Research Article

A Displacement Frequency Response Function-Based Approach for Locating Damage to Building Structures

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Frequency response function (FRF) data can provide considerably more information on damage in the desired frequency range as compared to modal data extracted from a very limited range around resonances. Among structural health monitoring techniques, FRF-based methods have the potential to locate structural damage. Conventional structural damage detection technology collects structural response data using contact systems, such as displacement or acceleration transducers. However, installing these contact systems can be costly in terms of labor, cost, and time. Several noncontact measurement technologies, such as optical, laser, radar, and GPS, have been developed to overcome these obstacles. Given the rapid advances in optical imaging hardware technology, the use of digital photography in structural monitoring systems has attracted considerable attention. This study develops a displacement FRF-based approach to locate damage to building structures. The proposed damage location index, *CurveFRFDI*, improves the sensitivity of *SubFRFDI*, which is a substructure FRF-based damage location index proposed by Lin et al. (2012). Moreover, the feasibility of applying the proposed approach to locate damage to building structures using displacement measured by a digital camera combined with digital image correlation techniques is also investigated in this study. A numerical example and an experimental example are presented to demonstrate the feasibility of using the proposed approach to locate damage to building structures for single and multiple nonadjacent damage locations.

1. Introduction

Vibration-based damage identification of structures refers to the in situ nondestructive sensing and analysis of the characteristics of a structure, including the structural response to external excitations, to detect changes that may indicate damage or degradation. The feasibility of applying various vibration characteristics, such as natural frequencies, mode shapes, mode shape curvatures, modal flexibility, and frequency response functions, to damage identification of structures has received considerable attention in the past few decades [1, 2].

Lee and Shin [3] identified two main advantages of using the frequency response function (FRF) data. First, modal data can be contaminated by modal extraction errors in

addition to measurement errors because they are derived datasets. Second, a complete set of modal data cannot be measured in all but the simplest structures. FRF data can provide considerably more information on damage in the desired frequency range as compared to modal data that are extracted from a very limited range around resonances. Some studies have shown that FRF-based methods are highly promising tools to detect damage to building structures [4–8]. Ni et al. [4] identified the seismic damage of a 38-story building using measured FRFs and neural networks. Their study used principal component analysis (PCA) to reduce dimension and eliminate the noise of measured FRFs. The PCA-compressed FRF data are then used as input to neural networks for damage identification. Furukawa et al. [5] developed a method to detect damage to building

structures that uses uncertain FRFs based on a statistical bootstrap method. Kanwar et al. [6] detected damage of reinforced concrete buildings using FRFs. Hsu and Loh [7] detected damage to building structures subjected to earthquake ground excitation using FRFs of intact and damaged systems as well as system matrices of the intact system to derive the damage identification equations. Lin et al. [8] proposed a substructure-based acceleration FRF approach that uses the index, *SubFRFDI*, to locate damage to building structures.

In addition to data analysis, data measurement is another problem that needs to be addressed in detecting damage to structures. Conventional structural damage detection technology collects structural response data by using contact systems, such as displacement or acceleration transducers. However, installing these contact systems can be costly in terms of labor, cost, and time [9]. Several noncontact measurement technologies, such as optical, laser, radar, and GPS, have been developed to overcome these obstacles [10]. The use of digital photography in structural monitoring systems has attracted considerable attention given the rapid advances in optical imaging hardware technology. Digital image correlation (DIC) is a measurement technique that extends the principles of photogrammetry to obtain full-field surface displacement measurements of an object using digital cameras. Several researchers have applied DIC to detect damage to building structures using dynamic responses [11–16]. Shih et al. [11, 12] developed a low-cost digital image correlation method to measure the dynamic response of shear buildings. The accuracy of the DIC method is sufficiently high for several applications. Combe and Richefeu [13] used the DIC technique coupled with geometrical rules to develop an approach to track the non-smooth trajectory of particles. Sieffert et al. [14] presented a digital correlation technique to capture the full-field displacement by using a high-speed camera of a full-scale structure tested on a shaking table. Lu et al. [15] presented a digital image processing approach with a unique hive triangle pattern by integrating subpixel analysis for noncontact measurement of structural dynamic response data. Hung and Lu [16] integrated this approach and GPU computing technique to save on computation time.

Many civil structures, especially high-rise buildings and long-span bridges, have low fundamental response frequencies. Measuring displacement directly, as opposed to integrating acceleration records (and potentially introducing significant error [17]), provides the opportunity to capture low-frequency response [18]. Detecting structural damage using displacement FRF should be more accurate than using acceleration FRF because the fundamental frequency of a structure becomes lower when a damage occurs. Based on these reasons, the objective of this paper is to enhance the work of Lin et al. [8]. In this paper, a damage location index (*CurveFRFDI*) based on *SubFRFDI* curvature and displacement FRF is used to locate damage to building structures to enhance the sensitivity of *SubFRFDI* to damage. The study also aims at investigating the feasibility of applying the proposed approach to locate damage to building structures using displacement measured by digital camera

combined with the DIC technique. Moreover, a numerical example and an experimental example demonstrate the feasibility of applying the proposed approach for locating damage to building structures.

