Research Article

A Modified Cubic Law for Rough-Walled Marble Fracture by Embedding Peak Density

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Received 5 April 2019; Revised 4 September 2019; Accepted 1 November 2019; Published 3 January 2020

Academic Editor: Timo Saksala

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The property of water flow through a single rock fracture is the base of describing the seepage characteristics of jointed rock mass. Five artificial tensile fractures of coarse-grained cylinder marble samples were made at about the midpoint of the long axis by using a self-made splitting mold. The upper and lower surfaces of the tensile fractures were scanned by a 3D laser scanner (OKIO) to obtain their 3D coordinates. Then, the Geomagic Studio Software and rock surface topography scan test software were used to obtain peak density values of each single fracture surface. To study the seepage characteristics of open fracture, 4 rectangular plastic spacers with the size of about 3 mm × 2 mm × 0.2 mm were put into the fracture when water flowed through the single rough fracture tests were conducted under different normal stresses using the self-developed radial flow system. According to the testing data, the relationships between the seepage characteristics of single rough rock fracture and the peak density of fracture surface were studied. It is discovered that the 3D fracture morphology had great influences on the seepage characteristics of the single rock fracture. A modified cubic law was put forward to present the relationship between the seepage characteristics of a rough rock fracture and peak density of two fracture surfaces. Comparison between the modified cubic law and the experimental data showed a relatively good agreement.

1. Introduction

Groundwater flow through jointed rock mass affects the stability of many engineering structures in civil engineering, mining engineering, hydropower engineering, petroleum engineering, and environmental engineering [1]. The water flow through a single rock fracture is usually the base of describing the seepage characteristics of jointed rock mass, which is why the water flow through a single rock fracture has been tested extensively in laboratory by a number of researchers.

In the early research on the seepage model for water flow through a single fracture, Lomize [2], De Marsily and Romm [1, 3], and Louis [4] firstly carried out the water flow test through two parallel plates and developed the so-called cubic law, which stated that the flow rate through the parallel plates had a cubic relation with the aperture of the plates. Thus, tiny change of the aperture may lead to major variation of the seepage flow rate. However, the surface of natural fractures is usually rough and undulant instead of smooth. If the aperture is large enough, the effect of rough and undulant fracture surface on the seepage characteristics of rock fractures may be slight [5] or negligibly low [6]. Singh et al. [7] found that water flowing through a single rough fracture in granite still obeyed the well-known “cubic law” even if the fractures were under the combination conditions of high \( b_p \) (maximum inlet water pressure, 25 MPa) and \( \sigma_3 \) (maximum confining pressure, 40 MPa). However, Konzuk and Kueper [8] summarized the research progress achieved on the seepage properties of rough rock joints and modified cubic law and suggested that the applicability of the local cubic law should be studied under the condition of different fracture surface three-dimensional morphology or abrupt aperture changes. Moreover, Raven and Gale [9] found the deviation of the relationship between the joint flow rate and the joint deformation from behaviour predicted by the
parallel plate model increased with the sample size and the
number of loading cycles increasing. Therefore, the validity
of the cubic law is suspicious [10, 11], and the cubic law has a
certain limitation in practical application.

Correspondingly, in order to still apply the cubic law in
analyzing seepage properties of rough rock fracture, various
researchers have tried to modify the cubic law by incorporating
some fracture roughness parameters. JRC (joint roughness
coefficient) is usually used to describe rock joint roughness in
rock engineering since the morphology of the actual rock joint
is usually very complex and it is difficult to completely describe
the profile feature of the rock joint according to the 10 standard
section lines [12]. Jaeger et al. [6, 10, 11] studied the effect of
rock joint roughness on the seepage properties of the joint from
different point of views and modified the cubic law by using
JRC to quantify the rock joint roughness so that the modified
cubic law could be applied in analyzing the seepage character-
istics of the rough rock joint. Neuzil [13] and Park and Hahn
[14] imported a function of aperture density distribution to
analyze the effect of joint surface roughness on the joint seepage
properties. Iwai [15] conducted the seepage experiment and
concluded that the way of the joint roughness affecting on the
water flow through a single joint was related to the contact ratio
of the rock joint surfaces. Subsequently, Zhou and Xiong [16]
put forward a modified cubic law by incorporating a joint
contact ratio. Zhao [17] proposed a new parameter, i.e., JMC
(joint matching coefficient), to describe the effect of the contact
state on the seepage characteristics of the rock joint qualita-
atively. Tsang and Tsang [18] proposed a channel model to
describe the seepage property of the rock joint based on the
integrated laboratory test and theoretical analysis. Lu et al.
[19] studied the channel flow phenomenon of seepage and
solute transport in a single joint using the discrete element
method. Brown et al. [20] studied the effect of joint roughness
on the joint seepage properties using the Reynolds equation
and the fractal model of joint surface morphology. A friction
factor was introduced in reference [21] as a function of two
independent variables, Reynolds number and relative rough-
ness, and was then formulated to describe the influence of
friction drag of the wall and local aperture changes on pressure
head distribution.

