A Method for Determining Feasibility of Mining Residual Coal above Out-Fashioned Goaf under Variable Load: A Case Study

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Out-fashioned goaf is the protective structure for mining the upper residual coal, and its stability is the core problem in mining the upper residual coal. According to the upward mining demand for No. 5 coal seam above the out-fashioned goaf in Baizi Coal Mine, a new method is proposed to determine the upward mining safety. According to the analysis of the actual situation of the mine, the coal pillar and suspended roof in the out-fashioned goaf are taken as the objects. Furthermore, a “coal pillar-suspended roof” system model based on the variable load induced by abutment pressure of upper coal seam mining is established. After the mechanical model was solved, the parameter acquisition method of the model was established. The basic parameters of Baizi Coal Mine were considered to determine the feasibility of mining residual coal above out-fashioned goaf. And the effects of variable load on the coal pillar and suspended roof stability were analyzed. The results show that the upper No. 5 coal seam in Baizi Coal Mine can be mined safely. Compared to the traditional method, which simplifies all the upper loads to uniform loads, the new method is safer. The system stability of the suspended roof and coal pillar is influenced by “a/L” and “L.” Axial stress curves of the coal pillar and suspended roof appear nearly parabolic with “a/L” varying. Their maximum values are obtained when the “a/L” value is around 0.5~0.6. In this situation, the combination structure is most easy to be damaged. The ratio $q_1/q$ has a linear relationship with all stresses of the system model. The failure sequence of the system model is determined by analyzing the relationship between the tensile strength of the suspended roof and compressive strength of the coal pillar. This study provides a reference case for coal resources upward mining under similar conditions.

1. Introduction

The amount of residual coal exceeds 1.2 Gt [1, 2], with a great majority resulting from the utilization of the out-fashioned coal mining method. It mainly includes the reamer-pillar coal mining, roadway pillar mining, and top-coal caving mining of the lane pillar. These goafs could be called as the out-fashioned goafs. According to statistical findings, it is observed that there are abundant coal resources left in out-fashioned goafs, and the recovery ratio is from 10% to 20% [3]. These out-fashioned mining methods are used for mining high-quality coal resources in many mines for a long time. In the 1950s to 1980s, in order to ensure coal supply and short-term economic benefits, massive lower high-quality thick coal seams have been mined in preference to the upper coal seams. Because of this fact, lots of coal resources above the out-fashioned goafs are abandoned, which is widespread in China, especially in Shanxi and Shaanxi provinces [4, 5]. If the upper residual coal remains unmined until the mine is closed, it will remain forever in the ground, resulting in wastage of resources. Currently, due to shortage of energy resources, we need to exploit them to improve the
recovery rate of coal resources. However, the upper residual coal distinguishes from the common coal in mining conditions. The upper residual coal is supported by pillars and suspended roofs in the out-fashioned goaf. When the upper residual coal working face goes across the coal pillar or the suspended roof, it is very likely to cause the coal pillar instability or the suspended roof rupture, causing the collapse of personnel or equipment. Hence, the stability of the coal pillar and suspended roof is the key technique problem in upper residual coal mining above the out-fashioned goaf. We must ensure the stability of the lower coal pillar and its suspended roof during mining upper coal seams.

Many researchers analyzed the integrity of the upper coal seam and its surrounding rock stability, proposing corresponding methods for determining the feasibility of upward mining. These methods mainly include the rational analytical method, the "three-zone" method, mathematical statistics method, and the method of equilibrium surrounding rock. Their determination formulas and application conditions are shown in Table 1.

These methods are proposed after statistically analyzing the field measured data, which are derived from longwall mining conditions. But, the out-fashioned mining differentiates from the longwall mining. In the out-fashioned goaf, compared with the longwall goaf, coal pillars with irregular size and shape were left to support the roof, which results in a limited roof caving or even no caving. The upper coal seam and its surrounding rock are integral and stable. So, if these methods are adopted to determine the feasibility of the mining upper residual coal seam above the out-fashioned goaf, plentiful coal resources will be wasted due to limitations on the applicability of these methods.

