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Research Article

Improved Nonprobabilistic Global Optimal Solution Method and Its Application in Bridge Reliability Assessment

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Utilizing the improved one-dimensional optimization algorithm conveniently solved the nonprobabilistic reliability index, however, only searching the part of probable failure points. Utilizing the global optimal solution method produced heavy computation, although it was capable of searching all probable failure points. This paper, based on these two methods, proposed an improved nonprobabilistic global optimal solution method. The presented method possessed the advantages of searching all probable failure points and generating less computation, by means of which the values of interval variables were determined based on the monotonicity of performance function to the corresponding variables. Without losing any probable failure points, this method was contributive to reducing root value point equations, lowering the computational complexity, and improving the computational efficiency. The effectiveness and feasibility of the presented method were verified by two examples. The proposed method was also introduced to build the nonprobabilistic reliability evaluation process of existing bridges. Taking the Longganhu bridge for example, the nonprobabilistic reliability index of it was calculated using the presented nonprobabilistic reliability evaluation process. The computed nonprobabilistic reliability index $\eta = 0.737 < 1$ indicated that the Longganhu bridge was unreliable and needed to be reinforced. Reinforcement measures were carried out for a hollow slab and bridge deck of the Longganhu bridge, respectively. The reinforced bridge was reevaluated as $\eta = 2.159 > 1$. The results showed that the bridge was reliable after reinforcement. The study illustrated the application and feasibility of the improved nonprobabilistic global optimal solution method and nonprobabilistic reliability evaluation process in reliability assessment and reinforcement of existing bridges.

1. Introduction

Substantial uncertainties appear unavoidably in practical engineering, such as structural engineering, geotechnical engineering, and environmental engineering. Reliability analysis is an effective tool to solve uncertainty problems. For example, reliability-based design has great advantages in dealing with uncertainties in geotechnical engineering [1], which provides a more consistent level of reliability and the possibility of accounting for load and resistance uncertainties separately [2]. The probabilistic reliability depends strictly on the probability density functions of the related parameters and requires large amounts of data. However, for most of engineering structures, it is difficult to obtain enough data and the probability density functions of related parameters. Wang et al. [3] presented a new analytic method to estimate reliability of fatigue crack growth prediction when the measured data were limited. Wang et al. [4] further improved the above analytic method combining the interval perturbation series expansion method and the first-passage principle. Actually, the reliability method for fatigue crack growth prediction with limited uncertainty information proposed by Wang et al. [3, 4] was consistent with the nonprobabilistic reliability concept presented by Ben-Haim [5], which effectively dealt with the reliability problems when only few or less statistical data were available [6].

The nonprobabilistic reliability index defined by interval model is the minimum norm of the coordinate vector in the standardized space, the computation of which is complex for the structures with the high nonlinear or implicit performance functions. Of course, for structures with the linear performance functions, the nonprobabilistic

reliability index was conveniently analytically expressed [7, 8]. For structures with the simple nonlinear performance functions, Guo et al. [9, 10] also suggested the definition approach, the transfer approach, and the optimization approach to solve the nonprobabilistic reliability index. For structures only having the hyperellipsoidal performance functions, Luo and Kang [11] also have successfully applied the design point method to conduct nonprobabilistic reliability analysis.

In order to simplify the search process and efficiently solve the nonprobabilistic reliability index, many engineers and researchers have contributed to the development and progress of nonprobabilistic reliability algorithm. The semianalytical approach proposed by Jiang et al. [8] and the one-dimensional optimization algorithm presented by Jiang et al. [12] were utilized to calculate the nonprobabilistic reliability index. Based on the semianalytical approach, Chen et al. [13] established a modified scheme to improve computational efficiency of the nonprobabilistic reliability index. Chen and Fan [14] proposed the improved one-dimensional optimization algorithm in order to reduce the workload for calculating the nonprobabilistic reliability index. Fan et al. [15] utilized the intersection line of the tangent plane and limit state function to linearize the performance function, and proposed the optimal searching algorithm. An evidence theory-based reliability analysis method was presented by Jiang et al. [16], introducing design optimization to obtain the nonprobability index and design point and to reduce the number of limit state functions for computation with the help of interval analysis method. Gradient projection method was suggested to solve the nonprobabilistic reliability index by creating search directions and establishing iterative computation formulation [17]. Chen et al. [6] and Fan and Chen [18] proposed the global optimal solution method and pointed out that the probable failure points only existed on extreme points and root points of the limit state function, avoiding the loss of probable failure points. Chen et al. [19] proposed a nonprobabilistic response surface limit method based on the interval model, which effectively avoided the tedious and oscillating iteration process and enhanced computational efficiency and precision.

