Research Article

Numerical Modelling of Interaction between Hydraulic Fractures and Natural Fractures by Using the Extended Finite Element Method

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Natural fractures usually develop in shale reservoirs. Thereby, in the process of hydraulic fracturing, it is inevitable that hydraulic fractures will intersect with natural fractures. In order to reveal the interaction mechanism between hydraulic-induced fractures and natural fractures, a two-dimensional fracture intersection model based on the extended finite element method (XFEM) is proposed, and the different types of intersecting criteria reported in the literature are compared. Then, the effects of natural fracture azimuth, fluid pressure in hydraulic fracture, and in situ principal stress difference on hydraulic fracturing are studied in detail. The results show that the fracture morphology is different under different criteria and working conditions. And the stress concentration phenomenon mainly concentrates on the tip in the obtuse angleside of natural fracture. Meanwhile, different fluid pressures in hydraulic fracture can also induce different intersection patterns. The obtained results in this work are of great benefit to understand the interaction mechanism between hydraulic fractures and natural fractures.

1. Introduction

Generally, shale reservoirs have very low porosity and permeability. Thus, hydraulic fracturing technique is widely employed to generate hydraulic/induced fractures and enhance the productivity of shale reservoirs. There are many discontinuities in shale reservoirs, such as natural fractures/cracks, bedding planes, and faults [1]. Natural fracture is probably one major reason for possible change in hydraulic fracture morphology from what it was originally expected. In other words, the interaction between hydraulic fractures and natural fractures arises commonly in naturally fractured low-permeability reservoirs [2]. Meanwhile, natural fractures have production capacity only when they are activated by induced fractures [3]. Thus, understanding the interaction mechanism between hydraulic fractures and natural fractures is of great significance for the stimulation of shale reservoirs.

Extensive theoretical works have been conducted to investigate the interaction mechanism between hydraulic fractures and natural fractures based on experiments [2, 4–6]. Gu and Weng [7] extended the Renshaw and Pollard results to apply to the nonorthogonal intersecting angle condition. Compared to theoretical and experimental methods, numerical methods can simulate the interaction process under more complex geological conditions. Numerous numerical works have been carried out on the basis of the discrete element method (DEM) [8] and the displacement discontinuity method (DDM) [9–11]. Zhang et al. [12] investigated the interaction mechanism between hydraulic fractures and natural fractures with a hybrid discrete-continuum method. Huang et al. [13] proposed different 3D perforation models of hydraulic fracture initiation and near-wellbore propagation. And, the pressure characteristics and performance of multistage fractured horizontal well have been analyzed with coupled flow and
geomechanics [14]. In addition, the full coupled multiscale numerical model has been introduced to investigate hydraulic fracturing complexity in naturally fractured rock masses [15]. Belytschko and Black [16] presented a minimal remeshing finite element method for crack growth, which is referred to as the extended finite element method (XFEM). Therein, the enrichment functions are added to the finite element approximation to account for the presence of the cracks. The XFEM allows the crack to be arbitrarily aligned within the mesh, overcomes the limitations inherent in the mesh shape, and can efficiently simulate fractures without remeshing. Dahi-Taleghani and Olson [17, 18] investigated mesh shape, and can efficiently simulate fractures without remeshing. Dahi-Taleghani and Olson [17, 18] investigated the intersection between hydraulic fractures and natural fractures based on a plane strain XFEM model. Xu et al. [19] developed the additional junction enrichment to describe branched elements. Wang et al. [20] established a fluid-solid coupled hydraulic fracturing model using the XFEM to simulate the interactions between hydraulic fractures and natural fractures and further investigate the formation of fracture network. There are a lot of criteria for the intersection. It is important to adopt the most appropriate criterion in different situations. That is, the intersection between hydraulic fractures and natural fractures still needs further study. Therefore, a two-dimensional crack intersection model based on the XFEM is proposed in this work. In addition, different types of intersection criteria reported in previous studies are used to numerically simulate the interaction in the fracturing process.

