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Research Article

Effect of Particle Shape on Repose Angle Based on Hopper Flow Test and Discrete Element Method

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The repose angle of granular material is an essential parameter to understand the microbehavior of the granular material and, then, to relate it with the macrobehavior. In this study, a self-design large-scale hopper flow test apparatus has been developed to measure the repose angle of the ballast using a fixed funnel method. Then, the numerical simulation using the realistic clump is compared with the experimental test to prove its validity. Meanwhile, the idealized clumps with custom shape parameters, including roughness of particle and ground, angularity, aspect ratio, and sphericity, were chosen to analyze the influences of particle shape on the repose angle. The results show that the angle of repose generally tends to increase with the increase of the friction coefficient of particles and the roughness of the ground. With the increase of the angularity from 0 to 4, the pile height and the repose angle increase. Meanwhile, the extended area decreases accordingly. For cuboid particles, with aspect ratio increasing from 1.0 to 1.67, the angle of repose increases firstly and then maintains a constant between aspect ratio 1.25–1.67. For ellipsoid particles, the angle of repose decreases, then reaches a minimum at aspect ratio around 1.3, and finally increases.

1. Introduction

Ballast is a granular material made of crushed natural rock, typically granite. The sharp edges of crushed stones enhance the interlocking between the particles. The main function of the ballast in a railway track is to spread the train load from the sleeper to the ground. It is well known that the repose angle of granular material is an essential parameter to understand the microbehavior of the granular material and, then, to relate it with the macrobehavior. For ballast, the critical state angle of shearing resistance or angle of repose is a function of the interparticle friction coefficient and the particle shape [1]. Therefore, some of the particle shape and interparticle properties of ballast can be calibrated using the numerical simulations with the repose angle of the ballast.

The measurement method of the repose angle should be selected based on predefined objectives and for a specific material and application. The existing methods measure both the static and dynamic angles of repose. However, for each method, test equipment of different size and scale is used. Generally, the tilting box method is suitable for cohesionless, well-graded materials with a grain size ≤10 mm. The angle of repose is measured as the tilting angle at which the material begins to slide [2, 3]. However, this method provides the coefficient of static (sliding) friction rather than the angle of repose. In the fixed funnel method, the granular materials are poured from a funnel at a certain height onto a selected base with known roughness properties. Nelson [4] measured the angle of repose of sulfathiazole materials for a pharmacology application. In this case, the angle of repose was found to be equal to the angle of internal friction of the material only when the grains had a uniform shape and size, and the uncertainty in the measurement was reported as 1.0°. Miura et al. [5] introduced a funnel-type device to determine the angle of repose in which the pile of soil was formed on a cylindrical pedestal with a depression. They studied the relationship between the angle of repose and the angle of internal friction, examining different factors such as the roughness of the base, density of the soil, mean grain size, grain shape, dilatancy, and lifting speed of the funnel. In addition, there are other methods, such as the hollow cylinder method, andtilting cylinder method [6]. Furthermore, the repose angle of two piles of granular material can be compared only if the piles have even, regular slopes, which are not exhibited for cohesive material or extremely angular particles. All the previously discussed methods of measurement assume that the granular material piles form perfect conical shapes, unlikely the case for ballast with grain size around 40 mm. Therefore, in this study, a special large-scale hopper flow test apparatus has been designed by the authors [7] to test the repose angle of the ballast.

Particle shape plays a key role in the behavior of granular material. It influences not only the physical state of the assembly (skeleton and porosity) but also the particle interaction, the stress-dilatancy response [8], the critical state [9], and the strength of biocemented glass beads [10]. In the past, various attempts have been made to characterize particle shape for railway ballast. However, due to the complexity and irregularity of the particle shapes, universally accepted effective shape characteristic parameters have not yet been established [11]. In the railway industry, various shape characteristics (i.e., flakiness, elongation, roughness, angularity, and surface texture) are used [12]. It should be noted that it is impossible to make a ballast sample with the same particle shape characteristics in the lab, so the discrete element method [13] can be used to simulate the hopper flow test. DEM simulation allows to generate the idealized shape particle defined by the authors and generates the same sample for each test. Rather than obtaining the angle of repose, this DEM is used to calibrate the numerical models, including the shape parameters. Chen et al. [14, 15] proposed to use the combination of spherical particles to generate irregular sphere combination and sphere cluster ballast particle model. The main feature of the particle cluster model is that it can easily control the number and combination mode of sphere composition to effectively simulate various shapes of ballast, and there is no internal force between the spheres of particle cluster. Zhu et al. [16] examined the influence of particle aspect ratio on 2D sand piles formed by means of a fixed height point like source and found a clear dip in the vertical pressure at the bottom and the extent of the dip increases with the increase of particle aspect ratio. However, it should be noted that the shapes used by different investigators are different, and the previous studies are largely comparative, not being able to result in a comprehensive quantitative picture about how particle shape affects heap properties.

This paper first briefly describes the large-scale hopper flow test apparatus, test procedure, and how to adequately measure the angle of repose for the ballast heap. Considering the limitation of the experimental materials used to study the influence of particle shape, DEM simulation of the large-scale hopper flow test using three-dimensional scanned realistic particles has been introduced and compared with the experimental results. Furthermore, idealized shape ballast models have been simulated to quantify the effect of aspect ratio, angularity, the friction coefficient of particles, and roughness of ground on the angle of repose. It should be noted that the study is an emphasis on the effect of particle shape parameters. Therefore the mass of the material, pouring height, and particle size in experimental test and simulations are kept the same and not discussed in this paper.