2. Damage Location Strategy

Lin et al. [8] presented a substructure-based FRF approach to locate damage to building structures. As presented in Figure 1, the i^{th} substructure is a structure containing the i^{th} – N^{th} floors (or degree of freedoms, DOFs) for an N -story building structure. The substructure-based FRFs of the j^{th} DOF in the i^{th} substructure can be simplified as follows:

$$\tilde{H}_j^{(i)}(\omega) = \frac{\ddot{X}_j}{\ddot{X}_{i-1}}, \quad j = i \text{ to } N, \quad (1)$$

where $\ddot{X}_j$ and $\ddot{X}_{i-1}$ are the Fourier transforms of $\ddot{x}_j(t)$ and $\ddot{x}_{i-1}(t)$, respectively; $\ddot{x}_j(t)$ and $\ddot{x}_{i-1}(t)$ are the absolute acceleration of the j^{th} and $(i-1)^{\text{th}}$ DOF, respectively. Theoretically, when the damage is assumed to have occurred in the columns between the i^{th} and $(i-1)^{\text{th}}$ DOFs, the substructure-based FRF is altered significantly in the i^{th} DOF as described by $\tilde{H}_i^{(i)}(\omega)$. For efficiency, only one substructure-based FRF, $\tilde{H}_i^{(i)}(\omega)$, is determined for each substructure to reduce the computational time. Damage location is based on the FRFs, $\tilde{H}_1^{(1)}(\omega)$, $\tilde{H}_2^{(2)}(\omega)$, $\dots$, $\tilde{H}_N^{(N)}(\omega)$, of all substructures.

The substructure-based FRF damage location index (*SubFRFDI*) for the i^{th} substructure is defined as follows:

$$\text{SubFRFDI}_i = 1 - \text{Exp} \left[\frac{\rho^2 \left(\sum_{\omega=a}^b \{ (NDF_i(\omega))^2 \} \right)}{n} \right], \quad (2)$$

where ρ , a , b , and n are working parameters and the $NDF_i(\omega)$ is expressed as follows:

$$NDF_i(\omega) = \frac{\bar{P}_i(\omega)}{\max \left[\left| \tilde{H}_{i,d}^{(i)}(\omega) \right|_{\max}, \left| \tilde{H}_{i,u}^{(i)}(\omega) \right|_{\max} \right]}, \quad (3)$$

and the absolute dissimilarity $\bar{P}_i(\omega)$ is defined as follows:

$$\bar{P}_i(\omega) = \left| \left| \tilde{H}_{i,d}^{(i)}(\omega) \right| - \left| \tilde{H}_{i,u}^{(i)}(\omega) \right| \right|, \quad (4)$$

where $\left| \tilde{H}_{i,d}^{(i)}(\omega) \right|$ and $\left| \tilde{H}_{i,u}^{(i)}(\omega) \right|$ are the magnitudes of $\tilde{H}_i^{(i)}$ in the damaged and undamaged states, respectively. These N dissimilarities, $\bar{P}_1(\omega) \sim \bar{P}_N(\omega)$, can be calculated correspondingly for a shear building with N floors. In equation (2), the coefficient ρ is a control value that scales the index between 0 and 1. The damage index is not sensitive to locate damage for a small ρ . However, the sensitivity of the damage index to locate damage will not increase for a large ρ . The value of the coefficient ρ is suggested to be 1–10, with 5 used in this work. The range of selected frequencies for calculating *SubFRFDI* is set to be $a \sim b$, where a is the starting frequency of zero and b is the end frequency, which is equal to the first modal frequency (undamaged state). The value n equals $(b-a)$ divided by sampling time. If the properties of a structural

FIGURE 1: (a) Original complete structure and (b) the i th substructure (reproduced from Lin et al. [8], under the Creative Commons Attribution license/public domain).

system do not change, then $SubFRFDI_i$ is close to zero. However, if the damage to the i^{th} floor in a shear building is severe, then the value of $SubFRFDI_i$ is high. Thus, $SubFRFDI_i$ can be regarded as the $SubFRFDI$ corresponding to location i (the i^{th} floor).

This study investigates the feasibility of applying displacement measured by digital camera combined with DIC technique to locate damage to building structures, and thus, the substructure-based FRF of the j^{th} DOF in the i^{th} substructure in equation (2) is modified to calculate $SubFRFDI_i$:

$$\tilde{H}_j^{(i)}(\omega) = \frac{\tilde{X}_j}{\tilde{X}_{i-1}}, \quad j = i \text{ to } N, \quad (5)$$

where $\tilde{X}_j$ and $\tilde{X}_{i-1}$ are the Fourier transforms of $\tilde{x}_j(t)$ and $\tilde{x}_{i-1}(t)$, respectively; $\tilde{x}_j(t)$ and $\tilde{x}_{i-1}(t)$ are the absolute displacement of the j^{th} and $(i-1)^{\text{th}}$ DOF, respectively.

