: Distribution of excess pore water pressure in three different situations ($t = 45$ h).

Overall, the dissipation trend for the measured values of excess pore water pressure at $t = 45$ h was relatively consistent with that of the theoretical values when considering a nonlinear distribution of soil voltage. However, an obvious difference was noted between the dissipation trend of the measured values and that of the theoretical values when considering the linear distribution of soil voltage. This result demonstrates the applicability of the electroosmotic consolidation equation based on a nonlinear distribution of the actual soil voltage. In addition, the figure shows that there was a large difference between the calculated value and the measured value when considering the nonlinear distribution near the anode; there are two main reasons for this phenomenon. First, the soil temperature near the anode clearly increased during the test, which indicates that electrolysis has a thermal effect on the electroosmosis treatment process. The change in temperature produced by this thermal effect leads to a change in the excess pore water pressure. Second, the geotextile wrapped around the anode outside the EKG plate changed from white to yellow (Figure 9).

This geotextile color change shows that the electroosmosis treatment process involves a more intense chemical reaction near the anode than near the cathode. Chemical reactions between ions can also cause changes in the excess pore water pressure. However, the influences of the temperature and ion reaction fields were neglected in the derivation. Thus, the measured value of the excess pore water pressure near the anode was quite different from that of the theoretical value.

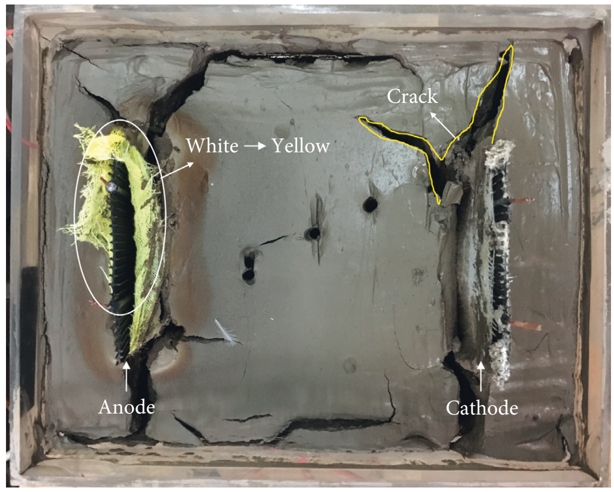

FIGURE 9: Changes in the phenomenological characteristics of the soil and electrodes after electroosmosis.

5.2. Analysis of Soil Average Moisture Content. In the process of electroosmosis, the increase in the effective stress of the saturated soil was equal to the dissipation of the excess pore water pressure, while the total stress remained unchanged. Consequently, the excess pore water pressure dissipation equation could be used to calculate the average moisture content of the soil at any time. Therefore, the following equation can be obtained:

$$\omega_t = \frac{a_v}{G_s}\left(u_t - u_0\right) + \omega_0, \tag{33}$$

where a_v is the compressibility coefficient, G_s is the specific gravity, ω_0 is the initial moisture content of the soil, u_t is the dissipation value of the excess pore water pressure at a certain time, and u_0 is the initial excess pore water pressure.

When collecting the results of previous electroosmosis experiments, it was found that Lefebvre and Burnotte performed laboratory electroosmosis experiments in 2002 [28]. Their experimental data are complete and have been cited in many studies. Therefore, the electroosmotic consolidation theory was further validated using their experimental data.

Lefebvre conducted an electroosmosis test on undisturbed soil acquired from the St. Lawrence Lowlands in Quebec, Canada. The basic parameters of the soil samples were provided in the original work of Lefebvre: $k_e = 2.7 \times 10^{-9}\,\mathrm{m^2 \cdot (s \cdot V)^{-1}}$, $k_h = 3 \times 10^{-10}\,\mathrm{m \cdot s^{-1}}$, $a_v = 1.5\,\mathrm{MPa^{-1}}$, $m_v = 0.54\,\mathrm{MPa^{-1}}$, $\omega_0 = 49\%$, $G_s = 2.79$, and $u_0 = 5\,\mathrm{kPa}$.

