

## Research Article

# Lyapunov Exponent-Based Study of Chaotic Mechanical Behavior of Concrete under Compression

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Chaos theory is advantageous in achieving a deeper understanding of the nonlinearity and randomness of concrete behavior. In this study, the experimental data of concrete under compression were examined and discussed using Lyapunov exponent. According to the value of the Lyapunov exponent, which was larger than 0, it could be quantitatively demonstrated that measured and fitted data exhibited chaotic features. Besides, the mechanical behavior of concrete could be predicted by deducing its evolution equation. Furthermore, the evolution and trends of the Lyapunov exponent indicated that the series with human intervention showed a stronger chaotic property, which led to the result that this kind of series might be more difficult to predict.

## 1. Introduction

Cement and concrete technology has become an indispensable basis for global development. Hence, it is necessary to obtain a deeper understanding of the nonlinearity and randomness of concrete behavior. The chaos theory provides a powerful tool for addressing the incongruities of nonlinear processes in concrete [1, 2]. A few researchers have attempted to apply this theory in the concrete field.

Firstly, applying chaos theory could be used to simulate the behavior of concrete using a typical chaotic model. For instance, the cellular automata method is one of the most widely applied methods [3, 4]. This model is used to simulate the transmission of heat [5–7] and corrosion procession [8], as well as the failure of the concrete product [9]. Also, the logistic model could describe the complex behavior accurately [10–12]. Some researchers developed this model and applied it to predict the damage of concrete material [13].

Secondly, the chaotic tools can be applied to predict data series using the method of chaos theory. Lyapunov exponents could characterize the chaotic phenomena quantitatively [14]. Several researchers implemented this index to analyze and predict the data series of structures [15] and acoustic signals [16]. The whole domain model (WDM) is usually used to analyze or reevaluate the time series [17].

Besides, one of the major applications of chaos theory is to analyze the chaotic system. By applying the earlier developed chaos-geometric and vibration blind source monitoring approach, the chaotic behavior of a cantilever beam can then be predicted [18, 19]. Sadhu and Hazra introduced a blind-source-separation-based method to detect the identification of damage instant, location, and the severity of damage of a vibrating structure [20].

As other similar complex systems, the mechanical behavior of concrete, which is observed as complex and random, is significantly influenced by the complex interactions. To predict complex and random behavior and to

prevent instability, several methods were presented and applied in engineering. Wang has succeeded in applying the differential transformation method to examine the nonlinear dynamic response of the aeroelastic system, based on which could improve the design of aircraft wings [21]. Besides, the chaotic tools can also provide some solutions to improve the stability and performance of the complex system [22, 23].

Finally, wavelet packet transform can be used to process the vibration signal and optimize the parameters of ball bearing [24–26]. Also, the methods of nonlinear analysis based on the wavelet expansions can help researchers to forecast the chaotic processes in the hydroecological system [27, 28].

Most of the studies mentioned above have introduced how to treating the quasi-endless time series using chaotic tools, for instance, the measurement data of chaotic quantum and laser systems and quantum devices [29, 30]. Otherwise, few researchers discussed the limited time series, e.g., the experimental data measured in the compressive tests of concrete. This type of time series also represents typically chaotic features [31].

This paper will be devoted to discussing the chaotic phenomena quantitatively in the concrete's experiments using the Lyapunov exponent and the WDM. Besides, as a newly-emerged cross-discipline, the application of chaos theory in concrete still requires further investigation. One of the problems that need solving is to describe the concrete chaotic behavior using an appropriate model. The purpose of this study is to fill up the research gap by applying the logistic evolution equation to describe the complex behavior of concrete, as well as by analyzing the chaotic experimental phenomena quantitatively.

## 2. Methods

**2.1. Evolution Equation.** The behavior of concrete under loads exhibits the features of complexity and randomness; hence, it could be assumed that concrete might be a chaotic system.

The concrete behavior at each moment is influenced by a group of variables related to time,  $t$  [32]. These variables describe the state changing with time, such as stress, strain, and displacement. These variables are denoted as  $x(t)$ ,  $y(t)$ ,  $z(t)$ ,  $\dots$ . Also, the behavior of concrete is influenced by several coefficients that do not change with the time, such as the material property. These coefficients are denoted as  $a, b, c, \dots$ .

The mechanical behavior of concrete is assumed as the simplest chaotic phenomenon, i.e., independent variables do not influence each other. Furthermore, the time interval of the record is constant. It could be considered that each condition is influenced by the previous moment. Thus, an evolution equation could describe this chaotic phenomenon. Considering variable  $x(t)$  as an example, the form of the evolution equation is given by

$$x(t+1) = f(\mu, x(t)), \quad (1)$$

where  $\mu$  represents the set of all coefficients,  $a, b, c, \dots$ .

**2.2. Lyapunov Exponent.** One of the characteristics of chaos is high sensitivity to initial values. The following is the explanation of sensitivity in detail. In concrete experiments, this sensitivity can be described qualitatively as follows: the series of measured data are plotted as a curve. Several curves are possible before the data are determined as shown in Figure 1. Each curve is considered as a path, and any path never overlap. At a certain time, two points in different paths might be considered close, particularly at the beginning. However, as the experiment progresses, the distance between these points increases after the same period [33]. This condition indicates that the behavior of concrete is strongly influenced by initial conditions, i.e., the sensitivity of concrete to initial values is high.

The Lyapunov exponent (LE) is used to measure the sensitivity to initial values. The LE is the average of a series of derivatives. Specifically, the distance between two points on different paths after a certain period is influenced by the derivative on each curve. In the evolution process of the paths, the derivatives at each moment change constantly. Thus, to determine the future distance between these two points, the average of all derivatives at each time should be considered; this means value is denoted as  $\lambda$ . Assuming that the behavior of concrete can be described by the differential form as  $x_{n+1} = F(x_n)$ ,  $\lambda$  can be calculated as follows [34]:

$$\lambda = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln \left| \frac{dF(x_i)}{dx_i} \right|. \quad (2)$$

The value of  $\lambda$  obtained from equation (2) is influenced by the form of function  $F(x)$  and not by the initial value of concrete behavior, which is defined as the LE of this series of data. The value of the LE can represent the extent of the separation caused by each iteration in the evolution process.

Based on the property of the derivative, in the case of  $\lambda < 0$ , the points would be on the same path after a certain period of evolution. Conversely, in the case of  $\lambda > 0$ , the points would be never farther apart.

In conclusion, the behavior is chaotic and cannot be determined quantitatively. That is, chaos would occur when  $\lambda > 0$ .

