Research Article

Statistical Investigation of Bearing Capacity of Pile Foundation Based on Bayesian Reliability Theory

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In order to improve the estimation accuracy of bearing capacity of pile foundation, a new forecast method of bearing capacity of pile foundation was proposed on Jeffrey's noninformative prior using the MCMC (Markov chain Monte Carlo) method of the Bayesian theory. The proposed approach was used to estimate the parameters of Normal distribution. Numerical simulation was used to produce pseudosamples. The parameter estimation of the maximum likelihood method and the Bayesian statistical theory was used to estimate the parameter estimation of the Normal distribution, which has been compared with the theoretical value of the pseudosample of Normal distribution. The result indicates that the forecast model of Normal distribution using the Bayesian method is better than that of the maximum likelihood method, and the performance of the proposed method was improved with increasing of pseudosample number. At last, the proposed method was applied to estimate the parameter of Normal bearing capacity distribution of pile foundation, which shows that the proposed method has a high precision and good applicability.

1. Introduction

With the development and improvement of science and technology, reliability theory has a wide application in civil engineering [1–5]. As a usual type of bridge foundation, the pile foundation plays an important role in the safety and economy of the structure design [6–10]. The bearing capacity of the pile foundation is considered as constant in the traditional deterministic design method while it is regarded as a random variable via the conventional analysis. From the point of statistical theory, probability analysis was utilized to consider the variability of the pile foundation bearing capacity, which is treated as a single random variable. The mechanism of pile foundation can be better revealed from the perspective of uncertainty. Based on the application of reliability theory, the guidance and codes could be improved better in civil engineering [11–18].

The accuracy of the statistical parameters of the bearing capacity of pile foundation has a great impact on the reliability analysis results. Several studies [19–26] have focused

on the bearing capacity of the Normal pile foundation distribution. However, the main disadvantage of the references is summarized as follows: (1) due to insufficient measured data, the statistical properties of pile foundation bearing capacity are usually assumed; (2) statistical parameters in Normal distribution are regarded as determined values not the random variables; (3) statistical parameters in Normal distribution are considered to be independent and irrelevant to each other.

To improve the accuracy of parameter estimation of the Normal distribution, the Bayesian method was used to estimate the parameters of Normal distribution in this paper and applied to the bearing capacity of the pile foundation. The method proposed in this paper improves the conventional methods in the following aspects: (1) the proposed method is based on the real data set; (2) the parameters of Normal distribution are considered as random variants in the proposed method; (3) the correlation of statistical parameters of Normal distribution can be considered in the proposed method.

An important issue in the Bayesian estimation is predicting the posterior distribution based on prior distributions in existing experience and historical statistical data sets. In this paper, the principle of the Normal distribution quantile is used to study the statistical parameters using the Bayesian theory. The Bayesian quantile estimation is based on the Bayesian reliability theory; however, the Bayesian reliability theory for quantile posterior estimation of Normal distribution is a complex, high-dimensional, and nonanalytical solution, so it is difficult to extract samples from the posterior distribution. However, the MCMC algorithms in the Bayesian analysis are adopted herein to solve the calculation problem. One of the advantages of the MCMC method is the combination of the Monte Carlo integration with the effective sampling technique. This method makes use of dynamic computer simulation technique rather than complex calculation to obtain the optimal solution. In this paper, the MCMC method based on Bayesian reliability is proposed to calculate the Normal distribution quantile, and the parameters of the Normal distribution are estimated. In order to demonstrate the application of the proposed method, the proposed method is applied to study the statistical characteristics of pile foundation bearing capacity.

2. Proposed Method

2.1. Normal Distribution and Its Quantile. The probability density function (pdf) and cumulative distribution function (cdf) of the Normal distribution [24] are given by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad (1)$$

$$F(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] dt, \quad (2)$$

where μ is the location parameter and σ is the scale parameter.

From equation (2), by taking the inverse function of the cumulative distribution function of standard Normal distribution and solving x , we obtain the expression for the quantile x_α under the desired percentage α :

$$x_\alpha = \mu + \Phi^{-1}(\alpha)\sigma, \quad (3)$$

where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative standard Normal probability distribution function.

2.2. Jeffrey's Noninformative Prior. In the Bayesian method, the location parameter and the scale parameter in the Normal distribution are regarded as random variables. The existing information was used for parameter estimation, and the joint probability prior distribution needs to be determined firstly. For the bearing capacity of the pile foundation, the prior distribution information of the location parameter and the scale parameter in the Normal distribution cannot be obtained based on the existing experience. Therefore, it is suitable to adopt the nonprior distribution information.

In the Bayesian approach, we regard μ and σ behaving as random variables with a joint pdf $\pi(\mu, \sigma)$. We shall investigate the estimation of x_α for Jeffrey's noninformative prior [27, 28].