2. Experimental Large-Scale Hopper Flow Test

In this large-scale hopper flow test, a self-design test apparatus, which is an approved patent, was established to measure the repose angle of coarse aggregate [7], which includes a cylindrical hopper connected with an open cone at the bottom, inserting plate slot at the bottom of the cone, support arms and adjustable height of the support, as shown in Figure 1. Among them, the diameter of the funnel is 0.5 m, the diameter of the cone outlet is 0.2 m, the gradient of the conical opening is 45°, and the distance between the outlet and the ground is 0.8 m determined by the preliminary tests. More details of the experimental results could be found in Chen et al. [17]. With the help of high-speed fixed focus camera and image processing software, the falling process can be fully recorded and the repose angle of coarse aggregate, pile height, and spreading area can be accurately measured. The test materials are selected from the railway ballast from Ezhou, which is the premium ballast certified by China Railway Corporation.

It should be noted that the aim of this research is to investigate the effect particle shape and not the particle size. Therefore, 120 kg uniform sized ballast sample with the mean size D_{50} of 40 mm is prepared, which is a typical size in practice. During the experimental process, the ballast sample was firstly added to the cylindrical hopper. Then the plate slot was taken out at the bottom of the cone and the ballast fallen out from the height of 80 mm. Once the ballast pile is stable, the front view of the pile was captured at the fixed position using the leveling fixed focus camera. Following the previous measurement method [18, 19], the authors then used digital image analysis techniques Image J to analyze and obtain the coordinates of a surface profile of the ballast pile slope, as shown in Figure 2. Subsequently, the surface profile was plotted and linearly approximated by using the least squares method, which can reduce the errors in the traditional direct measurements and increase the accuracy in the angle of repose values. It should be noted that each test was repeated at least three times for reducing the experimental errors. Figure 3 shows the repose angles of uniform graded ballast with a mean size of 40 mm. It can be seen that the measured repose angles are in a tolerated range from 37.1–39.3°, which means this hopper flow test apparatus, the total mass of ballast sample, the falling height, and the measurement method in the tests are suitable and reliable. These experimental results will be used to validate the following hopper flow test simulation results.

3. Numerical Simulation

3.1. Hopper Flow Test Simulation Using PFC3D. The DEM of hopper flow test is simulated to be consistent with the experimental test as present above. The dimensions of the hopper, the falling height, the gradient of the conical opening, the ballast gradation, and the total mass of the

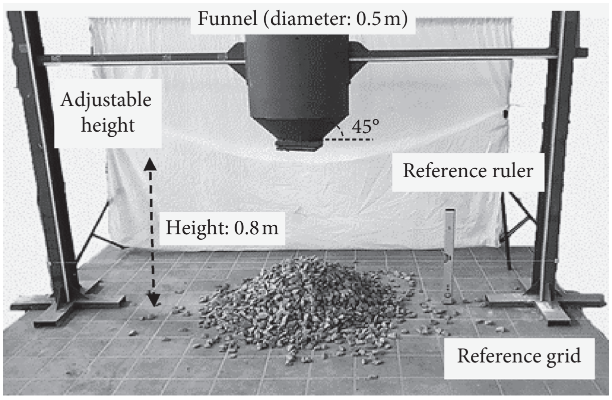


Figure 1: Large-scale hopper flow test equipment for ballast.

ballast sample are the same as those used in the laboratory tests. The DEM sample preparation procedure followed the experimental sample preparation. In the beginning, a uniform graded sample of clumps of size 40 mm was generated within the cylinder box without overlapping, which were given random orientations. The clumps were directly deposited in the hopper and cycled to equilibrium under a changing gravitational acceleration, which was reduced, gradually from $19.62\,\mathrm{m/s^2}$ to $9.81\,\mathrm{m/s^2}$. According to the preliminary trial, when the particle size is too large, the hopper outlet is prone to produce a stress arch, which will block and affect the particle falling. Therefore, the friction coefficient of the hopper is set to be 0.2 to make sure the particles are falling out continually. The whole sample was cycled to equilibrium under gravity, and then the bottom wall of the hopper was removed, which represents the baffle plate in the experimental tests. The pouring of the material was stopped when all the particles fell toward the ground wall and the heap reached a stable state, in which the velocity of all particles is smaller to $10^{-9}\,\mathrm{m/s}$. Then, the angle of repose was measured by the inverse tangent rule at which the average radius of the formed conical shape and the maximum height of the heaped ballast are measured, and then the repose angle is determined following the measurement method in the experimental test, as shown in Figure 2.

Figure 4 shows a heap of 120 kg elliptical clumps deposited from a hopper with a 20 cm circular hole and 0.8 m above the base wall. The spreading of the simulated particles demonstrates a realistic physical behavior of the ballast as shown in Figure 1. For these simulations, the micromechanical parameters used are listed in Table 1. The effect of the pebble-pebble and the pebble-facet friction coefficients on the repose angle would be further studied.