In the application of the analysis, the generalized index of concrete behavior is denoted as  $g$ . This index might include but is not limited to reaction, displacement, stress, or strain. This index is a time series denoted as  $g(t)$ , where  $t$  represents a moment,  $t \in \mathbb{N}^*$ . The time interval between two moments is denoted as  $\omega$ . Then, equation (2) could be rewritten approximately as follows, i.e., the  $j^{\text{th}}$  LE is equal to

$$\lambda_j = \frac{1}{j} \sum_{i=1}^j \ln \left| \frac{g(t_i) - g(t_{i-1})}{\omega} \right|, \quad j \in \mathbb{N}^*. \quad (3)$$

**2.3. Whole Domain Method.** It is difficult to predict the behavior with high sensitivity to initial values after considerable time; however, the prediction of short-term behavior according to a group of observation data is possible.

The series of chaotic data can be predicted based on the WDM proposed by Farmer and Sidorowich [35].

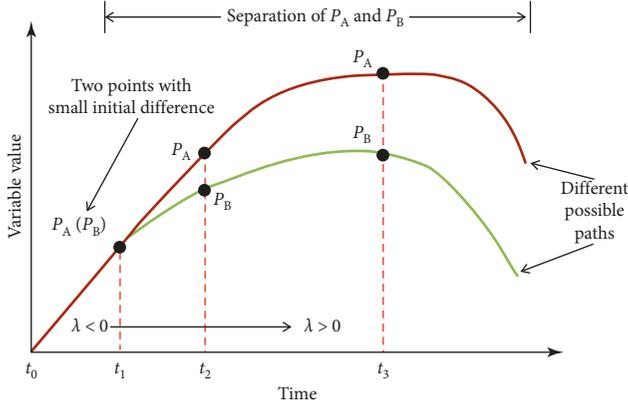


FIGURE 1: Schematic drawing of LE.

According to Takens theorem, as long as the embedding dimension and time interval are selected rationally, the trajectory obtained by a reconstructed phase space is equivalent to the trajectory of an actual system [36]. Thus, it can be demonstrated that a smooth map,  $f: \mathbb{R}^\xi \rightarrow \mathbb{R}^\xi$ , exists in the space of the  $\xi$ -dimension. The trajectory of the reconstructed phase space can be described by the following differential form:

$$Y(t + \omega) = f(Y(t)). \quad (4)$$

The form of the map  $f(\bullet)$  is obtained using the WDM to treat all elements of the series of experimental data. Then, the behavior of concrete can be predicted. In particular, the method is applied as follows: first, an approximate map is constructed, which is denoted as  $\hat{f}$ , where  $\hat{f}: \mathbb{R}^\xi \rightarrow \mathbb{R}^\xi$ . In this study, the form of  $\hat{f}$  was set as a polynomial, which is denoted as

$$\hat{f}(x) = a_1 x^s + a_2 x^{s-1} + \dots + a_{s+1} = [X][C], \quad (5)$$

where matrix  $[X]$  is a one-dimensional matrix of independent variables, and matrix  $[C]$  is a one-dimensional matrix of coefficients, which are given by the following equations:

$$[X] = [x^s \ x^{s-1} \ x^{s-2} \ \dots \ x \ 1], \quad (6)$$

$$[C]^T = [a_1 \ a_2 \ a_3 \ \dots \ a_{s+1}]. \quad (7)$$

Using all elements of series  $g(t), 1 \leq t \leq N$ , a linear equation group, i.e., equation (8), is obtained by substituting all values of the series into equation (5).

$$[A] = [B][C], \quad (8)$$

where matrix  $[A] = [g(2), g(3), \dots, g(N)]$ , and matrix  $[B]$  is denoted as

$$B = \begin{bmatrix} (g(1))^s & (g(1))^{s-1} & \dots & g(1) & 1 \\ (g(2))^s & (g(2))^{s-1} & \dots & g(2) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (g(N))^s & (g(N))^{s-1} & \dots & g(N) & 1 \end{bmatrix}. \quad (9)$$

It should be noted that the WDM is essentially a numerical method. Moreover, the series treated by WDM must

be finite. This results in two evident limits of application. The first is that the precise form of map  $f(\bullet)$  can never be obtained. The second is that it might be difficult to perform a long-term forecast. The longest period,  $t_{\max}$ , can be predicted by the WDM, which is approximately equal to  $1/\lambda_{\max} \cdot \lambda_{\max}$  indicates the maximum value of the LE obtained using equations (2) or (3).

### 3. Experiment

The design of experimental subjects followed the Chinese code for the design of concrete structures and the Chinese standard methods for the testing of concrete structures. The experimental subjects are represented as prisms in Figure 2, where the base of a prism is a square with an edge of 150 mm, and the height of the prism is 300 mm. Three groups of concrete prism samples with different gradations were prepared, and there were 30 samples in each group.

The mix proportions of concrete for each group were as follows: each cubic meter of concrete contained 195 kg of water, 325 kg of cement, 696 kg of sand, and 1184 kg of aggregates. The gradation of each group is represented in Table 1.

All samples were tested using the axial compressive load, which was controlled through displacement. The data of load and strain were measured by MTS and strain gages, respectively.

### 4. Results and Discussions

The analysis and discussions presented in this section are based on two types of data series, i.e., the original and raw experimental data and the fitted data of each sample group.

**4.1. Lyapunov Exponent.** The LE measurement method introduced in Section 2.2 was used to analyze the data measured in the experiments and simulation. The value of the LE describes the chaotic behavior quantitatively.

**4.1.1. Experimental Data.** Three prism samples were selected from each group. The time-stress curve and time-strain curve are shown in Figures 3 and 4, respectively. The LE series could be calculated as the time interval of recording which was 0.0001 s. For comparison, the corresponding LE curves are shown in the same images.

It can be observed from Figure 3 that the LE curves of different samples exhibit a similar form. These LE curves can be divided into three stages. (a) The first stage is before the loading. In this stage, the LE increases rapidly from 0 to 0.6 – 0.8 at the beginning and remains stable until the loading starts. This indicates that this data series enters the chaos state from the beginning of the measurement. (b) In the second stage, the LE increases considerably. The start and end of this process are approximately the same as the elastic stage of concrete. (c) In the third stage, the LE remains stable until the test is complete.

It should be noted that the LE of the sample selected from Group 3 decreases gradually in the final stage. This is



FIGURE 2: Mechanical experiment of the concrete prism.

