An Algorithm for Extracting Intuitionistic Fuzzy Shortest Path in a Graph

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1. Introduction

Graphs [1–4] are a very important model of networks. There are many real-life problems of network of transportation, communication, circuit systems, and so forth, which are modeled into graphs and hence solved. Graph theory has wide varieties of applications in several branches of engineering, science, social science, medical science, economics, and so forth, to list a few only out of many. Many real-life situations of communication network, transportation network, and so forth cannot be modeled into crisp graphs because of the reason that few or all of the arcs/links have the cost/weight which is ill-defined. The weights of the arcs are not always crisp but intuitionistic fuzzy (or fuzzy).

One of the first studies on fuzzy shortest path problem (FSPP) in graphs was done by Dubois and Prade [5] and then by Klein [6]. However, few more solutions to FSPP proposed in [7–10] are also interesting. Though the work of Dubois and Prade [5] was a major breakthrough, that paper lacked any practical interpretation even if fuzzy shortest path is found, but still this may not actually be any of the path in the corresponding network for which it was found. There are very few works reported in the literature on finding an intuitionistic fuzzy shortest path in a graph. Mukherjee [11] used a heuristic methodology for solving the IF shortest path problem using the intuitionistic fuzzy hybrid geometric (IFHG) operator, with the philosophy of Dijkstra’s algorithm. In [12], Karunambigai et al. in a team work with Atanassov, present a model based on dynamic programming to find the shortest paths in intuitionistic fuzzy graphs. Nagoor Gani and Mohammed Jabarulla in [13] also developed a method on searching intuitionistic fuzzy shortest path in a network. But all these algorithms have both merits and demerits (none is absolutely the best), as all these are greedy algorithms. In this paper we solve the intuitionistic fuzzy shortest path problem (IFSP) for a graph where the arc weights are intuitionistic fuzzy numbers, and then we reduce the method to the case of finding fuzzy shortest path in a graph. Our work here does also have the same kind of demerits (as in [12, 13]), but the major significance lies in the fact that we follow the concept of classical Dijkstra’s algorithm which is applicable to graphs with crisp weights and then extend this concept to graphs where the weights of the arcs are intuitionistic fuzzy numbers.

2. Preliminaries

A graph $G$ is an ordered pair $(V, E)$ which consists of two sets $V$ and $E$, where $V$ or $V(G)$ is the set of vertices (or, nodes), and $E$ or $E(G)$ is the set of edges (or, arcs/links). Throughout
in our work here we consider those graphs which are without loops. Graphs may be of two types: undirected graphs and directed graphs. In an undirected graph the edge \((i, j)\) and the edge \((j, i)\), if exist, are obviously identical unlike that in the case of directed graph. For a fast visit on the theory of graphs, one could see [1–4].

The intuitionistic fuzzy set theory of Atanassov [14–16] is now a well-known powerful soft computing tool to the world scientists. If \(X\) is a universe of discourse, an intuitionistic fuzzy set \(A\) in \(X\) is a set of ordered triplets 
\[
A = \{ (x, \mu_A(x), \nu_A(x)) : x \in X \},
\]
where \(\mu_A, \nu_A : X \rightarrow [0,1]\) are functions such that 
\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad \text{for all} \quad x \in X.
\]
For each \(x \in X\) the values \(\mu_A(x)\) and \(\nu_A(x)\) represent the degree of membership and degree of nonmembership of the element \(x\) to \(A \subseteq X\), respectively, and the amount 
\[
\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)
\]
is called the hesitation part. Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set if \(\pi_A(x) = 0\), for all \(x \in X\). For details of the classical notion of intuitionistic fuzzy set theory, one could see the book authored by Atanassov [15]. The concept of an intuitionistic fuzzy number is of importance for quantifying an ill-known quantity. Intuitionistic fuzzy numbers are the more generalized form of fuzzy numbers involving two independently estimated degrees: degree of acceptance and degree of rejection. In our work here throughout, we use the notion of triangular intuitionistic fuzzy numbers and the basic operations like IF addition \(\oplus\), IF subtraction \(\ominus\), “ranking” of intuitionistic fuzzy numbers, and so forth. Trivially, any crisp real number can be viewed as a fuzzy number or as an intuitionistic fuzzy number. There is no unique method for ranking the number of intuitionistic fuzzy numbers, because all the existing methods [17–19] are soft computing methods. Each method has got merits and demerits depending upon the properties of the application domains and the problem under consideration. However, if \(A_1, A_2, A_3, \ldots, A_n\) are \(n\) intuitionistic fuzzy numbers sorted in IF ascending order (in fact it is a kind of nonascending order, assuming that the IF equal intuitionistic fuzzy numbers take corresponding positions at random if there is no loss of generality) by any good predefined method, that is, if 
\[
A_1 < A_2 < A_3 < \cdots < A_n,
\]
then \(A_1\) and \(A_n\) are called, respectively, the IF-min and IF-max of these \(n\) intuitionistic fuzzy numbers. Almost all the existing methods [17–19] of ranking intuitionistic fuzzy numbers were developed independently, that is, not as extensions of the existing methods [20–26] of ranking of fuzzy numbers [27]. Although several authors [20–26] have reported several ranking methods of fuzzy numbers, all are having limitations too, that is, not an absolute method suitable for every application domain. However, if \(A_1, A_2, A_3, \ldots, A_n\) are intuitionistic fuzzy numbers sorted in fuzzy ascending order by a predefined method, that is, \(A_1 < A_2 < A_3 < \cdots < A_n\), then \(A_1\) and \(A_n\) are called, respectively, the fuzzy-min and fuzzy-max of these \(n\) fuzzy numbers.