3.2. Ballast Particle Shape Modeling. In this paper, three-dimensional scanning technology is used to obtain the three-dimensional image of real ballast particles. Then the surface file was imported into the discrete element numerical simulation program PFC^{3D} to establish the real ballast model, as shown in Figure 5. The modeling precision complex shape is determined by the ratio of the maximum and minimum sizes of including pebbles and the amount of overlap between the different pebbles. In principle, the more pebbles included in the particle cluster, the more accuracy the shape characteristics of the ballast reflect, but at the same time, the greater the corresponding computation time. In order to keep the calculation time reasonable which increases with the total number of spheres, the ballast particles modelled in the present study contain an average of 50 spheres per particle.

Barrett [20] reviewed various approaches to analyze particle shape in geology and sedimentology and expressed the shape of a particle in terms of three independent properties, namely form (overall shape), roundness (large-scale smoothness), and surface texture, as shown in Figure 6. It should be noted that each of these aspects of shape can itself be represented by more than one dimension. Form or aspect ratio reflects variations in the particle scale, while roundness or angularity reflects variations at the corners. Surface texture or roughness is a property of particle surfaces between and at the corners. To model the complex shapes of ballast particles and investigate the effect of particle shape on performance, four basic idealized shapes of clumps were simulated as shown in Figure 7. The eight-ball cubic clump and sixteen-ball cuboid are used, following Chen et al. [15]. The aspect ratio of the sixteen-ball cuboid clump is larger than the eight-ball cubic clump. The nine-ball ellipsoid clump is rounder than the cuboid clump. Meanwhile, the shape parameter, angularity, could be investigated by using the nine-ball tetrahedron model, which could add or remove the corner ball to increase or decrease the angularity. The surface roughness could be studied by using different friction coefficient of particle. It should be noted that the volumes of these idealized and realistic clumps are the same as a single sphere of radius 20 mm.

Simulation with real shape particle meant large numbers of particles and contacts involved to a point where the DEM model using the realistic particle became unmanageable as it would have required unreasonable calculation times. Moreover, it is hard to quantify each shape's realistic particles parameters. Taking into account these constraints, the numerical simulation using the realistic shape clump are compared with the experimental large-scale hopper flow test to prove its reasonability and validity. Furthermore, the idealized shape clump models with custom shape parameters were chosen to analyze the influences of shape parameters.

4. Simulation Results and Discussion

4.1. Comparisons of the Simulation Using Realistic Particles and Experimental Test. In this study, DEM of large-scale hopper flow test using real shaped particles has been simulated firstly. The sample of the real shaped particles has the same mass and uniform size distribution. This idealized shape model is expected to offer an irregular shape using the least number of spheres necessary to provide particle interlock. It should be emphasized that this paper chooses the nonbreakable clumps as ballast particles without considering the ballast particle crushing during the pouring process because the weight loss ratios of ballast sample after three tests range from 0.042% to 0.056%, which indicates the particle breakage could be ignored in the pouring process.

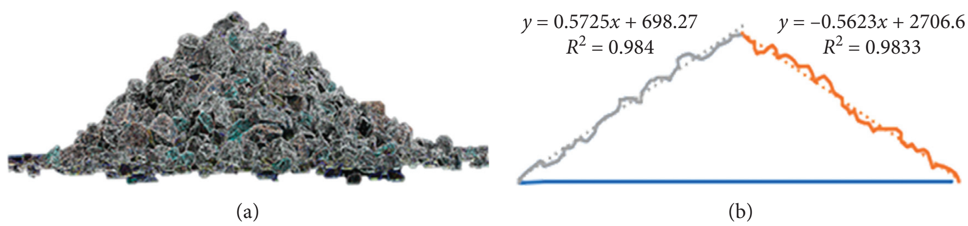


FIGURE 2: The measurement method of the angle of repose.

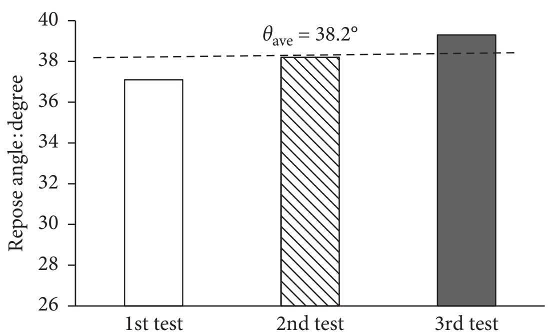


FIGURE 3: Experimental result of the repose angles from repeated tests. Note: θ_{ave} : average angle of repose.

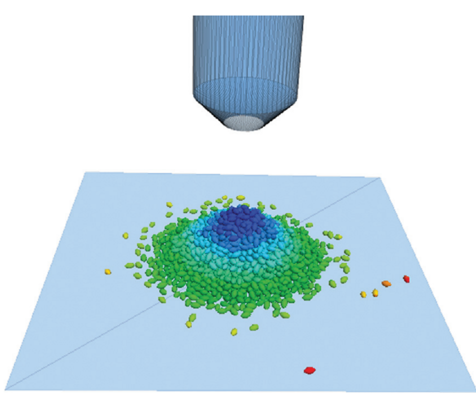


FIGURE 4: DEM hopper flow test model.