- (1) The logarithmic form of the maximum likelihood function is given by

$$L(\mu, \sigma) = \prod_{i=1}^n f(x_i) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right], \quad (4)$$

$$\ln L(\mu, \sigma) = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu)^2.$$

- (2) The Fisher information matrix is

$$\mathbf{I}(\mu, \sigma) = E\left(-\frac{\partial^2 \ln L}{\partial \mu \partial \sigma}\right). \quad (5)$$

- (3) The Jeffrey's noninformative prior of (μ, σ) is given by

$$\pi(\mu, \sigma) = [\det \mathbf{I}(\mu, \sigma)]^{1/2}. \quad (6)$$

For Normal distribution, the Fisher information matrix is

$$\mathbf{I}(\mu, \sigma) = -E \begin{bmatrix} \frac{\partial^2 \ln L}{\partial \mu^2} & \frac{\partial^2 \ln L}{\partial \mu \partial \sigma} \\ \frac{\partial^2 \ln L}{\partial \mu \partial \sigma} & \frac{\partial^2 \ln L}{\partial \sigma^2} \end{bmatrix}. \quad (7)$$

As obtained from Jeffreys, the joint prior distribution of the location parameter and the scale parameter is as follows:

$$\pi(\mu, \sigma) = \frac{1}{\sigma}. \quad (8)$$

2.3. MCMC Method. After joint prior distribution of the location parameter and the scale parameter is obtained, the posterior distribution of equation (3) can be calculated, and then the mean value of the posterior distribution is utilized to characterize the estimated value. The posteriori distribution is a complex, high-dimensional, and nonanalytical solution, so it is difficult to be clearly expressed. Therefore, the MCMC method [29] using dynamic computer simulation technique rather than complex calculation is adopted to obtain the optimal solution.

The basic principle of the MCMC algorithm is to transform a complex sampling problem into a series of simple sampling problems rather than extracting samples directly from complex posterior distributions. A Markov chain is constructed by sampling. The limiting distribution of the Markov chain is the actual distribution $f(\theta)$ of a

parameter θ . With the convergent Markov chain, parameters of the quantile model are then estimated:

$$\hat{E}(g(\theta) | x) = \frac{1}{n-m} \sum_{i=m+1}^n g(\theta_i). \quad (9)$$

The Metropolis-Hastings algorithm is intuitive and widely used in the MCMC method. The basic principle is as follows: a simple distribution with a posterior distribution approximation denoted as $q(\theta; \theta^{(i-1)})$, which is called the candidate generation density, is expected. Then, an initial value is selected. At the start of the i -th time iteration, the value of the parameter is set to $\theta^{(i-1)}$ based on which the i -th time iteration operates as follows:

Step 1: extract a sample θ^* from the candidate density $q(\theta; \theta^{(i-1)})$

Step 2: calculate the accepted probability $p(\theta; \theta^{(i-1)}, \theta^*) = \min\{((p(\theta = \theta^* | x)q(\theta = \theta^{(i-1)}; \theta^*)) / (p(\theta = \theta^{(i-1)} | x)q(\theta = \theta^*; \theta^{(i-1)}))), 1\}$

Step 3: accept $\theta(i) = \theta^*$ in the condition $p(\theta; \theta^{(i-1)}, \theta^*)$, and accept $\theta(i) = \theta^{(i-1)}$ in the condition $1 - p(\theta; \theta^{(i-1)}, \theta^*)$

Step 4: repeat Step 1 to Step 3 n times, and obtain the posterior sample $\theta^{(1)}, \theta^{(2)}, \dots, \theta^{(n)}$

Step 5: obtain $\hat{E}(g(\theta) | x) = (1/(n-m)) \sum_{i=m+1}^n g(\theta_i)$

2.4. Parameter Estimation. For two different percentages α_1 and α_2 obtained from the Metropolis-Hastings algorithm of the MCMC method, the corresponding quantiles can be calculated by the equation sets as follows:

$$\begin{aligned} x_{\alpha_1} &= \mu + \Phi^{-1}(\alpha_1)\sigma, \\ x_{\alpha_2} &= \mu + \Phi^{-1}(\alpha_2)\sigma. \end{aligned} \quad (10)$$

The estimated value of the Normal distribution statistical parameter can be obtained by solving equation (10):

$$\begin{aligned} \mu &= \frac{x_{\alpha_1} \cdot \Phi^{-1}(\alpha_2) - x_{\alpha_2} \cdot \Phi^{-1}(\alpha_1)}{\Phi^{-1}(\alpha_2) - \Phi^{-1}(\alpha_1)}, \\ \sigma &= \frac{x_{\alpha_2} - x_{\alpha_1}}{\Phi^{-1}(\alpha_2) - \Phi^{-1}(\alpha_1)}. \end{aligned} \quad (11)$$

In this paper, the optional normal distribution quantiles are selected as 0.95 and 0.99, respectively, to obtain the statistical characteristics of the bearing capacity of the pile foundation, according to the engineering practice based on the statistical characteristic requirement. The mean value and variance of the Normal distribution of the bearing capacity of pile foundation can be obtained, and the coefficient of variation can be further calculated, which will be applied in reliability analysis.