Analyzing the stability of the system composed of coal pillars and suspended roofs in the out-fashioned goaf is preferred to determine the feasibility of mining the upper residual coal seam above the out-fashioned goaf. A large number of scholars have been studying these relative issues. The residual coal pillar and its roof in the lower out-fashioned goaf were simplified to the mechanical model of beam and plate by using elastic theory. In the reference, the suspended roof was simplified as a simply supported beam and clamped beam [12]. And the limited spans were calculated to judge the feasibility of mining the upper coal above the out-fashioned goaf. If the maximum length of the suspended roof exceeds the limit span, the suspended roof would be judged to be unstable. Xie et al. [17] established a math analysis model of coal pillars and suspended roofs in goaf to study the out-fashioned goaf stability through researching the bearing character of the coal pillar. The suspended roof above the lane mining goaf was simplified as a consecutive deep seam mechanical model under uniformly distributed load [18]. From the mechanical model, two failure modes of antishearing and antistretching exist and the suspended roof stability determines the feasibility of upward mining. He et al. [19] established the interaction system model of "coal pillar-roof." In the system, the hard roof and coal pillar were, respectively, regarded as elastic slabs and continuous support spring. A strain-softening model considering the damage constituent was established to describe coal pillar mechanical characteristics. Furthermore, the model was utilized to study the instability mechanism of the "coal pillar-roof" interaction system in goaf. Finally, mechanical conditions and system stability were confirmed according to the engineering condition of Majiliang Coal Mine.

Furthermore, the influence of abutment pressure on the upper coal seam has been considered in upward mining. Li et al. [20] studied the damage evolution and failure mechanism of the suspended floor under concentrated load and uniformly distributed load and put forward a floor rock fracture criterion. Based on the experiment and field measurement in Qiuci Mine, Wu et al. [21] verified that abutment pressure induced by upper coal seam mining had specific effect on the stability of the lower goaf suspended roof in upward mining. Stability conditions of the suspended roof in lower goaf were calculated through simplifying abutment pressure as uniformly distributed constant load [22]. Based on elastic-plastic theory and practical engineering condition in Shirengou Iron Mine, China, a two-dimensional fixed beam mechanical model was built to analyze the stability of bi-level goafs and isolated pillars. And the result shows that a variety of stress transmit caused by incomplete overlapping of bilevel goafs gives rise to difference in different isolated pillar zones, causing stress and displacement mutation of goafs.

In the above studies, beam or slab models were built to study the stability of the system consisting of suspended roofs and coal pillars. And abutment pressure induced by upper coal seam mining was simplified to static additional load. The system stability of the "suspended roof and coal pillar" is a criterion determining whether the upper coal seam could be mined.

These research studies solve the problem of upward mining to some extent and promote the recovery and utilization of upper coal resources above the out-fashioned goaf [23]. However, coal mining is a dynamic process. With the advance of the upper coal seam working face, its abutment pressure will change periodically. The abutment pressure induced by upper coal seam mining simplified to static additional load is a gross oversimplification. The destructive effects on the coal pillar and suspended roof caused by shifting variable abutment pressure were ignored. This may lead to a decline in the accuracy of stability judgment, which may cause the judgment to be inconsistent with the actual engineering conditions, leading to the occurrence of suspended roof collapse accident, causing damage to personnel and equipment in upper coal seam mining.

In order to ensure the safety of upper residual coal above the out-fashioned goaf, a method for determining the feasibility of upward mining considering upward mining effects was proposed first. The method is used to judge the feasibility of upward mining by judging the stability of the out-fashioned goaf. Then, taking the coal pillar and suspended roof in out-fashioned goaf as an integral object, a stability mechanical model focusing on the variable load induced by abutment pressure of upper coal seam mining was established. And the parameter acquisition method of the model was established. Thirdly, the basic parameters of Baizi Coal...
Mine were considered to determine the feasibility of upward mining residual coal above the out-fashioned goaf. And the effects of variable load on the coal pillar and suspended roof stability were analyzed.

2. Engineering Situations

2.1. Geological Conditions. As shown in Figure 1, Baizi Coal Mine is located in Xunyi County, Shaanxi Province. The width of the coal mine is 1.5 kilometers from west to east, and the length is 2.0 kilometers from south to north [17].

As shown in Figure 2, No. 5 coal seam and No. 8 coal seam are commercial seams in Baizi Coal Mine. The coal seams and their roof and floor occurrence conditions are followed. The coalbed pitches from 1° to 5°. The buried depth of No. 5 coal seam is 115m to 190m. The thickness of No. 5 coal seam is from 0.8m to 2.5m, and the average thickness is 1.6m. And the thickness of No. 8 coal seam is from 0.8m to 5.5m, and the average thickness is 3.2m. The distance between No. 5 coal seam and No. 8 coal seam is from 20m to 40m, and its average distance is 25m. The detailed roof and floor occurrence condition distributions are shown in Figure 2.

2.2. Physical and Mechanical Parameters. The physical and mechanical parameters of the rock strata of the roof and floor are essential for determining the feasibility of ascending mining. The elasticity, volume weight, cohesion, and tensile strength of lithology are acquired in Table 2.

2.3. Mining Conditions. For historical reasons, No. 8 coal seam has been mined using the reamer-pillar coal mining method, which is one of the out-fashioned mining methods. The 4m wide coal was mined, and 4m wide coal pillars were left in No. 8 coal seam with this method. But the coal pillar is irregular in size and shape (Figure 3). The mining height of No. 8 coal seam is 2.4m, and the coal resources of No. 8 coal seam have been used up. So, No. 5 coal seam was planned to be mined. The longwall mining method was planned to utilize to mine No. 5 coal seam above No. 8 coal seam out-fashioned goafs. The mining engineering plan is shown in Figure 3.