In fact, the $SubFRFDI$ values corresponding to damaged and undamaged locations increase with the increasing of damage extent. Thus, $SubFRFDI$ is insensitive to damage with larger damage extent. Moreover, $SubFRFDI$ can locate single and multiple nonadjacent damages but is unable to locate multiple adjacent damages to building structures. Damage occurred at the location corresponding to a larger $SubFRFDI$ value than adjacent locations for single and multiple nonadjacent damage cases. That is, if the i^{th} floor is damaged, and the $(i-1)^{\text{th}}$ and $(i+1)^{\text{th}}$ floors are undamaged, $SubFRFDI_i$ is larger than $SubFRFDI_{i-1}$ and $SubFRFDI_{i+1}$, and the curve connecting $SubFRFDI_{i-1}$, $SubFRFDI_i$, and $SubFRFDI_{i+1}$ is open downward. Thus, damage occurred at the location corresponding to a negative $SubFRFDI$

curvature rather than a nonnegative $SubFRFDI$ curvature for single and multiple nonadjacent damages. The curvature of $SubFRFDI_i$, $K(SubFRFDI_i)$, can be simply defined as follows:

$$K(SubFRFDI_i) = SubFRFDI_{i+1} + SubFRFDI_{i-1} - 2SubFRFDI_i, \quad \text{for } i = 1, 2, \dots, N, \quad (6)$$

where $SubFRFDI_0$ and $SubFRFDI_{N+1}$ are both equal to zero. In this study, a damage location index based on $SubFRFDI$ curvature, $CurveFRFDI$, is used to locate damage to building structures to enhance the sensitivity of $SubFRFDI$ to single and multiple nonadjacent damages. The $CurveFRFDI$ for an N -story building structure is expressed as follows:

$$CurveFRFDI_i = \begin{cases} -K(SubFRFDI_i), & \text{if } K(SubFRFDI_i) < 0, \\ \frac{K(SubFRFDI_i)}{A}, & \text{if } K(SubFRFDI_i) \geq 0, \end{cases} \quad (7)$$

where A is a control value that scales $CurveFRFDI_i$ to a very small positive value for positive $SubFRFDI_i$ curvature. The value of coefficient A is large compared with $SubFRFDI$ curvature. It is suggested that $A \geq 100$, and A is set to be 100 in this study. The range of $CurveFRFDI_i$ is from 0 to ∞ . Like $SubFRFDI$, damage occurred at the location corresponding to a larger value of $CurveFRFDI$. Figure 2 shows the flow-chart of the proposed approach for locating damage to building structures.

FIGURE 2: The flowchart of the proposed damage location approach.

3. Digital Image Correlation

Digital image correlation (DIC) is an easy optical method to measure continuous deformation by tracking the position of the same physical points shown in a reference image and a deformed image. A rectangular subset of pixels is identified on the speckle pattern around a point of interest (POI) on a reference image and their corresponding location determined on the deformed image to achieve tracking.

This study employs the DIC approach presented in the work of Lu et al. [15] for displacement measurement in the experimental example. The approach used a unique hive triangle pattern by integrating subpixel analysis for non-contact measurement of structural dynamic response data. As shown in Figures 3(b) and 3(c), a fixed-size rectangular subset of pixels that make up hive triangle patterns is the region of interest (ROI), contained both within the reference

(source) and within the deformed (target) images and marked with a red color box. Meanwhile, in Figure 3(a), they are designated as I and I' in source and target images, respectively. The (x_0, y_0) and (x_1, y_1) are the coordinates of the points at the left-top of the ROI of the source image and the after deformed target images, respectively. Hence, u and v are the relative deformations of a particular point in image space. The coordinates of (x_0, y_0) and (x_1, y_1) are figured out in the following steps. Two images are first selected as S and T . The image S is an undeformed (source) image, and image T is a slightly deformed image (target) relative to image S . The difference between the two images, S and T , can be estimated through a simple difference method via pixel-wise computation, as shown in the following equation:

$$D(i, j) = |S(i, j) - T(i, j)|, \quad (8)$$

where i and j are the index of a pixel in the i th row and j th column of the source and target images based on the origin at the top-left corner. If the intensity of each pixel is from 0 to 255, the difference of each pixel between S and T is maximal when the equation value is 255, and S and T are exactly identical when the equation value is 0. If two images have the same background, the $D(i, j)$ of the pixels in the background area is close to zero. Therefore, if an ROI changes the position due to deformation, the $D(i, j)$ of these pixels covered in the ROI is relatively large.

Moreover, the coordinate, (x_0, y_0) , of the ROI of the source image can be computed using a mean-max method represented in equation (9). The method first calculates the means of the intensities of all columns and rows pixels in the source and one of the target images having slightly deformation. The index with the maximum mean values of the columns and rows indicates the x -axis and y -axis coordinates, respectively:

$$\begin{cases} x_0 \text{ is the index of } \max[\bar{X}_i] & \text{for } i = 1 \sim w, \\ y_0 \text{ is the index of } \max[\bar{Y}_j], & \text{for } j = 1 \sim h, \end{cases} \quad (9)$$

where $\bar{X}_i = \sum_{j=1}^h D(i, j)/h$ and $\bar{Y}_j = \sum_{i=1}^w D(i, j)/w$ in which w and h are the width and height of the source and target images in pixel. Particularly, the approach of Lu et al. employs an ideal hive triangle pattern as the ROI in calculating the correlation coefficient with the source image. The correlation coefficient used in the work is defined in the following equation:

$$CC = \frac{\sum \sum [(f - \langle f \rangle) \cdot (g - \langle g \rangle)]}{\{\sum \sum (f - \langle f \rangle)^2 \cdot \sum \sum (g - \langle g \rangle)^2\}^{1/2}}, \quad (10)$$

where f and g are the pixel value of ROIs for the source and target images, respectively. The sign $\langle \cdot \rangle$ denotes the mean operator and $\langle f \rangle$ and $\langle g \rangle$ are the means of ROIs in the source and target images, respectively. The coordinate, (x_1, y_1) , of the point at the left-top of the after deformed target images can be figured out based on the following equation:

$$(x_1, y_1) = \max[CC(I_{x_0, y_0}, I'_{x_1, y_1})], \quad (11)$$

FIGURE 3: (a) Simplified diagram of source and target images, showing displacement variation of image block. (b) Source image. (c) Target image (reproduced from Lu et al. [15], under the Creative Commons Attribution license/public domain).

where I and I' denote the aforementioned ROIs for source and target images, and CC refers to the correlation coefficient function to evaluate with the two ROIs. The relative displacement in the unit pixels (i.e., the pixel displacement), (d_x, d_y) , in the x -axis and y -axis coordinates can be estimated to be $(x_1 - x_0, y_1 - y_0)$. The precision of the pixel displacement, (d_x, d_y) , is an integer pixel value with original images. The actual length (L) of the pattern is a known quantity, and the pixel width (W) of the pattern has been evaluated by the measurement system. The actual displacement, (u, v) , can be calculated by using estimated pixel displacement, (d_x, d_y) , to multiply by a pixel ratio (R_p) from equation (12). At this point, the time-history displacements are estimated completely in the digital image measurement system:

$$R_p = \frac{L}{W}, \quad (12)$$

$$(u, v) = (d_x, d_y) \times R_p.$$

The subpixel analysis can improve the precision based on the subpixel estimation of a target image. The results of Lu et al. [15] indicated that the measurement system increases the precision to a pixel value of 0.1, or even 0.01, and the precision achieves 0.01 mm if R_p is less than 0.1 mm. Moreover, the maximum correlation coefficient is higher than 0.95 in the correct search and lower than 0.75 in the fail search. In the fail situation, another target image is select to evaluate the maximum correlation coefficient again.

By employing a noncontacted approach for measurement of structural dynamic response data, the record can be

divided into 9,000 image frames if the time history of displacements is recorded as a video that contains 90 seconds with 100 fps data. The first frame will be set as a source (reference) image, and other frames are the target images and are processed sequentially to obtain the coordinates (x_1, y_1) in each target image; consequently, the time history of displacements can be figured out. Figure 4 graphically presents a series of frames in a video and the corresponding computed time history of displacements.

4. Examples

To confirm the feasibility of the proposed approach for locating damage to building structures, two examples, a numerical example and an experimental example, are studied.

4.1. Numerical Example. As shown in Figure 5, a six-story shear plane frame structure is studied in this example to evaluate the feasibility of the proposed approach for locating damage. Each floor consists of one beam and two columns. The cross-sectional size of the beam is 240 mm $\times$ 30 mm. The length of the beam is 360 mm. The cross-sectional size of the column is 240 mm $\times$ 15 mm. The length of the column is 180 mm. All members (beams and columns) are assumed to be made from the same material, ASTM A992 steel, with a yield stress of $f_y = 3500$ kgf/cm². In this example, SAP2000 is used for structural analysis. The natural frequencies of the first six modes are 1.653 Hz, 4.975 Hz, 8.341 Hz, 11.472 Hz, 14.130 Hz, and 15.938 Hz. The damage in this example is

FIGURE 4: A series of frames in a video and the corresponding computed time history of displacements.

FIGURE 5: The six-story shear frame model.

simulated as reduced floor stiffness. Single and multiple damage locations are studied. Table 1 presents the simulated damage cases. The 1995 Kobe earthquake record is used as the external excitation, but normalized to 100 gal as the peak ground acceleration (PGA).

4.1.1. Damage Location Using SubFRFDI. Figure 6 compares the substructure-based FRFs for the 2nd and the 5th substructures ($\tilde{H}_2^{(2)}(\omega)$ and $\tilde{H}_5^{(5)}(\omega)$) for case Dam_2F15%. Significant changes were observed only in the substructure-based FRF of substructure 2, $\tilde{H}_2^{(2)}(\omega)$, which demonstrates the ability of the substructure-based FRF to characterize and locate damage.

Figures 7–15 present the comparison of the SubFRFDI values for damage cases listed in Table 1. Results show that although the damage locations of all cases shown in Figures 7–9 are predicted correctly by SubFRFDI, false detection could occur if the number of damage location is unknown. For example, the 1st and 3rd floors for case

Dam_2F&4F15% (see Figure 8(a)), the 2nd floor for case Dam_3F&5F15% (see Figure 8(b)), the 1st floor for case Dam_2F&3F15% (see Figure 8(c)), and the 1st floor for case Dam_2F&3F&4F15% (see Figure 9(b)) may be detected falsely because their corresponding SubFRFDI values are large. Figures 10–12 show that SubFRFDI values corresponding to damaged and undamaged locations increase with the increase in damage extent for cases of single and multiple damage locations. Thus, false detection could occur for large damage extent cases if the number of damage location is unknown.

