Review Article

Coherence and Disorder in Bilayer Quantum Hall Systems

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The quantum Hall bilayer at total filling factor $\nu = 1$ displays a number of properties akin to superfluidity, most clearly apparent in its very low dissipation in tunneling and counterflow transport. Theoretical descriptions in terms of quantum Hall ferromagnetism or thin-film superfluidity can be developed to explain these phenomena. In either case, merons can be identified as important low energy excitations. We demonstrate that a model in which puddles of merons induced by disorder, separated by narrow regions of interlayer coherence—a *coherence network*—can naturally explain many of the imperfect superfluid finite temperature properties that are observed in these systems. The periodic realization of this model shows that there can be low energy excitations beyond the superfluid mode. These are associated with transitions between states of different meron number in the puddles, where we argue that merons should be unbound at *any* temperature, and which can have important implications for the effect of quantum fluctuations on the system.

1. Introduction: Quantum Hall Bilayer as a Pseudospin System

The quantum Hall system at filling factor $\nu = 1$ supports a rich set of phenomena when discrete degrees of freedom can come into play at low energy. These are collectively known as quantum Hall ferromagnetism. Surprisingly, the spin degree of freedom in many such systems can be relevant in spite of the strong magnetic field in which the system is immersed, because the Landé g -factor is rather small in most two-dimensional electron gas systems based on GaAs. A description in terms of a spin-1/2 quantum ferromagnet turns out to be quite useful for this system, for example, suggesting that the basic charged excitations of this system are skyrmions [1, 2], excitations which carry a topological winding number in the spin and also turn out to be charged. Microscopic calculations [3–5] suggest these are indeed the low energy excitations in the clean limit, and several experimental results appear to confirm the presence of skyrmions [6, 7], although when disorder is taken into account such interpretations become less firm [8].

Quantum Hall ferromagnetism is also relevant in a very different context, the bilayer two-dimensional electron gas

with total filling factor $\nu = 1$ [9]. This system can be artificially fabricated using molecular beam epitaxy, resulting in two high quality layers of electron gas very close to one another. A direct mapping between this and the real spin-1/2 quantum Hall system may be established if one labels one layer as “up” and the other “down.” The layer index may thus be viewed as a pseudospin, and many of the ideas established for the spin behavior of the single layer quantum Hall system come into play for the bilayer, even if the real spin is fully polarized. (One can consider the situation in which both real spin and pseudospin are active degrees of freedom, leading to many possible states of the system [10]. In what follows we focus on the limit in which the Zeeman coupling polarizes the real spin. This has recently been realized experimentally, and qualitative features of the spin-polarized case are the same as at lower Zeeman couplings [11, 12].)

An important difference between the spin and the (bilayer) pseudospin degrees of freedom is the fact that interactions are not SU(2) invariant in the latter as they are in the former. This is because with finite layer separation d the Coulomb repulsion is larger for a pair of electrons in the same layer than it is for a pair in different layers with the same in-plane separation $\mathbf{r}$. The first and foremost

impact of this physics is that, in the absence of an external perpendicular electric field, the electron density will tend to be balanced between the layers. In the pseudospin language, this means that the effective spin-1/2 degree of freedom will favor an in-plane orientation. Even within this subspace of orientations, not all spin directions are equivalent. Interlayer tunneling energetically favors single particle states that are symmetric linear combinations of states in the two wells over antisymmetric combinations, which in the pseudospin language corresponds to spin oriented in the $\hat{x}$ direction. Real samples may be grown such that there is a wide range of possibilities for the scale of this term, from rather large so that all electrons are firmly in the symmetric state—essentially removing the layer degree of freedom from the problem—to very small, orders of magnitude below accessible temperatures. This latter situation has resulted in some of the most interesting and puzzling experimental observations on this system, which we will discuss in more detail below.