3. The Determination Method

Briefly, the method is to judge the feasibility of upward mining by judging the stability of lower out-fashioned goafs consisting of the coal pillar and their suspended roof. The stability of the coal pillar and suspended roof is determined by using the theoretical analysis method, based on structural mechanics and elastic theory [24]. Firstly, the pressure above

<table>
<thead>
<tr>
<th>Methods</th>
<th>Calculation formula and application condition</th>
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| Rational analytical method [6, 7] | $K > 6$ (single seam)  
$K > 6.3$ (multiple seam) |
| The “three-zone” method [6, 9–13] | Caving zone 1: $H_c = (h - S_s)/(b - l)$  
Caving zone 2: $H_c = 1000h/(c_1h + c_2)$  
Fracture zone 1: $H_f = 1000h/(c_3h + c_4)$  
Fracture zone 2: $H_f = m + \sum_i h_i$ (here (a) strong and hard: $c_1 = 2.1, c_2 = 16, c_3 = 1.2, c_4 = 2$; (b) medium strong: $c_1 = 4.7, c_2 = 19, c_3 = 1.6, c_4 = 3.6$; (c) soft and weak: $c_1 = 6.2, c_2 = 32, c_3 = 3.1, c_4 = 5$) |
| The method of equilibrium surrounding rock [14–16] | $H_P = M/((K_1 - 1) + h_P)$ (here $K_1, M$, and $h_P$ are, respectively, the bulking coefficient, mining thickness, and equilibrium surrounding rock thickness) |

**Table 1:** Stability determination methods of the upward mining of longwall goaf.

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the suspended roof is simplified as a constant load and the abutment pressure during mining the upper coal seam is simplified as a triangular distributed load, which is the main difference with other methods. Then, the mechanical model is established by using the theoretical analysis method; the model is utilized to study the stability of the system consisting of the coal pillar and their suspended roof of the lower out-fashioned goaf. The feasibility of mining the upper residual coal seam above the out-fashioned goaf is determined based on unstable conditions under combined action.

3.1. Model Establishment. Figure 4 provides a profile along the advancing direction of the upper longwall working face according to Figure 3. It indicates the relationship among the upper coal seam, lower residual coal pillar, and suspended roofs [25]. The suspended roofs bear the overburden load and varied distributed load induced by upper coal seam mining. An integral system model of the residual coal pillar and suspended roof is built to study the stability of the system model under combined action of overburden load and the lead abutment pressure of upper working face. In the system model, the roof is simplified as a girder which can bear axial force, shearing force, and bending moment, and coal pillar as elastic fixed support that only bears axial force, overburden load as uniformly distributed load, and the lead abutment pressure of upper working face as the triangular distributed load which is varied in length and scale. It is supposed that the coal pillar only bears axial force, and the simplified mechanical model of the suspended roofs and coal pillars is obtained (shown in Figure 4(b)).

3.2. Model Solution. The model was solved by using mechanics of materials and structural mechanics [24, 26]. The solution procedures are as follows: first, the results bearing only overburden load are calculated, and then, the results bearing only varied triangular load are calculated; finally, the final results utilizing the superposition principle are calculated.

The structure and load of the model are symmetrical when only overburden load acts. Half of the model was used to simplify the calculation and solve the mechanical model. The model before and after simplification is shown in Figure 5. According to the potential failure form of the coal pillar and suspended roof, the axial force and shearing force of the coal pillar and its roof need to be solved. So based on the static equilibrium, moment equilibrium, and deformation coordination conditions, the displacement equation of the simplified model is obtained as follows:

\[
\begin{align*}
&X_1 \left( \frac{f^3}{3E_2 I_2} + X_2 \left( \frac{f^2}{2E_2 I_2} + \frac{qL^2 l^2}{8E_2 I_2} \right) \right), \\
&X_2 \left( \frac{f^2}{2E_2 I_2} + X_3 \left( \frac{L}{E_2 I_2} + \frac{f}{E_2 I_2} \right) \right),
\end{align*}
\]

where \(X_1\) is the axial force of the suspended roof; \(M_2\) is the bending moment of the suspended roof; \(X_3\) is the maximal shearing force of the suspended roof; \(F_N\) is the axial force of the coal pillar; \(I_1\) is the inertia moment of the suspended roof; \(I_2\) is the inertia moment of the coal pillar; \(L\) is the distance of two adjacent pillars; \(l\) is the height of the pillar; \(q\) is the load intensity of the overburden load of the suspended roof.
The upper longwall goaf follows: \( q \) is the load intensity of the varied triangular load; and \( a \) is the range of the varied triangular load.