TABLE 1: Gradation of aggregates for concrete prism samples.

Sieving size (mm)	Separated mass retained (%)		
	Group 1	Group 2	Group 3
16	9	11	13
10	17	20	23
5	74	69	64

because the sample does not fail immediately, while test data are recorded continuously. After the failure of the sample, the measured stress becomes more stable, i.e., chaotic strength reduces. Moreover, it should be noted that this is one of the longest data series, with more than fifty thousand elements. (The other two series contain only 16,000 – 19,000 elements.) Therefore, the gradual decrease in the LE would be observed only if there is a long stable period after chaos.

It can be observed from Figure 4 that the LE of the strain series is approximately five at the beginning and remains stable until the end. The form of LE curves shows that though the strain curve appears regular, it is chaotic. This may be because the strain gages are highly precise, and the data measured always include small disturbances, which can be considered as measurement noise [37]. This property was observed for all other measured data series. However, the curves and LE are not presented in detail owing to the limitation of space.

**4.1.2. Fitted Data.** The experimental data of each group were fitted using the least-squares method; 65 fitted points were obtained. The three fitted time-stress and time-strain curves and the corresponding LE curves are shown in Figures 5 and 6, respectively.

The curves in the figures exhibit a similar form. It can be observed from Figure 5 that the LE increases rapidly at the beginning, followed by a stable stage. However, there is a

difference between the experimental and fitted data. For Groups 2 and 3, the second stage of the fitted LE curve, i.e., the growth period of the LE, is short. The fitted curve of the first group is similar to all LE curves obtained in the experiment in which it shows an evident increase in the LE.

The form of fitted strain curves was similar to the experimental curves. Furthermore, the stable LEs of the fitted stress and strain curves were significantly larger than the experimental value. This indicates that the fitted curve showed higher sensitivity. This was because the fitted series was treated using a numerical method. Hence, the stress-time curve represented a different chaotic property compared with the original and raw data.

In the meso-scale concrete can be considered as a three-phase composite consisting of the aggregates, the cement, and the interfacial transition zone between these two. The mechanical properties of each component, as well as the interaction among the components, directly influence the mechanical behavior of concrete in the macroscale [38].

It can also be assumed that the chaotic phenomenon shown in concrete's experimental data was caused by the mechanical properties of the three components and the interaction among them. Considering the concrete as a chaotic system, the mechanical properties of each component, the interaction among those components, the gradation and spatial distribution of aggregates, the boundary condition, and the load, etc., would then be considered as the initial values of this chaotic system. As the high sensitivity to the initial values is usually observed in a chaotic system, concrete is no exception. Although the data measured from the test were limited, they showed a typically chaotic feature.

In this experiment, the initial value is the gradation and spatial distribution of aggregates. Meanwhile, according to the randomness of the materials, the properties of materials would be slightly different. Thus, it can be estimated preliminarily that the slight variations of these initial values mentioned above cause the chaos represented in the experimental data measured.

According to the curve of the LE shown in Figures 3 and 4, it can be demonstrated that the chaotic phenomenon of the experimental data measured in the compressive test exists throughout the whole loading process. Furthermore, it can be considered that the chaotic phenomenon occurred in the elastic stage of the concrete sample's mechanical behavior.

## 4.2. Evolution Equation and Prediction

**4.2.1. Experimental Data.** An image of  $x(n+1)$  about  $x(n)$  was plotted to discuss the property of evolution of the experimental data. The series of  $x(1), x(2), x(3), \dots, x(n-1)$  was considered as the x-coordinate, and the series of  $x(2), x(3), x(4), \dots, x(n)$  was considered as the y-coordinate. Then, a group of scatter plots of stress and strain could be obtained, which are shown in Figures 7 and 8.

It is evident that the mapping between  $x(n)$  and  $x(n+1)$  is linear. Then, the form of  $\hat{f}$  can be set as  $ax+b$ , and the value of coefficients  $a$  and  $b$  can be obtained easily using equation (8). The evolution path moves from the lower left to

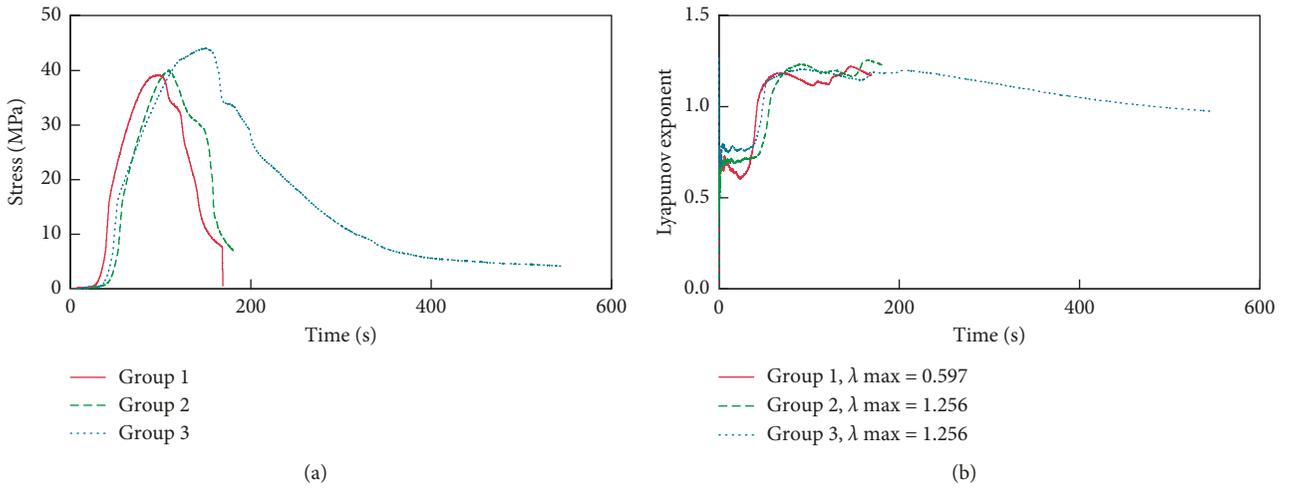


FIGURE 3: Stress-time curve and LE-time curve of experimental data.