Meanwhile, researchers proposed many mathematical models to expand the application of nonprobabilistic reliability theory. Wang and Qiu [20] proposed a probabilistic and nonprobabilistic hybrid reliability model based on probabilistic reliability model and nonprobabilistic set-based reliability models. Ni and Qiu [21] established a hybrid reliability model utilizing structural fuzzy random reliability and nonprobabilistic set-based models. Based on the probabilistic reliability model and interval arithmetic, Qiu and Wang [22] proposed a model to improve the interval estimation for the reliability index of probabilistic and nonprobabilistic hybrid structural system. An iterative hybrid structural dynamic reliability prediction model was developed by Fang et al. [23] under multiple-time interval loads with and without consideration of stochastic structural strength degradation. Considering various cases of uncertainties of structures, Wang et al. [24] built four hybrid models, including convex with random, convex with fuzzy random, convex with interval, and convex with other three types, and performed hybrid reliability analysis of structures with multisource uncertainties.

In addition, Chen et al. [25] presented the method and procedure of robust optimization design by dividing the optimization procedure into two stages based on the interval model, namely main optimization and sub-optimization. Wang et al. [26] proposed a nonprobabilistic reliability-based topology optimization method for detailed design of continuum structures, in which the unknown but bounded uncertainties existing in material and external loads were considered simultaneously. Wang et al. [27] proposed a nonprobabilistic time-variant reliability-based optimization strategy for closed-loop controller design of vibration reduction issues, which can be seen as the latest developments of nonprobabilistic reliability issues.

As stated by Chen et al. [6], Chen and Fan [14], and Fan and Chen [18], there were many advantages for improved one-dimensional optimization algorithm and global optimization method in solving the nonprobabilistic reliability index. Of course, disadvantages of these two methods were also reclarified here. The improved one-dimensional optimization algorithm only searched part of the probable failure points and faced with risk of losing the probable failure points. The global optimization method required a large amount of calculation, because there were $(1 + n!(n-1)!/2)$ extreme point equations and 2^{n-1} root value point equations to be solved for obtaining the nonprobabilistic reliability index in n-dimensional cases. In view of such situation, this paper proposed an improved nonprobabilistic global optimal solution method by combining improved one-dimensional optimization algorithm with global optimal solution method, developing the nonprobabilistic reliability evaluation process of existing bridges. Finally, two validation examples were accepted to verify the effectiveness and feasibility of the improved nonprobabilistic global optimal solution method for solving the nonprobabilistic reliability index, and an application example was selected to illustrate the feasibility and applicability of the nonprobabilistic reliability evaluation process in reliability assessment and reinforcement of existing bridges.

2. Nonprobabilistic Reliability Index Based on Interval Model

The limit state function (failure surface) representing structure failure is given as follows:

$$M = g(\mathbf{x}) = g(x_1, \ldots, x_n) = 0, \tag{1}$$

where $\mathbf{x} = \{x_1, x_2, \ldots, x_n\}$ are the basic variables of structures and $x_i \in X_i^I \quad (i = 1, 2, \ldots, n)$ are the interval variables.