The simultaneous equation (1) is solved, and the results are as follows:

\[
X_1 = \frac{qE_2I_2L^3}{\left(8E_1I_1I_2^2 - 4E_2I_2I_2' \right)},
\]

\[
X_2 = \frac{qL^3E_2I_2}{\left(24E_1I_1I_2^2 - 12E_2I_2I_2' \right)},
\]

According to the force balance in the vertical direction, utilizing the above results, the axial force of the coal pillar \( F_{NC} \) is calculated using the following formula:

\[
F_{NC} = \frac{qL}{2}.
\]

As shown in Figure 6, when the model only bears varied triangular load, the models before and after simplification are given. In order to obtain the axial stress, bending moment, and shearing stress of the suspended roof, the displacement equation of the simplified model is obtained as follows:

\[
\begin{aligned}
X_1' &= \frac{L^3}{\left(24E_1I_1I_1' + \frac{L^2I_2'}{8E_2I_2} \right)} = 0, \\
F_{NC} &= X_1' + \frac{q' a}{2},
\end{aligned}
\]

where \( X_3 \) is the maximal shearing force of the suspended roof and \( F_{NC} \) is the axial force of the coal pillar under uniform load.

The simultaneous equations (4) are solved, and the results are shown below:

\[
\begin{aligned}
X_1' &= \frac{2E_1I_1q' aL^2I'}{\left((3E_1I_1L + 8E_2I_2I_2')L^2 \right)}, \\
F_{NC}' &= \frac{2E_1I_1q' aL^2I'}{\left((3E_1I_1L + 8E_2I_2I_2')L^2 \right)} + q' a.
\end{aligned}
\]

Then, the methods of moment distribution are utilized to obtain the torque equilibrium equation to solve the moments of coal and the suspended roof:

\[
\begin{aligned}
M_1\left(\frac{L}{2E_1I_1} + \frac{I'}{E_2I_2} \right) + M_2\left(\frac{L}{2E_1I_1} + \frac{I'}{E_2I_2} \right) &= \frac{q' a(L-a)}{32E_1I_1} + \frac{q'I'(L-a)}{16E_2I_2}, \\
M_1\left(\frac{L}{E_2I_2} + M_2\left(\frac{L}{E_2I_2} + \frac{I'}{E_2I_2} \right) &= \frac{q' a(L-a)}{8E_2I_2}, \\
M_1 &= 2M_{ZR} = 2M_{ZL}.
\end{aligned}
\]

where \( M_{ZR} \) is the bending moment of the right suspended roof; \( M_{ZL} \) is the bending moment of the left suspended roof; and \( M_{M} \) is the bending moment of the coal pillar.

The simultaneous equations (5) and (6) are solved, and the result is shown as follows:

\[
\begin{aligned}
M_{ZL} &= \frac{q' a(L-a)}{2\left(8E_1I_2I_2' + 32E_1I_1I_2I_2' \right)} \\
M_{ZR} &= \frac{q' a(L-a)}{8E_1I_2I_2' + 32E_1I_2I_2'}
\end{aligned}
\]

As shown in Figure 6, we calculate the stress, bending moment, and shearing force of the suspended roof when it bears inverse varied triangular load. The mechanics equilibrium equations are given as shown below:

\[
\begin{aligned}
X_3'' &= F_N'' = \frac{q' a}{2}, \\
M_3'' &= \frac{q' a(L-a)}{4},
\end{aligned}
\]

where \( X_3'' \) is the maximal shearing force of the rock stratum of the roof; \( F_N'' \) is the axial force of the coal pillar under uniform load.
triangular load; and $M_M'$ is the bending moment of the coal pillar.

So far, the mechanical equation of the coal pillar and the suspended roof in the situation of bearing variable triangular load is obtained. According to equations (2), (5), (7), and (8), the superposition principle is utilized to calculate the results of the system model:

$$
X_1 = \frac{qE_1L_2l_3}{8E_1I_1l_2^4} - \frac{qL}{2} + \frac{qa}{2},
$$

$$
X_3 = \frac{2q'aE_1I_1L_2l_3}{3E_1I_1l_2^4} - \frac{qL}{2} + \frac{qa}{2},
$$

$$
F_N = \frac{2q'aE_1I_1L_2l_3}{3E_1I_1l_2^4} - \frac{qL}{2} + \frac{qa}{2},
$$

$$
M_M = \frac{q'a(L-a)L^2E_1I_1^2}{4} + \frac{q'a(L-a)L^2E_1I_1^2}{16E_1I_1l_2^4} + \frac{q'a(L-a)L^2E_1I_1^2}{64E_1I_1l_2^4} + 8E_1I_1l_2^4 + 32E_1I_1l_2^4, \quad (9)$$

where $M_M$ is the moment of the suspended roof; $M_M$ is the moment of the coal pillars; and $F_N$ is the combined axial force of coal pillars.