TABLE 1: Simulation parameters in DEM of the hopper flow test.

Parameters	Value
Ballast particle normal stiffness (N·m^{-1})	5×10^7
Ballast particle shear stiffness (N·m^{-1})	5×10^7
Wall normal stiffness (N·m^{-1})	10^8
Wall shear stiffness (N·m^{-1})	10^8
Ballast particle density (kg·m^{-3})	2550
Pebble-facet friction coefficient (in hopper)	0.2
Pebble-facet friction coefficient (ground)	0.5
Pebble-pebble friction coefficient	0.6

The pile height and the angles of repose from the front and side views could be determined as shown in Figures 8(a) and 8(b). The average spreading distance could be measured from the top view as shown in Figure 8(c). The average angle of repose is 39.9° and the average spread distance is around 105 cm, which is close to the angle of repose from the experimental test results. It can be concluded that DEM has a potential ability to simulate the large-scale hopper flow test. Providing similar conditions in the experiments and the simulations, the results of the following simulation using idealized shape particles could be compared with confidence.

4.2. Effect of Roughnesses of Particle and Ground on the Repose Angle. Both the roughness of particles and the sliding surface affect the angle of repose and friction coefficients of the particles [21]. A rough particle surface is critical to form a high interparticle friction force, which will increase the shear strength of the ballast. On the contrary, a smooth surface will create a low interparticle friction force which an easy rearrangement of particles will result. In this study, a nine-ball tetrahedron clump in Figure 7 is used as a ballast particle and the interparticle friction coefficient has been selected to represent the surface roughness. In order to study the effect of the surface roughness of particle, the effect of the ground roughness on the heap generation should be excluded. In this study, the friction coefficient of particle-particle ($f_{\text{p-p}}$) and the friction coefficient of particle-ground ($f_{\text{p-g}}$) are tested, ranging from 0.4–0.8 and 0.4–0.6, respectively, as shown in Figure 9.

Figure 9 shows that the effects of $f_{\text{p-p}}$ and $f_{\text{p-g}}$ on the angle of repose. It can be seen from Figure 9 that the angle of repose generally tends to increase with the increase of $f_{\text{p-p}}$ and $f_{\text{p-g}}$, which is in agreement with the conclusion by Miura et al. [5]. It could be explained that the ground roughness affects the arrangement of particles at the bottom in the initial pouring stage. $f_{\text{p-p}}$ is indeed an important factor affecting the particle interaction in the process of accumulation. It increases the interaction between particles and the angle of repose and maintains the particle pile more stable. Considering the repose angle of the experimental test, 38.2°, $f_{\text{p-p}}$ and $f_{\text{p-g}}$ in these simulations were set to be 0.6 and 0.5, respectively, which is proved to be reasonable. When both of $f_{\text{p-p}}$ and $f_{\text{p-g}}$ are larger than 0.6, the angle of repose tends to be above 38°, which means the ground and the interlocking between particles provide enough resistance to maintain the upper ballast pile.

4.3. Effect of Angularity on the Repose Angle. Angularity, or its inverse, roundness, is a measure of the sharpness of the edges and corners of an individual particle. A widely

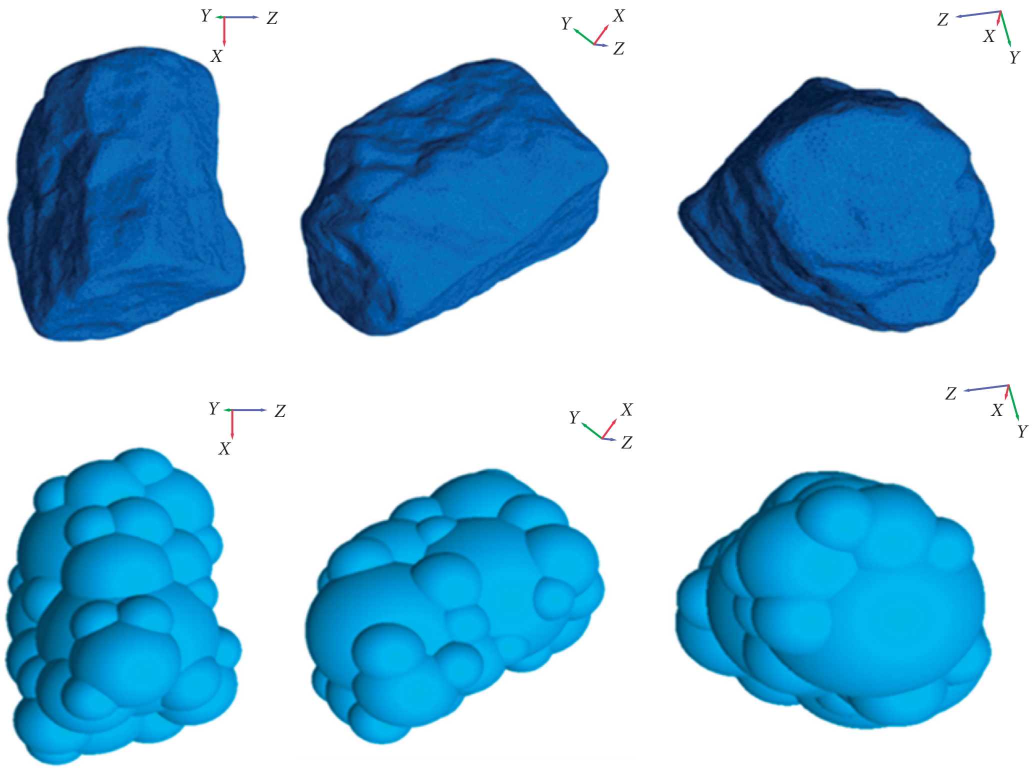


FIGURE 5: Realistic ballast model using 3D scanning technology and PDC^{3D}.

