Research Article

A Method to Improve the Seismic Performance of Steel Moment Resisting Frames Based on Eigenfrequency Optimization

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Steel moment resisting frames are a structural system used throughout the world, mainly for their ductility and the speed and ease of their construction. These buildings are usually designed per procedures based on seismic design codes, seeking to minimize the total cost of the building. To aid in better building designs, researchers have proposed different methodologies, which have been proven to be effective. However, their practical use has been limited by their low computational efficiency and their difficulty to implement by practicing engineers. This article proposes a method to improve the seismic performance of steel moment resisting frame buildings based on eigenfrequency optimization. The main advantage of the proposed method is its computational efficiency and that it is simple to implement. The method is demonstrated for a four-story and an eight-story building, whose seismic performance is compared to traditional building designs using nonlinear analyses and seismic fragility functions. The results show that the seismic performance improves significantly with the proposed method with respect to that of traditionally designed buildings, reducing their seismic fragility and increasing their overstrength. These findings and the computational efficiency of the method suggest that it is a viable alternative for use within engineering practice.

1. Introduction

Steel moment resisting frames (SMRF) are a structural system used throughout the world. This structural system is used in many countries mainly for the speed and ease of its construction and because of the satisfactory performance demonstrated in recent earthquakes. Traditionally, these buildings are designed by structural engineers according to the ductility, strength, and drift provisions of seismic design codes. These provisions change from code to code; for instance, if the North American and other international provisions are compared with the Eurocode 8 in terms of drift requirements, it is found that the latter is more stringent, and it can be evidenced in case of the stability coefficient, which often controls the design of SMRF and allows buildings to achieve a considerable overstrength [1].

Accounting for the need to produce designs within reasonable times, current design codes provide guidelines for designing SMRF based on elastic analyses, using factors to account for the nonlinear nature of the seismic response ($q$-factor for Eurocode 8 and $R$ factor for American codes). This practice has remained mainstream through many decades although it does not always lead to a conservative design [2]. Acknowledging these facts, current design codes are continuously under revision and they are updated as findings from research programs and experience gained from earthquake over the years become available [3].

The result of the application of the seismic codes is that most design offices use elastic models to design most SMRF buildings, which are analyzed using the equivalent lateral force or response spectrum analysis. The mainstream practice is to first calculate the section profiles that satisfy the drift requirement, then proceed to check the strength and ductility requirements, and finally design the member connections. In this design process, the experience acquired in previous projects plays an important role, as it serves as a
guidance for the engineer, who often uses it to check the design outcome. Frequently, most engineers aim to minimize the total cost of the building because of the increasing market competition. Because of this approach, the performance of most SMRF buildings is close to the minimum required by existing methods for optimizing the design of structural systems.

Reflecting on its ubiquity and to aid in better building designs for SMRF buildings, researchers have proposed different methodologies, which are based on two approaches: structural optimization and performance-based design (PBD). In structural optimization, several methods have been proposed [4] based on different optimization frameworks [5] with objectives such as weight minimization and performance optimization [6], for which they have used several evolutionary optimization algorithms. Among them, there are methods based on teaching-learning [7], ant colony [8], harmony search [9], bee colony [10], colliding body [11], dolphin echolocation [12], firefly algorithms with neural networks [13], and cuckoo search [14]. In performance-based design [15], SMRF are designed to meet certain seismic performance goals, requiring successive iterations that need a seismic performance evaluation. This approach has been applied to buckling-restrained frames [16, 17] and self-centering SMF with braces [18] and for designing SMRF with uncertainty quantification [19, 20].

Both approaches have demonstrated effectiveness in improving the seismic design of SMRF, and PBD has earned a recommendation per the ASCE as a pathway to impact structural engineering [21]. However, despite these benefits of structural optimization and PBD, their usage in engineering practice has been limited. Many reasons can explain this though there is a strong possibility that it is because these approaches use nonlinear models for the buildings. According to a survey in the US, nonlinear models are considered by 60% of engineers as time consuming/not practical [22] and have been identified as a factor that limits the practical application of a design approach [23].