In order to determine the stability of the model, the maximal axial stress and shearing stress should be calculated [26]. Considering the bending stress derived from force and bending moment, the maximal axial stress and shearing stress are acquired using the following equation:

$$
\begin{align*}
\sigma_{\text{max}} &= \frac{M_{\text{max}}}{W}, \\
\tau_{\text{max}} &= \frac{F_Q S_{Z_{\text{max}}}}{I_p b},
\end{align*}
$$

where $\sigma_{\text{max}}$ is the maximal axial stress; $\tau_{\text{max}}$ is the maximal shearing stress; $W$ is the section modulus in bending moment of the suspended roof, $W = bh^3/6$; $M_{\text{max}}$ is the maximal moment of beam, such as the moment of the suspended roof $M_Z; F_Q$ is the shearing force of the beam, the suspended roof $X_N$; $S_{Z_{\text{max}}}$ is the static moment of the suspended roof, $S_{Z_{\text{max}}} = bh^3/6$; and $I_p$ is the moment of inertia of the suspended roof.

The stress of axial and shearing is correlated with the moment of the beam, and the detailed relationship is shown in formula (11). The moment of the suspended roof is the moment of the beam. The axial stress and shearing stress of the suspended roof and axial stress of the coal pillar (formula (12)) are derived from formulas (9), (10), and (11):

$$
\begin{align*}
\tau_{\text{max}} &= \frac{2E_1I_1q'L(l-a)l'}{(3E_1I_1l_2^4 + 8E_2I_2l_2^4)} b_1 h_1 + \frac{qL}{2b_1 h_1} + \frac{qa}{b_1 h_1}, \\
\sigma_{\text{max}} &= \frac{1}{b_1 h_1} \left( \frac{qL^3E_1I_2}{(4E_1I_1l_2^4 - 2E_2I_2l_2^4)} b_1 + \frac{qE_2I_2l_3}{l_2} (8E_1I_1l_2^4 - 4E_1I_1l_2^4) b_1 + \frac{3q'(L-a)a(L^2E_1I_1^2 + 12l^2E_1I_1^2)}{4E_2I_2l_2(8E_2I_2L + 16E_1I_1l_2^4)} b_1 \right), \\
\sigma_{\text{max}} &= \frac{1}{b_2 h_2} \left( \frac{3q'(L-a)a(L^2E_1I_1^2 + 12l^2E_1I_1^2)}{2b_2 h_2} + \frac{q'a(L-a)(3E_2I_2L_2^4 + 12E_2I_2l_2^4)}{E_2I_2L_2(8E_2I_2L + 32E_1I_1l_2^4)} b_2 h_2 \right) + \frac{2q'aE_1I_1L_2l_3}{(3E_1I_1l_2^4 + 8E_2I_2l_2^4)} + \frac{q'a + qL}{4E_2I_2l_2L_2^4} \right),
\end{align*}
$$

\begin{align*}
(12) \\
(13)
\end{align*}
where $\sigma_{\text{max}}^R$ is the maximal shearing stress of the suspended roof; $\sigma_{\text{max}}^C$ is the maximal normal stress of the suspended roof; $b_1$ is the width of the transverse section of the suspended roof; $h_1$ is the height of the transverse section of the suspended roof; $\sigma_{\text{max}}$ is the bearing normal stress of the coal pillar; $b_2$ is the width of the transverse section of the coal pillar; and $h_2$ is the height of the transverse section of the coal pillar.

The shearing stress of the coal pillar has little influence on the strength stability of the out-fashioned goaf. So when we utilized the abovementioned results of the system stability of the coal pillar and the suspended roof, the effect of shearing stress on the coal pillar can be ignored.

3.3. Model Parameters Determination Method. The basic parameters of the determination model are acquired by engineering geologic reports, operational rules, and rock mechanical tests. In addition, the suspended roof, load, and compression strength of the coal pillar are determined by the following methods [27].

3.3.1. Suspended Roof. In the system model, the suspended roof is a key rock stratum. The suspended roof is the main structure that bears overburden load and abutment load. According to the theory of the key rock stratum [28], the key rock stratum is the stratum bearing the overburden load and varied distributed load induced by upper coal seam mining. The stability of the key rock stratum is a key question to studying ascending mining feasibility. So, the identification of the key rock stratum of No. 8 coal seam should be implemented. According the theory of key stratum, the method of identifying the key rock stratum is given in the following formula:

$$
q_1(x)_n = \frac{E_1d_1^2\sum_{i=1}^{n} \rho_i d_i}{\sum_{i=1}^{n} E_i d_i^2},
$$

$$
q_1(x)_n = q_1(x)_{n+1},
$$

where $q_1(x)_n$ is the load of the $n$th stratum; $q_1(x)_{n+1}$ is the load of the $(n+1)$th stratum; $E_i$ is the elasticity modulus of the first stratum; $E_i$ is the elasticity modulus of the $i$th stratum; $d_i$ is the thickness of the first stratum; and $\rho_i$ is the density of the $i$th stratum.