### 3. Graphs with IF Weighted Arcs

In most of the real-life problems of networks, be it in a communication model or transportation model, the weights of the arcs are not always crisp but intuitionistic fuzzy numbers (or, at best fuzzy numbers). For example, Figure 3 shows a public road transportation model for a traveler where the cost parameter for traveling each arc has been available to him as an intuitionistic fuzzy number.

But for such type of ill graph, there is no attempt made so far in the literature for searching an IF shortest path. In our method here, we solve this intuitionistic fuzzy shortest path problem (IFSPP) for graphs where we also use the notion of Dijkstra’s algorithm but with simple soft-computations without using any hybrid geometric operators, using only basics of Atanassov’s operators [15].

### 4. Intuitionistic Fuzzy Shortest Path in a Graph

In this section we solve the IFSPP for graphs where we use the philosophy of Dijkstra’s algorithm but with simple soft-computations with IF data. Consider a directed graph \(G\) where the arcs are of intuitionistic fuzzy weights (intuitionistic fuzzy numbers), as shown in Figure 1. Suppose that the subalgorithm IFW\((G)\) returns the intuitionistic fuzzy weight set \(W\) corresponding to each arc.

#### 4.1. IF Shortest Path Estimate \(d[v]\) of a Vertex \(v\) in a Directed Graph

As shown in Figure 2, suppose that \(s\) is the source vertex and the currently traversed vertex is \(u\). The IF estimate \(d[v]\) in graph \(G\) is computed using IF addition as follows:

\[
(\text{IF shortest path estimate of vertex } v) = (\text{IF shortest path estimate of vertex } u) \oplus (\text{intuitionistic fuzzy number weight corresponding to the arc from the vertex } u \text{ to the vertex } v) \quad (1)
\]

or

\[
d[v] = d[u] \oplus w_{uv}. \quad (2)
\]

#### 4.2. Intuitionistic Fuzzy Relaxation of an Arc in a Directed Graph

We extend the classical notion of relaxation to the...
Intermediary vertices between vertex \( s \) and vertex \( u \)

Arc between vertex \( s \) and vertex \( u \)

Fuzzy shortest distance between vertex \( s \) and vertex \( u \)

Fuzzy weight of the arc \( uv \)

**Algorithm 1**

IFISS \((G, s)\)
1. For each vertex \( v \in V[G] \)
2. \( d[v] = \infty \)
3. \( v \cdot \pi = \text{NIL} \)
4. \( d[s] = 0 \)

**Algorithm 2**

IF-RELAX \((u, v, W)\)
1. IF \( d[v] > d[u] \oplus w_{uv} \)
2. THEN \( d[v] \leftarrow d[u] \oplus w_{uv} \)
3. \( v \cdot \pi \leftarrow u \)

**Algorithm 3**

IFSP \((G, s)\)
1. IFISS \((G, s)\)
2. \( W \leftarrow \text{IFW}(G) \)
3. \( S \leftarrow \emptyset \)
4. \( Q \leftarrow V[G] \)
5. WHILE \( Q \neq \emptyset \)
6. DO \( u \leftarrow \text{EXTRACT-IF-MIN}(Q) \)
7. \( S \leftarrow S \cup \{u\} \)
8. FOR each \( v \in \text{Adj}[u] \)
9. DO IF-RELAX \((u, v, W)\)

**Figure 2:** IF estimation procedure for \( d[v] \) in a graph \( G \).

**Figure 3:** Diagram showing how the IF-RELAX algorithm works in a graph.

4.3. IF Shortest Path Algorithm (IFSP Algorithm) in a Graph.

We now present our main algorithm to find single source IF shortest path in a graph. We name this “intuitionistic fuzzy shortest path algorithm,” that is, in short by the title IFSP algorithm. In this algorithm we use the previously designed above subalgorithms and also the subalgorithm EXTRACT-IF-MIN \((Q)\) which extracts the node \( u \) with minimum key using a predefined IF ranking method and updates \( Q \) (see Algorithm 3).

**Example 1** (an example). Consider the following directed graph \( G \) where the IF weights (here they are intuitionistic fuzzy numbers) are shown against each link, as shown in Figure 4. We want to solve the single-source IF shortest path problem (IFSPP) taking the vertex \( A \) as the source vertex and the vertex \( D \) as the destination vertex.

Our algorithm computes the following results:

1. \( W = \{\omega_{AB} = \tilde{15}, \omega_{AC} = \tilde{3}, \omega_{BC} = \tilde{1}, \omega_{CB} = \tilde{4}, \omega_{CD} = \tilde{6}, \omega_{BD} = \tilde{2}\} \), and then
5. Conclusion

There are many real-life problems in the networks of transportation, communication, circuit systems, and so forth which are initially modeled into graphs and hence solved. In many of these directed graphs, in reality, the weights of the arcs are not always crisp but fuzzy numbers. In this paper we develop a new method to solve the intuitionistic fuzzy shortest path problem (IFSPP) from a source vertex to a destination vertex in a directed graph. The importance of our method lies in its potential to give solution in intuitionistic fuzzy environment, unlike any of the existing algorithms of IFSPP. Obviously, our algorithm does also work in case few or all of the weights are fuzzy numbers or crisp numbers, as a special case of IF numbers.

References