In addition, the applied voltage of the test was 5.25 V. Thus, A_1 was 7.72185, A_2 was −18.609, and A_3 was 15.6896. The electroosmosis test took 6 d to complete. By introducing the above parameters into equations (26) and (30), the distribution of the excess pore water pressure under linear and nonlinear potential distributions could be obtained; the calculation results are shown in Figure 10. The variation trend is consistent with the dissipation trend noted for the excess pore water pressure in the two cases obtained in Section 5.1.

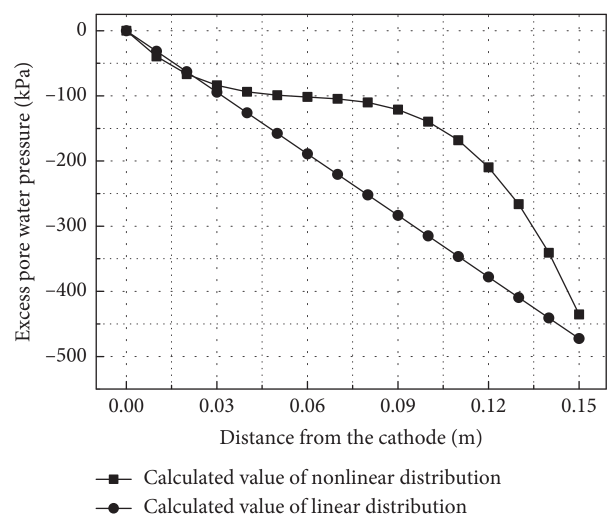

FIGURE 10: Distribution of excess pore water pressure based on the Lefebvre test data ($t = 6$ d).

Furthermore, according to the calculated excess pore water pressure, the average moisture content of the soil at $t = 6$ d could be obtained by selecting the excess pore water pressure at different distances from the cathode and using equation (33); the calculated results are shown in Figure 11. The average moisture content of the soil measured by the Lefebvre test data was 40%. The average moisture content of the soil was 41.68% under the assumption that the soil voltage was nonlinearly distributed. The average moisture content of the soil was 36.56% under the assumption that the soil voltage was linearly distributed. The average soil moisture content obtained under the assumption of a nonlinearly distributed soil voltage was closer to the measured value than that obtained when the soil voltage was assumed to be linearly distributed. These results further demonstrate that the electroosmotic consolidation theory established considering a nonlinear distribution of soil voltage is rational. The results obtained can be made more accurate using a consolidation equation considering the actual distribution of the soil voltage.

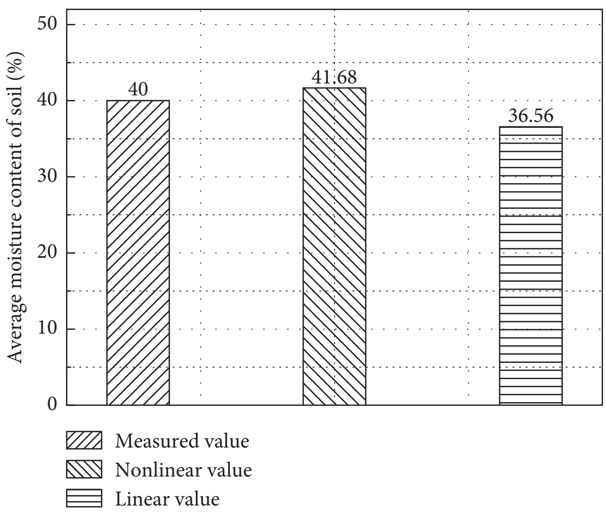

Figure 11: Average moisture content in the soil at $t = 6$ d.