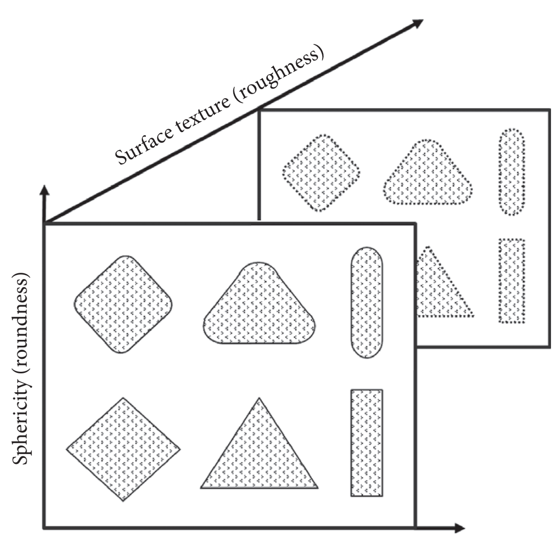


FIGURE 6: A simplified representation of form, roundness, and surface texture by three linear dimensions to illustrate their independence (modified after Barrett [20]).

accepted definition for the roundness is the ratio of the average radius of curvature of the corners and edges of a particle to the radius of the maximum inscribed circle. In this paper, the number of corners on the particles is used to represent the angularity. Previous research [22] shows that increased particle angularity increased the shear strength. However, ballast breakage increases and specimen stiffness decreases as well. Following Chen et al. [15], the tetrahedron particle is selected in this study, as shown in Figure 10. The radius of the small ball at each corner is equal to 1/17 of the radius of a large ball at the center of the tetrahedron particle. Therefore, it is considered that removing the small

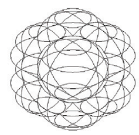
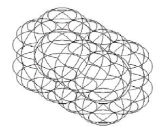
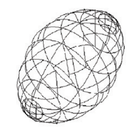
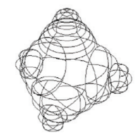
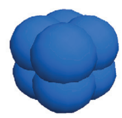
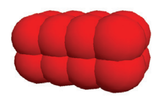
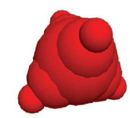
Particle shape	Cubic	Cuboid	Ellipsoid	Tetrahedron
Number of pebbles	8	16	9	9
CAD				
PFC^{3D}				

FIGURE 7: Idealized shape particles in these simulations.