A crucial concept that applies to the bilayer system and to quantum Hall ferromagnets in general is known as *spin-charge coupling*. Because of the strong magnetic field, at filling factor $\nu = 1$ electrons will tend to reside almost solely in the lowest Landau level. However, restricting the orbital degrees of freedom to a single Landau level constrains the way in which spatially varying spin configurations may be realized. For configurations in which the direction of spin varies slowly on the scale of the magnetic length $\ell_0 = \sqrt{\hbar c / eB}$, with B the magnetic field, one may show [9] that charge and spin densities are tied together by the relation [1, 2]

$$q(\mathbf{r}) = -\frac{1}{8\pi} \epsilon_{\mu\nu} \mathbf{m} \cdot \partial_\mu \mathbf{m} \times \partial_\nu \mathbf{m}, \quad (1)$$

where $\mathbf{m}(\mathbf{r})$ is a unit vector indicating the local direction of the spin. This relation may be applied somewhat more generally than just to states in the lowest Landau level; it works (with an appropriate overall constant) for any state with a quantized Hall conductivity σ_{xy} [13].

An effective energy functional for this pseudospin system which captures both the spin-charge coupling and the symmetry-breaking physics has the form [14]

$$\begin{aligned} E[\mathbf{m}] = & \frac{\rho_E}{2} \int d\mathbf{r} (\nabla m^z)^2 + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' q(\mathbf{r}) v(\mathbf{r} - \mathbf{r}') q(\mathbf{r}') \\ & + \int d\mathbf{r} V(r) q(\mathbf{r}) - \frac{\Delta_{\text{SAS}}}{4\pi\ell_0^2} \int d\mathbf{r} [m^x(\mathbf{r}) - 1] \\ & + \Gamma \int d\mathbf{r} (m^z)^2 - \frac{e^2 d^2}{16\pi\kappa} \int \frac{d\mathbf{q}}{4\pi^2} q m_{-\mathbf{q}}^z m_{\mathbf{q}}^z \\ & + \frac{\rho_A - \rho_E}{2} \int d\mathbf{r} (\nabla m^z)^2. \end{aligned} \quad (2)$$

The first three terms of the energy are SU(2) invariant contributions. The leading gradient term is the only one that appears in the nonlinear σ model for Heisenberg

ferromagnets, and ρ_E is the spin stiffness in the $\hat{x}$ - $\hat{y}$ plane. The second term describes the SU(2) invariant Hartree energy corresponding to the charge density associated with spin textures in quantum Hall ferromagnets. $v(r)$ is the Coulomb interaction screened by the dielectric constant κ of the host semiconductor. The third term incorporates interactions of the charge density with an external potential, for example, due to disorder. The fourth term describes the loss in tunneling energy when electrons are promoted from symmetric to antisymmetric states; here Δ_{SAS} is the single-particle splitting between symmetric and antisymmetric states.

The last three terms are the leading interaction anisotropy terms at long wavelengths. The term proportional to Γ implements the electrostatic energy cost of having a net charge imbalance between the layers. The $(\nabla m^z)^2$ term accounts for the anisotropy of the spin stiffness. Pseudospin order in the $\hat{x}$ - $\hat{y}$ plane physically corresponds to interlayer phase coherence so that $\rho_A - \rho_E$ will become larger with increasing d . The sum of the first and seventh terms in (2) gives an XY-like anisotropic nonlinear σ model. However, this gradient term is not the most important source of anisotropy at long wavelengths. The fifth term produces the leading anisotropy and is basically the capacitive energy of the double-layer system. The sixth term appears due to the long-range nature of the Coulomb interaction; its presence demonstrates that a naive gradient expansion of the anisotropic terms is not valid. ($\mathbf{m}_q$ is the Fourier transform of the unit vector field $\mathbf{m}$.) Equation (2) can be rigorously derived from the Hartree-Fock approximation in the limit of slowly varying spin textures [15], and explicit expressions are obtained for ρ_E (which is due in this approximation entirely to interlayer interactions), ρ_A (due to intralayer interactions), and Γ . Quantum fluctuations will alter the values of these parameters from those implied by the Hartree-Fock theory.

Equation (2) is an energy functional for an easy-plane ferromagnet. As such one expects this system to support vortex excitations. In this context these are called *merons*, of which there are two for each vorticity—one in which the local spin vector $\mathbf{m}$ points in the positive $\hat{z}$ direction in the vortex core center and the other with $\mathbf{m}$ in the negative $\hat{z}$ direction. Due to the spin-charge relation (1), this means that vortices and antivortices *each* may have charge $\pm e/2$. In clean systems at low temperatures, we expect vortices and antivortices to be bound into pairs. Such *bimerons* then become the basic charged quasiparticles of the system [14], carrying charge e . Interestingly, such a bimeron is topologically equivalent to a skyrmion, confirming our understanding of the close relation between the real spin and the bilayer pseudospin system.