6. Conclusions

A two-dimensional consolidation equation for the electroosmosis method was deduced based on the actual distribution of soil voltage in the process of electroosmosis. Analytical expressions of the excess pore water pressure and soil consolidation degree in the electroosmosis process were established. The consolidation equation was then verified by analyzing different test cases. The following conclusions were drawn from the analyses.

(1) The soil voltage distribution is nonlinear in the electroosmosis process. The distribution of the soil voltage can be accurately reflected using a cubic polynomial fitting.

(2) The proposed consolidation equation can reflect the actual consolidation situation more accurately than previously established consolidation equations because the proposed equation is based on the soil voltage distribution determined by electroosmosis testing. Moreover, the proposed consolidation equation improves consolidation theories of the electroosmosis method and provides a more reasonable reference for subsequent application of the electroosmosis method in practical contexts.

(3) The dissipation value of the excess pore water pressure calculated using the nonlinear distribution of soil voltage is quite different from the value measured at the anode. This phenomenon is mainly due to the chemical reaction and temperature field changes during the test process, which promote the dissipation of excess pore water pressure. If the theoretical derivation can be further improved, the two-dimensional excess pore water pressure equation for electroosmosis based on temperature effects and ion reaction effects will produce results closer to the measured values.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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References

[1] S. G. Kumar, G. Sridhar, and R. Radhakrishnan, "A case study of vacuum consolidation of soft clay deposit," *Indian Geotechnical Journal*, vol. 45, no. 1, pp. 51–61, 2015.

[2] C. Qi, J. Qi, L. Li, and J. Liu, "Stability analysis method for rock slope with an irregular shear plane based on interface model," *Advances in Civil Engineering*, vol. 2018, Article ID 2948286, 7 pages, 2018.

[3] A. R. Zheng and H. M. Zhu, "Experimental study on flocculation-accelerated deposition of dredger fill," *Chinese Journal of Geotechnical Engineering*, vol. 39, no. 2, pp. 145–148, 2017.

[4] X. B. Guo and J. Y. Fang, "Silt-removing technology using cutter suction dredger in dredging and reclamation project," *Port & Waterway Engineering*, vol. 28, no. 1, pp. 1–5, 2019.

[5] W. G. Luo, G. L. Yang, and J. Lu, "Research on dehydration-solidification construction technique of bottom silt in Moshui lake," *Engineering Journal of Wuhan University*, vol. 50, no. 2, pp. 187–192, 2017.

[6] J. Wang, R. Zhao, Y. Cai, H. Fu, X. Li, and X. Hu, "Vacuum preloading and electro-osmosis consolidation of dredged slurry pre-treated with flocculants," *Engineering Geology*, vol. 246, no. 1, pp. 123–130, 2018.

[7] H. Y. Lei, Z. Y. Qi, and Z. P. Zhang, "New vacuum-preloading technique for ultrasoft-soil foundations using model tests," *International Journal of Geomechanics*, vol. 17, no. 9, Article ID 04017049, 2017.

[8] Z. Zhang, Y. Chen, and G. B. Ye, "Performance evaluation of reinforced soft soil with surcharge and vacuum preloading," *Chinese Journal of Underground Space and Engineering*, vol. 14, no. 1, pp. 315–321, 2018.

[9] S. G. Chung, H. J. Kweon, and W. Y. Jang, "Observational method for field performance of prefabricated vertical drains," *Geotextiles and Geomembranes*, vol. 42, no. 4, pp. 405–416, 2014.

[10] Y. L. Tao, J. Zhou, and X. N. Gong, "Experimental study on transport process of electro-osmosis based on Hangzhou soft soil," *Journal of Central South University*, vol. 49, no. 2, pp. 448–453, 2018.

[11] S.-C. Chien, F.-C. Teng, and C.-Y. Ou, "Soil improvement of electroosmosis with the chemical treatment using the suitable operation process," *Acta Geotechnica*, vol. 10, no. 6, pp. 813–820, 2015.