For cases of multiple nonadjacent damage locations (cases Dam_2F&4F15% (Figure 8(a)), Dam_3F&5F15% (Figure 8(b)), and Dam_2F&4F&6F15% (Figure 9(a))), the damage locations can be predicted by the location (the lowest floor of the substructure) corresponding to the local maximum value of SubFRFDI. However, this criterion is not suitable for cases of multiple adjacent damage locations such as cases Dam_2F&3F15% (Figure 8(c)) and Dam_2F&3F&4F15% (Figure 9(b)).

TABLE 1: Damage scenarios of the numerical example.

Case	Reduced percentage of stiffness					
	k_1	k_2	k_3	k_4	k_5	k_6
Dam_2F5%	0	5	0	0	0	0
Dam_2F10%	0	10	0	0	0	0
Dam_2F15%	0	15	0	0	0	0
Dam_2F20%	0	20	0	0	0	0
Dam_2F25%	0	25	0	0	0	0
Dam_3F5%	0	0	5	0	0	0
Dam_3F10%	0	0	10	0	0	0
Dam_3F15%	0	0	15	0	0	0
Dam_3F20%	0	0	20	0	0	0
Dam_3F25%	0	0	25	0	0	0
Dam_4F5%	0	0	0	5	0	0
Dam_4F10%	0	0	0	10	0	0
Dam_4F15%	0	0	0	15	0	0
Dam_4F20%	0	0	0	20	0	0
Dam_4F25%	0	0	0	25	0	0
Dam_2F&4F5%	0	5	0	5	0	0
Dam_2F&4F10%	0	10	0	10	0	0
Dam_2F&4F15%	0	15	0	15	0	0
Dam_2F&4F20%	0	20	0	20	0	0
Dam_2F&4F25%	0	25	0	25	0	0
Dam_2F10%&4F15%	0	10	0	15	0	0
Dam_2F10%&4F20%	0	10	0	20	0	0
Dam_2F15%&4F10%	0	15	0	10	0	0
Dam_2F20%&4F10%	0	20	0	10	0	0
Dam_3F&5F5%	0	0	5	0	5	0
Dam_3F&5F10%	0	0	10	0	10	0
Dam_3F&5F15%	0	0	15	0	15	0
Dam_3F&5F20%	0	0	20	0	20	0
Dam_3F&5F25%	0	0	25	0	25	0
Dam_3F10%&5F15%	0	0	10	0	15	0
Dam_3F10%&5F20%	0	0	10	0	20	0
Dam_3F15%&5F10%	0	0	15	0	10	0
Dam_3F20%&5F10%	0	0	20	0	10	0
Dam_2F&3F15%	0	15	15	0	0	0
Dam_2F&4 F&6F5%	0	5	0	5	0	5
Dam_2F&4 F&6F10%	0	10	0	10	0	10
Dam_2F&4 F&6F15%	0	15	0	15	0	15
Dam_2F&4 F&6F20%	0	20	0	20	0	20
Dam_2F&4 F&6F25%	0	25	0	25	0	25
Dam_2F5%&4F15%&6F10%	0	5	0	15	0	10
Dam_2F10%&4F15%&6F5%	0	10	0	15	0	5
Dam_2F15%&4F10%&6F5%	0	15	0	10	0	5
Dam_2F&3 F&4F15%	0	15	15	15	0	0

Note. k_i ($i = 1 \sim 6$) represents the stiffness of the i^{th} floor.

The stiffness reduction of the 2nd floor (10%) is smaller than that of the 4th floor (15%), but $SubFRFDI_2$ is larger than $SubFRFDI_4$ for case Dam_2F10%&4F15% (see Figure 13(a)). However, the stiffness reduction of the 2nd floor (10%) is smaller than that of the 4th floor (20%) and $SubFRFDI_2$ is smaller than $SubFRFDI_4$ for case Dam_2F10%&4F20% (see Figure 13(b)). The results imply that the damage extent of the i^{th} floor is not absolutely larger than that of the j^{th} floor if $SubFRFDI_i$ is larger than $SubFRFDI_j$. Moreover, false detection could occur for cases Dam_2F20%&4F10%, Dam_3F15%&5F10%, Dam_3F20%&5F10%, and Dam_2F15%&4F10%&6F5% because $SubFRFDI_1$ (0.293) is a little larger than $SubFRFDI_4$ (0.286) for case Dam_2F20%&4F10% (see

Figure 13(d)), $SubFRFDI_2$ (0.400) is just a little smaller than $SubFRFDI_5$ (0.443) for case Dam_3F15%&5F10% (see Figure 14(c)), $SubFRFDI_2$ (0.592) is larger than $SubFRFDI_5$ (0.438) for case Dam_3F20%&5F10% (see Figure 14(d)), and $SubFRFDI_3$ (0.199) is just a little smaller than $SubFRFDI_6$ (0.214) for case Dam_2F15%&4F10%&6F5% (see Figure 15(c)). The results imply that $SubFRFDI$ is unable to locate damage for cases of two (three) damage locations when the damage extent (stiffness reduction) is equal to and larger than 10% (5%).