The entire space was divided into two parts by $g(\mathbf{x}) = 0$, namely, failure domain $(g(\mathbf{x}) < 0)$ and safety domain $(g(\mathbf{x}) < 0)$. According to the literature by Guo and Lv [7] and Jiang et al. [12], the nonprobabilistic reliability index η is defined as follows:

$$\eta = \frac{M^{c}}{M^{r}} \in \begin{cases} [1, +\infty), & \text{reliable and safe,} \\ (-1, 1), & \text{unreliable,} \\ (-\infty, -1], & \text{failure,} \end{cases} \tag{2}$$

where M^{c} and M^{r} represent the mean and dispersion of the performance function $M = g(\mathbf{x})$, respectively. The reliable structure requires $\eta > 1$, which indicates that fluctuation ranges of structure performances do not intersect with the failure domain.

The engineering practice has demonstrated that the limit state functions commonly described the different and complex forms and the high nonlinear characteristics. For some special structures, even the explicit functions were not presented for the nonprobabilistic reliability analysis. Therefore, solving the reliability index using Equation (2) could not be implemented. If the interval variable $\mathbf{x}$ was normalized by the mathematical transformation $x_i = x_i^c + x_i^r u_i$, the nonprobabilistic reliability index η is defined as follows [9, 12, 13]:

$$\begin{cases} \eta = \min\left(\|u\|_{\infty}\right) = \min\left\{\max\left(|u_1|, |u_2|, \ldots, |u_n|\right)\right\}, \\ \text{s.t.} \quad G(u_1, u_2, \ldots, u_n) = 0, \end{cases} \tag{3}$$

where u_i is the normalized interval variable and x_i^{c} and x_i^{r} are the mean and dispersion of the variable x_i, respectively.

Equation (3) demonstrated that η was the minimum distance from the origin of coordinates to the failure surface measured by infinite norm. Larger η means longer distance between the actual fluctuation region of structure performance and the failure region, and higher reliability level of structure.

3. Improved Nonprobabilistic Global Optimal Solution Method

The goal of this section was to propose the improved nonprobabilistic global optimal solution method, including the principle of presented method and procedure of utilizing it to search the nonprobabilistic reliability index.

Defining the following formula:

$$\lambda = \min\left(|u_i|\right), \quad (i = 1, 2, \ldots, n), \tag{4}$$

then there were n equations required to solve for finding λ, namely, the minimum of $|u_i|$. The definition of η expressed by Equation (3) demonstrated that the coordinates of the point in failure surface corresponding to the nonprobabilistic reliability index should satisfy $\eta = |u_1| = |u_2| = \ldots = |u_n| \le \lambda = \min(|u_i|)$ [8]. Therefore, only the range of λ was required in engineering practice, and one just needs to determine the monotonicity of performance function to all variables in this range. So this subsection will discuss the method solving λ. In practice, it was found that the maximum absolute value of the coefficients of variable u_i in formulas $x_i = x_i^c + x_i^r u_i$, that is, $\max|x_i^{r}|$, determined the value of λ [14, 28].

Let all interval variables u_i be equal to 0 except that corresponding to $\max|x_i^{r}|$. The following result can be obtained by solving the limit state function $G(0, \ldots, u_i, \ldots, 0) = 0$:

$$|u_i| = \lambda'. \tag{5}$$

Equation (4) indicates that λ' satisfies the following:

$$|u_i| = \lambda' \ge \lambda. \tag{6}$$

Because λ' is equal or greater than λ, one can replace λ by λ' in the preliminary computation. When $\max|x_i^{r}|$ was determined, λ can be obtained.

Now the improved nonprobabilistic global optimal solution method was proposed, and the procedure of it was outlined as follows:

Step 1. Determine minimum points. Solve all minimum points for $u_1, u_2, \ldots, u_n$ based on $G(u_1, u_2, \ldots, u_n) = 0$.

Step 2. Determine minimum points after dimensionality reduction. Considering $u_i = \pm u_j$ will reduce the dimensionality of $G(u_1, u_2, \ldots, u_n) = 0$, and the limit state function will be expressed as $G(u_i = \pm u_j, u_k) = 0$ ($k = 1, 2, \ldots, n$, but $k \ne i, j$). Obviously, $G(u_i = \pm u_j, u_k) = 0$ is the $(n-1)$-dimensional problem, and there are $n(n-1)$ limit state functions. All minimum points of $G(u_i = \pm u_j, u_k)$ need to be solved.