Contributing to the need of efficient and practical methods which can attract the attention of practicing engineers, this article proposes a method to improve the seismic performance of SMRF, whose novelty is a problem formulation that uses the natural frequencies as an objective function to optimize the seismic design, and the constraints to ensure the strength, capacity, and ductility requirements of design codes. This formulation brings two advantages to the proposed method: (1) it is easy to implement since it uses elastic models of the building and (2) it is computationally efficient and easy to integrate within engineering practice. The application of the method is demonstrated for two buildings, a four-story and an eight-story building. Both buildings are first designed per the traditional approach of engineering practice and then by the proposed method with the aid of the Borg-MOEA algorithm [24]. The method’s improvements in seismic performance are demonstrated using OpenSees [25] to compare the behaviour of this building against the same building designed in a traditional fashion. Further comparisons of the seismic performance are conducted based on seismic fragility functions that were developed based on the OpenSees results.

2. Eigenfrequency Optimization Method for the Design of Steel Moment Resisting Frames

Eigenfrequency optimization describes a problem where one or more natural frequencies, \( \omega_n \), of a structure or mechanism are optimized [26, 27]. This approach makes sense when the structures are subjected to dynamic loads whose response depends on one or more of its natural frequencies. Several problems involving dynamic loads in structural engineering have been solved using eigenfrequency optimization, such as maximizing the strength of RC column joints [28] and vibration reduction in trusses [29]. Additionally, this approach was used to optimize column dimensions using a surrogate membrane model of reinforced concrete moment frames, increasing the structure overstrength and ductility [30].

The performance of a building during an earthquake seems to be greatly affected by the smallest natural frequencies of the structure. The reliance of the seismic response on the natural frequencies has been leveraged to develop efficient structural analysis procedures for calculating the seismic response of buildings, such as the modal pushover analysis [31, 32]. Moreover, several standards use the dominating nature of the smallest natural frequencies to recommend seismic response methods such as the equivalent lateral force (ELF) and the modal response spectrum analysis, both using elastic models of the buildings [33, 34].

The natural frequencies of a structure are properties that can be optimized using eigenfrequency optimization, suggesting that it is a suitable approach to design buildings with improved seismic performance. Furthermore, the optimization of the natural frequencies of a structure has low computational costs since the objective function uses the elastic model of the building to make the calculation, which is significantly less complex than the nonlinear analyses required by existing methods for optimizing the design of SMRF.

2.1. Formulation of Eigenfrequency Optimization for Steel Moment Resisting Frames

Developing an efficient formulation of eigenfrequency optimization for SMRF has three challenges. The first one is that the elements sections in a SMRF are represented by discrete variables, given by the typical steel section profiles used in construction practice. Handling this problem requires expressing the element sections of the SMRF as a vector \( \mathbf{p} \) of integer indexes, each one corresponding to a section profile, which has associated the corresponding section properties (area, moment of inertia, etc.). A second challenge stems from the fact that the problem optimizes natural frequencies; therefore, it is necessary to introduce a constraint to the problem that limits the cost of the building, ensuring a competitive design. This is addressed by imposing a limit \( W_0 \) to the total weight of the building \( W(\mathbf{p}) \), which is a function of the profiles \( \mathbf{p} \) of the structural elements. A third challenge is ensuring that the resulting design satisfies design code requirements for resistance (e.g., strong column-weak beam) and functionality (e.g., deflexion control). To overcome this challenge, each element of the building (i.e., columns and beams) must be
limited between a minimum profile \( p_{i,\min} \) and a maximum profile \( p_{i,\max} \). In addition to fulfilling design code requirements, this scheme also contributes to limit the search space, avoiding the possibility of generating unviable solutions that would be discarded by the design code constraints.