4.1.2. Damage Location Using CurveFRFDI. Figures 16–24 present the comparison of the CurveFRFDI values for damage cases listed in Table 1. Results show that

FIGURE 6: The substructure-based FRFs in the second substructure for case Dam_2F15%. (a) The substructure-based FRFs of the 2nd DOF in the second substructure $\tilde{H}_2^{(2)}(\omega)$. (b) The substructure-based FRFs of the 5th DOF in the fifth substructure $\tilde{H}_5^{(5)}(\omega)$.

FIGURE 7: *SubFRFDI* values for cases Dam_2F15%, Dam_3F15%, and Dam_4F15%.

FIGURE 8: *SubFRFDI* values for (a) case Dam_2F&4F15%, (b) case Dam_3F&5F15%, and (c) case Dam_2F&3F15%.

FIGURE 9: *SubFRFDI* values for (a) case Dam_2F&4F&6F15% and (b) case Dam_2F&3F&4F15%.

FIGURE 10: The comparison of the *SubFRFDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in (a) the 2nd floor, (b) the 3rd floor, and (c) the 4th floor.

CurveFRFDI has higher sensitivity to damage than *SubFRFDI* for cases of single and multiple damage locations as compared Figures 16–24 with Figures 7–15. Nevertheless, results imply that *CurveFRFDI* is also unable to locate damage for cases of multiple adjacent damage locations. For

example, the 3rd floor for case Dam_2F&3F15% (see Figure 17(c)) and the 3rd floor for case Dam_2F&3F&4F15% (see Figure 18(b)) are falsely detected.

For most cases in Figures 19–21, the *CurveFRFDI* value corresponding to the damaged location increases with the

FIGURE 11: The comparison of the *SubFRFDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in (a) the 2nd and the 4th floors and (b) the 3rd and the 5th floors.

FIGURE 12: The comparison of the *SubFRFDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in the 2nd, the 4th, and the 6th floors.

increase in damage extent while that corresponding to the undamaged location is almost zero and varies slightly with the increase in damage extent (only in Figures 19(c) and 20(b), the *CurveFRFDI* value corresponding to the second floor wrongly increases with the damage extent). It implies that *CurveFRFDI* can locate damage regardless of intensity (extent) for most cases. Notably, detecting small extent

damage is an important issue for early warning of structural health monitoring. The excellent ability of *CurveFRFDI* to locate small extent damage can be investigated from case Dam_2F5% in Figure 19(a), case Dam_3F5% in Figure 19(b), case Dam_4F5% in Figure 19(c), case Dam_2F&4F5% in Figure 20(a), case Dam_3F&5F5% in Figure 20(b), and case Dam_2F&4F&6F5% in Figure 21.

FIGURE 13: *SubFRFDI* values for (a) case Dam_2F10%&4F15%, (b) case Dam_2F10%&4F20%, (c) case Dam_2F15%&4F10%, and (d) case Dam_2F20%&4F10%.

FIGURE 14: *SubFRFDI* values for (a) case Dam_3F10%&5F15%, (b) case Dam_3F10%&5F20%, (c) case Dam_3F15%&5F10%, and (d) case Dam_3F20%&5F10%.

4.2. Experimental Example. In this example, the dynamic responses of an eight-story downscale steel frame (Figure 25), subjected to the Chi-Chi earthquake for different values of PGA (50 gal, 100 gal, 200 gal, 500 gal, and 1200 gal) shaking table tests, are processed to demonstrate

the feasibility of the proposed approach for locating damage. Each floor of the steel frame has dimensions of 1500 mm in length, 1100 mm in width, and 1180 mm in height (Figure 26). Each floor consists of one slab and four columns. Each slab consists of two C 100 × 50 × 5 with length

FIGURE 15: *SubFRFDI* values for (a) case Dam_2F5%&4F15%&6F10%, (b) case Dam_2F10%&4F15%&6F5%, and (c) case Dam_2F15%&4F10%&6F5%.

FIGURE 16: *CurveFRFDI* values for cases Dam_2F15%, Dam_3F15%, and Dam_4F15%.

FIGURE 17: Continued.

FIGURE 17: *CurveFRFDI* values for (a) case Dam_2F&4F15%, (b) case Dam_3F&5F15%, and (c) case Dam_2F&3F15%.

FIGURE 18: *CurveFRFDI* values for (a) case Dam_2F&4F&6F15% and (b) case Dam_2F&3F&4F15%.

FIGURE 19: Continued.

(c)

FIGURE 19: The comparison of the *CurveFRFDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in (a) the 2nd floor, (b) the 3rd floor, and (c) the 4th floor.

(a)

(b)

FIGURE 20: The comparison of the *CurveFRFDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in (a) the 2nd and the 4th floors and (b) the 3rd and the 5th floors.