2. Intersection Model

The intersection of hydraulic fractures and natural fractures is a complex problem involving rock deformation, fluid flow and distribution at the joint point, and influence of in situ stresses, fluid pressure, and geometry of the interaction region as well as friction coefficient. The crossing criterion is described as follows [21]:

(1) When fluid pressure in hydraulic fracture is small, the induced fracture will be arrested due to the energy dissipation of induced fracture.

(2) If fluid pressure of hydraulic fracture is large enough to open the natural fracture, the hydraulic fracture will deflect to the natural fracture and propagate along the direction of natural fracture.

(3) If the natural fracture is very difficult to open, while fluid pressure in hydraulic fracture is large enough, the natural fracture will be penetrated, and the hydraulic fracture will propagate along the initial pathway.

(4) If the natural fracture is easy to open and fluid pressure in hydraulic fracture is large enough, the hydraulic fracture may penetrate the natural fracture while the natural fracture is opened simultaneously.

In this section, we present a brief review of several analytical criteria of interaction between hydraulic fractures and natural fractures and explain how they can be applied in the XFEM.

2.1. Blanton’s Criterion. Blanton’s criterion [4] considers that when hydraulic fracture reaches natural fracture plane, the fluid pressure at the intersection point will continue to rise until the induced fracture either opens or penetrates the natural fracture. If the required pressure for reinitiation is lower than the opening pressure, the induced fracture will penetrate the natural fracture and continue to propagate along the original direction. As shown in Figure 1, fluid pressure $p$ is imposed on the hydraulic fracture plane, $\theta$ is the angle of approach, $\sigma_{h\text{max}}$ and $\sigma_{h\text{min}}$ are the maximum and minimum in situ horizontal principal stresses, and $\sigma_n$ and $\tau$ are the normal and shear stresses acting on the natural fracture plane, respectively. Also, $\sigma_T$ is the normal stress component acting parallel to the natural fracture which depends on the in situ stresses, fluid pressure, and geometry of the interaction region as well as friction coefficient. The crossing criterion is described as follows:

$$p > \sigma_T + T_0,$$

where $T_0$ is the tensile strength of the rock. Accordingly, Blanton derived the following crossing criterion:

$$\frac{\sigma_{h\text{max}} - \sigma_{h\text{min}}}{T_0} > \frac{-1}{\cos 2\theta - b \sin 2\theta}$$

where $\sigma_{h\text{max}} - \sigma_{h\text{min}}$ is called the in situ horizontal principal stress difference.

Here, $b$ is defined as

$$b = \frac{1}{2c} \left[ v(x_0) - \frac{(x_0 - l)}{\mu_f} \right],$$

where $x_0$ is the point at which reinitiation will occur, $c$ is the zone of slippage in the natural fracture, $l$ is the half length of crack, and $\mu_f$ is the friction coefficient of the natural fracture plane. $x_0$ and $v(x_0)$ are given by

$$x_0 = \left[ \frac{(1 + c)^2 + e^{\pi(\omega r)}}{1 + e^{\pi(\omega r)}} \right]^{0.5},$$

$$v(x_0) = \frac{1}{\pi} \left[ (x_0 + l) \ln \left( \frac{x_0 + l + c}{x_0 + l} \right) + (x_0 - l) \ln \left( \frac{x_0 - l + c}{x_0 - l} \right) + c \ln \left( \frac{x_0 + l + c}{x_0 - l - c} \right) \right].$$
When the tensile strength of rock is 3.5 MPa, the chart of Blanton’s criterion as shown in Figure 2 can be obtained. In addition, Blanton’s criterion ignores the influence of fluid pressure on the induced stress at the natural fracture interface and assumes that the shear stress is a simple linear distribution along the natural fracture interface.