Step 3. Determine minimum points after further dimensionality reduction. The limit state functions can be converted into $(n-2)$, $(n-3)$, ... and 2-dimensional problems by successively reducing the dimensionality of $n(n-1)$ limit state functions $G(u_i = \pm u_j, u_k)$, using the method in Step 2 in this section. Then solve all the minimum points of the $(n-2)$, $(n-3)$, ... and 2-dimensional limit state functions in sequence.

Step 4. Determine the range of $|u_i|$. After determining $\max|x_i^{r}|$ based on the limit state functions obtained in Step 2 and Step 3 in this section, λ can be obtained. One needs to study the monotonicity of performance function for all variables in $[-\lambda, \lambda]$. If the monotonicity of performance function for most variables are difficultly recognized, let $M = G(0, \ldots, u_i, \ldots, 0) = 0$ and solve variable u_i. Therefore, the range of $|u_i|$ can be determined; that is, $|u_1| = |u_2| = \ldots = |u_n| \le \lambda = \min(|u_i|)$.

Step 5. Simplify limit state functions. If $(\partial M/\partial u_i) > 0$ for $u_i \in [-\lambda, \lambda]$, let $u_i = -u$ in the normalized limit state functions; if $(\partial M/\partial u_i) < 0$ for $u_i \in [-\lambda, \lambda]$, let $u_i = u$ in the normalized limit state functions. If the positive or negative sign of $(\partial M/\partial u_i)$ for $u_i \in [-\lambda, \lambda]$ is not determinate, one needs to plug $u_i = \pm u$ into the normalized limit state functions for computation.

Step 6. Solve limit state functions after simplification. All real solutions should be solved for all functions of one variable obtained in Step 5 in this section, but the plural solutions must be given up. The points corresponding to these real solutions are in practice the root value points.

Step 7. Determine the nonprobabilistic reliability index. The minimum of the absolute values of all solutions

according to minimum points and root value points is the final nonprobabilistic reliability index.

4. Nonprobabilistic Reliability Evaluation Process of Existing Bridges

According to the principles of structure design, the performance function of bridge structures is described as follows:

$$M = g(x) = g(x_1, x_2, \ldots, x_n) = R - S, \tag{7}$$

where R and S are the resistance and effects of actions of the bridge structures, respectively.

4.1. Parameters and Finite Element Model. To conduct the nonprobabilistic reliability evaluation process of existing bridges, the original key parameters should first be seriously distinguished and selected through the preliminary investigations of engineering examples and existing experiences. This was conductive to building the key parameter sets of existing bridges. Taking the simple-supported beam-bridge as an example, the key parameter sets mainly include the height of a hollow slab, thickness of web, width of bridge roadway, location and area of stranded wire, compressive strength of concrete, material density, dead load of structure, secondary dead load, and vehicle load.

As mentioned before, the performance function of bridge structures always appeared to be the high nonlinear characteristics and even there were no explicit functions for some complex bridges. Therefore, one can utilize the response surface method to build the performance function through the finite element analysis. This measurement can help engineers and researchers solve the nonprobabilistic reliability index step-by-step and smoothly conduct the nonprobabilistic reliability evaluation for the bridge structures.

4.2. Sensitivity Analysis. In bridge engineering, engineers and researchers have to face the arduous measurement task due to many key factors influencing the performance of bridge structures. As stated by Chen et al. [6], in order to obtain the nonprobabilistic reliability index using their analytical approach presented, there were $(1 + n!(n-1)!/2)$ limit state functions for solving the extreme points and 2^{n-1} limit state functions for solving the root value points for the n-dimensional problem. So the growth of the computational complexity was exponential with the number of dimensions. Therefore, it was necessary to conduct sensitivity analysis for all potential key parameters, by which the number of key parameters utilized in nonprobabilistic reliability evaluation of bridge structures might be reduced.