1198 mm, two C 100 × 50 × 5 with length 798 mm, one Plate 1500 × 1100 × 20, and four Plate 150 × 150 × 50. Each column consists of four H 100 × 100 × 6 × 8 with length 1060 mm, four Plate 1060 × 150 × 25, and eight Plate 150 × 150 × 25. Table 2 lists the steel types used to construct the eight-story steel frame. These series of shaking table tests were undertaken by the National Center for Research on Earthquake Engineering in Taiwan. The displacements response histories

of each floor are measured during the shaking table tests through linear variation differential transformation (LVDT) and a digital camera (Basler A504kc, sampling rate of 500 Hz) combined with the DIC approach presented in the work of Lu et al. [15], abbreviated as LVDT-measured data and DIC-measured data below. The damage in this example is simulated by reducing the cross section of certain columns as shown in Figure 27. Single and multiple nonadjacent

FIGURE 21: The comparison of the *CurveFRDI* for 5%, 10%, 15%, 20%, and 25% stiffness reduction in the 2nd, the 4th, and the 6th floors.

FIGURE 22: *CurveFRDI* values for (a) case Dam_2F10%&4F15%, (b) case Dam_2F10%&4F20%, (c) case Dam_2F15%&4F10%, and (d) case Dam_2F20%&4F10%.

damage locations are studied. Table 3 presents the simulated damage cases.

4.2.1. Damage Location Using LVDT-Measured Data. Figure 28 presents the comparison of the *SubFRDI* values of LVDT-measured data for cases of single damage location,

Dam50_1F and Dam100_1F. The damage locations of the two cases are predicted correctly by the *SubFRDI* of LVDT-measured data. Figure 29 shows the comparison of the *SubFRDI* values of LVDT-measured data for cases of two damage locations, Dam50_1F&3F, Dam100_1F&3F, Dam200_1F&3F, Dam500_1F&3F, and Dam1200_1F&3F. It

FIGURE 23: *CurveFRFDI* values for (a) case Dam_3F10%&5F15%, (b) case Dam_3F10%&5F20%, (c) case Dam_3F15%&5F10%, and (d) case Dam_3F20%&5F10%.

FIGURE 24: *CurveFRFDI* values for (a) case Dam_2F5%&4F15%&6F10%, (b) case Dam_2F10%&4F15%&6F5%, and (c) case Dam_2F15%&4F10%&6F5%.

shows that $SubFRFDI_1$ and $SubFRFDI_3$ are larger than others, and the damage locations (the 1st and the 3rd floors) of these cases are predicted correctly by the $SubFRFDI$ if the number of damage location is known in advance. Nevertheless, $SubFRFDI_3$ is much smaller than $SubFRFDI_1$ and

false detection of the 3rd floor may be occurred if the number of damage location is unknown.

Figure 30 presents the comparison of the *CurveFRFDI* values of LVDT-measured data for cases of single damage location, Dam50_1F and Dam100_1F. The damage locations

FIGURE 25: The eight-story steel frame.

FIGURE 26: A single floor of the eight-story steel frame: (a) front view and (b) side view.

of the two cases are also predicted correctly by the *CurveFRFDI* of LVDT-measured data. Figure 31 shows the comparison of the *CurveFRFDI* values of LVDT-measured data for cases of two damage locations, Dam50_1F&3F, Dam100_1F&3F, Dam200_1F&3F, Dam500_1F&3F, and Dam1200_1F&3F. Although *CurveFRFDI*₃ is much smaller than *CurveFRFDI*₁ and false detection of the 3rd floor may be occurred if the number of damage location is unknown, *CurveFRFDI* has higher sensitivity to damage than *SubFRFDI* as compared Figures 30–31 with Figures 28–29. To solve this difficulty, a threshold *CurveFRFDI* value is

suggested to be set to locate damage. The threshold *CurveFRFDI* value can be determined after numerous numerical simulations of damage scenarios for a certain building structure. For example, the threshold *CurveFRFDI* value can set to be 0.08 for this experimental example.

4.2.2. Damage Location Using DIC-Measured Data. Figure 32 presents the comparison of the *SubFRFDI* values of DIC-measured data for case Dam100_1F. Figure 33 shows the comparison of the *SubFRFDI* values of

TABLE 2: Steel types of the six-story shear frame.

Member		Steel type		
Type	Number	Type	Size (mm)	Number
Slab	8	C 100 × 50 × 5	Length: 1198	16
		C 100 × 50 × 5	Length: 798	16
		Plate 1500 × 1100 × 20	Length: 1500 Width: 1100 Thickness: 20	8
		Plate 150 × 150 × 50	Length: 150 Width: 150 Thickness: 50	32
Column	32	H 100 × 100 × 6 × 8	Length: 1060	32
		Plate 1060 × 150 × 25	Length: 1060 Width: 150 Thickness: 25	32
		Plate 150 × 150 × 25	Length: 150 Width: 150 Thickness: 25	64

(a)

(b)

(c)

FIGURE 27: The damaged columns.

TABLE 3: Damage scenarios of the experimental example.