With those considerations into account, equation (1) shows the formulation of the eigenfrequency optimization problem for SMRF, based on the stiffness \((K)\) and mass matrixes \((M)\) of a given structure. Here, \( \omega_p \) is defined as the \( n \)-th natural frequency of the building and \( p \) is an integer vector with the section profiles, which has associated the corresponding section properties:

\[
\begin{align*}
\text{max} & \quad \{\omega_p\}_{p=1,...,N}, \\
\text{subject to} & \quad p_{i,\min} \leq p_i \leq p_{i,\max} \\
& \quad W(p) \leq W_0 \\
& \quad [K - \omega_p^2 M] \varphi_p = 0.
\end{align*}
\]

This formulation seeks to maximize the set of \( N \) first natural frequencies of the structure, subjected to three constraints. In the first constraint, \( p_{i,\min} \) and \( p_{i,\max} \) are the minimum and maximum index values of each element represented in \( p \). The second constraint accounts for the volume limit previously discussed. The third constraint comes from the structural dynamic theory, and it means that \( \omega_p \) is a natural frequency of the structure.

### 2.2. Integration of Eigenfrequency Optimization in the Structural Design Workflow

Feasibility, understood as the degree to which the optimization procedure is easy or convenient to implement, is one critical aspect of an optimization method to succeed in engineering practice [23]. As demonstrated in equation (1), the proposed method uses the elastic properties of the building, so it is feasible for implementation in engineering practice. This section discusses the details for its integration.

Figure 1 shows the typical design process of SMRF. It starts with the evaluation of the gravity and seismic loads. After that, designers propose a set of initial dimensions for columns and beams, which are then used to check for the drift limit of the building. If the drift limit is not satisfied, a new set of dimensions is proposed until this condition is met. After that, designers proceed to check the strength and deflection requirements of design codes, and once these are met, the design is considered complete.

To assist engineers in designing SMRF with better seismic performance, the authors suggest applying the method herein proposed after all code requirements are satisfied, as illustrated by the dotted box in Figure 1. To achieve this goal, designers need to solve the problem from the previous section, considering the natural frequencies of the building as the objective function. In this regard, the authors recommend using the fundamental frequency as objective function for low-rise (i.e., less than 5 stories) buildings and adding more frequencies as the height increases. In addition, it is a good idea to consider \( W_0 \) as the weight of the design they have already completed and setting \( \varphi_{\min} \) as the index of the smallest section that satisfies the strength and deflection requirements.

This approach has several advantages. First, the design that results from the optimization satisfies not only the strength requirements but also the drift limit, as the method is maximizing the fundamental frequency (i.e., minimizing the fundamental period) and the drift limit is checked in the elastic range. Consequently, optimized building will have a smaller one than the traditionally designed building. The second advantage is that unlike existing methods, the proposed method does not need to perform any nonlinear analyses, significantly improving the computational performance. A third advantage of the proposed method is that the proposed formulation works for any type of buildings, regardless of their irregularity and whether they are two-dimensional and three-dimensional problems. In the latter case, the stiffness matrix must be formulated using the frame-type element which has 6 degrees of freedom on each element end. Similarly, the mass matrix must account for the additional degrees of freedom. In all cases, the method requires that the strength and ductility requirements must be met by the proper selection of the \( p_{i,\min} \) and the \( p_{i,\max} \) for the members profiles. The examples section provides strategies to achieve these requirements, ensuring that capacity design criteria such as strong column-weak beam are satisfied.

In this article, the software MATLAB® is used for creating functions with the implementation of this formulation. An example function for a two-dimensional 8-story building is available to download from this link. The function receives the indexes for columns and beams as inputs and returns the first two periods (associated to the eigenfrequencies) as outputs. In this study, the solver used is the Borg-MOEA [24], a state-of-the-art multiobjective optimization algorithm. The periods are selected as outputs because most algorithms are developed to minimize objective functions and not for maximization, like in eigenfrequency. Since the periods are inversely proportional to the frequencies, they are a suitable variable that is physically significant for the problem. That said, the eigenfrequency optimization can be solved using any multiobjective optimization algorithm.