2.2. W&T’s Criterion. Warpinski and Teufel [5] claim that the forming of intersection is determined by the net fluid pressure, $p_n$, which is defined as the fluid pressure in hydraulic fracture minus the minimum horizontal stress. Opening will take place if the induced pressure exceeds the far-field stress component acting normal to the natural fracture plane. The opening condition can be expressed as

$$\sigma_{h\max} - \sigma_{h\min} < \frac{2p_n}{1 - \cos 2\theta}$$

(5)

Beyond that, Warpinski and Teufel proposed a relationship expression that governs arrest mode as

$$\sigma_{h\max} - \sigma_{h\min} \geq \frac{2\tau_{0} - 2p_n\mu_f}{\sin 2\theta + \mu_f \cos 2\theta - \mu_f}$$

(6)

where $\tau_0$ is the shear strength.

The chart of W&T’s criterion has been plotted in Figures 3 and 4 corresponding to net fluid pressures of 0.7 MPa and 2.7 MPa, respectively.

It can be seen that there is a critical angle, denoted by $\alpha$, in the W&T’s criterion plate curve. When $\theta > \alpha$, shear slippage will not take place because normal stress on the natural fracture plane is big enough to stop the slippage. At that time, the hydraulic crack would either penetrate the natural fracture or be arrested. The critical angle can be derived from (6), which is given by
\[ \alpha = \arctan \left( \frac{1}{\mu_f} \right). \]  

(7)

2.3. R&P’s Criterion. With considering that the material is homogeneous and isotropic, Renshaw and Pollard [6] studied the interaction of a dry fracture with a frictional plane based on the elastic fracture theory. This criterion considers the case where induced fracture approaching natural fracture orthogonally. If the shear strength of the natural fracture is less than the induced tensile stress, the natural fracture will slip and the fracture will be arrested. The criterion can be expressed as

\[ -\sigma_{h\text{max}} \frac{T_0 - \sigma_{h\text{min}}}{T_0} > \frac{1 + \mu_f}{\mu_f}. \]  

(8)

Renshaw and Pollard propose that crossing will not take place if slippage occurs, which underestimates the discontinuity strength required to allow an induced fracture to cross it. Figure 5 is the chart of R&P’s criterion.

2.4. Modified R&P’s Criterion. In fact, the induced crack may approach the natural fracture with any angle, so it is necessary to improve the R&P’s intersection criterion and develop a more general applicability one. Sarmadivaleh and Rasouli [2] extended R&P’s criterion to the case of a noncohesive interface with a nonorthogonal approach angle. It can be expressed as

\[ -\sigma_n \frac{T_0 - \sigma_T}{T_0} > \frac{(1 - \sin(\theta/2)\sin(3\theta/2)) + (1/\mu_f \cos(\theta/2)) (|\sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)| + \alpha)}{(1 + \sin(\theta/2)\sin(3\theta/2))}, \]  

(9)

where \( \mu_f'' = \mu_f' + \mu_f \) and is called the total friction coefficient. Here, \( \mu_f' \) is the relative friction coefficient, denoted as follows:

\[ \mu_f' = \frac{\tau_0/\sigma_n}{((1 - \sin(\theta/2)\sin(3\theta/2)))/(1 - \sin(\theta/2)\sin(3\theta/2)) + (1/\mu_f \cos(\theta/2)) (|\sin(\theta/2)\cos(\theta/2)\cos(3\theta/2)| + \alpha))} - 1. \]  

(10)

3. Numerical Method

Here, we employ the XFEM [16] to simulate the studied problem. The element with no crack is regarded as normal element, which is the same as the standard FEM. Note that for crack-crossed elements, the Heaviside step function is introduced as enriched function to characterize the discontinuities of crack. That is,

\[ H(x) = \begin{cases} 
1, & \text{dist}(x) > 0, \\
-1, & \text{dist}(x) < 0,
\end{cases} \]  

(11)

where dist(x) is the shortest distance from x to crack. In order to determine whether it is positive or negative, n is defined as the unit normal vector to the crack. The distance is positive when x locates in the side where n points and it is negative when x locates in the other side [21].