Case	Damage location	External excitation
Dam50_1F	The 1 st floor	The Chi-Chi earthquake with PGA = 50 gal
Dam100_1F	The 1 st floor	The Chi-Chi earthquake with PGA = 100 gal
Dam50_1F&3F	The 1 st and the 3 rd floors	The Chi-Chi earthquake with PGA = 50 gal
Dam100_1F&3F	The 1 st and the 3 rd floors	The Chi-Chi earthquake with PGA = 100 gal
Dam200_1F&3F	The 1 st and the 3 rd floors	The Chi-Chi earthquake with PGA = 200 gal
Dam500_1F&3F	The 1 st and the 3 rd floors	The Chi-Chi earthquake with PGA = 500 gal
Dam1200_1F&3F	The 1 st and the 3 rd floors	The Chi-Chi earthquake with PGA = 1200 gal

FIGURE 28: *SubFRFDI* values of LVDT-measured data for (a) case Dam50_1F and (b) case Dam100_1F.

FIGURE 29: Continued.

FIGURE 29: *SubFRFDI* values of LVDT-measured data for (a) case Dam50_1F&3F, (b) case Dam100_1F&3F, (c) case Dam200_1F&3F, (d) case Dam500_1F&3F, and (e) case Dam1200_1F&3F.

FIGURE 30: *CurveFRFDI* values of LVDT-measured data for (a) case Dam50_1F and (b) case Dam100_1F.

FIGURE 31: Continued.

FIGURE 31: *CurveFRFDI* values of LVDT-measured data for (a) case Dam50_1F&3F, (b) case Dam100_1F&3F, (c) case Dam200_1F&3F, (d) case Dam500_1F&3F, and (e) case Dam1200_1F&3F.

FIGURE 32: *SubFRFDI* values of DIC-measured data for case Dam100_1F.

FIGURE 33: *SubFRFDI* values of DIC-measured data for (a) case Dam200_1F&3F and (b) case Dam500_1F&3F.

DIC-measured data for cases of two damage locations, Dam200_1F&3F and Dam500_1F&3F. Figure 34 presents the comparison of the *CurveFRFDI* values of DIC-measured data for case Dam100_1F. Figure 35 shows the comparison of the *CurveFRFDI* values of DIC-measured data for cases of two damage locations, Dam200_1F&3F and Dam500_1F&3F. The results also show that the *CurveFRFDI* has higher sensitivity to damage than *SubFRFDI* as compared Figures 34–35 with Figures 32–33. The results further indicate that applying the proposed approach to locate single and multiple nonadjacent

damages to building structures is feasible by using DIC-measured displacements.

The experimental example proves the feasibility of the proposed approach for location damage to building structures using DIC-measured displacement response. These data are supposed to be noise free or low noise corrupted. Future work should apply the proposed approach to measurements in the field (actual cases) to investigate its capacity to deal with DIC-measured displacement response corrupted by high noise.

FIGURE 34: *CurveFRFDI* values of DIC-measured data for case Dam100_1F.

(a)

(b)

FIGURE 35: *CurveFRFDI* values of DIC-measured data for (a) case Dam200_1F&3F and (b) case Dam500_1F&3F.

5. Conclusions

The current work develops an approach for locating damage to building structures. The proposed damage location index, *CurveFRFDI*, improves the sensitivity of *SubFRFDI*, a substructure FRF-based damage location index proposed by Lin et al. [8]. The feasibility of applying the proposed approach to locate damage to building structures using DIC-measured displacement is investigated in this study. A numerical example and an experimental example are presented to demonstrate the feasibility of using the proposed approach to locate damage to building structures. The following important conclusions are drawn from the results:

- (1) Using *SubFRFDI* can predict damage location accurately for cases of single damage location. However, *SubFRFDI* cannot locate damage for cases of multiple damage locations with large damage extent. *SubFRFDI* values corresponding to damaged and undamaged locations increase with the increase in damage extent for cases of single and multiple damage locations. Thus, false detection may be occurred for large damage extent cases if the number of damage locations is unknown.
- (2) In most cases, the *CurveFRFDI* value corresponding to damaged location increases with the increase in damage extent while that corresponding to undamaged location is almost zero and varies slightly with the increase in damage extent, which implies

that *CurveFRFDI* can locate damage regardless of intensity (extent) for most cases. Moreover, *CurveFRFDI* has higher sensitivity to damage than *SubFRFDI*. Thus, using *CurveFRFDI* can predict damage location more accurately than using *SubFRFDI* for cases of multiple damage locations with large damage extent.

- (3) For cases of multiple damage locations, some *CurveFRFDI* values corresponding to damaged location may be much smaller than others, making it difficult to detect those damage locations. To solve this difficulty, a threshold *CurveFRFDI* value is suggested to be set to locate damage. The threshold *CurveFRFDI* value can be determined after numerous numerical simulations of damage scenarios for a certain building structure.
- (4) Applying the proposed approach and DIC-measured displacements to locate single and multiple nonadjacent damages to building structures is feasible.
- (5) Both *SubFRFDI* and *CurveFRFDI* cannot locate damage for cases of multiple adjacent damage locations. How is the proposed approach applicable to the prediction of multiple adjacent damage locations should be investigated based on this research.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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