Since the fracture surface is rough, the cubic law with the
use of average aperture may not be able to describe the true
seepage characteristics of the rough rock fracture. Moreover,
as reviewed above, most of the modified cubic laws have used
either JRC or the fracture aperture distribution function to
characterize the influence of the fracture roughness on the
fracture seepage properties although neither JRC nor the
aperture distribution function are easy to obtain. Therefore,
the objective of this paper is to put forward a modified cubic
law to present the relationships between the seepage char-
acteristics and a 3D fracture morphology parameter of a
rough fracture by conducting water flow tests through a
single rough fracture.

2. Materials and Methods

2.1. Sample Preparation. Five cylindrical samples with a
diameter of 50 mm and a height of 100 mm were firstly
manufactured from white coarse-grained marble, then a
blind hole with a length of 60 mm and a diameter of 6 mm
was drilled from one end along the axis of each cylindrical
sample; finally, a self-designed splitting mold, as shown in
Figure 1(a), was used to divide each cylinder sample into two
halves at about the midpoint of the long axis. The splitting
mold consists of two identical parts, each of which consists
of an iron plate with a cylindrical groove and a wire with a
triangular cross section. The iron plate has a slot at the
midpoint of the long axis perpendicular to the axis and the
wire is fixed in the slot to split the sample with an edge.
Marble fracture surfaces of sample M1 and 5 single-frac-
tured samples are shown in Figure 1(b).

2.2. 3D Morphology Parameters of Rock Fracture Surface. The
upper and lower surfaces of the tensile fractures in a
coarse-grained marble were firstly scanned by a Tianyuan
OKIO-typed 3D laser scanner with CCD camera resolution
of $1.44 \times 106$ pixel and with measurement accuracy of up to
10 $\mu$m, as shown in Figure 1(c), and the distance between
adjacent points was about 16 $\mu$m. Marking points (black
rings on the surface of the sample, as shown in Figure 1(b))
are used to control the stitching of scanning data because
one time of scan is incomplete to obtain all the morp-
ho-logical data of a fracture surface. The coordinates of the
scanned surface will be written in ASCII or binary files in the
X, Y, and Z format, in which X, Y, and Z coordinates
represent the width, length, and height of the fracture
surface, respectively. Subsequently, the ASCII or binary files
were imported to the Geomagic Studio Software to complete
the encapsulation of the sample surface and were saved as
OBJ files, and encapsulated fracture surfaces of the sample
M1 were illustrated in Figures 1(d) and 1(e), respectively.
Finally, the OBJ files were imported to Rock Surface Top-
ography Scan Test Software (RSTST) by which 3D fracture
morphology parameters could be calculated. In the scanned
point cloud grid network, the point on the fracture surface is
selected as a peak if its height is higher than its adjacent eight
points. Peak density $S_{\text{peak}}$ is the peak number in the unit area,
and it can be obtained by the number of all the peak points
being divided by the projected area of fracture surface, as
shown in Table 1.