[12] E. Mohamedelhassan, "Laboratory model test on improving the properties of soft clay by electrokinetics," *ISRN Civil Engineering*, vol. 2011, Article ID 560834, 14 pages, 2011.

[13] H. Wu, L. M. Hu, and G. P. Zhang, "Effects of electro-osmosis on the physical and chemical properties of bentonite," *Journal of Materials in Civil Engineering*, vol. 28, no. 8, Article ID 06016010, 2016.

[14] J. Zhou, Y. L. Tao, C. J. Xu, X. N. Gong, and P. C. Hu, "Electro-osmotic strengthening of silts based on selected electrode materials," *Soils and Foundations*, vol. 55, no. 5, pp. 1171–1180, 2015.

[15] M. I. Esrig, "Pore pressure, consolidation and electrokinetics," *Journal of the Soil Mechanics and Foundation Division*, vol. 94, no. 4, pp. 899–922, 1968.

[16] T. Y. Wan and J. K. Mitchell, "Electro-osmotic consolidation of soils," *Journal of Geotechnical Engineering Division*, vol. 102, no. 5, pp. 473–491, 1976.

[17] J. Q. Su and Z. Wang, "Theory of two-dimensional electro-osmotic consolidation of soils," *Rock and Soil Mechanics*, vol. 25, no. 1, pp. 125–131, 2004.

[18] L. J. Wang, S. H. Liu, and Z. J. Wang, "A consolidation theory for one-dimensional large deformation problems under combined action of load and electroosmosis," *Engineering Mechanics*, vol. 30, no. 12, pp. 91–98, 2013.

[19] J. Wang, H. T. Fu, and C. Q. Cai, "Analyses of one dimensional electro-osmotic consolidation theory and test of soft clay under linear load," *Chinese Journal of Rock Mechanics and Engineering*, vol. 33, no. 1, pp. 179–188, 2014, in Chinese.

[20] Y. D. Zhou, A. Deng, Z. X. Liu, A. W. Yang, and H. Zhang, "One-dimensional electroosmosis consolidation model considering variable saturation," *Chinese Journal of Geotechnical Engineering*, vol. 39, no. 8, pp. 1524–1529, 2017.

[21] D. Jiao, X. N. Gong, and Y. Li, "Experimental study of consolidation of soft clay using electro-osmosis method," *Chinese Journal of Rock Mechanics and Engineering*, vol. 30, no. 1, pp. 3208–3216, 2011.

[22] A. R. Estabragh, M. Naseh, and A. A. Javadi, "Improvement of clay soil by electro-osmosis technique," *Applied Clay Science*, vol. 95, no. 1, pp. 32–36, 2014.

[23] C. Geng, "*Experimental study on electro-osmotic consolidation of marine dredger fill by using electric vertical drain*," M.S. thesis, China University of Geosciences, Beijing, China, 2016.

[24] Y. C. Hu, "*Experimental study on electro-osmotic consolidation of soft clay using EKG electrodes*," M.S. thesis, Wuhan University, Wuhan, China, 2005.

[25] Y. F. Zhuang and Z. Wang, "Study on interface electric resistance of electro-osmotic consolidation," *Rock and Soil Mechanics*, vol. 25, no. 1, pp. 117–120, 2004.

[26] Y. Li, X. N. Gong, and D. Jiao, "Experimental study on two-dimensional electroosmotic consolidation of soft clay," *Chinese Journal of Rock Mechanics and Engineering*, vol. 28, no. 2, pp. 4034–4039, 2009.

[27] Y. Li, "*Experimental and theoretical study on electro-osmotic consolidation of soft clay foundation*," M.S. thesis, Zhejiang University, Hangzhou, China, 2011.

[28] G. Lefebvre and F. Burnotte, "Improvements of electroosmotic consolidation of soft clays by minimizing power loss at electrodes," *Canadian Geotechnical Journal*, vol. 39, no. 2, pp. 399–408, 2002.