### 3. Example Applications

Two structures are used to demonstrate the application of the proposed method (Figure 2). The first one is a four-story building with three identical beam spans of 6 m. The second one is an eight-story building with five spans, all of which are 6 m length. All floors in both buildings have a story height of 3 m.

These buildings were designed per the LRFD in the AISC 360-16 following the customary engineering practice and then by the proposed method. Two section profile databases were used for this purpose. Tables 1 and 2 show the profiles used for beams and columns, which range from the IPE100 to the IPE600 and from HEA100 to HEA600, respectively. In both cases, the profiles were sorted in the ascending order of the moment of inertia about the sections’ strong axis, assigning the ID = 1 to the profiles IPE100 and HEA100, which have the smallest value of each group. The profiles
with the largest moment of inertia (IPE600 for beams and HEA600 for columns) had ID = 17 and ID = 19 assigned. This sorting is conducted to use the ID profiles of columns and beams that constitute the p vector described in the previous section, which is the optimization variable for the problem.

For both buildings, the objective function and constraints were coded in MATLAB® and the multiobjective evolutionary algorithm, Borg-MOEA [24] was used to solve the problem.

3.1. Results for the 4-Story Building. The proposed method (Figure 1) is applied to a four-story building (Figure 2(a)). This building is first designed according to the customary engineering practice for a base shear of 161.5 kN. The design results show that using profiles ID = 11 (HEA300) and ID = 10 (IPE100) for columns and beams satisfy all the drift, strength, and deflection requirements. This design has a mass of 7.28 ton, which is used as the W_0 limit for the problem formulation. Since this is a low-rise building, the fundamental frequency was considered as the only objective function.

A key step to formulate the optimization problem is defining the first constraint and choosing suitable values for p_{i,min} and p_{i,max}. As discussed in the previous section, these must be chosen such that the resulting optimized design complies with the resistance and functionality requirements.

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**Figure 1:** Typical design process of steel moment resisting frames. The dotted box represents the eigenfrequency optimization step proposed in this article.

**Figure 2:** Structures to be optimized. (a) 4-story SMRF. (b) 8-story SMRF.

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**Table 1: Sections for beams.**

<table>
<thead>
<tr>
<th>Profile</th>
<th>ID</th>
<th>Area (cm^2)</th>
<th>Inertia (strong axis) (cm^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPE100</td>
<td>1</td>
<td>10.3</td>
<td>171.1</td>
</tr>
<tr>
<td>IPE120</td>
<td>2</td>
<td>13.2</td>
<td>318.0</td>
</tr>
<tr>
<td>IPE140</td>
<td>3</td>
<td>16.4</td>
<td>541.1</td>
</tr>
<tr>
<td>IPE160</td>
<td>4</td>
<td>20.1</td>
<td>869.5</td>
</tr>
<tr>
<td>IPE180</td>
<td>5</td>
<td>23.9</td>
<td>1317.0</td>
</tr>
<tr>
<td>IPE200</td>
<td>6</td>
<td>28.5</td>
<td>1943.0</td>
</tr>
<tr>
<td>IPE220</td>
<td>7</td>
<td>33.4</td>
<td>2772.1</td>
</tr>
<tr>
<td>IPE240</td>
<td>8</td>
<td>39.1</td>
<td>3892.2</td>
</tr>
<tr>
<td>IPE270</td>
<td>9</td>
<td>45.9</td>
<td>5790.2</td>
</tr>
<tr>
<td>IPE300</td>
<td>10</td>
<td>53.8</td>
<td>8355.8</td>
</tr>
<tr>
<td>IPE330</td>
<td>11</td>
<td>62.6</td>
<td>11770.2</td>
</tr>
<tr>
<td>IPE360</td>
<td>12</td>
<td>72.7</td>
<td>16270.1</td>
</tr>
<tr>
<td>IPE400</td>
<td>13</td>
<td>84.5</td>
<td>23130.0</td>
</tr>
<tr>
<td>IPE450</td>
<td>14</td>
<td>98.8</td>
<td>33740.1</td>
</tr>
<tr>
<td>IPE500</td>
<td>15</td>
<td>116.0</td>
<td>48200.0</td>
</tr>
<tr>
<td>IPE550</td>
<td>16</td>
<td>134.0</td>
<td>67119.8</td>
</tr>
<tr>
<td>IPE600</td>
<td>17</td>
<td>156.0</td>
<td>92080.0</td>
</tr>
</tbody>
</table>
of the design code. For \( p_{i,\text{min}} \), a good strategy for beams is selecting the smallest profile that satisfies the serviceability (deflexion) requirements of the design code. One way to conduct this task is analyzing the floor system for the gravity loads and finding the profile for which the deflexions are closest to the code limit for the building. In this example, beam ID = 9 (IPE270) satisfies the strength and deflexion requirements; hence, they are used as the \( p_{i,\text{min}} \).