The sensitivity of one parameter in reliability evaluation of bridge structures was measured by the influence of varying this parameter on the performance function of bridge structures. As described by Equation (7), the performance of bridge structures was expressed by function M. The sensitivity of one parameter to the nonprobabilistic reliability index can be equivalent to the sensitivity of this parameter to performance function M.

Suppose that coefficient of variation of interval variable x_i was f_i. If x_i varied from $(1 - f_i)x_i^c$ to $(1 + f_i)x_i^c$, and the other interval variables were set as their means respectively, the value of the performance function was denoted as M_i. If all interval variables were set as their means respectively, the value of the performance function was denoted as M_0. Then, the sensitivity index K_i for interval variable x_i is defined as follows:

$$K_i = \frac{M_i - M_0}{M_0}. \tag{8}$$

$K_i < \xi < 5\%$ demonstrates that coefficient of variation of interval variable x_i has little influence on the nonprobabilistic reliability index η, so x_i is reasonably seen as the constant in nonprobabilistic reliability analysis of bridge structures.

4.3. Nonprobabilistic Reliability Evaluation Process. Based on the preliminary investigations and field measurements, the obtained data were firstly selected to build the finite element model (FEM). The performance function was further established utilizing the response surface method through the FEM analysis. With sensitivity analysis, the interval parameters were recognized and divided into interval variables and constants. Next, the improved nonprobabilistic global optimal solution method proposed in Section 3 was used to solve the nonprobabilistic reliability index of the existing bridges. The process of solving the nonprobabilistic reliability index was also with the aid of the MATLAB software. Finally, the reliability index obtained became the basis of reliability evaluation of the existing bridges, which also supplied the technical support for the reinforcement and reconstruction of them. The specific evaluation process is shown in Figure 1.

5. Illustrations

In this section, two validation examples are first given to verify the effectiveness and feasibility of the improved nonprobabilistic global optimal solution method presented for solving the nonprobabilistic reliability index. The Longganhu bridge, in Hubei Province, China, was chosen to demonstrate the feasibility and applicability of the nonprobabilistic reliability evaluation process in reliability assessment and reinforcement of the existing bridges.

5.1. Validation Examples

5.1.1. Example 1. Consider the following normalized performance function consisting of three interval variables u_1, u_2, and u_3:

$$\begin{aligned} M = {} & 567(0.6 + 0.072u_1)(2.18 + 0.109u_2) \\ & - 0.5(32.8 + 1.64u_3)^2. \end{aligned} \tag{9}$$

With the help of Steps 1–3 included in Section 3, all minimum points of the function described earlier were solved, namely, (−8.33, −20, and −20).

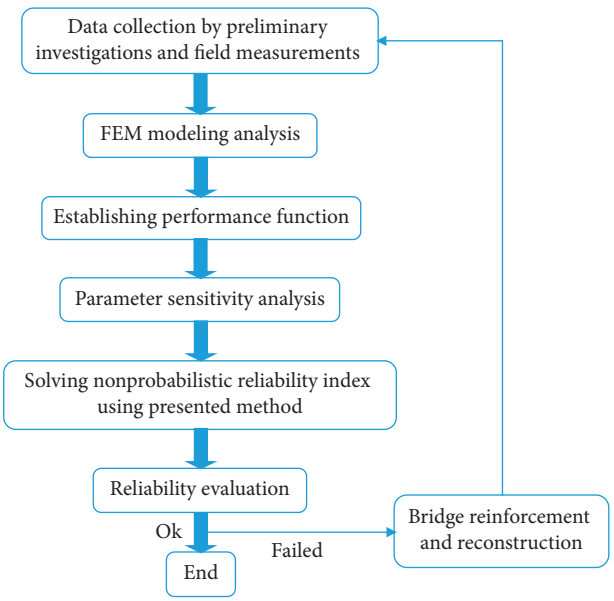

Figure 1: Reliability evaluation process.

