

## Review Article

# Outrigger and Belt-Truss System Design for High-Rise Buildings: A Comprehensive Review Part II—Guideline for Optimum Topology and Size Design

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This article is the second part of the series of the comprehensive review which is related to the outrigger and belt-truss system design for tall buildings. In this part, by presenting and analyzing as much relevant excellent resources as possible, a guideline for optimum topology and size design of the outrigger system is provided. This guideline will give an explanation and description for the used theories, assumptions, concepts, and methods in the reviewed articles for optimum topology and size design. Finally, this part ended up with a summary for the findings of the reviewed studies, which is useful to understand how different parameters influence the optimum topology and size design of a tall building with outrigger and belt-truss system.

## 1. Introduction

Building sector is blamed for being responsible for a significant amount of environmental emissions and resource depletion [1]. Annually, building sector uses around 40% of the materials entering the global economy [2] and emits around 33% of the global greenhouse gases [3]. Furthermore, in the building construction process, the manufacturing of building material contributes about 90% of the embodied energy and emission, while the construction and transportation contribute about 6% and 4%, respectively. [1] (see Figure 1). However, this negative impact of building construction on the environment becomes crucial in the situation of tall building construction due to the complexity, implementation difficulties, and the massive energy consumption of tall buildings compared with shorter buildings as a premium for height [4–6]. For the aforementioned issues and as a result of limited resources, financial uncertainty, energy shortage, urban sprawl, global warming, air pollution, and water shortage [5], optimum sustainable design of tall building structures should be achieved. One of

the main ways to obtain sustainable design for tall building structures is using structural optimization techniques (shape, topology, and size optimization), i.e., by finding the best material distributions (optimum topology) and amounts (optimum size) within a physical volume domain, design criteria such as safety, durability, serviceability, economic effectiveness (minimum necessary material), structural integrity, and functionality can be satisfied [7–9]. Also, the embodied energy during construction and operation phases will be minimized.

One of the most efficient structural systems for tall buildings is the outrigger and belt-truss system, which is a hybrid lateral load-resisting system, where in this system, the exterior structural system (e.g., frame tube and mega columns) and the interior structural system (e.g., steel-braced core and concrete core) [10, 11] are combined by rigid horizontal beams (shear links) called outriggers at specific levels and the perimeter columns of the exterior structural system are tied together by another rigid horizontal beam called belt truss at specific levels [9, 12, 13]. Many configurations of the outrigger system can be made by changing of exterior and interior structural system types and

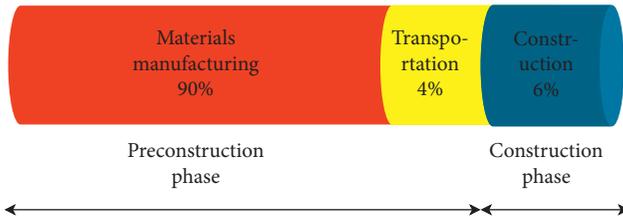


FIGURE 1: The embodied energy and emission by building phase.

structural system material, in addition to existence or absence of one of the system components. Therefore, from the definition of the system and due to its multiple possible configurations, the optimum topology and size design of the outrigger and belt-truss system is essential in order to obtain an efficient and successful design.

In this paper, the optimum topology and size design of the outrigger and belt-truss system is discussed. Definition of the topology and size terms of the outrigger system is given in Section 2. Section 3 clarifies the procedures of the optimization process. Sections 4–7 explain these procedures in detail. Summary of the findings of the reviewed articles in addition to the conclusion is presented in sections 8 and 9, respectively.

## 2. What are the Topology and Size of Outrigger and Belt-Truss System?

**2.1. Topology.** In general, the topology in structures refers to the material distribution within the structural elements (in continuum structures) or refers to spatial order and connectivity of the bars (in discrete structures such as trusses). However, the topology of the outrigger system, which is termed in this review, indicates to

- (a) The topology of outrigger system in overall structure, which in its turn means (i) the locations and number of outriggers or/and belt-truss and (ii) the various combinations of the system components (outrigger system configurations).
- (b) The topology of components themselves: (i) outrigger topology (truss shape, wall, etc.); (ii) belt-truss topology (truss shape, wall, etc.); (iii) interior structural system topology (core topology, e.g., steel frame-braced core or shear-wall core, etc.); and (iv) exterior structural system topology (e.g., moment-resisting frame, mega columns, etc.).

**2.2. Size.** The outrigger and belt-truss system size indicates the material amount of the specific material distributions (topology) of this system, i.e., cross sections of outrigger system components.

Topologies and sections of outrigger system components are mutually interacting, which makes the optimum design of this system a complicated indeterminate process.

In this paper, based on the aforementioned definitions of the topology and size of the outrigger and belt-truss system, many studies and articles were reviewed. Therefore, it should

be noted that some of the reviewed articles did not adopt these definitions, and they studied topics such as outrigger locations, outrigger numbers, and other issues, without clearly stating that the considered issue is a topology or size problem.

## 3. Procedures of Optimum Topology and Size Design for Outrigger and Belt-Truss System

Generally, in the reviewed articles, the adopted procedures of optimum topology and size design of the outrigger and belt-truss system were as follows: firstly, identifying the variables which should be optimized; secondly, modeling the structure; thirdly, formulating optimization problem according to chosen criteria, where the objective function, constraints and boundaries should be defined; and finally, solving the problem by using an appropriate searching technique to obtain the optimum solution (see Figure 2). However, the detailed application of these procedures is different between the initial and final design stages. That is because each of these stages requires different details and level of accuracy. The differences are mainly reflected in the adopted assumptions in modeling techniques and in the selected searching technique. These procedures will be explained and clarified in the next sections.

## 4. Design Variables (Decision Variables)

As the title of this review indicates the optimum topology and size design, mainly two types of design variables should be taken into consideration: topological variables and sizing variables. In the optimization process, the optimum values of these variables should be found to achieve the objective functions with considering the constraints and boundaries.

**4.1. Topological Variables.** The final values of the topological variables are most probably determined in the preliminary design stage. The primary topological variables of the outrigger and belt-truss system are as follows.

**4.1.1. Outrigger Locations.** It was the primary concern in several reviewed research studies [6, 14–22]. Finding the optimum value of this topological variable is considered an inherently complex problem because of the multiple factors which control the value of this variable like the stiffness of outrigger system components, space availability, the number of outrigger sets, lateral loads distribution, outrigger system type, and others. However, space availability is the main factor which controls outrigger placement, where this factor usually makes the problem of finding the real optimum locations of outriggers a purely academic problem because the potential locations of outriggers are typically limited to refuge and mechanical floors, as a result of (i) space planning which should be suited with architectural, leasing, and mechanical criteria and (ii) the quasiuniform distribution of these mechanical and refuge floors every 12 to 15 stories along the building height and near the building top, making those natural locations for outriggers.

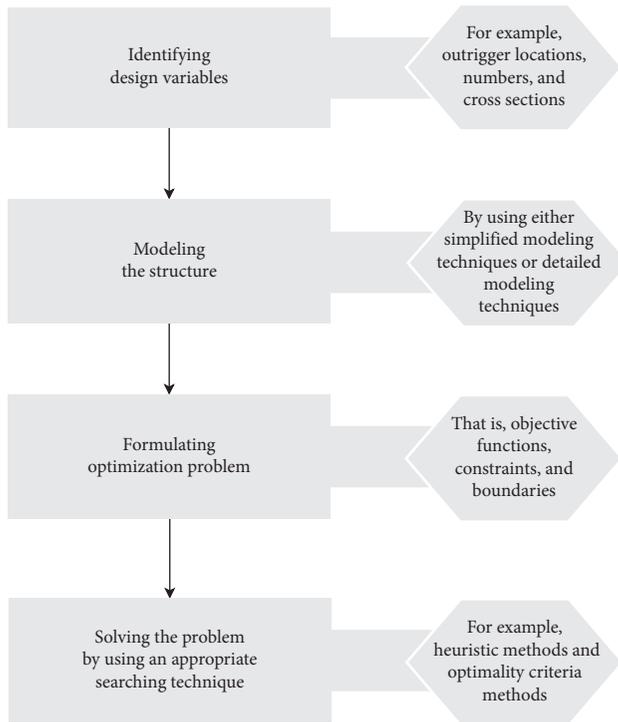


FIGURE 2: Procedures of optimum topology and size design for outrigger and belt-truss system.

Therefore, the optimization problem becomes to be determining the optimum locations of the outrigger system between those potential locations, e.g., the optimum location of outriggers in Suzhou Zhongnan center which is discussed in references [23–28]. However, acceptable performance can usually be achieved using these locations. In the situation that the mechanical levels are not appropriate for outrigger placement, such as in One Liberty Place in Philadelphia, the optimum locations of outriggers can be found considering that each outrigger should be of several stories high (super-diagonal run through occupied space) in order to guarantee that the obstructed width on any floor is less than one-quarter of the clear floor span. Tenants can conceal these braces behind partitions and cabinets with minimal influence on floor functionality. Another solution is using a virtual outrigger system. In all situations, any outrigger placement strategy affecting occupied spaces must consider its likely acceptability in the local leasing environment [29].

**4.1.2. Outrigger Numbers.** It is an important factor which determines the level of drift control and degree of stiffening, i.e., it drives the outrigger system effectiveness [6]. Outrigger number is also limited by space availability and the efficiency of the added outrigger, where the benefits of each added outrigger should be weighed against construction time and cost [22, 29]. To clarify that, more outrigger sets will lead to [29] (i) different optimum locations of outrigger sets; (ii) more opportunities for rotation restraint causing a reduction in drift; and (iii) more costs, time, effort for erection. In the situation that the total material quantity of outriggers is

unchanged: (i) more outrigger sets will lead to distribute the total material quantity across more outriggers resulting in more pieces for erection [29], while (ii) less outrigger sets will lead to involve fewer nontypical floors and minimize piece counts, but the members will be too heavy; therefore, high-capacity and more costly erection equipment is required. However, many studies referred that in most of the situations, except super tall buildings, using more than four outriggers levels is not efficient because the improved performance of the structure will be very small compared with construction time and cost [22]. A lot of the reviewed studies found the optimum value of this variable in different situations [15, 16, 20, 22, 30].

**4.1.3. Core Topology.** Core can be with different topologies, e.g., solid rigid walls, pure rigid frame, fully braced frame, partially and braced frame. This variable was investigated in several research studies [16, 31]. In these studies, different topologies of the core were evaluated in the optimization problem in order to minimize the total cost of the structure and optimize the performance.

**4.1.4. Outrigger and Belt-Truss Topologies.** Outrigger and belt truss can have different topologies from a fully solid wall or deep beam to any geometric shape of truss. These variables have a direct impact on the provided ductility or stiffness to the structure by the outrigger and belt-truss system. Several reviewed articles studied [9, 32–35] the optimum value of these variables by applying different methods.

**4.1.5. Other Topological Variables.** The material type (steel, concrete, or composite) and even the grade of strength for the same material (steel or concrete strength grade), which are used to design and construct the outrigger system, can also be considered as topological variables because each material has different behavior leading to different response of the structure and thus different optimum design. The article of Chen et al. [19] is an example of the studies that took this type of topological variables into consideration.

**4.2. Sizing Variables.** Sizing variables are critical variables to achieve the optimum design and obtain a reasonable structural scheme. This is because they have a very high impact on the performance and cost of the structure. Also, these variables should have practical values which make the construction process possible and easier. Usually, sizing variables take their approximate and initial values during the preliminary design stage, whereas they take their final values during the final design stage. These variables are represented by the section properties of outrigger and belt-truss system components, where the size of each component is represented by the property which reflects the main role of that component, e.g., cross section areas for columns and second moment of inertia for core walls and outriggers. In order to make the optimization process faster and easier, the number of sizing variables can be reduced by assuming that the

prementioned section properties are uniform or varying linearly or another known form along the building height. Several reviewed articles optimized these variables in their case studies [6, 15, 16, 19, 20, 30, 36].

**4.3. Other Related Variables.** In the preliminary design stage of high-rise buildings, the ratio of the core wall area to the whole floor area is a controlling factor, and it may be considered as an initial design variable, as in reference [37]. Usually, this variable is represented in the design problem by the distance from the edge of the core wall to the perimeter columns, where in practical engineering, this distance reflects the possible ratio of core wall area to the floor area [37].

## 5. Modeling Techniques

High-rise buildings are very complicated that even the detailed computational models of these buildings are considerable simplifications, and the analysis results will always be approximate, which will have at best an equivalent quality to the quality of the model and analysis method. However, modeling methods can be mainly classified into simplified (approximate) modeling methods and complex (detailed) modeling methods [38].

**5.1. Detailed Modeling Techniques.** In the final design stage, a high-fidelity model of the structure is required in order to conduct a three-dimensional analysis for taking full advantage of the special interaction between different elements of the structure, which can give a realistic simulation of the actual behavior of the building. Therefore, discrete modeling approach, represented by the detailed finite element method (FEM), is the best choice for the detailed modeling of the structure. In this approach, by using appropriate software, the structure is modeled as a three-dimensional model including all structural elements (core, columns, outriggers, belt truss, slabs, secondary beams, etc.). However, it takes more time than the simplified modeling approaches [16, 17, 35, 38–41]. Several reviewed articles, such as [6, 7, 9, 12, 14–17, 30, 36, 38–61], adopted this approach either as a primary modeling approach in their studies or to check the validity of the proposed simplified modeling methods.