3. Simulation

A simulation study was performed to compare the performance of the method proposed in this paper and the maximum likelihood method. 2000 random samples of sizes

$n = 10, 20, 30, 50, 100, 200, 300, 500$, and 1000 were generated from the Normal distribution. Since any Normal distribution data can be standardized to have a location parameter of 0 and scale parameter of 1, only samples with parameters $\mu = 0$ and $\sigma = 1$ were generated. The accuracy of the estimates is compared by using the following performance measures: root mean square error (RMSE) [30], Kolmogorov-Smirnov test (KS) [31], and coefficient of determination (R^2) [32]. The results of simulation study are presented in Table 1.

The following conclusions can be drawn from Table 1, compared to the maximum likelihood method, the estimates of μ and δ using the Bayesian quantile method are closer to the true value. Meanwhile, smaller RMSE and KS values as well as a larger R^2 value can be obtained by the Bayesian quantile method. The results indicate that the performance of the Bayesian quantile method is better than the Maximum Likelihood method in terms of RMSE, KS, and R^2 values for the estimation of Normal distribution, because the Bayesian quantile method considers the randomness and correlation of the location parameter and the scale parameter. In addition, an increase in the sample size of the simulated Normal distribution data generally results in the improvement of the different methods, and the difference between the maximum likelihood method and the Bayesian Quantile method can also be reduced with the increasing sample size.

4. Application

In this section, the parameter estimation methods in the previous sections are applied to real data-bearing capacity of the pile foundation. The data can be divided into bored pile and driving pile, including 107 bored piles and 151 driving piles.

Generally speaking, the bearing capacity of pile foundation is affected by many factors, such as different pile diameters, pile lengths, areas, soil layers, and construction methods. It is difficult to obtain a sufficient number of samples that can be considered to have the same preconditions for statistics. Therefore, the standard value of the bearing capacity calculated by the empirical formula of the bridge code is utilized in this study. The bearing capacity of the test pile is then normalized, and the test ratio of the dimensionless random variable is introduced by the measured value/standard value. The specific values of the test basis of each pile in this case can be referred to the literature [24].

The test ratio of the bored piles and driving piles bearing capacity is obtained by the Bayesian method proposed in this paper, which is compared to the results of the maximum likelihood method as listed in Tables 2 and 3, respectively. It can be summarized that, compared to the maximum likelihood method, the estimates of μ and δ using the Bayesian quantile method are better. Meanwhile, smaller RMSE and KS values as well as a larger R^2 value can be obtained by the Bayesian quantile method. The results indicate that the performance of the Bayesian quantile method is better than the maximum likelihood method in terms of RMSE, KS, and R^2 values for the estimate of Normal distribution. In conclusion, the Bayesian method performs better for the

TABLE 1: Comparison of the estimation methods.

n	Parameter	Maximum likelihood method		Bayesian quantile method	
		μ	σ	μ	σ
10	Mean	0.1222	1.1567	0.0941	1.0811
	RMSE	0.2653	0.3577	0.1981	0.2713
	KS		0.3533		0.2457
	R^2		0.8329		0.8897
20	Mean	0.0757	1.1023	0.0511	1.0588
	RMSE	0.1835	0.2453	0.1448	0.1857
	KS		0.2635		0.1977
	R^2		0.9013		0.9233
30	Mean	0.0533	1.0757	0.0478	1.0421
	RMSE	0.1532	0.2134	0.1135	0.1577
	KS		0.1526		0.0997
	R^2		0.9552		0.9753
50	Mean	0.0437	1.0532	0.0392	1.0387
	RMSE	0.1152	0.1562	0.0826	0.1281
	KS		0.1299		0.0873
	R^2		0.9632		0.9873
100	Mean	0.0365	1.0484	0.0278	1.0326
	RMSE	0.0761	0.1392	0.0518	0.0985
	KS		0.0831		0.0653
	R^2		0.9767		0.9915
200	Mean	0.0265	1.0361	0.0212	1.0148
	RMSE	0.0571	0.0765	0.0471	0.0512
	KS		0.0674		0.0521
	R^2		0.9811		0.9945
300	Mean	0.0185	1.0299	0.0132	1.0102
	RMSE	0.0366	0.0571	0.0276	0.0327
	KS		0.0524		0.0499
	R^2		0.9911		0.9976
500	Mean	0.0055	1.0053	0.0035	1.0021
	RMSE	0.0231	0.0372	0.0183	0.0161
	KS		0.0313		0.0272
	R^2		0.9952		0.9998
1000	Mean	0.0015	1.0010	0.0004	1.0002
	RMSE	0.0131	0.0212	0.0071	0.0009
	KS		0.0113		0.0065
	R^2		0.9987		0.9996

TABLE 2: Parameter estimates, RMSE, KS, and R^2 for the drilled pile foundation.