2.3. Flowing Test Device. Figure 2(a) depicts the test system
for studying the seepage characteristics of the splitting
fracture, which includes three parts, i.e., the water supplying
system, the loading-seepage-measurement system, and the
water collecting system. The inlet water head is controlled by
the height of the water tank relative to the position of the
rock fracture. The loading-seepage-measurement system
includes normal loading system, normal deformation
measurement system, and water switch. The water collecting
system mainly includes heat-shrink tubing (collecting water
outflow from the rock fracture), discharging tube, beaker,
and balance. The upper and lower parts of heat-shrink
tubing were tightened by the pipe clamp after their shrinkage
by heating, and the middle part of the heat-shrinkable tubing
near the fracture plane outlet had no pressure on the sample
side. Vaseline was filled between the heat-shrinkable tube and the side surface of the sample within the specified range of the two pipe clamps to prevent water from flowing out through the upper or lower part of the heat-shrinkable tube.

The normal load is applied on the sample through a self-developed creep testing device UCT-2, which includes upper pressure head, lower pressure head, and feed water plate, as shown in Figure 2(a). The normal stress can then be calculated according to the pump pressure and the cross-sectional area of the rock sample. The normal displacement is measured by three equidistant dial indicators attached to the rods which are fixed to the upper plate. The measuring heads of the three dial indicators touch the ring plate which is fixed to the sample. Thus, the reading variation of the three dial indicators can reflect the normal displacement between the upper plate and the ring plate, the average of which is taken as the normal deformation of the rock fracture approximately under lower normal stress levels.

2.4. Test Procedure. The single-fractured specimen in the radial flow test is shown in Figure 2(b). Four rectangular plastic spacers with the size of about 3 mm × 2 mm × 0.2 mm were placed in the fracture to make sure the fracture remains open and their size was very small compared with that of the fracture so that the four rectangular plastic spacers had little effect on the water seepage through the fracture. During the radial flow test, the water was supplied from the inlet hole and then flowed along the blind hole reaching the rock fracture. After that the water seeped radially from the centre of the specimen through the rock fracture and finally left the fracture flowing into the breaker through the discharging tube. During the radial flow test, a constant normal stress was applied on the rock fracture, which was set as 1∼6 MPa, respectively. Under each normal stress, the inflow water head were all set as 22 m. The readings of three dial indicators are recorded once normal stress or water head changed.

3. Results and Discussion

3.1. The Flow State of Fluid through a Rock Fracture. Reynolds number $R_e$ can describe the flow state of fluid and reflect the influence of parameters such as flow velocity, viscous coefficient of fluid, and shape of seepage passage. By calculating Reynolds number, the flow state of water through a rock fracture can be judged. The calculation formula is as follows:

$$R_e = \frac{\nu L \rho}{\mu},$$

where $R_e$ is Reynolds number; $\nu$ is characteristic velocity, m/s; $\mu$ is dynamic viscous coefficient of water; the temperature
of water is about 10°C; \( v = 1.31 \times 10^{-3} \) kpa·s; \( L \) is characteristic length, and for fracture flow, its value is twice the equivalent hydraulic aperture [22].

In the test, the seepage flow starts from the inner hole boundary of the sample and flows radially along the fracture surface to the outer circumference. The flow velocity and Reynolds number are different at the inner and outer boundary of the fracture surface. According to the testscheme, when the initial normal stress is 2 MPa and the head difference is 22 m, the flowrate and Reynolds number calculated are the largest. The inner Reynolds number \( R_{ei} \) and the outer Reynolds number \( R_{eo} \) are calculated as shown in Table 2. From Table 2, it can be seen that under initial normal stress of 2 MPa, the inner Reynolds number \( R_{ei} \) is greater than the outer Reynolds number \( R_{eo} \) and its maximum value is 9.54, which is far less than the critical Reynolds number 500. Therefore, the flow state of water through the rock fracture can be considered as laminar flow. In addition, the permeability of intact rock is generally very small and its permeability coefficient is generally less than \( 10^{-7} \) cm/s. The minimum rock fracture permeability coefficient in this paper is \( 5.38 \times 10^{-3} \) cm/s, which is about 4 orders higher than that of intact rock; thus, the influence of rock permeability on rock fracture seepage test results can be neglected.