**5.2. Simplified Modeling Techniques.** Even though the detailed modeling techniques such as the finite element method (FEM) can provide a high-fidelity model, they cannot be a substitute for the simplified methods. This is because of the nature of the simplified methods which may offer a better understanding of the structural system behavior than FEM. Also, simplified modeling methods are fast and yield reasonably accurate design parameters if used correctly and a suitable set of assumptions is chosen. The multiple features of the simplified methods make the use of these methods vital during the initial design stage in order to determine the general design characteristics and initial parameters such as the dimensions of members in rational values and during the

final design stage in order to check the computer analysis results manually [12, 38, 40, 58, 59, 61–63].

For the outrigger and belt-truss system in tall structures, several simplified modeling techniques have been developed in the past decades [38], as follows.

**5.2.1. Continuum Approach.** In the literature, it has also been termed “shear connection method,” “continuous medium method,” or “continuous connection method” [22]. In this approach, the primary noncontinuous (discrete) structure, by using a set of assumptions, is modeled as an elastically equivalent continuous structure such as beam model (one-dimensional analytical model based on beam theory) [38, 53]. Usually, the continuum approach can be applied to model the low and medium-rise buildings up to 20 stories [64–69]. However, several reviewed articles used this approach to model the outrigger and belt-truss system in tall buildings [12, 62, 70]. In these articles, based on the studied configuration of the outrigger and belt-truss system and the purpose of the study, the concept of the continuum approach is applied as follows:

- (i) In a multi-outrigger-braced structure or pair of coupled shear walls that is stiffened by a heavy beam and outrigger: the structure is simplified by assuming that all horizontal connecting elements (the set of outriggers and the coupling beams) are smeared along the building’s height in order to produce an equivalent continuous connecting medium between the vertical elements (equivalent uniform bracing system or continuous distribution of lamina with equivalent stiffness) [22, 50, 71, 72].
- (ii) A high-rise building with a combined system of single tube (framed tube), core, and outrigger and belt-truss system: in accordance with the classical beam theory and by employing the orthotropic box beam analogy method, the framed tube structure is modeled as a cantilevered hollow section beam of orthotropic plates (equivalent membranes with equivalent properties). This model consists of three main components [59, 60, 62]: (i) two web panels (parallel to the lateral loads) represent the web frames; (ii) two flange panels (perpendicular to the lateral loads) represent the flange frames; (iii) four columns at the four corners of the perimeter frame represent the corner columns (see Figure 3). These structural elements are interconnected to each other along the joints of the panels and connected to the floor slabs at each floor level [62]. The interaction between the outrigger system and shear core under lateral loads on framed tube system is modeled as rotating springs (bending spring with constant rotational stiffness) at outrigger-belt truss location along the height of the structure. Therefore, the effect of outrigger, belt truss, and shear core on the framed tube can be considered as a concentrated moment applied at belt truss and outrigger location. This moment acts in the

opposite direction of the rotation created by lateral loads [55, 56, 73–75].

- (iii) A high-rise building with a combined system of multiple tubes (such as tube-in-tube), core, and outrigger and belt-truss system: the structure is modeled in the form of two parallel cantilevered hollow beams which are constrained at the outrigger and belt truss locations by rotational springs. These beams are forced to keep equal lateral displacements, and the amount of lateral loads carried by each of them is a function of each beam relative stiffness [76] (see Figure 4).

In general, the tall buildings with the outrigger and belt-truss system can be idealized, using continuum approach, as a flexural and shear cantilevered beam with rotational springs positioned at the outrigger and belt truss location [36, 61, 77, 78].

**5.2.2. Discrete Approach.** In high-rise buildings, the behavior of the structural system (core, mega column, outrigger, and belt-truss) cannot be simulated with reasonable accuracy by continuum models. This is because the deflected shape of such a structural system is often characterized by curvature reversals, which cannot be predicted by only beam theory (used in continuum approach) because the diagram of bending moment of a cantilever beam under lateral loads in one direction can never ever change the sign [79]. Also, the analysis and design of the structural system (core, outrigger, and belt-truss) are not simple, because the distribution of forces between the outriggers and core depends on the relative stiffness of each element [29]. Therefore, the outrigger-braced structure is not strictly amenable to the continuum modeling and should be considered in its discrete arrangement [22]. In the reviewed articles, special discrete models for tall buildings with the outrigger and belt-truss system have been developed by several researchers [22, 36, 51, 80, 81], such as

- (i) Two-dimensional frame model: in this method, the model is mainly based on the plane frame element that simplifies and models the whole structure into shear-deformable beam elements or classical beam elements. Therefore, the core is considered as a vertical cantilever (Euler–Bernoulli beam), the outriggers can be represented by horizontal beams cantilevered from the core, and exterior columns are also considered as vertical beam elements with only axial stiffness [18, 22, 36–38, 63]. In the situation of damped outrigger system (linear viscous dampers), this method allows modeling the dampers between the end of outriggers and the perimeter columns [82, 83].
- (ii) Simplified finite element model: in this method, the structure is modeled, by appropriate software, as a two-dimensional model with only the primary

structural elements (core, columns, and outriggers) [6, 9, 15, 21, 44].

**5.2.3. Special Modeling Methods.** These methods can be used in conjunction with prementioned methods to study the behavior and response for the part of the outrigger and belt-truss system structure such as the inelastic behavior of the steel braces or connections. The special modeling methods which are used in the reviewed papers are

- (i) Inelastic modeling of steel braces: it can be categorized into the following broad categories [14]: (i) phenomenological models, which are based on simplified hysteretic rules that simulate the cyclic axial forces-deformations relationship of braces; (ii) continuum finite element models, which subdivide the brace into smaller elements to simulate the accurate brace behavior; (iii) physical theory models, where the brace hysteretic behavior is modeled with two elements connected by a generalized plastic hinge for simply pinned braces, implemented using distributed or concentrated inelasticity model.
- (ii) Fiber-based mechanical idealization, which is able to reproduce the contribution of brace members and connection systems, accounting for the all potential failure modes. By comparison with experimental tests, this idealization is proven to be a viable and promising approach when used for seismic response assessment and predicting the nonlinear dynamic behavior of systems such as the outrigger system and super-tall mega-frame structures [35, 47, 84–89].
- (iii) Simplified mechanical model of outriggers: in order to understand the behavior of outrigger components, the outrigger's mechanical behavior is decomposed into [35]: (i) rigid frame model, which mainly considers the nonlinear property of the bottom and top chords, and (ii) truss model, which simulates the nonlinear behavior of the diagonal member. By superposition of the two models, the overall behavior of the outrigger can be obtained. The decomposition is shown in Figure 5.

**5.3. Additional Assumptions Related to Modeling Techniques.** Modeling of the tall building with the outrigger and belt-truss system will not be complete by only using the aforementioned modeling techniques. Therefore, an additional set of assumptions should be considered in order to describe the boundary conditions of the model, the behavior of model components, and other considerations, which are essential in establishing the structural model. This set of assumptions directly influences the accuracy of analysis results. For that, in the detailed modeling techniques, these assumptions should simulate the reality as much as possible, e.g., representing all changes in the cross sections along the building height, modeling wind loads by using wind tunnel records and seismic loads by using time-history records, and

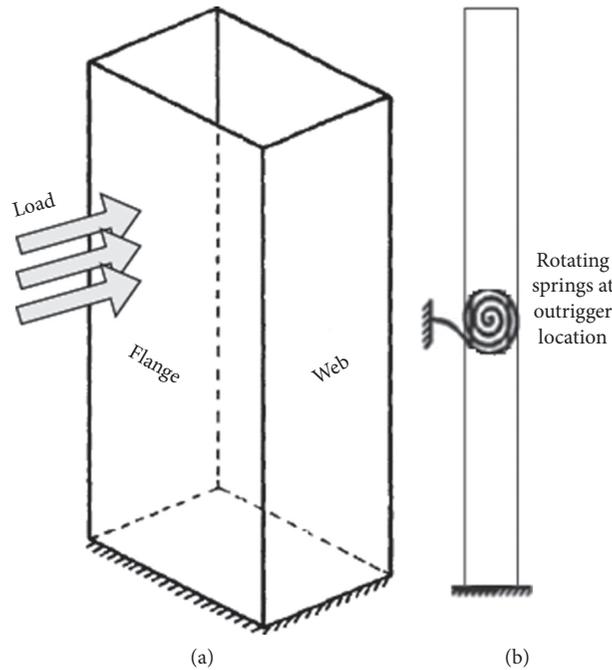


FIGURE 3: (a) The equivalent hollow section beam. (b) The equivalent impact of the core and outrigger on the hollow section beam.

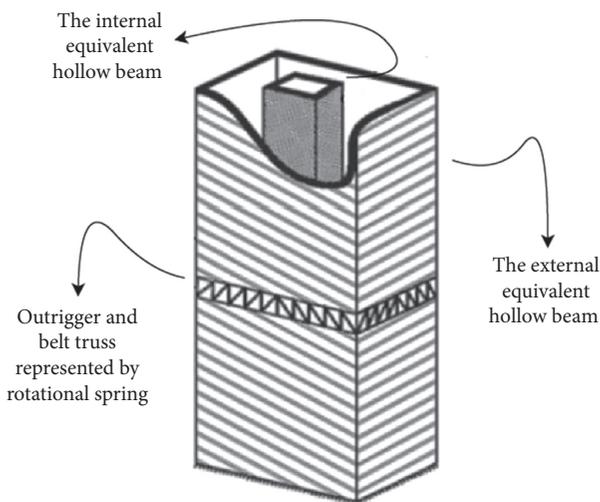


FIGURE 4: The equivalent model of combined system of multiple tubes, core, and outrigger and belt truss [60].

modeling the boundary conditions and the connectivity of the structural elements in precise manner. In the simplified modeling techniques, only the primary dominant parameters on the structural response are considered in the model, while the secondary parameters are ignored. Hence, the adopted assumptions for simplified modeling of the outrigger and belt-truss system in the reviewed articles can be classified as follows.

### 5.3.1. Load Modeling Assumptions

- (i) Gravity loads (dead loads, live loads, etc.): in general, at the preliminary design stage, since the

studied issue is the optimum topology design of outrigger system, the gravity loads are not considered in the most of the simplified analysis methods. However, in the final design stage, gravity loads must be considered in detail.

- (ii) Lateral loads (wind and seismic loads): in contrast to vertical loading (gravity loads), wind and earthquake load effects on a building increase exponentially with building height [53]. For preliminary sizing, the predominant action among wind and earthquake should be assessed [42]. Calculation of the seismic and wind loads according to the available standards yields different lateral loads distributions. For example, ACSE 7-10 [90] offers three different procedures to design wind on cladding and components: (i) simplified procedure; (ii) analytical method; (iii) wind tunnel procedure. However, seismic and wind loads can be represented by approximate, well-known, easy load distribution over the height of the building [19, 37, 81]. Wind loads distribute in trapezoidal form, uniform form, or sometimes parabolic form; in the trapezoidal distribution form, such forces can be considered as a combination of upper (inverted) triangularly distributed loads and uniformly distributed loads over the building height [19, 20, 36, 51, 80, 91], while the earthquake loads acting on a high-rise building often distribute in triangular form and concentrate at the top of the building (equivalent static load) [17, 19, 36, 51, 91].
- (iii) Special lateral loads (blast loads): blast is a pressure disturbance arising from a sudden release of energy, due to terrorist or accidental explosions [61, 92]. Considering this type of load in tall building design

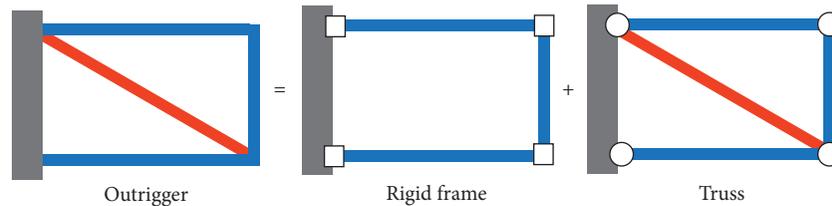


FIGURE 5: Simplified mathematical model of outriggers [35].

is important to check the safety of the building against progressive collapse. Several reviewed papers studied this phenomenon, such as [39, 61, 93]. However, there are high uncertainties of blast load calculations [39]. The two main factors impacting the magnitude of blast loads on the building are (i) the standoff distance from the structure to the blast source and (ii) the bomb size or charge weight [61] (see Figure 6). In order to present a blast wave relationship for high explosive, the Hopkinson–Cranz method (cube root scaling method) can be used [92], while for predicting blast loads, special expressions can be used, such as the equation introduced by Brode [94], the relationship introduced by Newmark and Hansen [95], or expression introduced by Mills [39, 96]. Also, another method to determine the blast loads is called TM5-1300, which is developed in 1990 by the US Department of Defense, which was used by several studies such as Kulkarni and Sambireddy [97]. The mode of blast load distribution along the building height depends on the size of the studied structure. In the situation of tall buildings, the blast wave magnitude varies significantly across the structure's surface if the blast source is near the building due to the unequal scaled distance of different heights on the façade [98]. If the blast source is far from the building, the distribution of blast load is approximately uniform [39]. For detailed modeling of blast loads, time-history of blast pressure wave for each story along the building's height should be developed by an empirical and theoretical based approach, such as Friedlander's equation, which is usually described as an exponential function [39, 99].