3.3.2. Load. The loads of overburden and abutment pressure are the key parameters [5]. Solving the vital parameters has two steps. Firstly, the pressure of overburden and abutment should be obtained. The methods of obtaining the pressure are similar to the stimulated test and field measurements [9, 29]. And then the load can be calculated by

$$
q = P_q \times L,
$$

$$
q' = q_z \times L,
$$

$$
q_z = 0.637(P_Q - P_q) \left[ \arctan \frac{x_0l_0}{z \sqrt{4x_0^4 + l_0^2 + 4z^2}} \right] + \frac{l_0z \sqrt{4x_0^4 + l_0^2 + 4z^2} - \sqrt{l_0^2 + 4z^2}}{2x_0(l_0^2 + 4z^2)},
$$

where $q$ is the load of the overburden; $q'$ is the load of the abutment pressure; $P_q$ is the mine ground pressure when mining upper coal seam; $P_Q$ is the mine ground pressure when periodic weighting of ascending mining occurs; $x_0$ and $l_0$, respectively, are the width and length of the load of overburden; and $z$ is the depth of floor rock strata.

3.3.3. Compression Strength of Coal Pillar. The compression strength of the coal pillar correlates with uniaxial compressive strength and coal pillar size [30]. The uniaxial strength of the laboratory coal sample was converted into the uniaxial strength of the critical cube by using the Hustrulid method. The Hustrulid method estimates the size effect of the coal pillar in the field. The formula of the Hustrulid method is shown below:

$$
\sigma_m = \sigma_c \sqrt[3]{\frac{D}{d^2}},
$$

where $D$ is the sample diameters; $d$ is the sample height; $\sigma_c$ is the compression strength of the coal sample; and $\sigma_m$ is the uniaxial strength of the critical cube.

Furthermore, the strength of the coal pillar in the field is calculated using the relationship of uniaxial strength of the critical cube and coal pillar [31]. The representative strength converting formula is as follows:

$$
\sigma_{s1} = \sigma_m \left[ 0.778 + 0.222\left( \frac{W}{h} \right) \right],
$$

$$
\sigma_{s2} = \sigma_m \left( \frac{W}{h} \right)^{0.446},
$$

$$
\sigma_s = \min\{\sigma_{s1}, \sigma_{s2}\},
$$

where $W$ is the width of the coal pillar; $h$ is the height of the coal pillar; and $\sigma_s$ is the uniaxial compression strength of the coal pillar.
4. Results and Discussion

4.1. Parameters. The study data of Baizi Coal Mine [32] are utilized to verify the validity of theoretical formulas in the following. According to engineering situation, the fundamental data are followed. The distance of two adjacent coal pillars \( L \) is 4 m; the mining height of No. 8 coal seam \( h \) is 2.4 m. Physical and mechanical parameters, derived from Table 2, are shown in Table 3.

The key stratum of No. 8 coal seam, the load of overburden and abutment pressure, and uniaxial compression strength of the coal pillar are vital for the feasibility determination of mining upper coal seam. So, the vital parameters of the suspended roof, load, and coal pillar are solved in the following.

Firstly, combining the geological data in Table 1 and the key stratum determination formula (13), the key stratum is determined. The cohesion of siltite in No. 8 coal seam roof is 12.2 MPa; the tensile strength is 5.1 MPa. The siltite is the key rock strata. So, the siltite is the suspended roof. Parameters of the key rock strata were determined. The thickness of siltite \( h_1 \) is 6.7 m. So, the parameters of the suspended roof are shown in Table 3.

Furthermore, the vertical pressure data of No. 5 coal seam floor, measured by stress sensor, are collected and analyzed. We find that the pressure of No. 5 coal seam floor is equal when the first pressure is approaching; the vertical pressure \( P_v \) is 2.67 MPa. No. 8 periodic pressure significantly occurs in the procedure of mining upper coal seams. The maximum pressure \( P_Q \) is 4.3 MPa. According to formula (14), the load intensities \( q \) and \( q' \) are calculated in Table 3.

Finally, the compression strength of the coal pillar correlates with uniaxial compressive strength and coal pillar size. The test specimens are round; specimen diameter is \( D = 5 \) centimeters, and specimen height is \( d = 10 \) centimeters. The laboratory experiment was carried out on the coal sample of No. 8 coal seam to get the uniaxial compressive strength \( \sigma_s \) is 15.7 MPa [17]. The uniaxial strength of the critical cube is converted by using formula (17) and formula (18). The compression strength of the coal pillar \( \sigma_s \) is 11.8 MPa.