3.2. The Basic Theory of Radial Flow in a Single Fracture. The cubic law was derived from the Navier–Stocks equation based on the smooth parallel plates model as follows:

\[ \mu = -K J, \]

\[ K = \frac{g}{12v} b^2, \]

where \( \mu \) is flow velocity, \( K \) is hydraulic conductivity, \( J \) is hydraulic gradient, \( v \) is kinematic viscosity coefficient, \( g \) is acceleration of gravity, and \( b \) is mechanical aperture.

\[ v = 1.31 \times 10^{-6} \text{ kpa·s} \] at room temperature of 10°C and \( g = 9.8 \times 10^3 \) kg/m³.

Based on the seepage theory for groundwater flow, the radial flow rate can be calculated using the following equation:

\[ Q = -K \frac{dH}{dr} 2\pi rb, \]

where \( Q \) is flow rate, \( r \) is radius, and \( H \) is water head.

Equation (4) can be integrated to become

\[ \frac{Q}{\Delta H} = 2\pi \frac{Kb}{\ln(R/r_0)}, \]

where \( R \) is the outer boundary radius, \( r_0 \) is the internal boundary radius, and \( \Delta H \) is head difference.

Combining equation (3), equation (5) is written as

\[ Q = \frac{\pi g}{6v \ln(R/r_0)} b^3. \]

Equation (6) can be simplified as follows:

\[ \frac{Q}{\Delta H} = C b^3, \]

where

\[ C = \frac{\pi g}{6v \ln(R/r_0)}. \]

Equation (7) is used to describe the radial flow through smooth parallel plates. Since natural fractures are usually rough, equivalent hydraulic aperture \( b_e \) can be used to replace the aperture \( b \) in equation (7), that is,
\[
\frac{Q}{\Delta H} = Ch^3_e.
\] (9)

3.3. The Relations between Equivalent Hydraulic Aperture and Fracture Closure. Fracture aperture directly affects the seepage flow, but it is difficult to measure the fracture aperture directly. On the other hand, the fracture closure can be easily measured. The fracture closure reaches the maximum value when the equivalent hydraulic aperture \( b_e \) calculated by equation (9) is zero theoretically. Figure 3 shows the relationship between the equivalent hydraulic aperture \( b_e \) and fracture closure \( \Delta b \) obtained from the water radial seepage through the fractures of 5 specimens under six normal stresses described in Section 2.

It can be seen from Figure 3 that the equivalent hydraulic aperture \( b_e \) decreases linearly with the fracture closure \( \Delta b \) increasing as a whole. That is, the equivalent hydraulic aperture \( b_e \) has a negative linear correlation with the fracture closure \( \Delta b \). Thus, when the equivalent hydraulic aperture \( b_e \) decreases and reaches zero, the maximum mechanical aperture \( (b_m)_{\text{max}} \) which is defined as the maximum normal closure \( \Delta b \) of the rock fracture can be obtained. Therefore, the mechanical aperture \( b_m \) under different normal stresses can be calculated by subtracting normal closure \( \Delta b \) from maximum mechanical aperture \( (b_m)_{\text{max}} \). The relationship between the equivalent hydraulic aperture and the fracture closure is

\[
b_e = p_1\Delta b + p_2,
\] (10)

where \( p_1 \) and \( p_2 \) are fitting parameters.

3.4. The Relationship between the Flow Rate Per Head and Mechanical Aperture. Figure 4 depicts the relationship between the flow rate per head and the mechanical aperture obtained from the water radial seepage through the fractures of 5 specimens tests under different normal stresses. As can be seen from Figure 4, the flow rate per head increases with the mechanical aperture increasing, which can be well fitted by the power function in the following equation:

\[
\frac{Q}{\Delta H} = p_3 b_m^n,
\] (11)

where \( p_3 \) is a fitting parameter and \( n \) is the exponent.