For embedded elements, Westergaard function is introduced as enriched function, which can be expressed as

\[ \phi_n(r, \theta) = \sqrt{r} \left\{ \sin \frac{\theta}{2} \cos \frac{\theta}{2}, \sin \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{\theta}{2}, \sin \frac{\theta}{2} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \right\}, \]  

(12)

where r and \( \theta \) are the local parameters in crack-tip polar coordinates.
where $\theta_{tip}$ is the crack-tip angle in the orthogonal coordinate system.

In the framework of XFEM, the node displacement approximation is [9]

$$\mathbf{u}^h(x) = \sum_{i \in N} N_i(x) \mathbf{u}_i + \sum_{i \in N_{emb}}^4 \phi_i \mathbf{b}_i, \quad (14)$$

where $J^i(X)$ and $J^{II}(X)$ are the signed distance functions. Also, $q_I^1$ and $q_{II}^K$ are the enriched DOFs of main crack and the branch crack, respectively.

The enrichment of the element containing a junction is as follows:

$$u^h(x) = \sum_{I=1}^4 N_I(X)u_I + \sum_{J=1}^4 N_J(X)[H(f^I(X)) - H(f^I(X_J))]q_I^J + \sum_{K=1}^4 N_K(X)[H(f^{II}(X_K)) - H(f^{II}(X_K))]q_{II}^K. \quad (15)$$

The enrichment of the element containing an intersection is defined as follows:

$$u^h(x) = \sum_{I=1}^4 N_I(X)u_I + \sum_{J=1}^4 N_J(X)\{H(f^I(X_J)) - H(f^I(X_J))\}q_I^J + \sum_{K=1}^4 N_K(X)\{J(X) - J(X_K)\}q_{II}^K. \quad (16)$$

where $J^I$, $c^1_K$, and $c^2_K$ are the enriched DOFs of main crack and two branch cracks, respectively.

### 4. Results and Discussion

#### 4.1. Effect of Natural Fracture Azimuth

In this section, we study the influence of natural fracture azimuth on the hydraulic fracture propagation. The model is $100 \text{ m} \times 100 \text{ m} \times 1 \text{ m}$ in size. A linear elastic and isotropic block is considered with Young’s modulus of 25 GPa and Poisson’s ratio of 0.20, and the tensile strength $T_0$ is 1.0 MPa. The in situ principal stress difference is set as 2.0 MPa, and fluid pressure in hydraulic fracture is 10 MPa. With seeding

$101$ nodes each side, the model is divided into 10000 elements. The initial induced crack, with a length of 5 m, is set at the left boundary. Three cases of different natural fractures orientations are studied. As shown in Figure 6, natural fractures with horizontal angles of 30°, 45°, and 60° are set in the block.

The results based on Blanton’s criterion are illustrated in Figure 7. In this case, hydraulic fracture opens one side of the natural fracture and reinitiate in the opposite plane. That is, the offset of hydraulic fracture occurs in the $y$ direction. And the induced fracture’s offset will be larger if the natural fracture azimuth is bigger. When the in situ principal stress difference is 2.0 MPa, the induced crack is
impossible to penetrate the natural fracture regardless of the crack angle. The propagation behavior of hydraulic fracture in this work agrees well with that of previous study [9].

As we can see from Figure 8, when the angle of intersection is 30°, the stress concentration phenomenon mainly concentrates on the upper tip of the natural fracture. And the induced stress at the lower tip of the natural fracture is smaller as it is suppressed by the fluid pressure. The impact of squeezing will be reduced when the intersection angle gradually increases. When the intersection angle reaches 135°, the stress concentration phenomenon mainly occurs at the lower tip of the crack, which is exactly contrary to the 45° case. In Wang et al.’s study [9], the stress distribution along the natural fracture becomes asymmetrical and the stress concentration is more significant in the vicinity of the upper tip of the natural fracture.