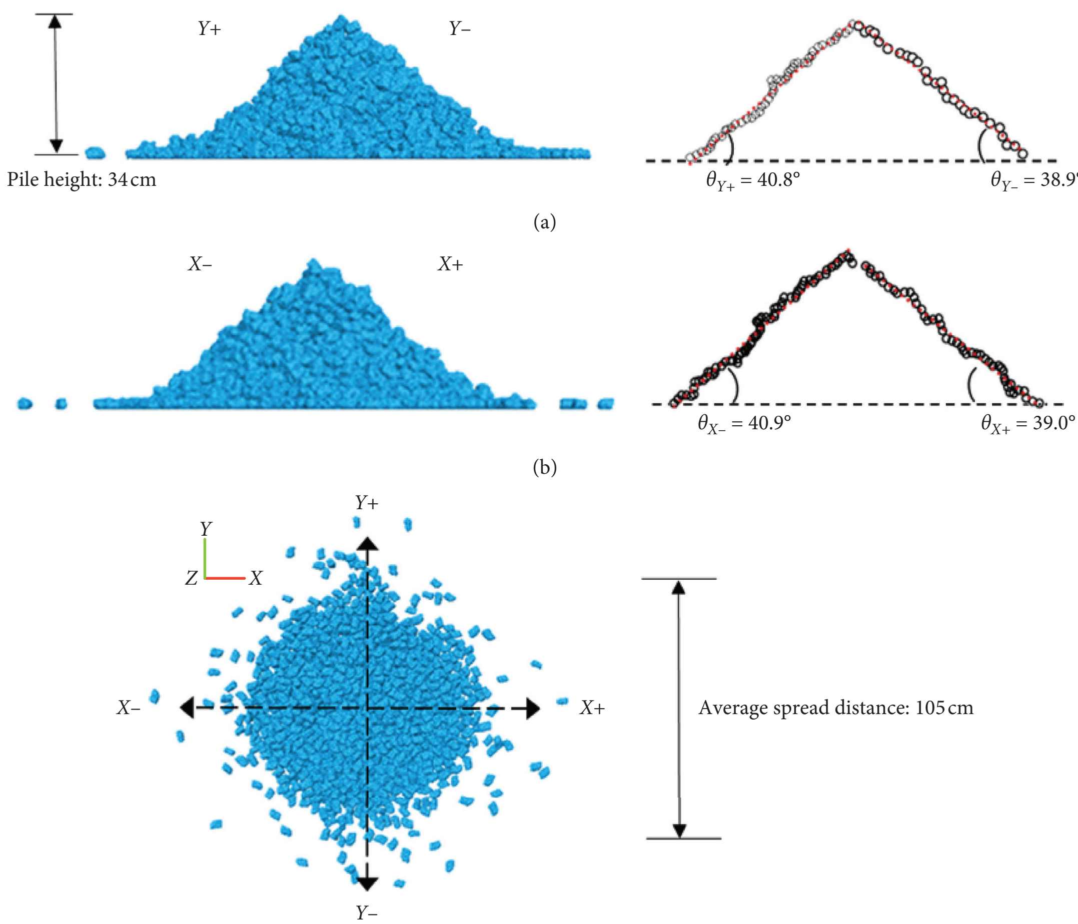


FIGURE 8: The ballast heap using realistic particles. (a) Side view. (b) Front view. (c) Plan view.

ball at the corner of the tetrahedron will not affect its overall shape characteristics, and the influence of angularity on the angle of repose could be studied. Figure 10 shows the five simulated particles with angularity from 0 to 4, and the same volume as a sphere of radius 20 mm. Figure 11 shows the measurement of pile properties for the four-ball clumps, which has a smaller average repose angle, but a wider falling area.

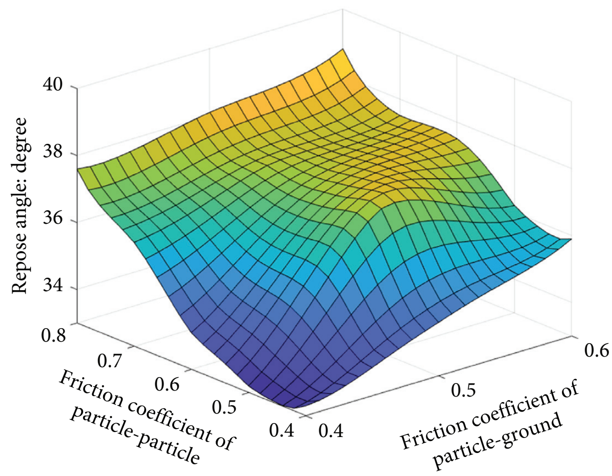


Figure 9: Effect of $f_{\text{p-p}}$ and $f_{\text{p-g}}$ on the angle of repose.

Figure 10: Idealized shaped clump used to represent the angular particles.

It can be seen from Figure 12 that with the increase of the angularity, the pile height of the tetrahedron clumps increases; meanwhile, the expansion degree, which is the average radius of the falling area, decreases accordingly. This phenomenon is consistent with the experimental test that is the higher the pile height, the more concentrated the falling area. Moreover, Figure 13 shows that the repose angle increases from 33.5 to 38.5° with the increase of the number of edges and corners from 0 to 4, which can be explained that more angular particles will tend to form a higher voids ratio and increase the possibility of interlocking between the particles, and therefore the repose angle formed will be larger.

4.4. Effect of Aspect Ratio and Sphericity on the Repose Angle. Particle shape is commonly represented by sphericity, which is defined as the ratio of surface area between a sphere and particle of the same volume. For the sphericity, cuboid and ellipsoid types of ballast models with the same aspect ratio and volume have been simulated for comparative analysis, which represents nonspherical and spherical particles. Meanwhile, in order to investigate the effect of aspect ratio, each type is simulated with three different aspect ratios, which include 1.0, 1.25, and 1.67, respectively, as shown in Figure 14. The aspect ratio can be expressed as the ratio of the particle length to the particle width. The variation of the angle of repose with aspect ratio is shown in Figures 15 and 16. The relationship between the angle of repose and aspect ratio reveals that spheres have the lowest angle of repose, which is approximately 29°, and the cuboid particles with 1.67 aspect ratio have the highest angle of repose, which is approximately 39°.

For cuboid particles, with aspect ratio increasing from 1.0 to 1.67, the angle of repose increases first, then maintains a constant between aspect ratio 1.25–1.67. For ellipsoid particles, the angle of repose decreases, then reaches a minimum at aspect ratio around 1.3, and finally increases. The relationship is similar to the variation of coordination number with aspect ratio for particle packing [23]. Thus, cuboid and ellipsoid particles follow different variation trends of angle of repose with aspect ratio. Such a difference may be caused by the sphericity difference of cuboid and ellipsoid particles. On contrast, the tetrahedron particle without corner (non-angularity) in Figure 10 and the elliptical particles with aspect ratio 1.0 (sphere) have a similar sphericity, and the repose angles of them are 33.2° and 33.7°, respectively. Therefore, the aspect ratio and sphericity affect the angle of repose together and the laws need to be further studied.