**5.3.2. Boundary Condition Assumptions.** In the reviewed articles, the connectivity of outrigger and belt-truss components was assumed as follows:

- (i) Outrigger to core connection: it can be assumed as a rigid connection, as adopted in [18–20, 22, 36–38, 41, 51, 52, 61, 71] studies, or a pinned connection, as adopted in references [9, 74, 75].
- (ii) Outrigger to exterior column connection: it can be assumed as a hinged (pinned) connection in order to ensure that exterior columns carry only axial forces. This assumption was adopted in references [9, 17, 71].

- (iii) Outrigger to floor structure (slab) connection: the outriggers are connected to the structural floors only at the locations of exterior columns and shear walls to allow the flexural deformations to take place in the outriggers, i.e., the slab is not part of the outrigger structure. This assumption will lead to identical rotations in the exterior columns and shear wall at all floor levels. This assumption was adopted in most of the reviewed references.
- (iv) Core to base connection: usually, it is assumed to be a rigid connection, as indicated by most of the reviewed studies.
- (v) Exterior columns to base connection: it can be assumed as a rigid connection, as adopted in references [12, 40, 53–56, 59, 60], or a pinned connection, as adopted in references [6, 7, 18, 19, 22, 36, 38, 49, 51, 52, 61].
- (vi) The horizontal members except the outrigger (floor beams or girders) connections: for simplicity, they are assumed to be connected through hinged connections at both ends, i.e., pinned to the core and columns to preclude the frame action, so that the entire horizontal loads are resisted by (i) core bending; (ii) the bending and axial deformation of the exterior columns acting as cantilevers. This assumption was adopted in references [51, 58, 81, 100]. However, in order to enhance the structure resistance to lateral drift, the external spandrel beams are connected with moment connections at mega columns.
- (vii) Connections within the outrigger and belt-truss members: usually, they are assumed to be rigid due to the high stiffness of the outrigger and belt truss. However, references such as [39, 55, 59] assumed the members' connections of outriggers to be rigid and members' connections of belt truss to be pinned.

In most of the reviewed research studies, the foundations were assumed to be fixed foundations. However, nonfixed foundations were also considered in some studies such as Hoenderkamp study [52]. In this study, the foundations under exterior columns such as piled foundations, which act in a vertical direction only, were modeled as linear springs with a translational linear stiffness. While the foundation of the core is only subjected to a bending moment under lateral loads, i.e., the net axial loads on the core foundation due to lateral loads are zero if

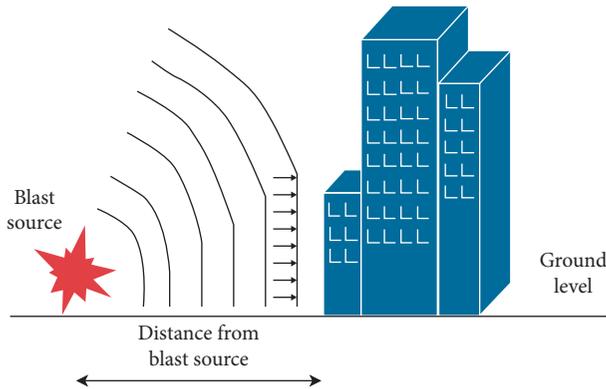


FIGURE 6: Blast load on buildings [61].

the core positioned at the center of a symmetric structure. So, core foundation can be modeled as rotational spring with a rotational stiffness [52].

**5.3.3. Assumptions for Modeling the Behavior of Outrigger and Belt-Truss System Components.** In the analysis of the outrigger system, the reviewed studies used various assumptions for the behavior of outrigger and belt-truss system components (the considered deformations in the analysis). These assumptions classified and summarized as the follows.

- (i) Interior structural system (core): in the situation of the concrete core, this core is assumed to be rigid against shear and flexible for bending. This means if the core is with fixed foundation, only bending deformation considered in the core's rotation and shear deformation is neglected, i.e., in bending, the plane sections remain plane so that the rotations at the face of the shear wall are identical to those at the wall center line. Thus, the horizontal deflection of the concrete wall could be represented by a single flexural (bending) stiffness parameter, as adopted in references [22, 37, 51], while if the core is with nonfixed foundation, additional rotation should be considered in the total core rotation due to core's foundation rotation, as adopted in reference [52].

In the situation of steel braced-frame core, this core in contrast to concrete core is assumed to be flexible for bending and shear, i.e., the assumption "plane sections remain plane" is not appropriate for braced frames. Thus, the simple wide column behavior applied to concrete shear walls cannot be adopted for trusses. This means bending deformation, as a result of axial strain in the columns and shear-racking deformation due to strain in the diagonal members, is considered in the steel braced-frame core's rotation. So, the deflected shape of a truss could be represented by bending and racking shear stiffness parameters. This assumption was adopted in references [36, 38, 57, 58].

- (ii) Exterior structural system: in the situation of exterior mega columns, these mega columns are assumed to be rigid against flexure. This means if the mega columns are pin-connected to fixed foundations, only axial deformation is considered. Thus, columns could be represented by a single parameter which represents the axial stiffnesses of the columns. This assumption was adopted in references [22, 36–38, 41, 51], while if the mega columns are pin-connected to nonfixed foundations (piles), the total column's axial displacement consists of column's axial displacement because of axial deformation of the column, in addition to vertical displacements in the column's foundation, as adopted in reference [52].

In the situation of an exterior framed tube, this frame is assumed to be flexible to bending and shear. This means the shear and bending deformations of the frame members are taken into account. This assumption was adopted in references [12, 53, 54, 57, 58, 60, 62].

- (iii) Outriggers: in the situation of concrete wall or truss outriggers, the behavior of the outrigger can be assumed rigid against shear and bending, which means there is no deformation of outrigger considered in the outrigger's rotation. Also, the behavior can be assumed rigid against shear and flexible to bending, which means bending deformation of outrigger is considered in the outrigger's rotation. Thus, the resulting single-curvature behavior is represented by a single flexural (bending) stiffness parameter. These assumptions were adopted in references [22, 41].

In the situation of steel truss, the riggers are attached to the exterior columns and core only, allowing double curvature in the outrigger to take place. This forced double curvature will increase the flexural stiffness of outrigger because in truss structures, the bending moment, arising from the axial forces in the truss cords, cannot vary linearly over the rigger's length. This phenomenon has stiffening impacts on the truss. However, its behavior is assumed to be flexible to bending and shear. This means bending and shear-racking deformations are considered in the outrigger's rotation. Therefore, the resulting double-curvature behavior of outriggers could be represented by bending stiffness parameter and racking shear stiffness parameter of the riggers (the rigger's racking shear stiffness is obtained by the summation of the racking shear stiffnesses of all bracing segments of the rigger). This assumption was adopted in references [36–38, 51, 52, 57, 58, 91, 101].

- (iv) Belt truss: belt trusses are usually assumed to be infinitely rigid in order to ensure full participation of all exterior columns in resisting the axial forces

arising from the overturning moment, as assumed in references [18, 22, 41, 61, 74, 75, 100].

- (v) Floor structures or diaphragm: in the preliminary design stage, rigid diaphragm (in-plane rigid or axially rigid floor structures) is somehow considered an acceptable assumption, because the floor structures (slabs) subject to insignificant deformations in their planes. Besides, the high in-plane stiffness of the floor slabs will restrict any tendency of the slabs to deform out of plane, which prevents any motion perpendicular to their planes. This means the floor structures will cause identical rotations in the core (steel braced-frame or shear walls) at the wall-truss interfaces and façade columns (or framed tube) at outrigger levels. This assumption was adopted in references [18, 36, 41, 51, 57, 60, 61, 74, 75]. In the situation of detailed models of the outrigger and belt-truss system, it is recommended that the diaphragms' in-plane deformations adjacent to the outrigger should be considered when determining the displacements and inner forces of the structure [30], i.e., floor structures are assumed to be semirigid diaphragms.

**5.3.4. Section Property Assumptions.** In simplified modeling techniques, the sections of the horizontal structural elements such as outriggers and belt trusses can be assumed constant for all outrigger and belt truss levels along the building height [57, 58]. On the other hand, for the vertical structural elements, e.g., columns and core, the section property assumptions can be assumed as the follows.

- (i) Uniform (constant-section model): in this assumption, the section properties are not changed along the height of the building, where only the properties of the lowest region of the actual structure are taken into consideration for uniform structure modeling. As a result, the complexity of the model and preliminary design problem will be significantly reduced. However, the accuracy of the obtained results will be sufficient for the preliminary design stage because the criteria of concern for initial design (e.g., the moment at the core base, axial forces in the columns, and drift at the top) are predominantly affected by the section properties of the lowest region of the structure. This assumption was adopted in references [22, 45, 57, 58, 60].
- (ii) Nonuniform: considering nonuniform properties will lead to more accurate modeling of the structure (e.g., it might provide a realistic mass and stiffness distribution for the investigation of first natural frequency) because in practical structures, there is a reduction up the height in the sectional area of the columns and the inertia of the core. This assumption was adopted in references [17, 45, 55, 81]. Usually, the core's flexural rigidity and the perimeter columns' axial rigidity (also the flexural rigidity of the

perimeter columns when it is not negligible in comparison with that of the core) are subject to similar variations along the structure's height. Thus, the sections properties change along the height of the building in one of the following methods: (i) step-wise variation in stiffness, in which the changes in the stiffness occur at outrigger location, so the core's flexural rigidity, and the perimeter columns' axial rigidity are constant in the region between two adjacent outrigger levels.; (ii) linear and square variation in stiffness, in which the core's flexural rigidity and perimeter columns' axial rigidity vary linearly (or another way of variation) with respect to height; (iii) constant-stress method, in which the sections of wall and columns adjusted per the applied axial force along the building height, so equal axial stresses can be developed in these members.

Although the constant-stress and constant-section models are not suitable for high-rise buildings, they represent an extreme case of the wall and column sectional profiles used in the high-rise building structures.

#### 5.3.5. Assumptions of Material and Geometrical Behavior

- (i) Material behavior: in most of the reviewed studies, the behavior of the structural material was assumed to be a linear elastic behavior, which means the structural material is homogeneous and obeys Hook's law [60, 74, 75]. While the nonlinear elastic behavior or plastic behavior (the stresses are not proportional to the strains [102]) is usually considered in the detailed study, e.g. the study of Brunesi et al. [47].
- (ii) Geometrical behavior: the majority of the reviewed articles assumed the relationship between the displacement and strain to be linear (geometric linearity). However, Lee et al. [58] and Brunesi et al. [47] in their studies took geometric nonlinearity into account, which means the relationship between the displacement and strain is nonlinear [102].

#### 5.3.6. Assumptions Related to the Shear Lag Phenomenon.

Usually, this phenomenon is taken into account in hollow box girders and in tubular buildings such as the combined system of a core, framed tube, and outrigger system [12]. Therefore, if the structural system is a combined system of core, mega columns, and outriggers, this phenomenon will not be considered.

The factors behind the shear lag phenomenon are the flexural and shear flexibilities of the frame members against lateral loads, which mean that the axial stresses in the frame columns due to overturning moment cannot be determined by "plane sections remain plane" assumption as in rigid frame members [62]. These flexibilities, under lateral loads, lead to a nonlinear distribution of axial stresses and strains in flange and web frames of the framed

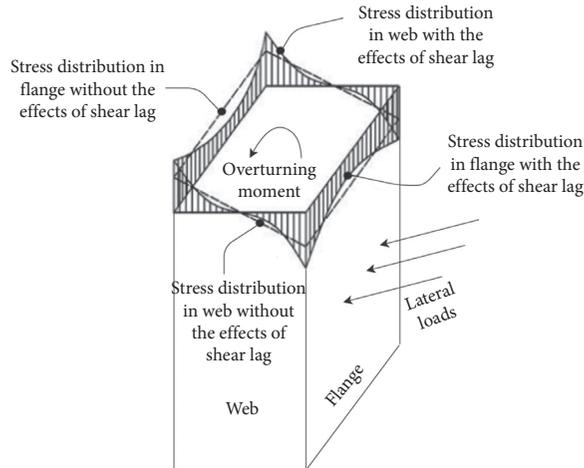


FIGURE 7: Stress distributions in flange and web frames of the framed tube [53].