Method	Parameter		RMSE	KS	R^2
	μ	σ			
Maximum likelihood method	1.1042	0.1952	0.0677	0.0462	0.8826
Bayesian quantile method	1.1322	0.1897	0.0329	0.0165	0.9283

TABLE 3: Parameter estimates, RMSE, KS, and R^2 for the driven pile foundation.

Method	Parameter		RMSE	KS	R^2
	μ	σ			
Maximum likelihood method	1.1645	0.1985	0.1122	0.1175	0.9218
Bayesian quantile method	1.1586	0.2017	0.0728	0.0816	0.9577

parameter estimation of the Normal distribution of pile foundation bearing capacity.

According to different parameter estimation methods, the cumulative distribution function curves for the drilled and driven pile foundations are shown in Figures 1 and 2,

respectively. It can be seen that the Bayesian method provides better fit because its curve is closer to the empirical cdf curve. Therefore, the method based on the Bayesian reliability method is recommended for the research of pile foundation bearing capacity.

FIGURE 1: The cumulative distribution function curves according to different parameter estimation methods for the drilled pile foundation (note: the horizontal ordinate is the random variable, and the longitudinal coordinate is the cumulative distribution function).

FIGURE 2: The cumulative distribution function curves according to different parameter estimation methods for the driven pile foundation (note: the horizontal ordinate is the random variable, and the longitudinal coordinate is the cumulative distribution function).

Based on the statistics of the bearing capacity of pile foundation, the safety factor of pile foundation is calculated based on the probability method. The limit state function of bearing capacity of pile foundation is as follows:

$$Z = P - K(G + Q), \quad (12)$$

where P is the bearing capacity of pile foundation, G is the dead load, Q is the live load, and K is the safety factor.

The statistics of random variables in equation (12) is shown in Table 4. The safety factor of pile foundation can be calculated using the inverse reliability method based on the target reliability index of 3.5. The results of the safety factor of pile foundation are shown in Table 5 in the condition of different coefficients of variation of random variables of bearing capacity of pile foundation. Comparison of the safety factor was conducted in view of different coefficients of variation of random variables, probabilistic method, and deterministic method. It can be indicated that probability statistics of bearing capacity of pile foundation have a major effect on the safety of pile foundation. Ignoring the probability statistics of the bearing capacity of pile foundation could overestimate the degree of safety for the bearing capacity of pile foundation. In the case study, using the Bayesian reliability method to estimate the statistics of bearing capacity of pile foundation was recommended though there were not enough samples in actual engineering.

5. Conclusion

The forecast method of bearing capacity of pile foundation was proposed based on the Bayesian reliability theory. Numerical simulation was used to produce pseudosamples. The Bayesian statistical theory and maximum likelihood parameter estimation method were used to estimate the parameters of the Normal distribution, and the performance of two methods was compared. The following conclusions can be drawn.

The results of the numerical simulation study indicate that the Bayesian forecast model of Normal distribution is better than the maximum likelihood estimation, and the performance could be improved with the increasing of pseudosample number. The difference between the maximum likelihood method and the Bayesian method is very

TABLE 4: Statistics of random variables.

Random variable	Distribution type	Mean value μ	Standard deviation σ	Coefficient of variation ($\delta = \sigma/\mu$)
P	Normal	1876 kN	1407 kN	0.15
G	Normal	831 kN	16.62 kN	0.02
Q	Extreme I	310 kN	62 kN	0.2

TABLE 5: Safety factors of bearing capacity of pile foundation.

Safety factor	Coefficient of variation ($\delta = \sigma/\mu$)			Deterministic method
	0.10	0.15	0.20	
K	1.57	1.41	1.36	1.8

little for large sample sizes, but it is slightly more for smaller sample sizes.

The application to the real data set of pile foundation bearing capacity indicates that the estimation accuracy of the Bayesian method is better than that of the Maximum Likelihood method. Therefore, the Bayesian method is more appropriate for estimating the parameters of Normal distribution of pile foundation bearing capacity.

The Bayesian method proposed in this paper is recommended in order to improve the estimation accuracy of pile foundation bearing capacity reliability analysis in practical engineering.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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