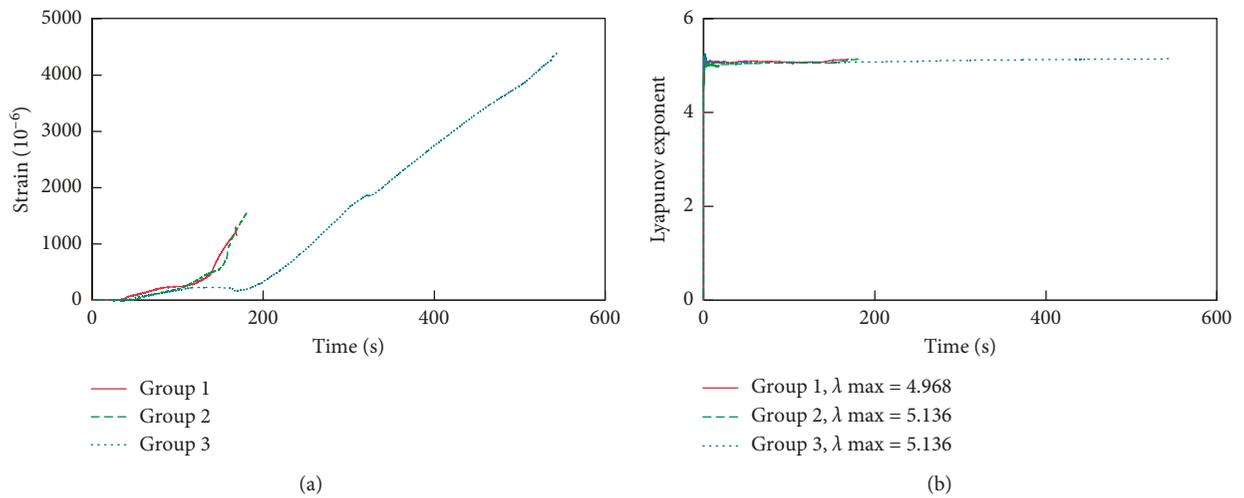


FIGURE 4: Strain-time curve and LE-time curve of experimental data.

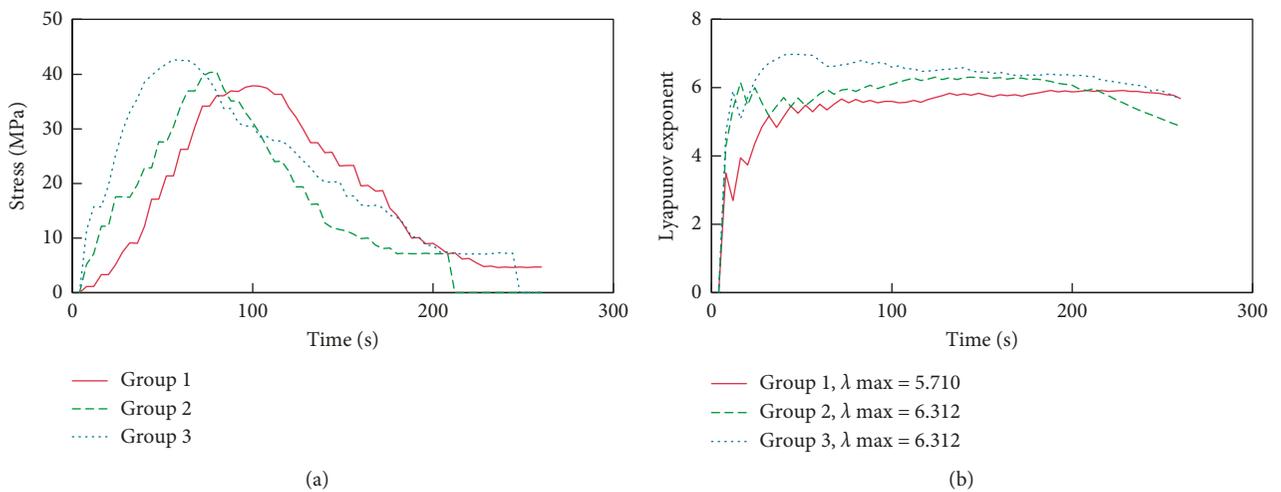


FIGURE 5: Stress-time curve and LE-time curve of fitted data.

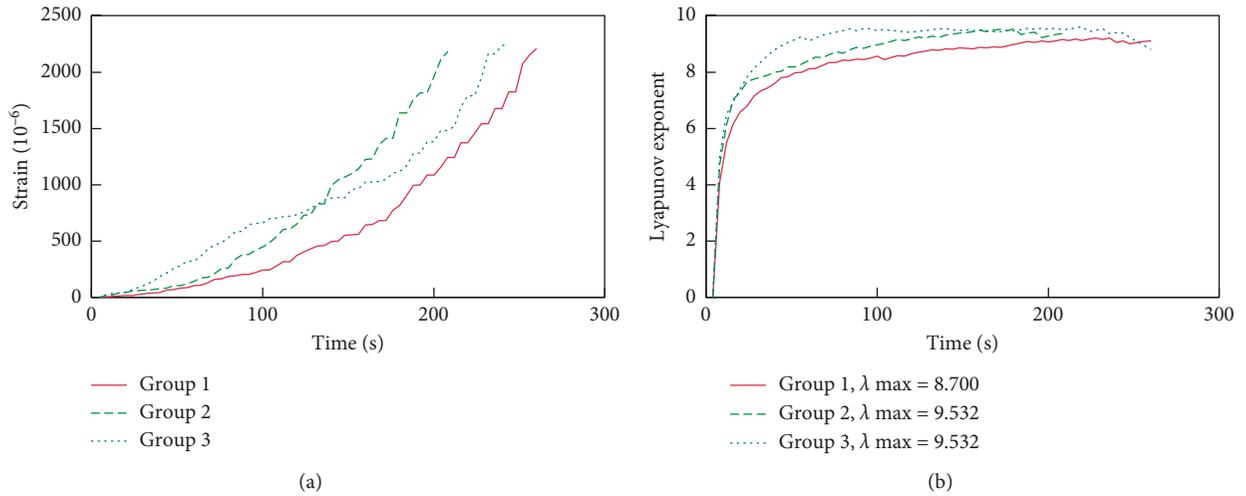


FIGURE 6: Strain-time curve and LE-time curve of fitted data.