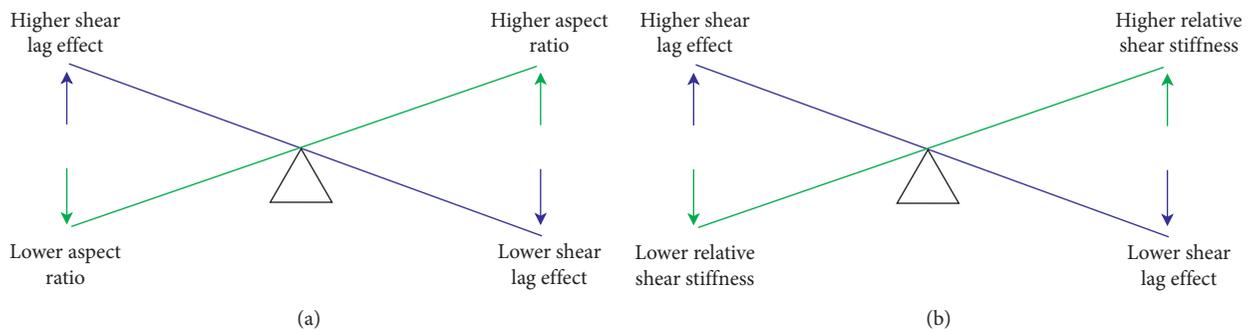


FIGURE 8: (a) Shear lag against aspect ratio. (b) Shear lag against relative shear stiffness.

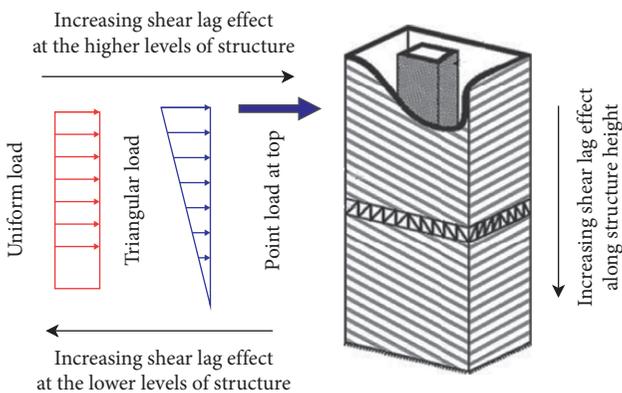


FIGURE 9: Shear lag effect against load distribution along the building height.

tube [12, 73, 103] (see Figure 7). This phenomenon can be influenced by several parameters, such as the relative shear stiffness, aspect ratio of the framed tube structure, and the distribution of lateral loads. The interactions between shear lag and such parameters are clarified in Figures 8 and 9.

In order to consider shear lag in the simplified modeling techniques, the following set of assumptions were used in the reviewed papers:

- (i) Neglecting the out-of-plane actions (out-of-plane deformations) of the frame panels (web and flange). This means the three-dimensional system will be analyzed as an equivalent plane frame; thus, the distributions of the bending stresses or axial displacements of the web panels are independent from that of the flange panels and the shear lag coefficient of a particular frame panel is only dependent on the elastic properties of that panel and not on any other panels [104, 105]. It is considered to be a reasonable assumption because (i) the out-of-plane deformations of the frame panels are small and insignificant; (ii) the main interactions between the flange and web panels consist of vertical shear forces; (iii) the shear lag in specific panel is clearly more related to this panel's properties compared with other panels. This assumption, also, could substantially reduce the complexity of the problem and the amounts of data and computation required; thus, simpler formulas for shear lag evaluation can be obtained.
- (ii) Assuming the order of the polynomial function which represents the distribution of axial displacements or bending stresses across the width of the web and flange. For web panels, it could be assumed

as a third order (cubic function) as in reference [62], fifth-order polynomial as in reference [60], or other order. For flange panels, it could be assumed as a second order (quadratic function such as the function of parabolic or hyperbolic cosine shape) as in reference [53], fourth-order polynomial as in reference [60], or other order.

Depending on such previous assumptions, Kwan introduced in his study a set of design charts, where the shear lag coefficient is plotted against the relative shear stiffness parameter. These charts can be used to evaluate the shear lag effects [62].

**5.3.7. Other Assumptions.** In addition to previous assumptions, there are more assumptions adopted in the reviewed articles for simplified modeling, such as the following:

- (i) Several studies considered a symmetric plan of the structure to ensure the structure cannot twist, i.e., there is no torsion due to the symmetric distribution of the load over the width of the structure [57, 61, 74, 75]. Therefore, the three-dimensional building can be analyzed as an equivalent two-dimensional (planar) model.
- (ii) In general, reviewed studies assumed that the outrigger and belt-truss members would not subject to buckling, so the postbuckling effect is not considered. However, in detailed studies such as Tavakoli et al. [39], the situation in which the outrigger and belt-truss members buckle, i.e., postbuckling effect, is taken into account.
- (iii) In the analysis of phenomena such as differential shortening, a set of assumptions, related to the predicted sources of differential shortening and the planned construction sequence, should be suitable and simulate the reality of the considered case [45, 46, 106].
- (iv) For the sake of simplicity, stress concentrations, which occur at the belt-truss location, are neglected [59, 60].
- (v) In order to model the mass of the outrigger and belt-truss system, which is essential in modal analysis of the structure, some of the reviewed articles considered the mass of the outrigger as a lumped mass at the outrigger level [55]. In the situation that the outriggers are relatively light, their masses can be neglected for simplifying the analysis [107].
- (vi) In the continuum modeling approach, the resulting continuous beam which has multiple degrees of freedom can be modeled with single degree of freedom by using some assumptions [108], such as (i) the dynamic response and first mode shape are completely determined by the first frequency; (ii) in the first mode, the energy absorption for the continuous beam and the simulated single degree of freedom system is considered to be equal, so the equivalent stiffness, mass, and force of the single-degree-of-freedom system can be predicted.

- (vii) Most of the reviewed articles assumed that the studied model is vertically regular. However, just a few studies took the vertical irregularity into account, as in reference [109].

## 6. Formulating the Optimization Problem

The optimization problem for finding the optimum topology and size for the outrigger and belt-truss system should be formulated to achieve and satisfy single or multiple objective functions in addition to a set of constraints, within practical limits and boundaries for the changing of the design variable value. An example of the formulation of the problem, as presented in references [23–26], is clarified in Figure 10.

**6.1. Optimization Problem Parts.** In the structural optimization problems (e.g., topology and size), the problem is defined by four parts: (i) decision variables (design variables), which is explained in Section 4; (ii) an objective function; (iii) constraints; and (iv) bounds on the design variables. Both of the constraints and boundaries establish the design space of the formulated problem.

**6.1.1. Objective Function.** Objective function is a mathematical relationship between the design variable and the targeted criterion of the design. In most situations, for topology and size optimization, this criterion is an economic one. However, in some studies for research purposes or other reasons, one or more of strength and serviceability criteria were used to formulate the objective function. This function can be formulated by theorems, laws, and relationships of solid mechanics, strength of material, and geometry. A lot of the reviewed research studies considered a single-objective function in the formulated optimization problem such as [22, 37, 49], while few articles considered multiple objective functions such as [15, 18, 21, 110]. In all situations, the objective function could not be used as constraint at the same time [15].

**6.1.2. Constraints.** Constraints are mathematical equations or inequalities, which link between design variable and performance criteria. The constraints are important to guarantee that the obtained optimum design satisfies safety, strength, serviceability, and constructability criteria. Formulation of the constraints can be done, also, by using theorems, laws, and relationships of solid mechanics, strength of material, and geometry.

**6.1.3. Boundaries.** In the optimization problems, upper and lower bounds for the design variables should be defined. These bounds, in structural optimization, usually represent logical bounds such as the outrigger location is set between the bottom and the top of the building [18, 19, 21], bounds related to the availability and industrial fabrication of the structural members section in order to guarantee the ease of installation, or maybe bounds related to mathematical logic of the optimization problem, e.g. in the reference [111],

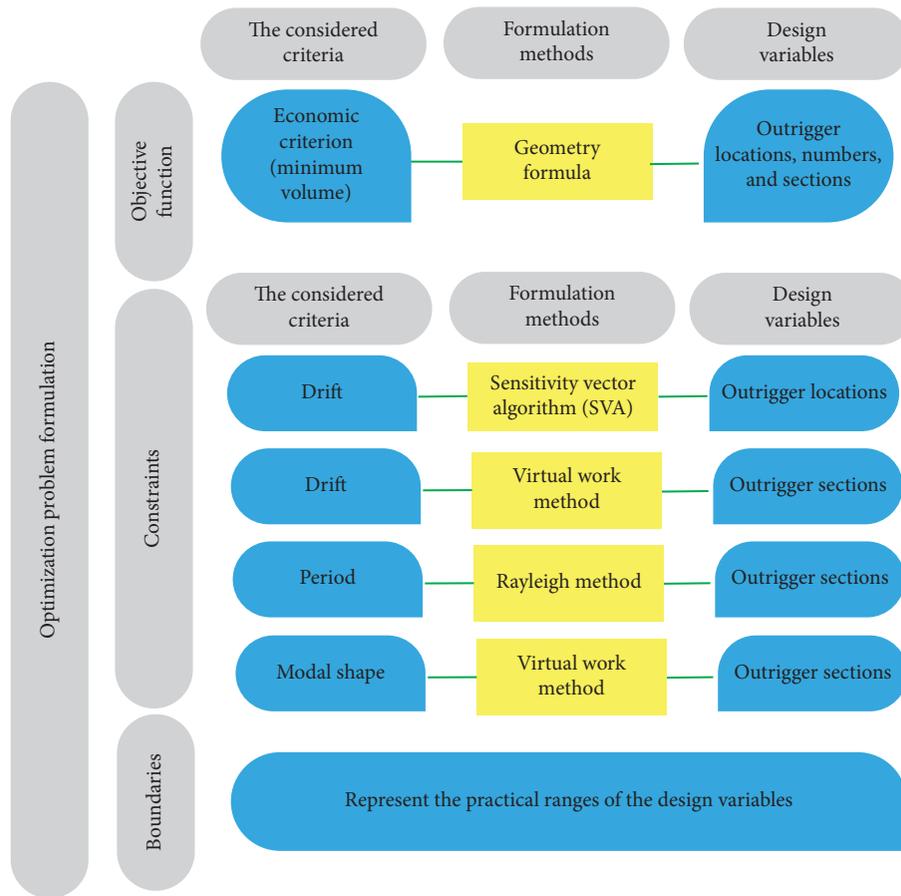


FIGURE 10: Problem formulation example as presented in references [23–26].

where a non-zero lower bound was adopted for the design variable to ensure the numerical stability during the optimization process. Furthermore, in the problem of the optimum topology of outrigger system, a spatial bound for the available locations of outriggers should be considered, where most potential locations of the outriggers are the mechanical floors to get more rentable space [21].

**6.2. Criteria of Optimization Problem Formulation.** As mentioned above, the constraints and objective functions of the outrigger and belt-truss system optimization problems were mainly formulated depending on the variety of criteria. The considered criteria in the reviewed articles with illustration to their significance are discussed in the following sections.

**6.2.1. Economic Criteria.** It was adopted as an objective function in the studies [15, 16, 20, 112, 113]. This function is defined in order to minimize material costs. In other words, minimizing the volume of the structural members (e.g., core wall, external columns, outriggers and belt-trusses which compose the outriggers system) is controlled by the performance criteria and constraints [20, 112, 113].

**6.2.2. Performance Criteria.** The structural performance of the designed building, using optimization techniques,

should satisfy the performance limitations of the design code [112, 113]. The performance criteria are classified as follows.

(1) *Strength Criteria.* In most design cases of high-rise buildings, strength criteria are considered as constraints and not as objective functions because in the design of tall buildings, the strength criteria are less violated than serviceability criteria which most probably control the design. Strength criteria are as follows:

- (i) Bending stress because of the overturning moment at the core base [16, 20, 21, 35].
- (ii) Shear Stress at the base: it can be considered as a constraint or not considered at all because it rarely violates safety standards in comparison with the bending stress [20].
- (iii) Column axial forces (axial stress): especially in the situation of a tubular structure, the axial stress in a frame tube is a critical factor. This is because of the shear lag phenomenon, which causes a concentration of the stresses in corner columns compared to inner columns leading to a reduction in the lateral stiffness of the structure. Moreover, the magnitude of the axial stress is affected by the relative shear stiffness of the frame tube, the stiffness and locations of outriggers and belt trusses, and the aspect ratio of the structure [62]. Therefore, the axial stress should

be smoothed and decreased to be in the standards' limitations [16, 19, 53, 60].

- (iv) Other criteria, e.g., static to seismic load ratios in the critical braces at different floor levels, are considered in the study [47].

(2) *Serviceability Criteria.* Generally, serviceability criteria govern the design of tall buildings, as long as occupants' comfort in high-rise structures mainly depends on these criteria to be satisfied. So, it is prevalent to categorize the design methods of high-rise buildings as displacement-based [16]. Basically, serviceability criteria clarify the sufficiency of the structural system's stiffness (or resilience) and mass in order to satisfy occupants' comfort criterion under lateral loads [35]. Serviceability criteria are as follows:

- (i) Horizontal deflections: it is considered in two forms, either top drift or interstory drift. Controlling the top drift under lateral loads is one of the main concerns in high-rise buildings' design. So, several research studies considered the top drift as an objective function in the optimization problem. Top drift is mainly dependent on structure height and the stiffness of core and columns in addition to the stiffness and location of outriggers [6, 14–16, 18, 20, 21, 55, 61, 80]. However, in real structure design, in addition to engineering practice and design codes, interstory drift is considered the most significant target for controlling the design of high-rise building structures compared with top drift [14, 18, 19, 36, 37, 41, 80, 114]. However, in the preliminary design stage and for seeking simplicity, the top drift can be used instead of interstory drift [16, 18].
- (ii) Fundamental period (frequency of vibration, or natural mode shapes): the natural or fundamental period of free vibration is used to obtain the design base shear. Also, reducing the natural period (which is mainly affected by the stiffness and location of outriggers in addition to the stiffness of the core and columns) is an important factor for occupants' comfort because it is the dominant period in the response to wind and seismic-induced vibrations in high-rise buildings [40, 55, 61, 80, 101].
- (iii) Response acceleration: it is one of the predominant parameters which affect human perception to vibration and motion, where it brings the feel of the building drift to the human notice. The structure acceleration is usually requested to be below a certain limit in the design process in order to avoid occupants' discomfort under lateral loads. For instance, a structure response of acceleration up to  $0.5 \text{ m/s}^2$  makes difficulties to walk naturally and leads to lose the balance when standing at the top of such structures. Moreover, this can adversely affect the nonstructural components of the building such as furnishings and other equipment, as well as the occupants' safety [42, 47, 115].