For columns, \( p_{i,\text{min}} \) must be selected simultaneously with the beams \( p_{i,\text{max}} \), ensuring that the strong column-weak beam requirement is met. In this example, column ID = 10 (HEA280) and beam ID = 11 (IPE330) are selected for the columns \( p_{i,\text{min}} \) and beams \( p_{i,\text{max}} \). Based on engineering experience and to avoid having an unnecessary large space, ID = 12 (HEA320) and is set as the upper bounds for columns (i.e., \( p_{i,\text{max}} \)).

Table 3 shows the results of the building design per the traditional method and the one proposed in this article. This result was achieved using the Borg algorithm configured to perform 100000 function evaluations, which required 21 minutes of computing time running on a single core of a Ryzen 7 1700 processor at 3.4 GHz. To ascertain the convergence of the Borg algorithm, this process was repeated five times with different seed values, obtaining the same results.

The results (Table 3) show that the proposed method did not change the building’s beam configuration, leaving all floors with the initial ID = 10 (IPE300) profile. This result may stem from the additional constraint imposed to the objective function that beams within a same floor must use the same section profile. This constraint was included to accommodate construction practices, where it is common for all beams within a same floor to use the same section. Regarding columns, the method assigns ID = 12 (HEA330) sections to columns of the first two stories and ID = 10 (HEA280) to columns in the top half of the building. The material consumption for this building is 7.22 ton, a 0.8% reduction compared to the traditional alternative. In terms of ratio of the column inertia to beam inertia, column ID = 12 to beam ID = 10 in the first two floors of the optimized design is 2.74, a 25% increase compared to the 2.18 in the traditional design. On the other hand, for floors 3 and 4, this ratio decreases to 1.63. In all cases, these ratios satisfy the code constraints.

### Table 2: Sections for columns.

<table>
<thead>
<tr>
<th>Profile</th>
<th>ID</th>
<th>Area (cm²)</th>
<th>Inertia (strong axis) (cm⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEA100</td>
<td>1</td>
<td>21.2</td>
<td>349.2</td>
</tr>
<tr>
<td>HEA120</td>
<td>2</td>
<td>25.3</td>
<td>606.0</td>
</tr>
<tr>
<td>HEA140</td>
<td>3</td>
<td>31.4</td>
<td>1033.1</td>
</tr>
<tr>
<td>HEA160</td>
<td>4</td>
<td>38.8</td>
<td>1672.8</td>
</tr>
<tr>
<td>HEA180</td>
<td>5</td>
<td>45.3</td>
<td>2509.9</td>
</tr>
<tr>
<td>HEA200</td>
<td>6</td>
<td>53.8</td>
<td>3692.0</td>
</tr>
<tr>
<td>HEA220</td>
<td>7</td>
<td>64.3</td>
<td>5410.2</td>
</tr>
<tr>
<td>HEA240</td>
<td>8</td>
<td>76.8</td>
<td>7763.1</td>
</tr>
<tr>
<td>HEA260</td>
<td>9</td>
<td>86.8</td>
<td>10449.9</td>
</tr>
<tr>
<td>HEA280</td>
<td>10</td>
<td>97.3</td>
<td>13669.9</td>
</tr>
<tr>
<td>HEA300</td>
<td>11</td>
<td>112.5</td>
<td>18260.1</td>
</tr>
<tr>
<td>HEA320</td>
<td>12</td>
<td>124.4</td>
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</tr>
<tr>
<td>HEA340</td>
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</tr>
<tr>
<td>HEA360</td>
<td>14</td>
<td>142.8</td>
<td>33090.0</td>
</tr>
<tr>
<td>HEA400</td>
<td>15</td>
<td>159.0</td>
<td>45070.0</td>
</tr>
<tr>
<td>HEA450</td>
<td>16</td>
<td>178.0</td>
<td>63720.0</td>
</tr>
<tr>
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<td>17</td>
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<td>86969.9</td>
</tr>
<tr>
<td>HEA550</td>
<td>18</td>
<td>211.8</td>
<td>111900.1</td>
</tr>
<tr>
<td>HEA600</td>
<td>19</td>
<td>226.5</td>
<td>141199.9</td>
</tr>
</tbody>
</table>