The next discussions in this subsection were to solve the root value points. According to Step 4 presented in Section 3, the coefficient of variable u_3 was 1.64 larger than those of variables u_1 and u_2. Then, let $u_1 = 0$ and $u_2 = 0$; the limit state functions were simplified as follows:

$$M = G(u) = G(0, 0, u_3) = 0. \tag{10}$$

$u_3 = 3.45$ was obtained by solving Equation (10). The range of interval variables u_1, u_2, and u_3 was determined:

$$|u_1| = |u_2| = |u_3| \leq \lambda = 3.45. \tag{11}$$

According to Step 5 presented in Section 3, the derivatives of M to three variables u_1, u_2, and u_3 in $[-3.45, 3.45]$ were conducted and satisfied as follows:

$$\begin{aligned} \frac{\partial M}{\partial u_1} &> 0, \\ \frac{\partial M}{\partial u_2} &> 0, \\ \frac{\partial M}{\partial u_3} &< 0. \end{aligned} \tag{12}$$

Therefore, there were the following results for simplifying the limit state function:

$$\begin{aligned} u_1 &= -u, \\ u_3 &= -u, \\ u_3 &= u. \end{aligned} \tag{13}$$

Substituting the results expressed by Equation (13) into the original limit state functions as shown in Equation (9) would make the simplified limit state functions available as follows:

$$36.88 - 29.22u + u^2 = 0. \tag{14}$$

It was convenient to solve Equation (14), and there were two solutions, namely, 1.1556 and 56.7732, to be obtained. So two root value points of the original limit state functions were determined: (−1.1556, −1.1556, and 1.1556) and (−56.7732, −56.7732, and 56.7732).

By comparing the absolute values of minimum points and root value points, the most probable failure point was obtained as (−1.1556, −1.1556, and 1.1556), which was a root value point. The nonprobabilistic reliability index was finally determined as $\eta = 1.1556$, which was in accordance with that using the global optimal solution method by Fan and Chen [18]. There were four quadratic equations (root value point equations) necessarily to be solved using the global optimal solution method, and six solutions were obtained. However, only one quadratic equation (root value point equation) was necessarily to be solved using the method presented in this paper. The computational complexity was reduced 75%.

5.1.2. Example 2. One performance function including three interval variables u_1, u_2, and u_3 was considered:

$$M = \left(u_1 - \frac{7}{4}\right)^2 + \left(u_2 - \frac{4}{5}\right)^2 - 4u_3 + 5.5. \tag{15}$$

According to Steps 1–3 included in Section 3, all minimum points of the above function were solved, namely, (1.75, 0.8, and 1.375), (1.4048, 1.4048, and 0.8), and (1.275, 1.275, and 1.4878). In addition, the range of u_1, u_2, and u_3 was determined as $[-2.3, 2.3]$, in which the derivatives of M to every variable were conducted and satisfied as follows:

$$\begin{aligned} \frac{\partial M}{\partial u_1} &< 0, \\ \frac{\partial M}{\partial u_2} &> 0, \\ \frac{\partial M}{\partial u_3} &< 0, \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{\partial M}{\partial u_1} &< 0, \\ \frac{\partial M}{\partial u_2} &< 0, \\ \frac{\partial M}{\partial u_3} &< 0. \end{aligned} \tag{17}$$

Therefore, there were the following results for simplifying the limit state function:

$$\begin{aligned} u_1 &= u, \\ u_2 &= -u, \\ u_3 &= u, \end{aligned} \tag{18}$$

$$\begin{aligned} u_1 &= u, \\ u_2 &= u, \\ u_3 &= u. \end{aligned} \tag{19}$$

Substituting the results expressed by Equations (18) and (19) into the original limit state functions as shown in Equation (15) would make the simplified limit state functions available as follows:

$$4.6 - 1.05u + u^2 = 0, \tag{20}$$

$$4.6 - 4.55u + u^2 = 0. \tag{21}$$

It is obviously found that Equation (20) has no solution and Equation (21) has two solutions, namely, 3.0329 and 1.5171. So two root value points of the original limit state functions were determined: (3.0329, −3.0329, and 3.0329) and (1.5171, 1.5171, and 1.5171).