4.2. Judgment. The maximal shearing stress of the suspended roof is \( \tau_{max} = 1.595 \) MPa, and it is under the shearing strength of suspended roof \( \tau = 8.92 \) MPa. The maximal tensile stress of the suspended roof is \( \sigma_{max} = 3.98 \) MPa, and it is under the uniaxial tensile strength of the suspended roof \( \sigma_{1} = 5.1 \) MPa. The maximal axial stress of the coal pillar is \( \sigma_{max} = 10.2 \) MPa, and it is under the axial strength of the coal pillar \( \sigma_2 = 11.1 \) MPa. These data mean that the system strength of the suspended roof and coal pillar is more than the stress on the system. Hence, the system of the coal pillar and suspended roof is stable which is determined by this method. And the results corresponded with the practical engineering projects.

4.3. Effects of Variable Load. The varying values of \( a/L \) are used to measure the influence of load movement on axial force and shear force in the model. The range of varied triangular load \( \tau \) is associated with the distance of the adjacent coal pillar. According to formulas (11) and (12) and the Baizi coal mine’s conditions, when \( a/L_0 \) is 0.1, 0.2, 0.3, . . . , 1, the effect of the varied triangular load for the maximal axial stress and shearing stress of determined formulas on system stability is analyzed. With varying \( a/L \), the maximal stress of rock and coal pillar is shown in Figure 7.

As shown in Figure 7, the variation trend of axial stress of the residual coal pillar and suspended roof is similar. As \( a/L \) increases, these stresses first go up and then go down. But with the increase of \( a/L \), the peak value of shear stress in the coal pillar appears later than that in the suspended roof. Synchronous changes in the position and size of variable loads are the reason.

As shown in Figure 7, in order to analyze the effects of the varied load, we used \( (q + q') \) as the uniform load calculation stress of the model when \( a/L = 0 \). The maximal shear and axial stress of the suspended roof as well as the maximal axial stress of the coal pillar are calculated as 1.07 MPa, 0.026 MPa, and 3.6 MPa. Without considering the varied load, the maximal stress of the model is much smaller than when considering the varied load. This indicates that whether the load movement considered has a great influence on the stress when the load value is the same. Obviously, the results obtained by this method are more secure.

In general, the compressive strength of coal and rock mass is vital in all strength [33]. The shear strength is greater than the tensile strength. Furthermore, the shearing stress of the suspended roof is small and increases slowly. So, the suspended roof is not subject to shear failure. Whether the suspended roof tensile failure or coal pillar compression failure occurs firstly? The calculated stress in the pillar is greater than that of the rock. However, for coal and rock mass, failure sequence cannot be ensured from the curves. It is determined by the relationship between the tensile strength of the suspended roof and the compressive strength of the coal pillar.

4.3.1. Load Range Effect for Suspended Roof. From practical ascending mining engineering project cases [19, 21, 22], it is not difficult to find that the system strength stability of the residual coal pillar and suspended roof can be mainly influenced by the parameters \( L \) and \( a/L \). Hence, the regulations of system stability are studied through the control variables approach. The influence of regulations of the stability of the suspended roof is studied in detail firstly in the following.

According to formulas (11) and (12) and Table 3, the relationship of stress of the suspended roof and coal pillar with \( a/L \) and \( L \) was analyzed. When \( L = 4, 8, 12, 16, 20 \) meters and \( a/L \) is 0.1, 0.2, . . . , 1.0, the shear stress and axial stress of the suspended roof and coal pillar are, respectively, calculated and displayed in Figures 8–10.

As shown in Figure 8, the shearing stress of the suspended roof has a positive correlation with \( a/L \). When \( L \) is larger, the shearing stress curves of the suspended roof are steeper. It indicates that when the length of the suspended roof is longer, the system stability of the
suspended roof and coal pillar is lower, whereas other facts are equal.

In practical mining engineering projects, when the length of the suspended roof gets wider, the system strength stability will be destroyed more easily. And the shearing stress of the suspended roof increases with the increase of the varied load range on the suspended roof. Based on the aforementioned analysis results, engineers in coal mine can calculate the maximal length and the varied load range to determine whether the suspended roof can be fractured.

The relationship of axial stress of the suspended roof with \( a/L \) is shown in Figure 9. The curves show a parabolic shape with downward opening. For different \( L \), the axial stress curves are higher and steeper with increasing \( L \). All axial stress curves of the suspended roof have a positive correlation with \( L \). When \( L \) increases, the curves converge more. It is indicated that the axial stress increases more and more quickly than the shear stress. For all curves in Figure 9, when \( a/L \) is about 0.5, the axial stress curves reach maximum. These results can instruct practical engineering operations. For practical mining engineering projects, when \( a/L \) equals to about 0.5, the stability of the system of the coal pillar and suspended roof will be destroyed more easily. Hence, this situation should be avoided in the process of ascending mining.