For $\Delta_{\text{SAS}} = 0$, it is clear that above the Kosterlitz-Thouless temperature the meron pairs will unbind. Renormalization group calculations and simulation studies suggest that such unbinding can still occur if Δ_{SAS} is sufficiently small, either from thermal fluctuations [16] or disorder [17, 18]. The presence of unbound merons in the system has profound physical implications and may explain a number of remarkable phenomena that have been observed in experiments, as we now explain.

2. Analogy with Two-Dimensional Superfluidity

The analogy with easy-plane ferromagnetism suggests a different way to interpret the energy functional in (2). If Γ is sufficiently large then out of plane fluctuations will be strongly suppressed, and in a first approximation one may ignore m_z as a dynamical degree of freedom. Writing $m_x + im_y = e^{i\theta}$, to lowest order in gradients the energy functional may be written in the simple form

$$E_{SF} = \int d\mathbf{r} \left[\frac{\rho_s}{2} (\nabla\theta)^2 - \frac{\Delta_{SAS}}{4\pi\ell_0^2} \cos\theta \right] \quad (3)$$

in the absence of an external potential $V(\mathbf{r})$. For $\Delta_{SAS} = 0$ (i.e., negligible tunneling), this has exactly the form expected for a two-dimensional thin film superfluid, with θ the condensate wavefunction phase and ρ_s an effective two-dimensional “superfluid stiffness.” In this case one expects the system to have a linearly dispersing “superfluid mode” which is analogous to the spin wave of an easy-plane ferromagnet. The presence of such a mode has been verified in microscopic calculations using the underlying electron degrees of freedom [19]. This suggests the possibility that one might observe some form of superfluidity in this system. To see exactly what this means, it is convenient to consider momentarily a wavefunction for the groundstate of the system in terms of the electron degrees of freedom

$$|\Psi_{ex}\rangle = \Pi_X \left[u_X + v_X c_{T,X}^\dagger c_{B,X} \right] |\text{Bot}\rangle, \quad (4)$$

where $|\text{Bot}\rangle$ represents the state in which all the single particle states of the bottom layer in the lowest Landau level, created by $c_{B,X}^\dagger$, have been filled. For a state with uniform density and equal populations in each well, $u_X = v_X = 1/\sqrt{2}$. More generally, one can represent an imbalanced state, obtained physically with an electric field applied perpendicular to the bilayer, by taking $u_X = \sqrt{\nu_T}$ and $v_X = \sqrt{\nu_B}$, with $\nu_T + \nu_B = 1$. The constants ν_T and ν_B represent the filling fractions in each of the layers, and the situation where $\nu_T \neq \nu_B$ turns out to be quite interesting, as we will discuss below.

Equation (4) is an excellent trial wavefunction for the groundstate, provided the layer separation d is not too large [20]. It shows that in the ideal (clean) limit, this system has a coherence much akin to that of a superconductor and that the condensed objects in the groundstate are *excitons*, particle-hole pairs with each residing in a different layer. This immediately implies that the superfluidity in this system will be in *counterflow* transport, where electric current in each layer runs in opposite directions.

Something much like this has been observed in experiments where electrical contact is made separately with each layer [21, 22]. In such studies current flows in opposite directions in the two layers. Such a current can be sustained by exciton flow, since the charge of the two constituents of the exciton have opposite sign. By measuring the voltage drop in a single layer along the direction of current, one may learn about dissipation in this exciton flow. In the experiments [21, 22], the dissipation is nonvanishing at any temperature $T > 0$, but apparently extrapolates to zero as $T \rightarrow 0$.

Another type of experiment takes advantage of the fact that Δ_{SAS} , while very small (typically several tens of microKelvin), is not zero. When the last term in (3) is included, the energy functional has a form very similar to that of a Josephson junction, suggesting that this system supports a Josephson effect [23]. In tunneling experiments, where one separately contacts to each layer such that current must tunnel between them, the tunneling I - V is nearly vertical near zero interlayer bias [24], which appears very similar to a Josephson I - V characteristic.