In summary, the most probable failure point was (1.4048, 1.4048, and 0.8), which is a minimum point. So the nonprobabilistic reliability index was $\eta = 1.4048$. This result was in accordance with that using the global optimal solution method by Fan and Chen [18]. However, there were four quadratic equations (root value point equations) necessarily to be solved using the global optimal solution method and only two quadratic equations necessarily to be solved using the improved nonprobabilistic global optimal solution method. The computational complexity was reduced 50%.

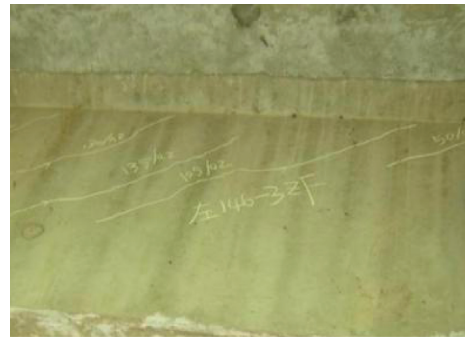

Figure 2: Diagonal cracks on the web.

Figure 3: Longitudinal horizontal cracks in the bottom slab.

5.2. Application Example

5.2.1. Reliability Evaluation before Reinforcement. The Longganhu bridge is one large-span bridge in the expressway link of the Jiujiang yangtze river bridge from Huangshi to Huangmei in Hubei Province, China, which is the part of the trunk line of the Hurong expressway. The Longganhu bridge has the overall length of 9.634 km and 480 spans. The superstructures of the Longganhu bridge are the prestressed concrete hollow slab with a cantilever, actually the simply supported continuous bridge deck system.

The prestressed simply supported hollow slab of one bridge span with the length of 20 m was chosen for nonprobabilistic reliability analysis. In-situ measurements demonstrated that there were the following diseases for the superstructures of this bridge: (1) diagonal cracks on the web of a hollow slab girder; (2) diagonal cracks on the web and in bottom slab of a hollow slab girder; (3) longitudinal horizontal cracks of girder; (4) longitudinal cracks, pockmarks, and pothole of bridge deck. These diseases are clearly seen in Figures 2 and 3.

Here, the nonprobabilistic reliability evaluation of Longganhu bridges was performed based on the prestressed simply supported hollow slab structures using the nonprobabilistic reliability evaluation process presented in Section 4. The following detailed procedures of nonprobabilistic reliability evaluation were outlined as follows:

Step 1. Determine parameters utilizing actual measurement method. The range of total area (A_s) of stranded wire was [10360, 11480] mm^2, range of hollow slab height (h) was [1380, 1420] mm, range of web thickness (d_1) was [1380, 1420] mm, range of vehicle load (F) was [220, 260] kN, range of tensile strength design value (f_{pd}) of stranded wire was [1.2, 1.32] kN/mm^2, range of cross section area (A) of the hollow slab was $[3.3, 33.41] \times 10^5$ mm^2, range of thickness (d_2) of bridge deck pavements was [130, 170] mm, range of dead load (q_1) of crash barrier was [8.02, 12] N/mm, distributed coefficient of lateral load (a) was [0.264, 0.283], and range of bulk density of reinforced concrete (ρ_1) was $[24, 28] \times 10^{-6}$ N/mm^3.

Step 2. Carry out FEM analysis and establish performance function. The finite element model for the prestressed simply supported hollow slab of one bridge span with a length of 20 m of the Longganhu bridge was built, which is illustrated in Figure 4. The midspan cross section was selected as the control section for analyzing the superstructure. Meanwhile, the response surface method was utilized to construct the performance function.