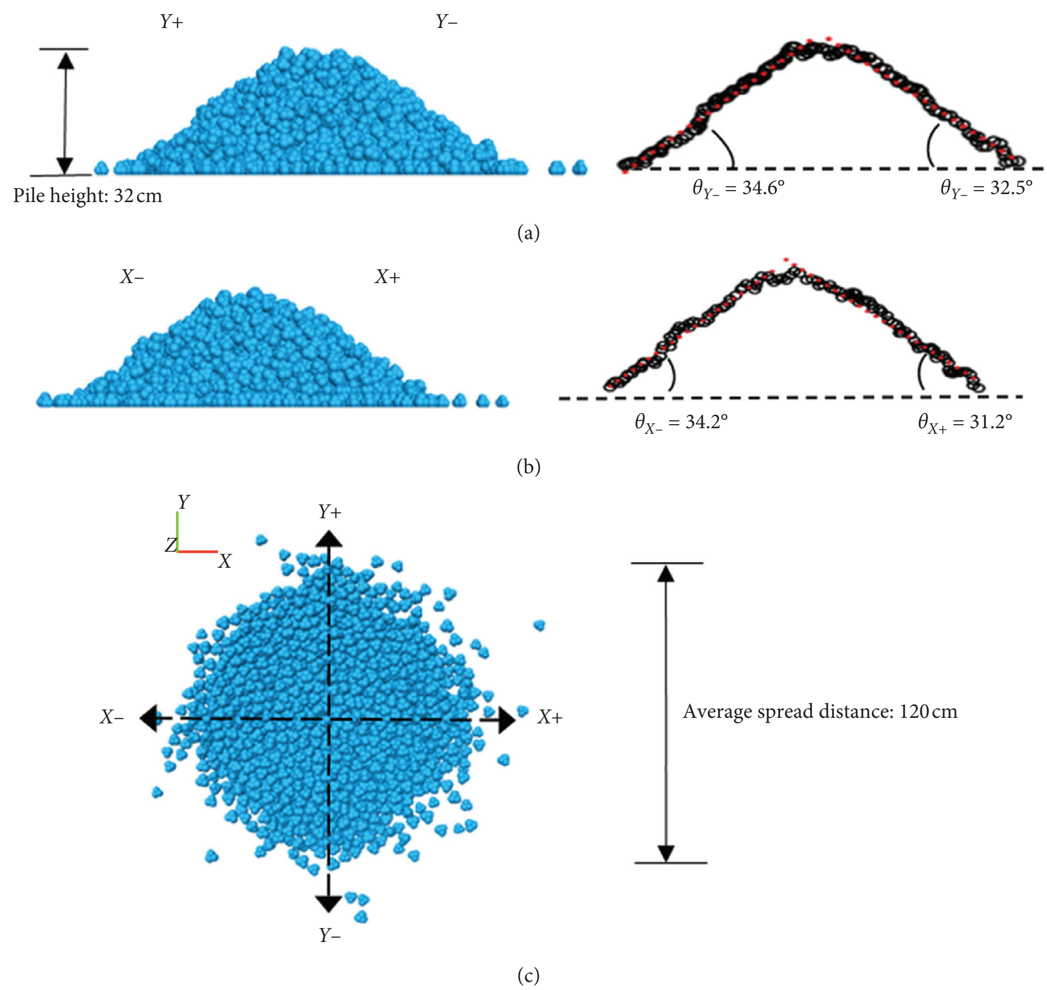


FIGURE 11: Ballast heap using four-ball tetrahedron clumps. (a) Side view. (b) Front view. (c) Plan view.

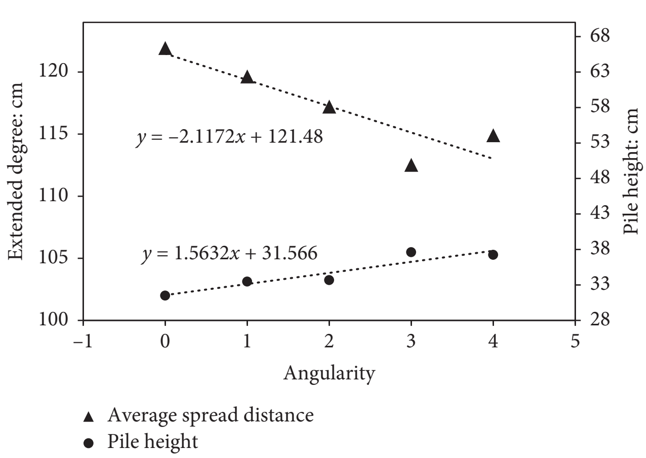


FIGURE 12: Pile height and extended degree change with the angularity.

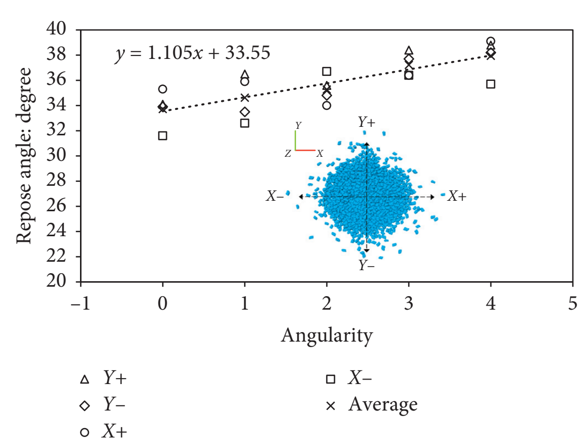


FIGURE 13: Angle of repose varies with angularity.