Another similarity between Josephson junction physics and the quantum Hall bilayer is the existence of a critical current. Josephson junctions can pass dissipationless currents only up to a limit that scales as $\sqrt{\Delta_{SAS}}$ in the model of (3). Above this current, a voltage sets in, and the system behaves dissipatively. Again, behavior reminiscent of this has been observed experimentally [25], although in the quantum Hall bilayer one should note carefully that the critical current separates a low from a high dissipation regime, rather than a zero from a nonzero dissipation regime. A number of theories have addressed this issue in the clean [26–28] and dirty [29] limits, which give very different behaviors with respect to how the critical current should scale with respect to the sample area. Interestingly, it is the latter which seems to agree best with experiment.

While these results look quite similar to what one might expect for exciton superfluidity, it is important to recognize that these results clearly are *not* genuine superfluid behavior. If the condensate could truly flow without dissipation, one would expect zero dissipation at any finite temperature below the Kosterlitz-Thouless transition, where vortex-antivortex pairs unbind. In experiment this truly dissipationless flow appears to emerge, if at all, only in the zero temperature limit. Similarly, the Josephson effect should be truly dissipationless, whereas in experiment there is always a measurable tunneling resistance at zero bias. The superfluidity in this system is *imperfect*. What kind of state can be nearly superfluid in this way? The answer likely involves disorder, which as mentioned above can cause the meron-antimeron pairs to unbind at arbitrarily low temperature. We next discuss a model which seems to capture much of the physics found in experiment.

3. The Coherence Network Model

One important way in which skyrmions and merons of the $\nu = 1$ quantum Hall system are different than those of more standard ferromagnets is that they carry charge. This means that they couple to electric potential fluctuations due to disorder. In these systems, disorder is ubiquitous because electrons are provided to the layers by dopants, which leave behind charged centers. The resulting potential fluctuations are extremely strong, and in fact divergent at long length-scales. In the Efros picture, the system screens nonlinearly by creating large puddles of positive and negative charge, separated by narrow strips of incompressible Hall fluid with local filling factor near $\nu = 1$ [30, 31]. For the bilayer system, the charge flooding the puddles should take the form of merons and antimerons, whose high density

FIGURE 1: Representation of coherence network. Links and nodes separate puddles of merons (circles). Meron charge and electric dipole moments indicated inside circles, as are strings of overturned phase connecting meron-antimeron pairs. Reproduced from [18].

spoils the interlayer coherence. The coherence however will remain strongest in the regions separating the puddles, even though some meron-antimeron pairs will likely straddle them. Thus one forms a network structure for the regions where the coherence is strong, and these should dominate the “superfluid” properties of the system. A schematic picture of the system is illustrated in Figure 1.

The key assumption in this model is that with such dense puddles, merons are able to diffuse independently through the system. This is supported by a renormalization group analysis, which suggests that there exists a state in which disorder enters as an effective temperature, so that one would likely be above any meron-antimeron unbinding transition for such strong disorder [18]. Motion of the merons is then limited by energy barriers for them to cross the coherent links between puddles. The tendency for dissipationless counterflow to emerge only at zero temperature now becomes very natural. When condensed excitons flow down the system, these produce a force on the merons perpendicular to that current [32]. The resulting meron current is limited by the activation energy to hop over the coherent links and vanishes rapidly but only completely when the temperature drops to zero. This meron current induces a voltage drop in the direction of the exciton current via the Josephson relation, rendering the counterflow current dissipative. True superfluid response in this system can only occur at zero temperature.

Dissipation in the tunneling geometry also emerges naturally in this model [18]. Since the current flows into (say) the top layer on the left and leaves via the bottom layer on the right, the current in the system must be decomposed into a sum of symmetric “coflow” and antisymmetric counterflow (CF). The former is likely carried by edge currents which are essentially dissipationless in the quantum Hall state. To obtain the correct current geometry, the CF current must point in opposite directions at the two ends of the sample. Thinking of the network as a Josephson array, the current of excitons—that is, CF current—is proportional to $\nabla\theta$. In order to inject CF currents in opposite directions at each end

of the sample, the phase angle at the sample edges should be rotated in the *same* direction. This means that the phase angle throughout the system will tend to rotate at a uniform rate, which is limited by the $(\Delta_{\text{SAS}}/4\pi\ell_0^2)\cos\theta$ term in (3). This is most effective at the nodes of the network, where the coherence is least compromised by the disorder-induced merons.