Step 3. Carry out sensitivity analysis for parameters. Sensitivity analysis was carried out for the parameters given in Step 1 in this subsection. The results indicated that the total area (A_s) of stranded wire, hollow slab height (h), web thickness (d_1), and tensile strength design value (f_{pd}) of stranded wire can significantly influence the bearing capacity of the bridge structures. Therefore, these four parameters were accepted as the key interval variables for nonprobabilistic reliability evaluation of the Longganhu bridge. The other parameters given in Step 1 in this subsection were taken as the constants, the values of which could be determined as the means of them respectively.

Figure 4: Finite element model for a hollow slab.

Step 4. Solve the nonprobabilistic reliability index. Through normalizing four key interval variables obtained in Step 3 in this subsection, the normalized limit state function was built based on the performance function established in Step 2 in this subsection. Using the improved nonprobabilistic global optimal solution method presented in Section 3 to solve the nonprobabilistic reliability index for this normalized limit state function, the final result was $\eta = 0.737$.

Step 5. Nonprobabilistic reliability evaluation. Because $\eta = 0.737 < 1$, one deservedly judged that the midspan cross section was unreliable, and some measures had to be taken to reinforce it.

5.2.2. Reinforcement Measure and Performance Evaluation. With the aid of status analysis and related computation, the following reinforcement and maintenance measures were implemented for the Longganhu bridge:

(1) The prestressed steel cables were anchored outside of the hollow slab to increase the thickness of the web to 30 cm, except the ranges within 5 m of the ends of the hollow slab. Webs on the two sides of the hollow slab were fully thickened with a thickness of 18 cm. In addition, the prestressed steel cables, $3\phi^j 15.24$, were arranged in the thickened webs on both sides of the hollow slab to improve the shear capacity and bending resistance ability of the hollow slab.

(2) Steel belt (R235) with a thickness of 8 cm was chosen to paste on the bottom side of the base slab in the ranges within 50~300 cm of the ends of the hollow slab. The cracks in the hollow slab were dealt with using the mud-jack method. The steel belt was constructed using the grouting methods.

(3) Due to the additional concrete pouring for hollow slab, many grouting holes were dug, and the bridge deck was in need of repair.

After reinforcement of bridges, the ranges of related parameters were determined again, and the nonprobabilistic reliability index for the reinforced bridge was solved using the presented method in this paper. Final value of the nonprobabilistic reliability index was obtained; that is, $\eta = 2.159 > 1$, which indicated that the bridge is reliable. The results of reassessments showed that the reinforcement and maintenance measures adopted were practical and effective for the bridge.

6. Conclusions

This paper presented an improved nonprobabilistic global optimal solution method to solve the nonprobabilistic reliability index. Through reducing the dimensionality of limit state function and the number of root value equations, the presented approach greatly improved computational complexity of solving the nonprobabilistic reliability index. Of course, exponentially reducing the number of root value equations would not bring the loss of probable failure points. Through judging the monotonicity of performance functions to all interval variables, the multivariate equations were transformed into single-variable equations to solve the root value points. This improvement was verified by two validation examples to be feasible and effective for the acquirement of the nonprobabilistic reliability index.

In addition, this paper proposed the nonprobabilistic reliability evaluation process of existing bridges and successfully applied it in nonprobabilistic reliability assessment and reinforcement of the Longganhu bridge. Finite element analysis and response surface method were significantly conductive to establishing the performance function. Sensitivity analysis for all potential key parameters reduced the number of interval variables in nonprobabilistic reliability evaluation of bridge structures and also reduced the computations. The result of the nonprobabilistic reliability evaluation for the Longganhu bridge was $\eta = 0.737 < 1$, which indicated that the Longganhu bridge was unreliable and needed to be reinforced. After reinforcement measures, the Longganhu bridge was reevaluated as $\eta = 2.159 > 1$ and was reliable.

In summary, the presented approach was to efficiently solve the nonprobabilistic reliability index for complex performance function and to practically evaluate reliability of engineering structures. The study would provide technical support for the nonprobabilistic reliability evaluation and reinforcement of existing bridges.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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