The values of \( p_3 \) and \( n \) and correlation coefficient \( R^2 \) are listed in Table 3 for the five fractures tested in the radial seepage tests. The correlation coefficients \( R^2 \) for all five fractures are above 0.94. The range of the exponent \( n \) is 2.94–3.17, i.e., close to 3, which is consistent with the general cubic relationship between the flow rate per head and the mechanical aperture. As the mechanical aperture increases, the increasing rate of the flow rate per head increases gradually. It is because the contact area between the fracture surfaces becomes larger and larger as the normal closure increases, which results in fewer flow paths and smaller flow rates. The fitting parameter \( p_3 \) in equation (11) reveals the influence of the fracture roughness on the flow rate, which is unequal to the coefficient \( C \) in equation (9) derived according to the smooth parallel plate model. Correspondingly, a correction coefficient \( \xi \) (Table 3) is introduced to modify equation (9), which becomes

\[
\frac{Q}{\Delta H} = C\xi b_m^3.
\] (12)

After that, the relationships between the flow rate per head and the mechanical aperture are fitted again using equation (12) with the regression parameter \( \xi \) and correlation coefficients \( R^2 \) are listed in Table 3. As can be seen from Table 3, the correlation coefficients \( R^2 \) are all above 0.91.

Further, the relationship between the regression parameter \( \xi \) and the average peak density is shown in Figure 5. As can be seen from Figure 5, there is a power relationship between \( \xi \) and the average peak density \( (S_{\text{av}})_{\text{ave}} \) (which is one of the 3D morphology parameters), and the correlation coefficient is 0.91.
If the fitting equation between $\xi$ and the average peak density ($S_{pd}$)ave illustrated in Figure 5 is substituted into equation (12), the following equation can be obtained:

$$\frac{Q}{\Delta H} = C \times 10^{8} (S_{pd})^{16.83}_{ave} b_{m}^{3}.$$  \hspace{1cm} (13)

Equation (13) indicates that the relationship between the flow rate per head and the mechanical aperture is a modified cubic law containing the 3D morphology parameter $S_{pd}$. It should be pointed out that whether the proposed modified cubic law here can be used for a different rock fracture size needs a further study.

### 3.5. The Relationship between Mechanical Aperture and Normal Stress

However, as mentioned before, the mechanical aperture is a parameter which cannot be easily obtained, but it is easy to obtain the normal stress during the fracture seepage test. Therefore, it is rather useful to build a relationship between the flow rate and the normal stress from the practical point of view.

Figure 6 depicts the relationship between the mechanical aperture and the normal stress obtained from the fracture seepage tests. As shown in Figure 6, the mechanical aperture decreases linearly with the increase of the normal stress and the decreasing rates are rather similar for all five specimens. Thus, the linear relationship between the mechanical aperture and the normal stress $\sigma$ can be denoted using the following equation:

$$b_{m} = p_{4}\sigma + p_{5},$$  \hspace{1cm} (14)

where $p_{4}$ and $p_{5}$ are linear regression parameters.

The regression parameters $p_{4}$ and $p_{5}$ and the correlation coefficient $R^2$ for each specimen are shown in Table 3. Theoretically, the 3D morphology parameters affect the relationship between the mechanical aperture and the normal stress. Thus, the 3D morphology parameters should have effect on the fitting parameters $p_{4}$ and $p_{5}$ in equation (14) too. The relationship between the fitting parameter $p_{4}$ and the average peak density ($S_{pd}$)ave and that between the parameter $p_{5}$ and ($S_{pd}$)ave are illustrated in Figure 7, respectively. It can be seen from Figure 7 that both $p_{4}$ and $p_{5}$ are exponentially related to ($S_{pd}$)ave with the correlation coefficients $R^2$ of 0.97 and 0.98, respectively, and $p_{4}$ increases with the increase of ($S_{pd}$)ave while $p_{5}$ decreases with the increase of ($S_{pd}$)ave. Correspondingly, the average peak density ($S_{pd}$)ave of the fracture surfaces influences the size of the mechanical aperture directly. The greater the average peak density is, the smaller the mechanical aperture is under the same normal stress.