Generally, the induced acceleration of high-rise building is determined by the dynamic properties under the given lateral loadings. For decreasing the building acceleration, several methods can be used [27], such as (i) changing the natural vibration period, which is a straightforward method, but costly; (ii) installing supplementary damping systems (such as damped outriggers); this will be a cost-effective solution in the situation that the acceleration response exceeds the comfort criteria with large margins; (iii) using structural optimization principle (such as modal shape updating method); this will be a powerful and cost-effective solution in the situation that the acceleration response exceeds the comfort criteria with small margins (below 20% over the codes' limit), due to the local impact nature of the modal shape updating method, which reduces the acceleration by locally calibrating the modal shape near the floor where maximum building acceleration occurs.

*6.2.3. Energy Criterion.* Energy criterion is also considered as a reliable measure in engineering problems. Energy is equivalent to the product of displacement and force. Moreover, it is a more comprehensive criterion than stress (strength criteria) or displacement (serviceability criteria), when computing the optimum topology of the outrigger and belt-truss system in a high-rise building structure. The structure under lateral external loads stored the work done by these loads as strain energy in its structural members such as outriggers and belt trusses which stored a portion of this energy. Hence, depending on the required behavior from the outrigger and belt-truss system, the optimum topology and size of the system can be calculated either for minimum external work of the system (i.e., minimum compliance or maximum global stiffness) if rigid behavior is required [111] or for maximum absorbed energy (maximum energy-dissipating capacity) by outriggers and belt trusses if ductile behavior is required [35, 74, 75]. In some research such as [63], maximizing the damping ratio represents the energy criterion in the optimization problem.

*6.2.4. Constructability Criteria.* It is usually used as a constraint in the optimization problem in order to ensure that the obtained optimum design is compatible with the practical standards of high-rise building construction. For example, in two adjacent stories, this constraint can prevent the construction of columns with larger sections depths, above the columns with lower sections depths [16].

*6.2.5. Special Criteria.* In fact, these criteria are the same prementioned criteria, but they are considered to measure the effect of special phenomena, such as time-dependent phenomena (differential shortening and foundation dishing) and accidental phenomena (explosion leads to progressive collapse), on the structural elements from safety, serviceability,

and economic viewpoints. However, these phenomena will be discussed in detail in a special review by the authors.

(1) *Differential Axial Shortening (DAS) and Foundation Dishing (FD)*. As known, in tall buildings, the differences in the stress between core and exterior columns lead to different vertical shortening and settlements (especially for structures built on compressible soil) between them which accumulate along the height of the structure and can develop adverse impacts on the structural and nonstructural members [45, 46]. It is important to understand axial shortening and foundation dishing behavior and reduce their effects to (i) satisfy safety criteria (they can lead to redistribution for members forces, so stresses in outriggers may increase 2 to 4 times [106]) and (ii) avoid serviceability failures in tall buildings such as cracking of partitions and slabs and damage to lift guide rails and plumbing [106]. Basically, DAS occurs due to time-dependent phenomena such as the short-term elastic shortening, in addition to long-term inelastic shortening (creep and shrinkage), which is an inevitable problem in concrete and composite tall buildings. Therefore, with the increasing height of the structures, these phenomena required special consideration in design and construction.

(2) *Progressive Collapse*. This phenomenon occurs as a result of a sudden loss for one or more of primary structural members such as columns due to blast or other reasons. When this phenomenon is considered in the design, the main goal will be to find the optimum topology of the outrigger system to prevent or mitigate the progressive collapse and redesign the cross section of the adjacent elements to the expected failed column to achieve safety requirements [7, 39]. Progressive collapse issue and related problems such as structure response to the blast loading were studied in several articles [116–121].

6.3. *Formulation Methods*. In order to formulate the objective functions and constraints of the optimization problem, a variety of methods can be used. These methods basically based on the strength of material methods in addition to the mechanics of rigid body methods (statics and dynamics), and specifically on the analysis methods of indeterminate structures. By using these methods, the characteristics of the structure (e.g., period and stiffness) and response of the structure (e.g., moments, forces, deformations, or displacements) can be written as a function of the design variable. The appropriate method is chosen according to the considered criteria and design variables. In the reviewed papers, the most used methods were (i) the force method (also called “flexibility method of analysis,” “method of consistent deformation,” or “flexibility matrix method” [36, 37]), which was adopted in references [12, 22, 38, 51, 53, 73, 114, 122]; (ii) the virtual work principle, which was adopted in references [23–26]; (iii) the Rayleigh method, which was adopted in references [23–26]; (iv) the principle of minimum elastic potential energy, which was adopted in references [40, 53, 55, 59–62]; and (v) the

general moment-curvature relationship which was adopted in references [50, 72].

## 7. Searching Optimum Solution Techniques

In general, the formulated structural optimization problems (shape, size, and topology) are constraint problems, which can be converted to unconstrained problems by replacing the constraints with a penalty term in the objective function. Also, structural optimization problems might be either multiobjective optimization problems or single-objective optimization problems. Moreover, these problems can be taxonomized into discrete (often integer) or continuous problems from types of variables viewpoint (continuous or discrete variables) or into linear, quadratic, convex, or nonlinear problems depending on the nature of the objective function and constraints in addition to the smoothness of the functions (differentiable or nondifferentiable). In order to solve the prementioned problems, different types of searching techniques can be employed. The appropriate searching techniques should be chosen based on the nature and complexity of the formulated problem which varies between design stages and the studied cases. The following sections will illustrate the adopted searching techniques in the reviewed articles.

7.1. *Direct Searching Technique*. Direct searching technique is mainly based on the principle of calculus and other mathematical procedures to find the common solution for a set of equations. This technique can be employed in the simple problems of the preliminary design stage, where less parameters and criteria are considered. By using this technique, a closed-form solution can be obtained. Several reviewed articles adopted the direct searching technique, such as references [22, 53, 60, 74, 75].

7.2. *Trial-and-Error Searching Technique*. Trial-and-error searching technique is usually used in the relatively simple problems which do not have a lot of parameters. In this technique, the mathematical model of the structure is analyzed, using an analysis tool (see Section 7.7), for all the possible optimum values of the design variables or only the expected one, and then the analysis results are checked and compared based on the considered criteria in order to pick up the optimum value of design variable (see Figure 11). If the design space is quite small, all the possible values of design variables are checked in this technique (exhaustive search method), while if the design space is quite big, only the expected optimum solutions, which are chosen based on the designer’s experience or guidance of previous studies, are checked. The trial-and-error method is used in several studies such as references [6, 9, 14, 30, 39, 45, 49].

7.3. *Optimality Criteria (OC) Techniques*. Optimality criteria technique is an indirect method of optimization which satisfies a set of criteria related to the structure’s behavior. They are suitable for problems with a few constraints and

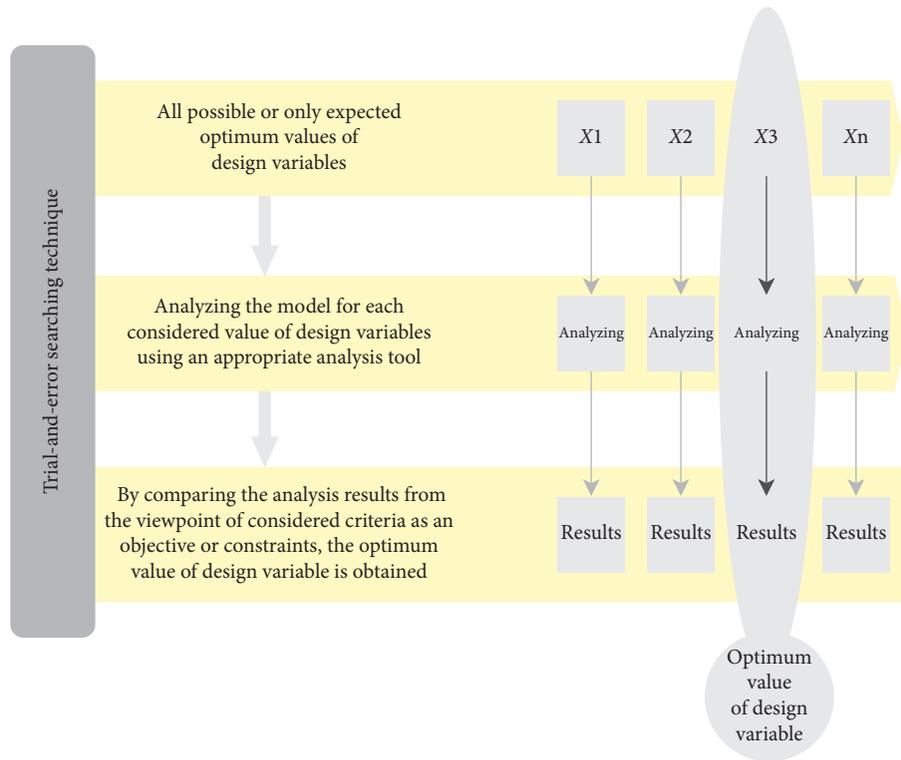


FIGURE 11: Trial-and-error searching technique.

a large number of design variables such as tall building design problems. OC can be remarkably efficient in the situation of sizing and continuous topological optimization problems [8, 31, 123]. Also, OC can be used to solve mixed-discrete-continuous nonlinear programming (mixed-integer nonlinear programming (MINLP)) [8, 17], which are common in the optimum design of the outrigger system because it includes continuous and discrete design variables. These techniques are often based on the Kuhn–Tucker optimality condition [124]. In the situation of topology optimization problem, several optimality criteria methods can be used in order to reformulate the problem and convert it from topology problem to sizing problem, such as the level set method [125–127], solid isotropic material with penalization (SIMP) [111, 128, 129], homogenization [128, 130, 131], and growth method for truss structures [8]. After converting the topology problem to sizing one, a set of criteria for the optimal design is derived, and then a recursive algorithm is applied to resize the design variables for indirectly satisfying the OC. In other words, firstly, sensitivity analysis (gradient-based algorithms, e.g., Lagrange multiplier and augmented Lagrange multiplier) is employed to guide design variables updates [111, 132], then by using analysis tool (see Section 7.7), the structure is reanalyzed for the updated values of the design variables, and finally, the satisfaction of the obtained results from the analysis to the considered criteria and end-searching conditions should be checked (see Figure 12). Several reviewed studies adopted the OC method to solve the topological and sizing optimization problems of outrigger system such as references [17, 23–26, 111, 132].