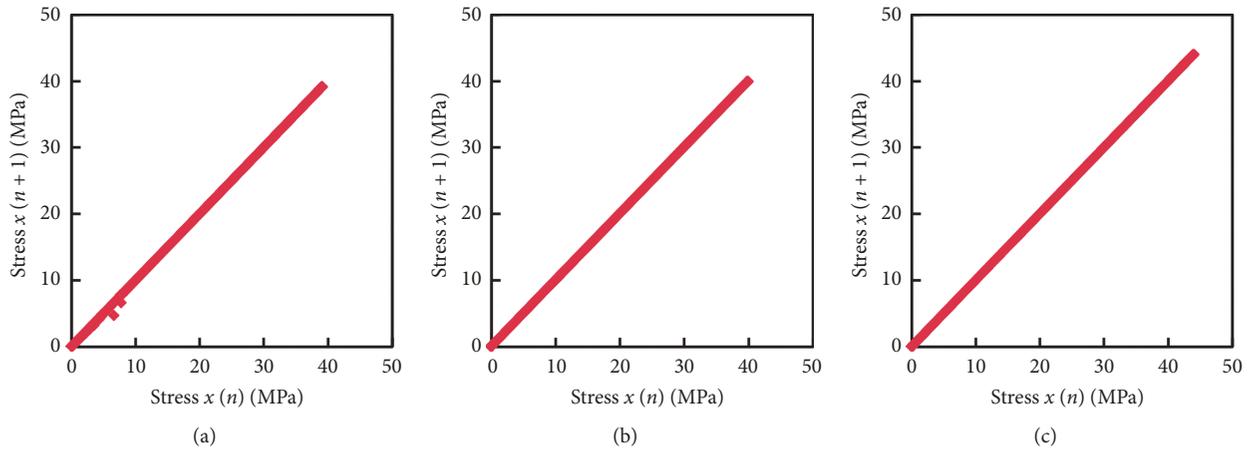


FIGURE 7: Evolution of each experimental stress series. (a) Sample of Group 1. (b) Sample of Group 2. (c) Sample of Group 3.

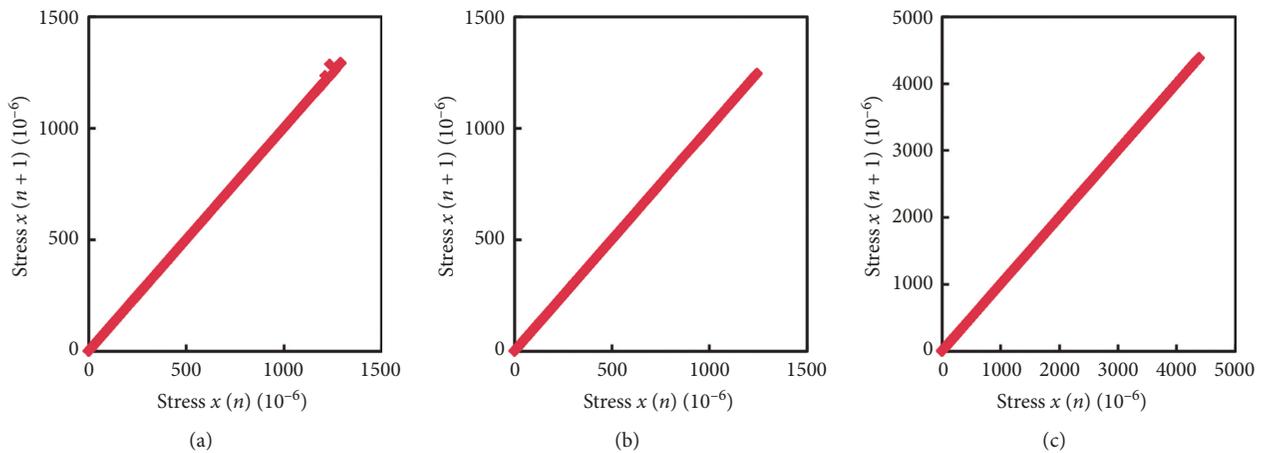


FIGURE 8: Evolution of each experimental strain series. (a) Sample of Group 1. (b) Sample of Group 2. (c) Sample of Group 3.

TABLE 2: Coefficients of linear evolution equations.

	Ascending		Descending	
	Slope	Intercept	Slope	Intercept
Measured stress Group 1	1.000014	0.003699	1.000106	-0.007845
Measured stress Group 2	1.000121	0.002308	1.000124	-0.007110
Measured stress Group 3	0.999991	0.003164	0.999902	0.000267
Measured strain Group 1	1.000064	0.018264	1.000161	0.038356
Measured strain Group 2	1.000189	0.009607	1.000232	0.017043
Measured strain Group 3	0.999998	0.015202	1.000013	0.078633
Fitted stress Group 1	0.981863	1.928350	0.973973	-0.531977
Fitted stress Group 2	0.909646	3.978237	0.892836	0.596810
Fitted stress Group 3	0.790530	7.919761	0.965617	-0.075783
Fitted strain Group 1	1.043743	6.194828	—	—
Fitted strain Group 2	1.040320	15.300724	—	—
Fitted strain Group 3	1.026596	15.149465	—	—