Substituting the relationship between $p_{4}$ and ($S_{pd}$)ave and that between $p_{5}$ and ($S_{pd}$)ave in Figure 8 into equation (14), the following equation can be obtained:

$$b_{m} = - \left( 0.026 + 2.73e^{-13.54(S_{pd})_{ave}} \right) \sigma + 7.26 \times 10^{4}e^{-38.27(S_{pd})_{ave}} + 0.42.$$  \hspace{1cm} (15)

If equation (15) is substituted into equation (13), the following equation is obtained:
Figure 7: Relationship between fitting parameters $p_4$, $p_5$, and $(S_{pd})_{ave}$ (a) $p_4$ vs $(S_{pd})_{ave}$; (b) $p_5$ vs $(S_{pd})_{ave}$.

Figure 8: Continued.
In equation (16), \((S_{pd})_{ave}\) can be obtained by adopting 3D laserscanner and Rock Joint Morphology Test Software and all other parameters can be obtained easily, too, during the fracture seepage test. Thus, equation (16) provides an important new relationship between the flow rate and the normal stress taking the influence of the fracture roughness into account through incorporating the 3D morphology parameter \(S_{pd}\).

### 3.6. The Amendment of Equation (16).

Figure 8 depicts the relationships between the flow rate per head and the normal stress obtained both experimentally from the fracture seepage tests and theoretically according to equation (16). There exists obvious deviation between the theoretical prediction using equation (16) and the experimental data from the fracture seepage tests. Specially, the theoretical prediction using equation (16) is larger than the experimental data for specimens M1 and M4 and is lower than the experimental data for other specimens, which may be because equation (16) contains total errors of equations (10)–(16). From Table 1, we can see that \((S_{pd})_{max}\) of specimens M1 and M4 are also larger than other specimens so that the deviation between the theoretical prediction using equation (16) and the experimental data may have some relation with \((S_{pd})_{max}\). Thus, an amendment coefficient \(k\) is introduced, which is the average ratio between the flow rates per head obtained experimentally from the fracture seepage test and theoretically using equation (16). The relationship between \(k\) and the maximum peak density of the fracture surface \((S_{pd})_{max}\) for the five specimens is shown in Figure 9. It can be seen from Figure 9 that the amendment coefficient 

\[
k = 5.42e^{-366.46(S_{pd})_{max}}
\]

\[
R^2 = 1.00
\]

The relationships between the flow rate per head and the normal stress obtained theoretically according to equation (17) are also illustrated in Figure 8. It can be seen from Figure 8 that after including a 3D morphology parameter, i.e., the peak density, the cubic model in equation (17) predicts the relationship between the flow rate per head and the normal stress in the rock fracture seepage test well. Therefore, equation (17) may be used in numerical simulation or theoretical prediction on the flow rate per head in
the rock fracture seepage test under different normal stresses with the peak density taken into account.

4. Conclusions

(1) There is a linear relationship between the equivalent hydraulic aperture and the fracture closure during the rough rock fracture seepage tests under various normal stresses. The equivalent hydraulic aperture decreases with the increase of the fracture closure. The total normal closure can be set as the maximum mechanical aperture, and then the mechanical aperture under different normal stresses can be calculated by subtracting the normal closure from the maximum mechanical aperture.

(2) There is a power relationship between the flow rate per head and the mechanical aperture, and the index \( n \) is close to 3. Thus, the relationship between the flow rate per head and the mechanical aperture meets approximately the cubic model. Since the mechanical aperture cannot be measured easily from the rock fracture seepage test, a correction coefficient \( \xi \) is introduced to relate the mechanical aperture to the average peak density \((S_{pd})_{ave}\) which is a 3D morphology parameter to be measured easily.

(3) The mechanical aperture has a linear relationship with the normal stress during the rough rock fracture seepage tests, and the mechanical aperture decreases with the increase of the normal stress.

(4) Equation (13) is a modified cubic law containing the 3D morphology parameter \((S_{pd})_{ave}\) of rock fracture, which can describe the influence of rock fracture roughness on the cubic relationship between the flow rate per head and the mechanical aperture. Equation (17) is suitable to predict the relationship between the flow rate per head and the normal stress of rock fracture after including a 3D morphology parameter \((S_{pd})_{max}\).

Data Availability

The research data used to support the findings of this study are included within the article. Request for more details should be made to the corresponding author.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

This research was funded by the China National Natural Science Foundation (grant no. 51109076).

References