**7.4. Heuristic or Intuitive Techniques.** Heuristic intuitive searching techniques are derived from observations of engineering processes or biological systems. These techniques cannot always guarantee optimality but can provide viable efficient solutions. Several heuristic methods can be employed for topology optimization problems such as (i) computer-aided optimization (CAO) [133, 134]; (ii) fully stressed design [124]; (iii) evolutionary structural optimization (ESO) [135, 136]; (iv) soft kill option; (v) isoline/isosurface topology design (ITD); (vi) sequential element rejection and admission (SERA); and (vii) bidirectional ESO (BESO) [137, 138]. Genetic algorithm (GA), as an example of the evolutionary algorithms, is a search technique based on the natural selection and mechanics of natural genetics (the evolutionary theory of Darwin, namely, the principle of “Survival of the Fittest”) [19, 21]. It was firstly developed by Holland [31]. GA can be applied to a wide range of design problems, especially problems with discrete geometric, topological, and sizing variables [16]. Therefore, it has gained wide popularity and has demonstrated its advantages over the conventional gradient-based optimization techniques, where GA simultaneously explores the entire design space; thus, it has more ability to reach the global optimum [31, 139]. A GA begins its search with a population of solutions which are usually created randomly within specified upper and lower bounds of each variable. Thereafter, the GA procedure enters into an iterative operation of updating the population (design variable) and reanalyzing the structure using analysis tool until one or more prespecified termination criteria are met (see Figure 13). The four main operators for updating population are selection, crossover, mutation, and elite preservation

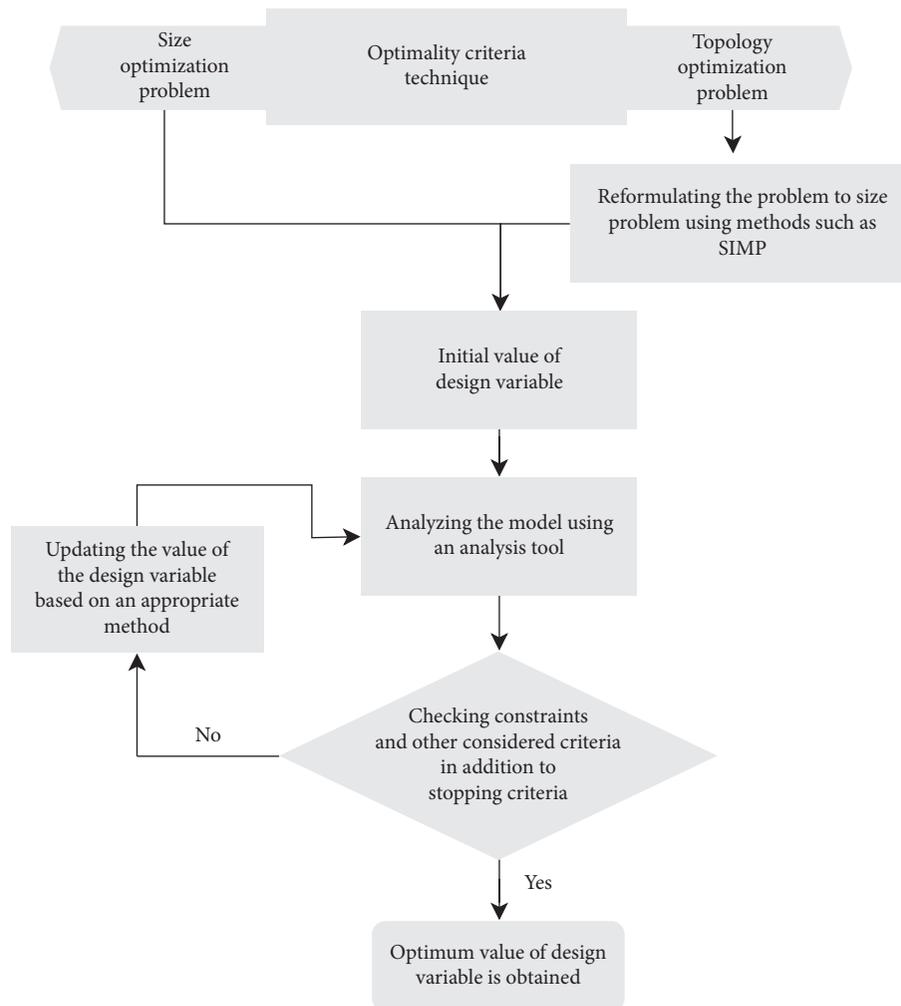


FIGURE 12: Optimality criteria technique.

[19, 20, 140, 141]. GA was used in several review studies, such as references [16, 20, 31].

**7.5. Hybrid Techniques.** OC and heuristic methods have some deficiencies. In discrete topological optimization problems, applying the OC method is very difficult. Moreover, the convergence to the global optimum is not always guaranteed by using the OC method [142], while GA, as a type of heuristic methods, is considered a zero-order method that usually requires many functional evaluations in order to achieve the convergence. Thus, as the complexity and scale of a building structure increase, the required computational effort increases as well, which leads to difficulties in applying GA to practical large-scale problems. Another shortcoming of the GA method is that because GA is a stochastic technique, it is usually incapable of determining the precise global optimum and converges at best to a near optimum [143]. Therefore, to overcome the problems associated with OC and GA while maintaining their merits, hybrid methods have been developed. The developed OC-GA method by Chan et al. [31] is one of the

examples of hybrid methods, which introduces OC as a local search into GA [31]. Also, the proposed method by reference [110] that combines the advantages of gradient-based optimization and an elite population in order to solve the multiobjective optimization problem is another example of the hybrid methods.

**7.6. Techniques for Multiobjective Optimization.** Usually, the global optimum design cannot be obtained by considering only one criterion as an objective function. Therefore, multiobjective optimization (MO) should be dealt by designers in many situations. Sometimes, problems arise when the objectives are in conflict, i.e., the optimal solution of one objective function is different from that of the other. By solving these problems, a set of trade-off optimal solutions are obtained, which are known as Pareto frontier optimal solutions [110, 139]. This means multiple options of structural schemes are available, so designers and clients can choose a scheme flexibly according to their engineering experience, space planning of structures, and financial situation. As a result, in MO, higher-level qualitative

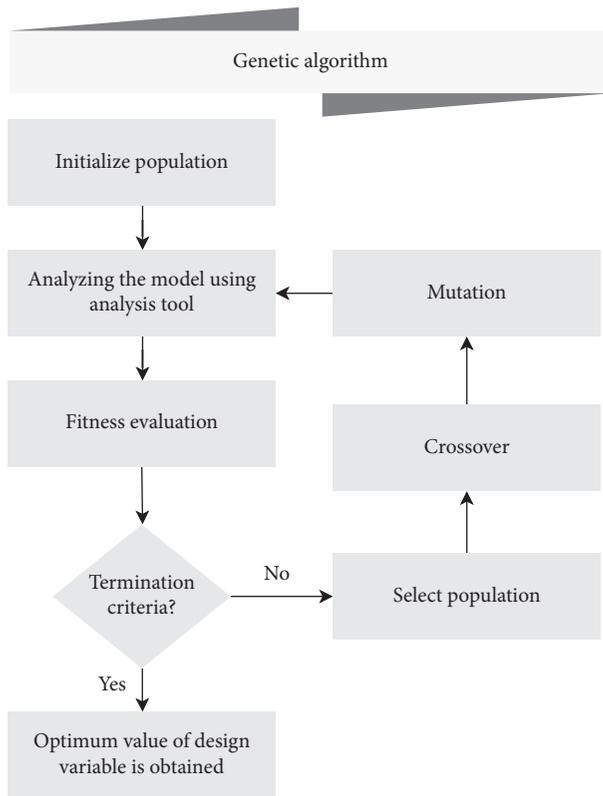


FIGURE 13: Genetic algorithm.

considerations are required to make a decision [18]. Most of the prementioned algorithms can be adopted to solve this type of problems such as genetic algorithm. References [15, 18] are examples of reviewed studies which discuss multiobjective topology and size optimization problems for the outrigger system in high-rise buildings.

**7.7. Analysis Tools.** As seen from the previous sections, some of the searching techniques are engaged with analysis tools. In the reviewed articles, many analysis tools were adopted. These tools are based on several analysis methods, which can be summarized as follows.

**7.7.1. One-Dimensional Displacement-Based Finite Element Method.** This method can be used in the analysis of simplified finite element models. It can predict the lateral drift of structures such as wall frame with outriggers and belt trusses under horizontal loads. In this method, the generalized displacements are expressed over each element as a linear combination of the one-dimensional Lagrange interpolation function [57]. Lee et al. [58] used this method considering the geometric nonlinearity. In this study, the nonlinear algebraic equations of the present theory were linearized by using the Newton–Raphson iterative method.

**7.7.2. Linear Static Analysis.** It is also termed as “equivalent static method.” It is based on formulas given in the codes of

practice and requires less computational effort. In this method, the dynamics of the building are considered in an approximate manner. Usually, it is a preferable analysis method to analyze the regular structures with limited height or to analyze the tall buildings in the preliminary design stage for determining the predominant action between lateral loads (seismic or wind action) to be used during preliminary sizing [42]. The procedures of these methods are summarized in the following steps [144, 145]: (i) computing the design base shear for the whole building; (ii) distributing the computed design base shear along the height of the building; and (iii) distributing the obtained lateral forces at each floor levels to the individual lateral load-resisting elements. This method was employed in reference [47].

**7.7.3. Nonlinear Static Analysis.** It is also called “pushover analysis,” which is described in the guidelines of U.S. Federal Emergency Management Agency FEMA-273 [146] and FEMA-356 [147]. It is an improvement over linear dynamic or static analysis because it takes inelastic behavior of structure into consideration. In this method, the permanent gravity loads are firstly applied, and then by the gradual application of the lateral load (different lateral load patterns can be used to represent different inertia forces [14]), the damage pattern and deformation of structure can be estimated. Usually, pushover analysis is used for seismic analysis in which the structural behavior is characterized by using a capacity curve that demonstrates the relation between the displacement of the roof and the base shear force [144]. In fact, there are different pushover analysis procedures; each one has certain disadvantages and limitations such as [148–155] (i) invariant load distribution in the traditional approaches; (ii) inability to safely account for higher modes effects in modal or multimodal adaptive solutions procedures; (iii) uncertainties in the combination of different modal contributions; (iv) underestimation of plastic hinges rotations. However, pushover analysis has played an important role in the development of the PBSO method (performance-based seismic design method) in codes and guidelines (e.g., FEMA-356, 2000; ATC-40, 1996) [147, 156]. This method was employed in references [14, 115, 157–160].

**7.7.4. Linear Dynamic Analysis.** It can be performed by the response spectrum method. In this method, the peak response of the structure under earthquake loads is directly obtained from the seismic response. This is accurate enough for structural design applications. The primary difference between linear dynamic and linear static analysis is the forces’ level and distribution along the structure height [144, 145]. This method was employed in references [42, 47, 48, 100].

**7.7.5. Nonlinear Dynamic Analysis.** Time-history analysis is a type of nonlinear dynamic analysis. It is a step-by-step analysis of the dynamic response of a structure under specified loading which may vary with time. This method can describe the actual behavior of the structure under

different load conditions, e.g., earthquake. Therefore, it is an important technique for seismic analysis of structure, particularly when the evaluated structural response is nonlinear. Basically, this analysis is based on the direct numerical integration of the differential equations of motion by considering the elastoplastic deformations of the structural elements. Thereby, to perform such an analysis, a representative earthquake time-history is required for a structure to be evaluated [144, 145, 161]. This method was employed in references [39, 47, 48].

Another type of nonlinear dynamic analysis is the incremental dynamic analysis (IDA). It is an extension of a single time-history analysis, in which the seismic “loading” is scaled in increasing order. IDA was adopted by the guidelines of FEMA [162, 163] as the state-of-the-art method to determine global structural collapse capacity. This analysis has been developed in several different forms [164–170] to estimate the probabilistic distribution of structural response in the terms of engineering demand parameters (e.g., peak floor acceleration, peak interstory drift, shear, or moment) by conducting nonlinear dynamic analyses of the structural model under a set of ground motion records, each scaled to multiple intensity levels (which is designed to force the structure all the way from elasticity to final global dynamic instability). Thus, curves of response versus intensity level can be obtained [171, 172]. Thereby, the designer will have [172] (i) better understanding of the structural implications of different ground motion levels (rare, basic, and frequent earthquakes); (ii) better understanding of the changes in the nature of the structural response as the intensity of ground motion increases (i.e., how stable or variable all response parameters are from one ground motion to another); and (iii) the ability to estimate the dynamic capacity of the global structural system. This method was adopted in reference [42, 121, 173].

**7.7.6. Modal Analysis.** The static analysis, quasistatic analysis, and dynamic analysis provide a one-to-one relationship between a particular input (e.g., the applied force) and the structure response (e.g., displacement). In contrast, modal analysis provides an overview of the limits of the response of a system. In other words, for particular input (e.g., applied load of certain frequency and amplitude), what are the limits of the structure response (e.g., what and when is the maximum displacement). Therefore, modal analysis is the gathering of a variety of techniques, whose primary aim is obtaining the dynamic characterization of structures (e.g., period, modal constants, natural frequencies, and damping loss factors) from measured vibration data. Since the measured vibration data can be either in the form of impulse response function or frequency response function, the modal analysis can be conducted either in time domain or in frequency domain [174, 175]. Basically, in the reviewed articles, two types of modal analysis were employed:

- (i) Linear modal analysis (LMA): LMA is usually applied to a MDOF system (multi-degree-of-freedom system) in order to break it into several independent

SDOF systems (single-degree-of-freedom systems). As a result, several independent vibration modes can be obtained and combined linearly to gain the final vibration mode [175]. This analysis was adopted in most of the reviewed articles, such as [100].

- (ii) Nonlinear modal analysis (NLMA): NLMA is used due to the fact that a linear modal analysis sometimes cannot adequately assess the contribution of higher vibration modes to seismic response. These higher modes, in a specific case, represent a significant source of additional stresses in high-rise building prototypes. This analysis was adopted in studies such as [42].

**7.7.7. Special Analysis Methods.** Studying phenomena such as differential shortening or progressive collapse in tall buildings with the outrigger and belt-truss system requires special analysis methods. The used methods in the reviewed papers were as follows.

(1) *Progressive Collapse Analysis.* This analysis can be conducted by applying the alternate path method (APM), which is discussed by UFC09 code. APM is well applicable for investigating the structure ability to bridge the located failed elements. The main philosophy of APM is bridging the loads across the local failed area and redistributing it to the adjacent elements. APM was adopted in reference [7].

(2) *The Long-Term Analysis Methods.* Several long-term analysis methods are available, such as the effective modulus method, rate of creep method, step-by-step method (construction sequence analysis), and age-adjusted effective modulus method. In general, the step-by-step method is considered the most precise method for analyzing and designing the outrigger system under time-dependent phenomena such as column shortening, due to its ability to address any creep function and any complexity of the high-rise building structure [46]. In [46], the analysis of tall buildings with the outrigger system under differential settlement of foundations (foundation dishing) was conducted in three stages: (i) simplified analysis that takes the static forces balances into consideration; (ii) incomplete interaction analysis that only takes the interaction of the foundation and soil or the foundation and superstructure into consideration; and (iii) complete interaction analysis that takes the interaction of the integrated SFS (subgrade-foundation-superstructure) system into consideration.