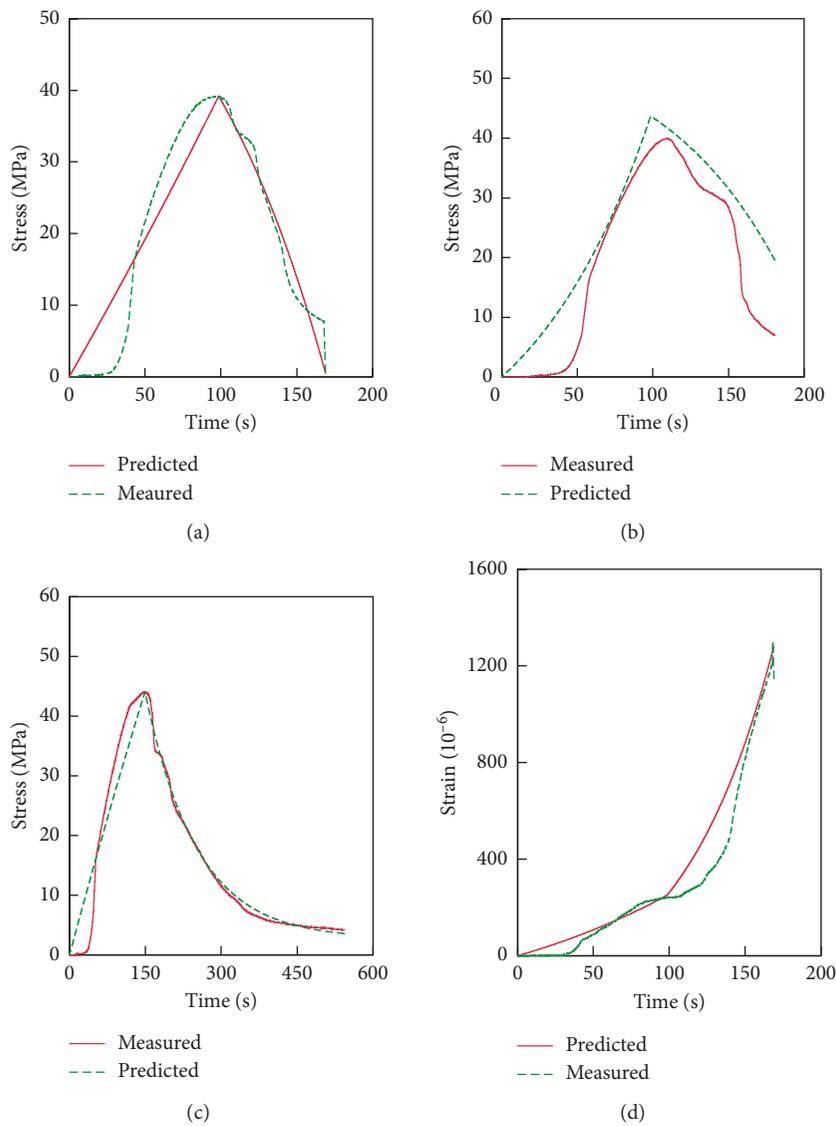


FIGURE 9: Continued.

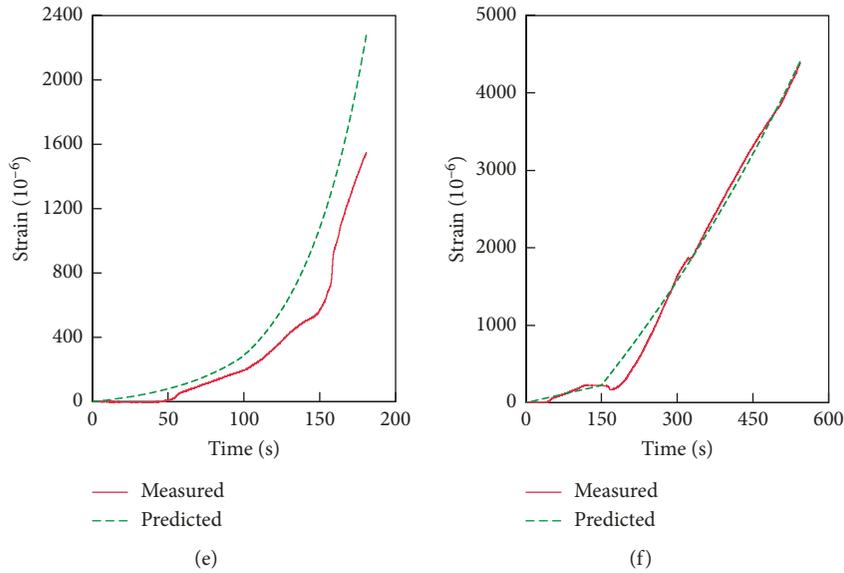


FIGURE 9: Prediction of each experimental stress and strain series. (a) Stress of Group 1. (b) Stress of Group 2. (c) Stress of Group 3. (d) Strain of Group 1. (e) Strain of Group 2. (f) Strain of Group 3.

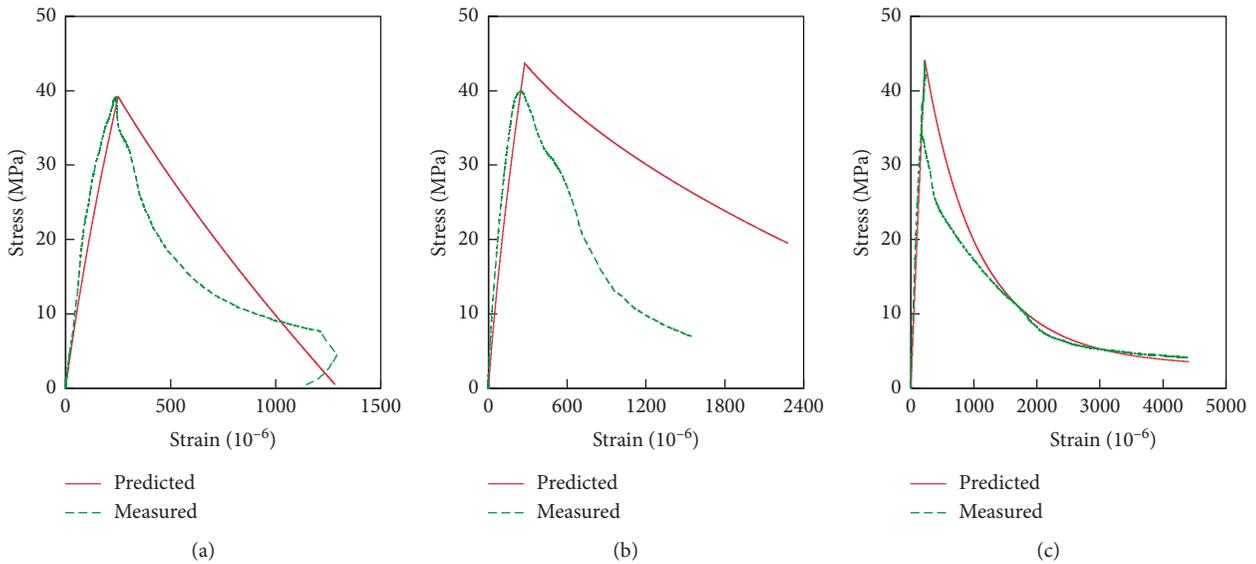


FIGURE 10: Predicted and measured stress-strain curves. (a) Sample of Group 1. (b) Sample of Group 2. (c) Sample of Group 3.

the upper right first, and then, it returns through the same path from the upper right to the lower left repeatedly. In other words, the evolution equations change continuously. It is assumed there are only two evolution equations, which represent the path before and after the peak value of stress. Then, the evolution equations of these two phases are calculated, and the coefficients are shown in Table 2. The predicted results of stress and strain are shown in Figure 9. Based on the predicted results, the stress-strain curves of each sample are plotted, which are shown in Figure 10.

It should be noted that the evolution equations obtained based on the WDM could predict the general behavior of concrete. There are still several differences between the

measured and predicted series, which indicate that it is difficult to fully predict chaotic behavior.

**4.2.2. Fitted Data.** The scatter plots of fitted data are shown in Figures 11 and 12, and the predicted curves are shown in Figures 13 and 14. The coefficients are shown in Table 2. As the fitted series were obtained using the least square method, the strain series was more regular. Thus, only one evolution was calculated for each curve.