The dynamics of a typical node with phase angle θ may be described by a Langevin equation

$$\Gamma \frac{d^2\theta}{dt^2} = \sum_{\text{links}} F_{\text{link}} - \gamma_0 \frac{d\theta}{dt} - h \sin\theta + \xi(t). \quad (5)$$

The quantities F_{link} represent the torque on an individual “rotor” (*i.e.*, θ variable) due to its neighbors, transmitted through the links. Γ is the effective moment of inertia of a rotor, proportional to the capacitance of the node, $h = \Delta_{\text{SAS}}/4\pi\ell_0^2$, ξ is a random (thermal) force, and γ_0 is the viscosity due to dissipation from the other node rotors in the system. For a small driving force, the node responds viscously, and the resulting rotation rate has the form $\gamma\dot{\theta} = \sum F_{\text{link}}$. The Josephson relation $V = (\hbar/e)(d\theta/dt)$ then implies that the viscosity γ is proportional to the tunneling conductance σ_T of the system. For $k_B T \gg h$ one may show the viscosity for an individual node to be [33]

$$\gamma = \gamma_0 + \Delta\gamma = \gamma_0 + \sqrt{\frac{\pi}{2\Gamma}} \frac{h^2}{(k_B T)^{3/2}}. \quad (6)$$

As each node contributes the same amount to the total viscosity, the *total* response of the system to the injected CF current obeys

$$I_{\text{CF}} \propto N_{\text{nodes}} \Delta\gamma \frac{e^2 V_{\text{int}}}{\hbar} = \sigma_T V_{\text{int}}. \quad (7)$$

Note that because the nodes respond viscously, the tunneling conductance is proportional to the area of the bilayer. This is a nontrivial prediction of the model discussed here, which has been confirmed in experiment [34]. The proportionality of the tunneling conductance to Δ_{SAS}^2 is another nontrivial prediction which appears to be consistent with experimental data, and which contrasts with the result one expects in the absence of disorder, for which $\sigma_T \propto \Delta_{\text{SAS}}$.

4. Drag Experiments and Interlayer Bias

When an electric field is applied perpendicular to the layers, the density in the two layers becomes imbalanced. The effect of this can be incorporated into the model, (2), by replacing $\Gamma \int d\mathbf{r} (m^z)^2$ with $\Gamma \int d\mathbf{r} (m^z - m_0)^2$, with $m_0 = \nu_T - \nu_B$. The imbalance has interesting consequences for merons: since the pseudospin field $\mathbf{m}$ does not drop back into the $\hat{x} - \hat{y}$ plane as one moves out from the center of meron, the spin-charge relation (1) indicates that the four types of merons will have four different charges. These charges are specifically given by $q_{s,T(B)} = -s \sigma \nu_{B(T)}$, where $s = \pm 1$ is the vorticity of the meron, and the $T(B)$ subscript reflects the layer in which the magnetization at the core of the meron—its polarization—resides. The index σ indicates a sign associated with the

polarizations: $\sigma = 1$ for $\mathbf{m}$ in the meron center oriented in the top layer, $\sigma = -1$ for merons where it resides in the bottom layer.

The connection between polarization and charge has very interesting consequences for another type of transport experiment specific to bilayers, known as drag. In these experiments, one drives a current through only a single layer and measures voltage drops either in the drive layer or the drag layer. Within the coherence network model, the activation barrier for merons to hop across incompressible strips will clearly depend on the relative orientation of the meron polarization and the applied bias. Naively one would think that at low temperature, transport will be dominated by only the smallest activation energy, so that a measurement of resistance will reveal an activation energy that is symmetric around zero bias, which drops as the bias increases.

But this is not what is seen in experiment. The activation energy as measured in the drive layer is highest when the density is biased into the drive layer and decreases monotonically as the imbalance is changed so that more density is transferred to the drag layer. In the drag layer, the measured voltage drops turn out to be much smaller than in the drive layer, and are symmetric, but *increase* as the layer is imbalanced [35].