### Table 3: Design results for the 4-story building.

<table>
<thead>
<tr>
<th></th>
<th>Traditional design</th>
<th>Optimized design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Story</td>
<td>Column ID</td>
<td>Beam ID</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

3.2. Results for the 8-Story Building. The application of the proposed method to the 8-story building (Figure 2(b)) has one important difference compared to the 4-story building. Being a midrise building that has twice the stories and two additional beam spans compared to the four-story building, selecting only one frequency as objective function may not be the most appropriate option, as it could neglect the potential impacts of higher frequencies on the building seismic performance. Consequently, the first two frequencies are selected as objective functions, turning this into a multiobjective optimization problem.

Like the four-story building, the first step is to design the building according to the customary engineering practice. This building was designed for a base shear of 755 kN. The section profiles that satisfy all design code requirements for this building are ID = 17 (HEA900) for columns and ID = 11 (IPE330) for beams. This initial design has a mass of 34.1 ton that is used to calculate \( W_0 \) for the optimization formulation (equation (1)).

To define the section constraints of the optimization problem, the smallest profile that satisfies the deflexion requirement for beams is ID = 100 (IPE300); hence, it is used to define the beams’ \( p_{i,\text{min}} \). For the upper limit for beams, \( p_{i,\text{max}} \) ID = 13 (IPE400) is selected, together with ID = 15 (HEA400) as \( p_{i,\text{min}} \) for columns. These profiles ensure that the column-to-beam inertia ratio of the building is at least 1.95. Lastly, profile ID = 19 (HEA600) is selected as the upper bound for the column size.

The optimization results obtained using the Borg-MOEA [24] algorithm after 1000000 function evaluations are shown in Table 4. Since the problem has two objectives, it does not have a single solution. Instead, the optimization provides a set of designs which are the nondominant solutions of the problem. This means that each solution has one frequency smaller when individually compared to the others. For instance, solution 1 has a smaller \( w_1 \) than solution 3, but \( w_2 \) is smaller for the solution 3 compared to solution 1.

The analysis of the designs allows identification of a heightwise pattern for the structural elements. In terms of the columns configuration, all designs start with section ID = 19 (HEA600) at the bottom two stories of the building, which is the section profile with the largest moment of
3.3. Computational Performance. As stated in the Introduction section, the novelty of the method lies on the problem formulation based on eigenfrequency optimization, which offers important benefits in terms of computational performance. These are achieved without incorporating improvements and/or innovations in the solution technique. For the two buildings used as example, the results were obtained using the Borg-MOEA [24] algorithm, which is a state-of-the-art method to solve multiobjective optimization problems. Nevertheless, the formulation lends itself to other solution techniques based on evolutionary algorithms.

For the 5-story building, 21 minutes of computational time using one core of a Ryzen 7 1700 processor were necessary to obtain a solution. For the 8-story building, it took 42 minutes with the same conditions, with a total of 1000000 function evaluations. These computational times compare favorably to others reported in literature, where for two-dimensional problems, it may take between 12 hours and 1.5 weeks depending on the problem formulation [35].