The scatter plots and predicted curves show less evident linearity compared with the original and raw data. This may be because the fitted data are a kind of data series with

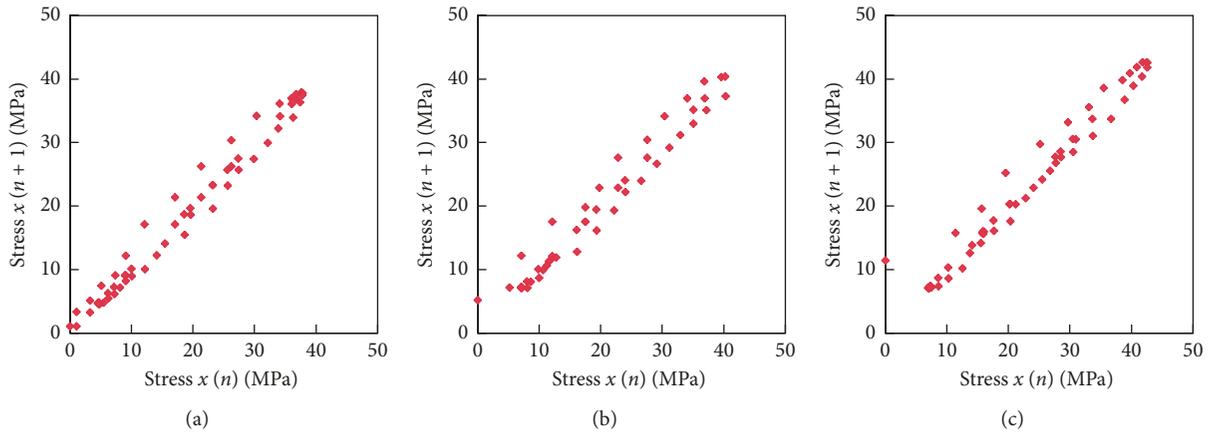


FIGURE 11: Evolution of each fitted stress series. (a) Fitted result of Group 1. (b) Fitted result of Group 2. (c) Fitted result of Group 3.

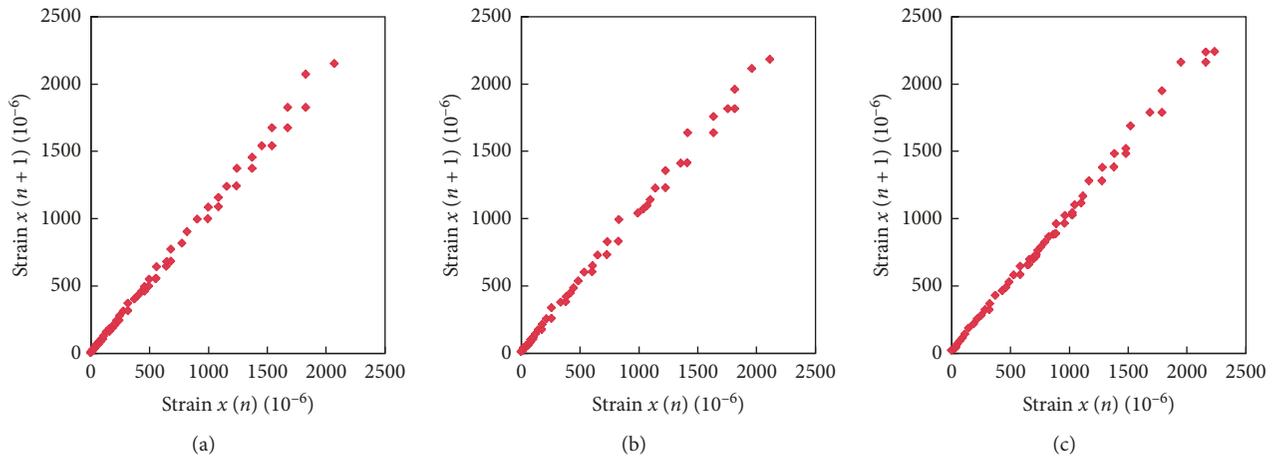


FIGURE 12: Evolution of each fitted strain series. (a) Fitted result of Group 1. (b) Fitted result of Group 2. (c) Fitted result of Group 3.

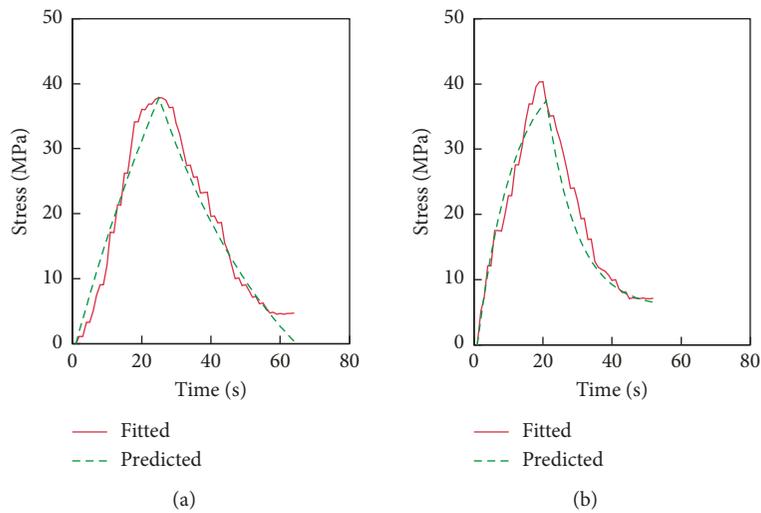


FIGURE 13: Continued.