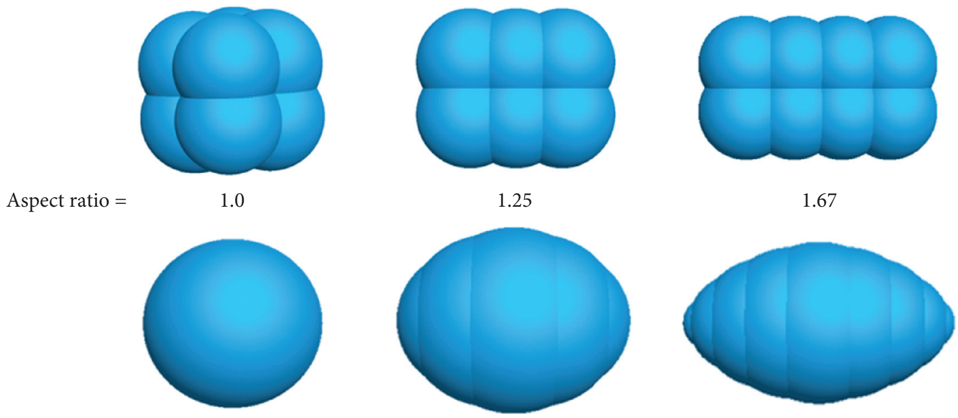


Figure 14: Arrangement of shape test for DEM numerical simulation.

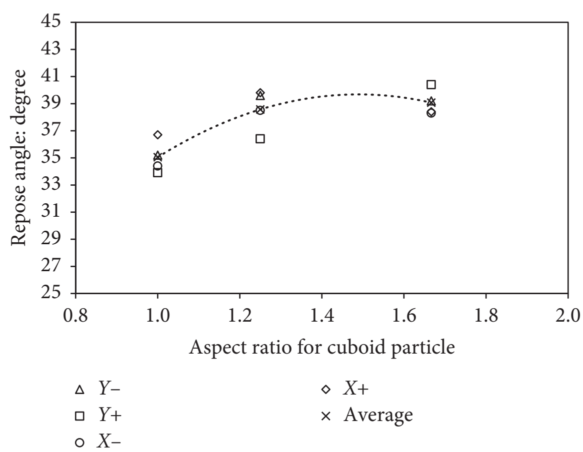


Figure 15: Angle of repose varies with the aspect ratio of cuboids.

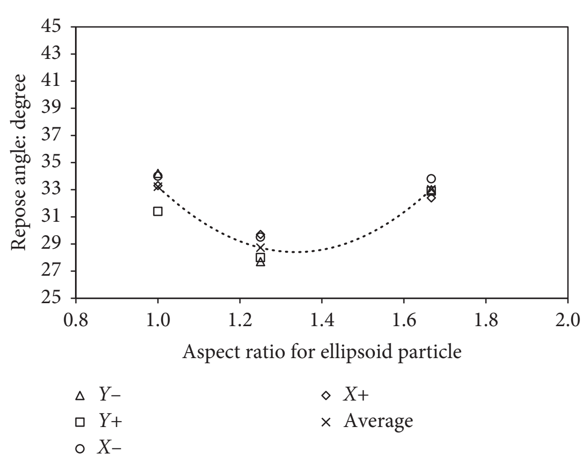


Figure 16: Angle of repose varies with the aspect ratio of the ellipsoid.

5. Conclusions

Both experimental large-scale hopper flow tests and DEM simulation using scanned realistic shape particles have been presented. Then several idealized shape particles with different shape parameters were introduced to quantify the effect of aspect ratio, angularity, the friction coefficient of particles, and roughness of ground on the angle of repose. The main conclusions are as follows:

(1) The measured repose angles of uniform graded ballast with a mean size of 40 mm are in a tolerated range from 37.1 to 39.3°, which proves the proprietary equipment of large-scale hopper flow test for coarse aggregate is suitable and reliable.

(2) The average angle of repose of DEM simulation using realistic particles is 39.9°, which is close to the angle of repose from the experimental test results. It can be concluded that DEM has a potential ability to simulate the large-scale hopper flow test. Providing similar conditions in the experiments and the simulations, the results of using an idealized shape particle could be compared with confidence.

(3) The angle of repose generally tends to increase with the increase of the friction coefficient of particles and the roughness of the ground. With the increase of the angularity, the pile height of the tetrahedron clumps increases; meanwhile, the expansion degree, which is the average radius of the falling area, decreases accordingly. The repose angle increases from 33.5 to 38.5° with the increase of the number of edges and corners from 0 to 4.

(4) For cuboid particles, with aspect ratio increasing from 1.0 to 1.67, the angle of repose increases first, then maintains a constant between aspect ratio 1.25–1.67. For ellipsoid particles, the angle of repose decreases, then reaches a minimum at aspect ratio around 1.3, and finally increases. Cuboid and ellipsoid particles follow different variation trends of the angle of repose with aspect ratio. Such a difference may be caused by the sphericity difference of cuboid and ellipsoid particles. The aspect ratio and sphericity affect the angle of repose together and the laws need to be further studied.

Generally, an angular ballast with higher aspect ratio and friction coefficient, lower sphericity, or roundness would has a larger angle of repose.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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