## 8. Summary of the Reviewed Studies’ Findings

The findings of the reviewed articles illustrate how the design variables (topology and size) impact the response of the structures, how modeling assumptions and other steps in optimization process influence the accuracy of the obtained optimum values of topologies and size, and how these design variables are mutually interacting, in addition to general instructions such as one related to place the outriggers in the optimum locations. However, these findings can be classified and summarized as follows.

### 8.1. The Impact of Design Variables on the Structure's Response

#### 8.1.1. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Response (Drift, Interstory Drift, and Displacement)

- (i) Increasing the number of outrigger levels decreases the drift and interstory drift. But with each additional level, the efficiency of the drift reduction gradually decreases [7, 13–15, 17, 18, 20, 21, 30, 41, 43, 51, 58, 71, 81, 110, 111, 176].
- (ii) In the situation of flexible outrigger, for effective control of the displacement and drift, it is recommended to put the outrigger at the upper part of the structure [7, 17, 18, 20, 45, 57, 61, 80, 81, 111, 177, 178].
- (iii) In the situation of rigid outrigger, it is recommended to lower its position to reduce the drift [45, 57, 61, 177].
- (iv) A small changing of the outrigger level does not affect its performance in reducing the drift [41, 50, 51, 72, 81].
- (v) The outrigger at the top story is not sufficient to reduce the drift and interstory drift. So, it is not a desirable state [14, 18, 43, 48, 54, 61, 74, 80, 107].
- (vi) The optimum location in terms of interstory drift is lower than that of top drift [41, 61].
- (vii) According to reference [173], the structure can have smaller maximum interstory drift as well as exceeding probabilities if the improved viscously damped outriggers are located at the lower part of the structure while the BRB outriggers are installed at the upper part of the structure.

#### 8.1.2. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Response (Core Bending Moment and Restraining Moment)

- (i) Increasing the number of outriggers reduces the base core moment [18, 20, 21, 48, 51, 179].
- (ii) The effective way to reduce the base core moment is to place the outrigger near the base of the structure [6, 20, 36, 38, 48, 50, 57, 71, 72, 80, 81, 111, 177, 179].
- (iii) In the case of the flexible outrigger, placing the outrigger at a lower position is not a desirable way to reduce base moment [57, 80, 107].
- (iv) Increasing the number of outriggers reduces the contribution of the core in resisting the bending moment [20].

#### 8.1.3. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Response (Base Shear)

- (i) Generally, the outrigger increases the base shear [14, 42, 43, 47].
- (ii) The contribution of the higher modes in the base shear is so sensitive for a small relative change of the outrigger location [47, 100].

- (iii) Placing flexible outrigger at a high level increases the contribution of the second mode in the base shear [42, 100].

#### 8.1.4. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Response (Axial Forces in Core and Mega Columns)

- (i) Outrigger system increases the axial forces in the perimeter columns [6, 43, 47, 180].
- (ii) Adding a belt truss increases the axial column forces [180].

#### 8.1.5. Design Variables (Stiffness of Outrigger) vs Structure's Response (Drift, Interstory Drift, and Displacement)

- (i) Reducing the stiffness of the outrigger truss increases the top drift [19, 21, 36, 37, 50, 57, 71, 72, 80, 81, 177].
- (ii) Too flexible outrigger induces an unknown stress state in the core, and its effect on the drift is not clear [57, 61, 80, 177, 178].
- (iii) The relation between the stiffness of the outrigger and the drift is nonlinear [17].
- (iv) The diagonal member is more sensitive than the chord member in controlling the drift and displacement [24, 26, 32, 181].
- (v) At the high levels, the chord member is more sensitive than the diagonal in controlling the drift [181].
- (vi) Increasing the depth of the outrigger trusses significantly reduces the drift [177].

#### 8.1.6. Design Variables (Stiffness of Outrigger) vs Structure's Response (Core Bending Moment and Restraining Moment)

- (i) Increasing the stiffness of the outrigger increases the restraining moment of the core and thus decreases the base core moment [21, 36, 38, 50, 51, 57, 72, 80, 107, 177].

#### 8.1.7. Design Variables (Stiffness of Core Walls) vs Structure's Response (Drift, Interstory Drift, and Displacement)

- (i) Increasing the stiffness of the core is not the optimum solution to reduce the drift [16].
- (ii) Increasing the stiffness of the core reduces the top drift and interstory drift [16, 19, 30, 50, 72, 74, 81].

#### 8.1.8. Design Variables (Stiffness of Perimeter Columns) vs Structure's Response (Drift, Interstory Drift, and Displacement)

- (i) Increasing the stiffness of the perimeter columns reduces the top drift and interstory drift [16, 19, 29, 30, 50, 52, 72, 74, 81].

- (ii) Increasing the columns' sections to reduce the interstory drift is better than increasing the core sections [30].

#### 8.1.9. Design Variables (Stiffness of Perimeter Columns) vs Structure's Response (Core Bending Moment and Restraining Moment)

- (i) Increasing the stiffness of the exterior column decreases the core bending moment [29, 52].

#### 8.1.10. Design Variables (Core to Column Relative Stiffness) vs Structure's Response (Drift and Base Moment)

- (i) Increasing the flexural rigidity ratio of core-to-column reduces the drift. But with the value of this ratio more than 3, the increasing efficiency in reducing the drift is insignificant [19, 50, 72].
- (ii) The flexural rigidity ratio of core-to-column does not have too much influence on the drift reduction efficiency [182].

### 8.2. The Impact of Design Variables on the Dynamic Characteristics of the Structure

#### 8.2.1. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Dynamic Characteristic (Period, Frequency, and Mode Shapes)

- (i) Increasing the outrigger number decreases the period of the structure [14, 26, 80, 101, 115].
- (ii) Placing the outriggers at high levels increases the period, i.e., reduces the frequency [14, 23, 26, 80, 101, 107].
- (iii) For reducing the period of a structure, increasing the number of outriggers is less efficient than increasing the members' size [26].
- (iv) The contribution of the higher vibration modes is sensitive to small alterations in the location of belt truss [100].

#### 8.2.2. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Dynamic Characteristic (Acceleration)

- (i) Placing the outriggers at lower levels increases the acceleration [43, 47].
- (ii) According to reference [183], the maximum floor acceleration belongs to the far-fault records and occurs near the outrigger level.

#### 8.2.3. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Dynamic Characteristic (Energy Dissipation)

- (i) Placing the outriggers around the midheight of the building increases damping ratio [63].
- (ii) With the increasing viscous damping coefficient, the optimal position of damped outriggers goes down at the first mode of vibration [63].

- (iii) For higher energy dissipation, higher outrigger location is required in the situation of small dampers [63].

- (iv) From the energy dissipation viewpoint, the importance of the optimum location of the outrigger trusses is less than the importance of the damper size [63].

#### 8.2.4. Design Variables (Number and Location of Outrigger and Belt Truss) vs Structure's Dynamic Characteristic (Stiffness and Mass/Weight)

- (i) More outrigger levels increase the lateral stiffness of the structure, i.e., reduce the structure ductility, but the efficiency of each additional outrigger in improving the stiffness is less [14, 18, 48]. Therefore, some reviewed studies recommended that using more than four outrigger levels is not efficient to increase structure stiffness [22, 77, 78].
- (ii) Increasing the number of outriggers decreases the volume of the core and increases the volume of the perimeter columns (because the axial force in the perimeter columns increases too), especially when the number increases from 1 to 2 outriggers [16, 17, 20].
- (iii) Increasing the number of outriggers decreases the weight of the structure, especially from 2 outriggers to 3 outriggers [81].

#### 8.2.5. Design Variables (Stiffness of Outrigger System Components) vs Structure's Dynamic Characteristic (Period and Frequency)

- (i) Increasing the stiffness of the outrigger and columns reduces the fundamental period of the structure, i.e., increases the frequency [40, 59, 80, 101, 107].
- (ii) The diagonal member in outrigger truss is more sensitive in controlling the period than the chord member [24, 26, 181].

#### 8.2.6. Design Variables (Stiffness of Outrigger System Components) vs Structure's Dynamic Characteristic (Mass/Weight)

- (i) The importance of diagonal members equals eight times that of chord members in order to control the weight. Thus, increasing the diagonal cross section decreases the weight of the structure [23, 24, 26, 181].
- (ii) Increasing the stiffness of the outrigger reduces the building weight [16].

#### 8.2.7. Design Variables (Stiffness of Outrigger System Components) vs Structure's Dynamic Characteristic (Energy Dissipation)

- (i) Increasing the axial rigidity of the columns increases the dissipated energy [42, 63].

- (ii) Increasing core to column stiffness ratio decreases the dissipated energy [63].
- (iii) Web member in the truss (diagonal) dissipates 96% of the input energy [35].
- (iv) According to reference [184], when the plasticity can be extended anywhere in the core walls over the height, the contribution of the BRB outriggers to inelastic energy demand is reduced.

### 8.3. Optimum Topology and Size of Outrigger and Belt-Truss System for Different Criteria

- (i) For controlling the interstory drift and displacement, it is recommended to put the outrigger between 0 and  $0.5H$  from the top of the structure [6, 41, 48, 53, 57, 61].
- (ii) In order to resist both the drift and progressive collapse (PC), it is recommended to place the belt trusses within the lower and upper thirds of the structure [7].
- (iii) For single outrigger, the optimum location to increase the stiffness is between  $0.5H$  and  $0.55H$  [100].
- (iv) The minimum number of outriggers to control the period is three outriggers [23, 26].
- (v) The optimum location of single outrigger in order to reduce the period is at  $0.5H$  [59, 107].
- (vi) The optimum number for satisfying the drift and the period is five outriggers [23, 26].
- (vii) Placing the outrigger at  $0.5H$  leads to a reduction in differential shortening in columns, where at outrigger level, the value of differential shortening in the columns decreased approximately to zero [45, 106, 115].
- (viii) According to reference [110], as the number of outriggers increases, the values of lateral displacement and differential axial shortening decrease.
- (ix) According to reference [110], the optimal locations of outriggers for differential axial shortening are higher than the optimal locations for maximum lateral displacement.
- (x) The optimum value of the flexural rigidity ratio between core and perimeter columns is three [19, 41, 50, 72, 74, 176, 182].
- (xi) The optimum value of the flexural rigidity ratio between core and perimeter columns is 3.7 to achieve 5% damping ratio [63].
- (xii) The optimum value of the relative stiffness between core and outrigger is four [19, 41, 50, 72, 74, 176, 182].
- (xiii) The outrigger geometry arrangement which delivered maximum stiffness does not always yield the highest ultimate load capacity [32].
- (xiv) The diagonal member locations evaluated using topology optimization differ from the assumed member line designed by engineers [33].
- (xv) It is recommended to use symmetric truss shape [32].
- (xvi) For braced core, up to 25 stories, zipper column core will be more efficient as it provides sufficient lateral stiffness and more space for nonstructural components [16].
- (xvii) According to reference [185], using two half-story outrigger system along the height instead of a one-story outrigger is more effective, while the number of braced bays for outrigger arms is the same for both of them. This decreases the values of the interstory drift, roof displacement, fundamental time period, and core base moment. This result is also valid for a four-half-story outrigger system instead of a two-story outrigger system.
- (xviii) According to reference [186], using the distributed belt wall system as virtual outriggers is effective in reducing lateral drift of tall buildings as much as the conventional belt and outrigger structure.
- (xix) According to reference [187], the semiactive outrigger damping system can reduce both displacement and acceleration responses considerably compared to the conventional outrigger system.
- (xx) According to reference [188], the outrigger tuned inertial mass electromagnetic transduce (TIMET) improves energy generation efficiency to a large extent with the input energy to the structure kept low.
- (xxi) According to reference [189], for all modes, the parallel system of dampers results in lower amplitude of vibration and achieved more efficiently compared to the damper is in series, until the parallel system arrives to 100% of damping for the third mode.

In the reviewed articles, the obtained optimum topologies in terms of optimum locations of the outrigger and belt-truss system are clarified in Tables 1–11.

### 8.4. The Impact of the Modeling Assumptions on the Optimum Topology and Size of Outrigger System

- (i) The assumption related to load distribution (uniform or nonuniform) does significantly affect the optimum location of outriggers [17, 41, 80, 81, 182]. However, the optimum location in the situation of uniformly distributed load is a bit higher than triangular and parabolic distributed loads [14, 19, 37, 74].
- (ii) According to reference [19], when the concrete grades increase from C30 to C80, the maximum interstory drift of the optimal schemes of the

TABLE 1: RC core walls with one steel outrigger truss.