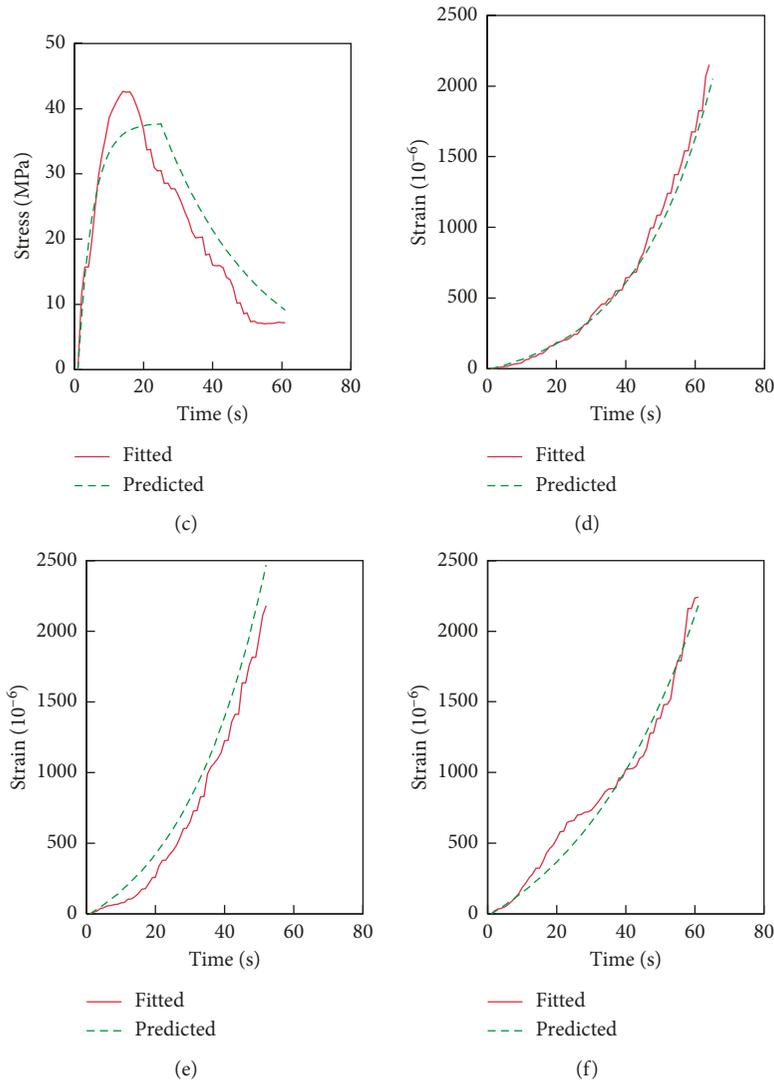


FIGURE 13: Prediction of each fitted stress and strain series. (a) Stress of Group 1. (b) Stress of Group 2. (c) Stress of Group 3. (d) Strain of Group 1. (e) Strain of Group 2. (f) Strain of Group 3.

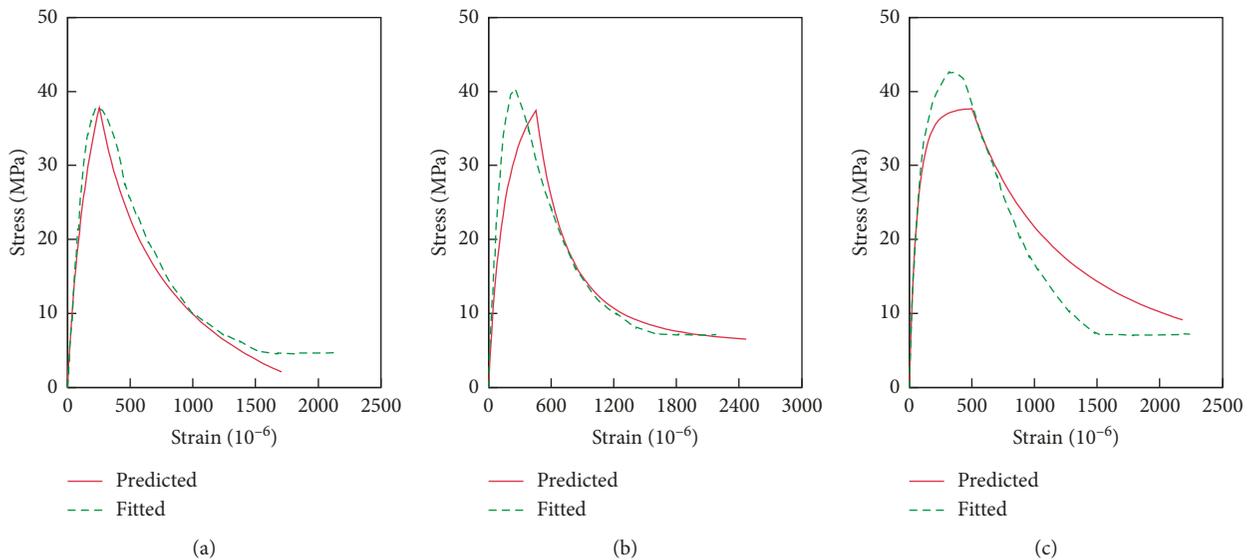


FIGURE 14: Predicted and fitted stress-strain curves. (a) Sample of Group 1. (b) Sample of Group 2. (c) Sample of Group 3.

human intervention. This kind of data could exhibit stronger chaos, which would be more difficult to predict.

However, the effect of prediction is better than the application in the original and raw data. This can be caused by two reasons. The first is that the number of fitted elements is less than the number of measured elements. Limited by the property of chaos, the series with more elements would be more difficult to predict. The second reason is that evolution forms keep changing. Only an approximate solving method is applied in this study. It can be observed from Table 2 that there is a considerable difference between each evolution coefficient, particularly the slope. This is caused by the sensitivity to initial values.

## 5. Conclusions

**5.1. Summary and Conclusion.** This study analyses the typical chaotic phenomenon in the experimental data of concrete tests. The following conclusions can be drawn:

- (1) The value of the LE of the experimental data of the concrete compressive test series was always larger than 0. This demonstrated that the behavior of concrete is highly sensitive to initial values. The values of the LE of fitted data were larger than those of the original and raw data, which indicated higher sensitivity.
- (2) The general behavior of concrete during the experiment could be predicted based on the WDM. All evolution equations were linear; this was in good agreement with the test results.
- (3) The fitted data did not show good agreement with the typical evolution equation. This may be caused by the stronger chaotic property in the fitted data, and this property may be due to human intervention, i.e., the fitting process.
- (4) The prediction of simulation results was accurate. This condition indicated the limit of the WDM, i.e., the series with more elements would be more difficult to predict.

Furthermore, this study could provide some essential information that will help improve efficiency, productivity, and competitiveness for structure engineers, contractors, and concrete manufacturers. This approach can also guide the study of cementitious materials and quasi-brittle materials.

**5.2. Future Work.** The interesting phenomenon presented in this paper was incidentally observed when we treated the experimental results, and this phenomenon could be observed in almost all concrete experiments. However, only a preliminary qualitative and quantitative analysis was performed. A deeper understanding of this phenomenon should be obtained. In addition to the observations of this study, several phenomena would be more evident and typical in the original and raw data obtained from concrete experiments. Therefore, only our data could be discussed in

this manner. More experiments should be performed to study this phenomenon.

## Data Availability

The experimental results used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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