In terms of convergence, the proposed method in combination with the Borg-MOEA algorithm showed a good performance. This was measured for the four-story and the eight-story buildings by selecting five different sets of initial population to start the searching process. In all cases, the same result was obtained, regardless of the starting point. Since the novelty of the proposed method relies on its formulation, the computational efficiency, and ease of formulation which it confers to the optimization problem, no further tests were conducted as the Borg-MOEA is an algorithm that has been used to solve more complex problems [36, 37].

3.4. Application of the Method for Three-Dimensional Buildings. As discussed previously, the formulation of the proposed method also works for three-dimensional buildings. However, there are two important considerations. The first one is that three-dimensional problems require the use of an appropriate stiffness matrix, where each element end must have 6 degrees of freedom. In this regard, the authors recommend using the Timoshenko stiffness matrix. Correspondingly, the mass matrix must account for the additional degrees of freedom. The strength, ductility, and capacity requirements of design codes can be met by proper selection of the $p_{\text{min}}$ and the $p_{\text{max}}$, as demonstrated in the examples above.

The second consideration is the selection of the number of frequencies. Due to the almost unlimited number of possible three-dimensional buildings, it is not easy to provide a thumb-rule to select the number of frequencies. Nonetheless, the authors recommend a similar approach to the one used by design codes to select the number of modes in a response spectrum analysis, ensuring that number of modes achieves a minimum percentage of the building mass.

4. Seismic Performance Evaluation

In order to have a comprehensive view of the implications of the proposed method in the seismic performance, OpenSees [25] was used to create fiber models for the traditionally designed buildings design and the ones designed with the proposed method. Beam and columns are modeled using force-based elements with fibers of steel. The model uses five integration points, with the steel modeled considering $E_s = 200$ GPa, $F_y = 350$ MPa, $F_u = 450$ MPa, and a rupture deformation $\varepsilon_{\text{rup}} = 14\%$. The foundation is modeled as rigid, and the gravity loads for the model are calculated based on the expected loads and using the combination $1.05D + 0.25L$. Corotational effects are included to account for contribution of the geometric stiffness matrix at large deformations. The seismic performance is evaluated using pushover analysis through displacement control, with a load pattern proportional to the first mode of vibration of the building. The buildings are pushed to a 10% roof drift ratio (i.e., the ratio between roof displacement and the buildings height) in 1 mm increments.

4.1. Evaluation of the 4-Story Building. The pushover results for the traditionally designed building and the building designed per the proposed method are shown in Figure 3. The X-axis shows the roof drift ratio (RDR) of the buildings.
as percent, and the Y-axis shows the ratio ($V_s/V_{design}$) between the base shear $V_s$ in the pushover and the design base shear $V_{design}$ of the building.

The results from this analysis show that the buildings have almost identical performance up to 1.5% roof drift, suggesting that the seismic performance for small earthquakes will be similar. After this point, the building designed per the proposed method shows better performance, exhibiting a higher overstrength (maximum $V_s/V_{design}$). On the other hand, both buildings have similar values for the postpeak slope. To further investigate the seismic performance improvements of the proposed method, the SPO2-FRAG software [38] is used to generate fragility functions at the life safety (3% RDR) and the collapse fragility (4.5% RDR) levels.

Compared to the traditionally designed building, the seismic performance at the life safety and collapse levels for the building designed following the procedure presented is improved. An acceleration $S_a = 0.625$ g is required for a 10% of probability of exceedance for the traditionally designed building at the collapse prevention level. For the building designed according to the method herein proposed, $S_a = 0.70$ g ($T_1 = 0.94$ s) is needed to reach this probability, representing a 12% increase. The results for the life safety level (Figure 4) show similar improvements in the building designed following the proposed method. Considering again, a 10% of exceedance probability and an acceleration $S_a = 0.50$ g is needed for this building, which is 10.2% higher than in its traditionally designed counterpart (0.454 g).