Ref	$H$ of the building (m)	$X/H$ from the bottom (stories)	Main criterion
[18]	260	0.55 (2)	Minimizing top drift
[176]	$H$	0.6 (1)	
[57]	288	0.4 (1)	
[49]	210	0.6 (1)	
[45]	280	0.73 (2)	
[48]	210	0.73 (2)	Minimizing interstory drift
[19]	260	0.65 (1)	
[17]	280	0.64 (4)	Structure weight
[45]	280	0.725 (2)	Decrease the differential shortening
[111]	201	0.313 (1)	Compliance minimization under wind loading
[57]	288	0.3 (1)	Core base moment
[107]	42	0.5 (1)	Frequency

Note.  $H$  in the tables refers to the building height, while the numbers between brackets refer to the height of the outrigger trusses (e.g., one or two stories high).

TABLE 2: RC core walls with one RC outrigger.

Ref	$H$ of the building (m)	$X/H$ from the bottom (stories)	Main criterion
[177]	175	0.72 (1) flexible	Minimum top drift
[177]	175	0.57 (2) infinite rigid	
[13]	210	0.53 (2)	
[20]	400	0.61 (2)	Minimize the structure weight

TABLE 3: Steel braced core with one steel outrigger truss.

Ref	$H$ of the building (m)	$X/H$ from the bottom (stories)	Main criterion
[36]	87	0.68	Top drift
[15]	70	0.6 (1)	Weight optimization
[16]	136	0.55- to 0.65 (1) X zipper core	
[7]	150	The lower third of the building	Progressive collapse

TABLE 4: RC core walls with two RC outriggers.

Ref	$H$ (m)	$X1/H$	$X2/H$	Main criterion
[81]	$H$	0.42 (1)	1 (1)	Top drift
[41]	180	0.45 (1)	1 (1)	
[180]	90	0.5 (1)	1 (1)	
[13]	210	0.55 (2)	1 (2)	Minimize the structure weight
[20]	400	0.41 (2)	0.73 (2)	
[106]	$H$	0.33 (2)	0.66 (2)	Minimize the differential shortening
[63]	$H$	0.46 (1)	1 (1)	Ductility

TABLE 5: RC core walls with two steel outrigger trusses.

Ref	$H$ (m)	$X1/H$	$X2/H$	Main criteria
[18]	260	0.32 (2)	0.69 (2)	Top drift
[51]	87	0.48 (1)	0.79 (1)	
[58]	288	0.50 (1)	0.88 (1)	
[49]	210	0.53 (1)	1.00 (1)	
[6]	90	0.50 (1)	1.00 (1)	
[190]	$H$	0.28 to 0.56BRBs outriggers	0.7 to 0.8BRBs outriggers	Interstory drift
[19]	260	0.54 (1)	0.77 (1)	
[17]	280	0.50 (4)	0.79 (4)	Structure weight
[26]	598	0.44 (2)	0.66 (2)	Decrease the differential shortening
[45]	280	0.56 (2)	0.83 (2)	
[111]	201	0.24 (1)	0.43 (1)	Compliance minimization under wind loading.
[18]	260	0.32 (2)	0.69 (2)	Base core moment
[6]	90	0.23 (1)	1.00 (1)	
[21]	200	0.17	0.49	Minimize the core base moment with drift under $H/400$
[6]	90	0.33 (1)	1.00 (1)	Decrease the column forces

TABLE 6: Steel braced core walls with two steel outrigger trusses.

Ref	$H$ (m)	$X1/H$	$X2/H$	Main criteria
[38]	160	0.50 (2)	1.00 (2)	
[179]	240	0.33 (2)	0.67 (2)	
[43]	105	0.50 (3)	1.00 (3)	
[47]	120	0.50 (1)	1.00 (1)	
[14]	35 stories	0.4 to 0.68 (1)	1.00 (1)	Drift
	30 stories	0.35 to 0.66 (1)	1.00 (1)	
	25 stories	0.3 to 0.6 (1)	1.00 (1)	
	20 stories	0.2 to 0.5 (1)	1.00 (1)	
[7]	150	0.33 (1)	1.00 (1)	
[16]	153	0.49 (2)	0.67 (2)	Weight optimization
[14]	35 stories	0.34 to 0.65 (1)	1.00 (1)	Fundamental period
	30 stories	0.3 to 0.63 (1)	1.00 (1)	
	25 stories	0.2 to 0.52 (1)	1.00 (1)	
	20 stories	0.2 to 0.45 (1)	1.00 (1)	
[14]	$H$	<b>0.3 to 0.6 (1)</b>	<b>1.00 (1)</b>	Increases stiffness, thereby resisting higher forces during seismic excitation

TABLE 7: Three outrigger structures.

Ref	Core type	Outrigger (stories)	$H$ (m)	$X1/H$ (from the bottom)	$X2/H$	$X3/H$	Main criteria
[30]	RCC	Steel (1)	236	0.26	0.48	0.71	
[42]	Steel	Steel (1)	180	0.33	0.67	1.00	
[49]	RCC	Steel (1)	210	0.42	0.58	1.00	Drift-interstory drift
[15]	Steel	Steel (1)	70	0.30	0.60	0.80	
[58]	RCC	Steel (1)	577.5	0.34	0.60	0.86	
[111]	RCC	Steel (1)	201	0.16	0.28	0.42	Compliance minimization under wind loading.
[18]	RCC	Steel (2)	260	0.23	0.47	0.76	Pareto optimal solutions (drift with moment)
[21]	RCC	Steel (2)	200	0.06	0.26	0.46	Minimize the core base moment (drift under $H/400$ )
[20]	RCC	RCC (2)	400	0.33	0.55	0.80	
[15]	Steel	Steel (1)	140	0.3	0.50	0.80	Minimize the structure weight
[21]	RCC	Steel (1)	$H$	0.05	0.18	0.32	
[17]	RCC	Steel (4)	280	0.46	0.65	0.85	

TABLE 8: Four outrigger structures.

Ref	Core type	Outrigger (stories)	$H$ (m)	$X1/H$ (from the bottom)	$X2/H$	$X3/H$	$X4/H$	Main criteria
[20]	RCC	RCC (2)	400	0.29	0.47	0.64	0.84	
[15]	Steel	Steel (1)	175	0.24	0.36	0.60	0.84	Minimize the structure weight
[17]	RCC	Steel (4)	280	0.42	0.55	0.71	0.89	
[111]	RCC	Steel (1)	201	0.16	0.42	0.55	0.70	Compliance minimization under wind loading
[18]	RCC	Steel (2)	260	0.18	0.36	0.56	0.80	Pareto optimal solutions (drift with moment)
[21]	RCC	Steel (2) fixed OT stiffness	200	0.07	0.27	0.47	0.67	Minimize the core base moment with drift under $H/400$
		More rigid	240	0.05	0.18	0.32	0.59	
		More flexible	240	0.19	0.366	0.558	0.8	
		Rigid $\omega = 0.2$		0.32	0.48	0.66	1.00	Minimize the top drift
[80]	RCC	Flexible $\omega = 0.9$	$H$	0.47	0.59	0.72	1.00	Minimize the base moment in core
		Rigid $\omega = 0.2$		0.11	0.20	0.31	1.00	
		Flexible $\omega = 0.9$		0.22	0.30	0.43	1.00	
[15]	Steel	Steel (1)	105	0.20	0.40	0.60	0.80	Optimal roof displacement
			140	0.30	0.50	0.65	0.85	
			175	0.24	0.52	0.68	0.84	
[18]	RCC	Steel (2)	260	0.18	0.36	0.56	0.80	Pareto optimal solutions (drift with core moment)

" $\omega$ " in Table 8 indicates the relative rigidity between core, column, and outriggers.

TABLE 9: Five outrigger structures.

Ref	Core type	Outrigger (stories)	H (m)	X1/H (from the bottom)	X2/H	X3/H	X4/H	X5/H	Main criteria
[17]	RCC	Steel (4)	280	0.41	0.54	0.65	0.78	0.91	Structure weight
[18]	RCC	Steel (2)	260	0.14	0.29	0.45	0.62	0.83	Pareto optimal solutions (drift with moment)
[21]	RCC	Steel (2) fixed stiffness	200	0.07	0.27	0.47	0.66	0.86	Minimize the core base moment with drift limit under H/400.
		More rigid	240	0.05	0.18	0.32	0.45	0.58	
		More flexible	240	0.06	0.2	0.336	0.491	0.752	

TABLE 10: Six outrigger structures.

Ref	Core type	Outrigger (stories)	H (m)	X1/H (from the bottom)	X2/H	X3/H	X4/H	X5/H	X6/H	Main criteria
[21]	RCC	Steel (2) more rigid	240	0.05	0.18	0.32	0.45	0.59	0.72	Minimize the core base moment with drift limit under H/400.
		More flexible	240	0.114	0.247	0.381	0.514	0.647	0.83	
[18]	RCC	Steel (2)	260	0.12	0.24	0.37	0.51	0.67	0.85	Pareto optimal solutions (drift with moment)

TABLE 11: Seven outrigger structures.

Ref	Core type	Outrigger (stories)	H (m)	X1/H (from the bottom)	X2/H	X3/H	X4/H	X5/H	X6/H	X7/H	Main criteria
[21]	RCC	Steel (2) more rigid	240	0.05	0.18	0.32	0.45	0.58	0.71	0.85	Minimize the core base moment with drift limit under H/400.
		More flexible	240	0.052	0.185	0.319	0.452	0.585	0.719	0.852	
[18]	RCC	Steel (2)	260	0.11	0.21	0.32	0.44	0.56	0.70	0.86	Pareto optimal solutions (drift with core moment)

structure becomes increasingly smaller, but the optimal outrigger locations of the structure are almost unchanged, while according to references [22, 176], in case of using different materials for core, outrigger, and peripheral columns, the concrete strength has a direct influence on the alpha coefficient and this coefficient affects the optimum location of outrigger.

- (iii) Neglecting the shear deformations of the outriggers and core overestimates the shear rigidity of the rotational springs and core leading to underestimate the lateral deflection of the outrigger-braced structures [38, 57].
- (iv) The effect of outriggers' shear deformation is more important than the core's one on the behavior of outrigger-braced structure [38].
- (v) For steel trusses or braced core systems, the shear deformation should not be neglected because it will considerably affect the results of preliminary design. But in the situation of RC core and outrigger, shear deformations could be neglected [37].
- (vi) Good accuracy of slab stiffness contribution will be achieved if the typical floor slab is modeled as a shell element [41].
- (vii) Assuming that the belt trusses and outrigger arms are infinitely rigid leads to overestimate

the deflection and moment reductions [36, 38, 177].

- (viii) The employed method of analysis [30, 42, 47, 48]:
  - (i) response spectrum analysis (RSA) is unable to predict the plastic hinges positions;
  - (ii) there is a disagreement from 10% to 20% between RSA and time-history analysis (THA);
  - (iii) nonlinear time-history analysis (NLTHA) reduces the base moment comparing with RSA;
  - (iv) the obtained optimum location of outrigger using RSA is higher than that of NLTHA.
- (ix) According to reference [190], the differences between the results of spectral analysis and nonlinear response history analysis are because the spectral analysis calculation is based on the modal superposition using elastic mode shapes.
- (x) According to reference [190], the vibration periods of the accurate models (member-by-member models) are slightly longer than the simplified models (discrete mass models).
- (xi) According to reference [190], the BRB outrigger responses and roof drift peaks obtained from the simplified and accurate models are in close approximation.
- (xii) The proposed simplified modeling method in reference [191] accurately predicts the axial force

ratio and horizontal displacement of the joint with average errors 1.7% and 4.9%, respectively.

### 8.5. The Mutual Interactions between Design Variables

- (i) In general, the optimum location of outrigger from drift viewpoint is lower for the stiffer outrigger truss than for the flexible outrigger truss [16, 19, 37, 38, 41, 50, 57, 72, 74, 81, 176, 177].
- (ii) The optimum location of the outrigger goes up with the increasing core flexural rigidity [19, 41, 50, 72, 74, 176, 182].
- (iii) The optimum locations of the outriggers can be influenced a lot by the relative stiffness of the core and columns [182].
- (iv) With increasing the stiffness of perimeter columns, the optimum location of the outrigger fluctuates around an optimum point [63].
- (v) If the relative stiffness of core, column, and outriggers ( $\omega$ ) increases, the optimum location will go up [19, 21, 41, 50, 72, 74, 80, 176, 182].
- (vi) If the number of outriggers increases, the cross sections of these outriggers and core wall will decrease, whereas the cross section of the perimeter column will increase [20].
- (vii) If the cross section of diagonal members of outrigger increases, the number of outriggers to achieve similar performance will decrease [23, 24, 26, 181].
- (viii) If the cross section of diagonal members of outrigger increases, the stiffness of the structure will increase, and the optimum location of the outrigger will be lower [23, 37, 181].

## 9. Conclusion

In this paper, a variety of theories and methods, which were used in the reviewed articles, are presented in the form of manual or procedures for optimum topology and size design of outrigger and belt-truss system in both initial and final design stages. Also, summary of the findings of the reviewed articles is given, where these findings are classified and presented in a way that illustrates the effects of different parameters on the optimum topology and size design of the outrigger system. Moreover, this review paves the way for composing a special standard or code to design tall buildings with the outrigger and belt-truss system. Future directions should concentrate on improving more effective methods, approaches, and algorithms for optimum topology and size design of the outrigger system. In addition, more reviews related to other design issues of outrigger and belt-truss system should be conducted, such as differential shortening and lock-in forces, etc.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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