![Figure 6: Initial fracture distribution. (a) Initial crack distribution, 30°. (b) Initial crack distribution, 45°. (c) Initial crack distribution, 60°.](image)

4.2. Effect of In Situ Principal Stress Difference. In order to examine the influence of the in situ principal stress difference on hydraulic fracture propagation, we compare the results of two cases with the in situ principal stress difference of 2 MPa and 4 MPa. Natural fractures with horizontal angle of 60° are set in the block. The shape of induced fracture after the intersection is displayed in Figure 9. Different from before, the offset of hydraulic crack does not occur. Hydraulic fracture penetrates the natural fractures without changing the propagation direction when the in situ principal stress difference is 4 MPa. When the principal stress difference increases, hydraulic fracture requires a larger driving fluid pressure to obtain a larger net fluid pressure to open natural fractures. But at this time, the net fluid pressure is enough to induce re initiation occurring on the other side of natural fracture. Therefore, the hydraulic fracture will be inclined to penetrate the natural fracture when the in situ principal stress difference increases to a certain degree.

![Figure 7: Fracture propagation morphology under Blanton’s criterion. (a) Final fracture, 30°. (b) Final fracture, 45°. (c) Final fracture, 60°.](image)

4.3. Effect of Fluid Pressure. In order to examine the influence of fluid pressure on the hydraulic fracture propagation, the intersection criterion is changed into the W&T’s criterion. Natural fractures with horizontal angle of 30° are set in the block. And the in situ principal stress difference is set...
Figure 8: Von Mises stress field under different natural fracture azimuth conditions. (a) von Mises stress (step = 3), 30°. (b) von Mises stress (step = 3), 45°. (c) von Mises stress (step = 3), 60°. (d) von Mises stress (step = 3), 135°.

Figure 9: Intersection results based on Blanton’s criterion under different in situ principal stress difference conditions. (a) In situ principal stress difference is 2 MPa; final fracture, 60°. (b) In situ principal stress difference is 4 MPa; final fracture, 60°.
The shapes of induced fracture of different cases are shown in Figure 10. We compare the results of four cases with the fluid pressure of 2.5 MPa, 2.8 MPa, 3.5 MPa, and 4.5 MPa. The shear strength and the friction coefficient are set as 0.7 MPa and 0.6 MPa.

It can be found from Figure 10 that there are four intersection modes after the intersection of hydraulic crack and natural fracture. When the fluid pressure is 2.5 MPa, the net fluid pressure is too small to offset the normal stress in the natural fracture surface. At this point, the hydraulic crack will be arrested when it encounters the natural fracture. When the fluid pressure increases to 2.8 MPa, the induced stress is big enough to make the hydraulic fracture penetrate the natural fracture. When the pressure continues to increase to 3.5 MPa, the condition of opening is satisfied at this time, and the hydraulic crack is deflected into the direction of natural fracture. The induced stress can simultaneously meet the conditions of penetration and opening when the fluid pressure increases to 4.5 MPa. Thus, two branch cracks generate at this time.

5. Conclusions

In this paper, we compare several intersection criteria and simulate the interaction process between hydraulic fractures and natural fractures based on the XFEM. The effects of natural fracture azimuth, fluid pressure, and in situ principal stress difference on hydraulic fracturing are investigated in detail. The main conclusions are summarized as follows:

1. Blanton’s criterion is applicable to the situation where the natural fracture interface has a high normal stress. Also, the W&T’s criterion considers that the natural fracture friction coefficient and shear strength are also the influencing factors of crack propagation. The modified R&P’s criterion takes into account the cohesive effects of natural fracture.

2. The stress concentration phenomenon mainly concentrates on the tip in the obtuse angle side of natural fracture. Also, the induced fracture’s offset becomes larger with the increase of the natural fracture azimuth.

3. Hydraulic fracture is inclined to penetrate the natural fracture when the in situ principal stress difference increases to a certain extent.

4. Different fluid pressures in hydraulic fracture may trigger different intersection patterns, such as penetration mode, arrest mode, offset mode, and simultaneity mode.
Data Availability
The data used to support the findings of this work are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References