4.3.2. Load Range Effect for Coal Pillar. The relationship curves of axial stress of the coal pillar with \( a/L \) are shown in Figure 10. The curves show a parabolic shape with downward opening which have a similar trend with the curves of axial stress of the suspended roof. The axial stress of the coal pillar has a positive correlation with the value of \( L \). Hence, the distance of two adjacent coal pillars should be given priority consideration.

Furthermore, when all values of \( a/L \) is about 0.5, the axial stress curves reach maximum. These results can instruct practical engineering operations. For practical mining engineering projects, when \( a/L \) equals to about 0.5, the strength stability of the system of the suspended roof and
coal pillar is used to determine the feasibility of upward mining above the out-fashioned goaf.

4.3.3. Load Magnitude Effect for the System Model. With all other conditions being equal, we changed the value of variable load q′ (such as 0.2q, 0.4q, 0.6q, 0.8q, and 1.0q). And the stress curves of the system model are derived with the varying ratio of q′/q (Figure 11).

Figure 11 shows that the ratio of q′/q has a linear relationship with all stresses of the system model. The slope of shearing stress curves is the smallest. The ratio of q′/q has least effect on shearing stress of the suspended roof, which is consistent with the conclusion that shearing force does not affect roof failure [34]. When the ratio of “a/L” is the same, the slopes of the curves between axial stress of the suspended roof and coal pillar are very close, indicating that the axial force of the suspended roof and coal pillar increases nearly synchronously with the change of ratio of q′/q. The failure sequence of the system model depends on the relationship between the tensile stress of the suspended roof and the compressive strength of the coal pillar. According to the linear relationship shown in Figure 11, the failure sequence of the system model is calculated using the following formula:

\[ \frac{\sigma_s}{\sigma_t} = f \left( \frac{q'}{q} \right) = \frac{k_c (q'/q) + \sigma_{sI}}{k_r (q'/q) + \sigma_{tI}} \]  \hspace{1cm} (19)

where \( \sigma_{sI} \) and \( \sigma_{tI} \) are, respectively, the axial stress of the suspended roof and coal pillar when \( q' \) equals to zero; \( k_c \) and \( k_r \) are, respectively, axial stress curve slopes of the suspended roof and coal pillar; \( \sigma_s \) is the compression strength of the coal pillars; and \( \sigma_t \) is the tensile strength of the suspended roof.

For cases of varying ratio of \( (q'/q) \), the relationship between \( f \left( \frac{q'}{q} \right) \) and the ratio of \( \sigma_s/\sigma_t \) are compared to determine whether the coal pillar or suspended roof will be damaged first, which is more advantageous for field technicians.

5. Conclusions

A method for determining the feasibility of upward mining residual coal above the out-fashioned goaf was studied, and the following conclusions were obtained:

1. Variable additional load is mainly considered to simulate the influence of abutment pressure caused by upper coal seam mining. On this basis, the suspended roof and coal pillar in the lower out-fashioned goaf are taken as objects to establish the mechanical model. The mechanical model solution and parameter acquisition method were also established. The practical engineering data of Baizi Coal Mine were utilized to analyze the relationship of stress with “a/L” and “L.”

2. Compared with the system without variable load, the ultimate stresses obtained by the system model considering the variable load are much higher. Therefore, compared with this method, the traditional method, which simplifies upper loads to uniform loads, leaves a greater safety factor to ensure the safe mining of the upper coal seam above the out-fashioned goaf.

3. Through the analyses of determining formula of key parameters, the system stability of the suspended roof and coal pillar is influenced by “a/L” and “L,” and the effect of “a/L” is significant for axial and shearing stress. Axial stress curves of the coal pillar and suspended roof appear nearly parabolic with
varying “a/L.” Their maximum values are obtained when the values of “a/L” are 0.5–0.6. In this situation, the combination system consisted of the coal pillar and suspended roof is most easy to be damaged. With the increase of the distance L between the two adjacent coal pillars, the stress of the model increases, and the axial stress of the coal pillar increases the fastest. The ratio of q'/q has a linear relationship with all stresses of the system model. The failure sequence of the system model is determined by analyzing the relationship between the tensile strength of the suspended roof and compressive strength of the coal pillar.

(4) Under the action of variable load caused by No. 5 coal seam mining in Baizi Coal Mine, the suspended roof and coal pillar of the lower out-fashioned goaf remain stable, which is consistent with the results of engineering practice. This study provides a reference for the mining of coal resources under similar conditions.

Data Availability

The method was validated by the data provided in Ref. [32].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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