Summing up the results for this case, the building designed according to the proposed method has improved performance over the traditional one at the life safety and the collapse prevention levels. These benefits come without any additional costs in materials and were achieved after less than 30 minutes of computational time.

### 4.2. Evaluation of the 8-Story Building

The pushover results for the 8-story traditionally designed building and the four designs obtained using the proposed method are shown in Figure 5.

Several observations can be drawn from these results. To start, all the buildings designed per the proposed method have a higher overstrength ratio (largest $V_s/V_{design}$ value) than the traditional design. In addition, these four buildings reach this value close to a 3.5% RDR, which is also higher than the 3.1% RDR at which it occurs for the traditional building. Comparing the four designs obtained per the proposed method, solutions 1, 2, and 4 show similar performance levels, as demonstrated by their almost overlapping pushover curves. On the other hand, the
performance observed for solution 2 is slightly worse, albeit by small margin. When RDR smaller than 1.5% is considered, the behaviour observed for all buildings is similar, suggesting that they will exhibit similar performance when subjected to small earthquakes.

Overall, the pushover results for the eight-story building show similar benefits to those observed for the 4-story building. Moreover, the improvements for the overstrength ratio and its associated RDR are slightly higher than the ones in the four-story building, suggesting that the use of two frequencies as objective functions instead of one has brought better solutions. These findings support the argument that the designs obtained for the eight-story building will have similar or better improvements compared with the traditional design in the life safety and collapse fragility.

5. Conclusions and Future Work

A method to design steel moment resisting frames has been proposed based on eigenfrequency optimization. Compared to existing alternatives, the proposed method has the advantages of being computationally efficient and feasible to implement. These benefits stem from the fact that the method uses the information of elastic models of the buildings. The application of the method is demonstrated for a four-story and an eight-story building, whose seismic performance is evaluated using nonlinear fiber models which are used to develop seismic fragility functions. The findings of this study support the following conclusions:

1. The proposed method produces designs of steel moment frames that have superior seismic performance than buildings designed following the customary engineering practice. The results from the seismic performance for a four-story building show that the building designed using the proposed method is 12% less susceptible to collapse than its traditionally designed counterpart.

2. The method is computationally efficient, as it allowed us to obtain a design in less than half hour of computing time for the four-story building and in less than one hour for the eight-story building. In addition, since the method uses elastic models, it is feasible to implement by practicing engineers without major efforts, suggesting that it can be integrated in the design workflow of engineering offices.

3. Eigenfrequency optimization is a viable framework to optimize the design of steel moment frames, which lends itself to be implemented and solved using evolutionary approaches like genetic algorithms.

4. The method can be used for buildings with more complex dynamic responses as it allows incorporation of more than one natural frequency as objective function. The application for an eight-story building suggests that this practice may bring additional benefits in the seismic performance compared to when only one frequency is considered. For this building, the pushover results show that the designs that result from the method application have higher overstrength than the traditional design. In addition, they also showed that the postpeak range starts at a larger roof drift ratio than in the traditional design.

The buildings used to demonstrate the application of the method in this study were regular buildings, whose elevation can be represented by two-dimensional frames. However, the proposed method can also be used to optimize the seismic design of three-dimensional buildings with minimal changes compared to two-dimensional buildings. The implementation details and recommendations are also provided in this work.

Future research on this topic can be pursued in several topics. One interesting area is evaluating the benefits of the proposed method for other building configurations, particularly for high-rise buildings where several frequencies are needed to capture the dynamic response and which would be required as objective functions. Considering the potential to be a computationally efficient method, it is a worthy topic to investigate the method application to optimize three-dimensional buildings with different types of irregularities. Finally, it is worth exploring the potential benefits of using other solution techniques such as simulated annealing, ant colony, and a wide family of evolutionary algorithms instead of the genetic algorithms used in this work.

Data Availability

No data from third party sources were used to support this study. The results obtained come from numerical simulations conducted by the authors.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
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