A careful analysis of the situation requires a method for determining voltage drops in individual layers, not just the interlayer voltage difference, which is what the Josephson relation applied above actually reveals. This can be accomplished [36] by adopting a “composite boson” description of the $\nu = 1$ quantum Hall state [1, 37]. The idea is to model electrons as *bosons*, each carrying a single magnetic flux quantum in an infinitesimally thin solenoid. The Aharonov-Bohm effect then implements the correct phase (minus sign) when two of these objects are interchanged [37]. By orienting the flux quanta opposite to the direction of the applied magnetic field, on average the field is canceled, and in mean-field theory the system may be modeled as a collection of bosons in zero field. The quantum Hall state is then equivalent to a Bose condensate of these composite bosons. For the coherent bilayer state, there is an additional sense in which the bosons are condensed: they carry a pseudospin with an in-plane ferromagnetic alignment.

Because merons carry physical charge, they will carry a quantity of magnetic flux proportional to this charge. In analogy with a thin-film superconductor [32], this means that a net current in the bilayer (i.e., a coflow) creates a force on the meron perpendicular to the current. This has to be added to the force due to a counterflow component. Together, these yield a net force which may be shown to be [36]

$$\mathbf{F}_T = \frac{es}{2}\Phi_0[(1+\sigma)\mathbf{J}_B - (1-\sigma)\mathbf{J}_T] \times \hat{z}, \quad (8)$$

where $\mathbf{J}_{T(B)}$ is the current density in the top (bottom) layer. As is clear from this expression, merons with only one of the possible polarizations are subject to a force in a drag experiment, since one of the two current densities vanishes.

FIGURE 2: Expected activation energy Δ as a function of relative density imbalance between the layers, $\Delta\nu/\nu_T$. For voltage drop measured in the drive layer, the results are asymmetric with around $\Delta\nu = 0$, as shown explicitly for $d = 0.8l_0$ (see text). For measurements in the drag layer, the data is reflected around $\Delta\nu = 0$; the measured result would follow the higher of the two activation energies, yielding a result symmetric in bias. Inset: activation energy at zero bias versus layer separation. Reproduced from [36].

The force $F_{s,\sigma}$ on merons of vorticity s and polarization σ will cause them to flow with a velocity $u_{s,\sigma} = \mu_{s,\sigma} F_{s,\sigma}$, where $\mu_{s,\sigma}$ is an effective mobility, which we expect to be thermally activated, with a bias dependence of the activation energy as discussed above. The resulting motion of the vortices induces voltages in two ways. The first is through the Josephson relation for the interlayer phase, yielding the relation [36]

$$\Delta V = \Delta V_T - \Delta V_B = -\frac{2\pi\hbar}{e} y_0 \sum_{s,\sigma} n_{s\sigma} s u_{s\sigma} \quad (9)$$

for the voltage drops between ΔV_σ between two points a distance y_0 apart along the direction of electron current, in layer σ , where $n_{s,\sigma}$ is the meron density. The second is due to the effective magnetic flux moving with the merons, which induces a voltage drop between electrons at different points along the current flow that is independent of the layer in which they reside. This contribution is given by [36]

$$(\nu_T \Delta V_T + \nu_B \Delta V_B) = -\frac{\hbar}{e} y_0 \sum_{s,\sigma} n_{s\sigma} q_{s\sigma} u_{s\sigma}. \quad (10)$$

In a drag geometry we have, for example, $J_B = 0$ and $\mathbf{J}_T = (I/W)\hat{y}$, with I the total current and W the sample width. Combining (9) and (10), we obtain $\Delta V_B = 0$ and

$$\frac{\Delta V_T}{I} = \frac{y_0}{W} \hbar \Phi_0 (n_{1,-1} \mu_{1,-1} + n_{-1,-1} \mu_{-1,-1}). \quad (11)$$

Notice that the final result depends on the mobility of *only*

merons with polarization $\sigma = -1$. It immediately follows that the voltage drop in the drive layer is asymmetric with respect to bias, precisely as observed in experiment.

In order to explain the voltage drop in the drag layer ($\Delta V_L \neq 0$), we must identify how forces on the $\sigma = +1$ merons might arise. A natural candidate for this is the attractive interaction between merons with opposite vorticities, which in the absence of disorder binds them into pairs at low meron densities. Assuming that driven merons crossing incompressible strips will occasionally be a component of these bimerons, a voltage drop in the drag layer will result. The mobility of such bimerons is limited by the energy barrier to cross an incompressible strip. These strips are likely to be narrow compared to the size scale of the constituents of the bimeron [36], so we expect the activation energy to be given approximately by the maximum of the activation energies for merons of the two polarizations $\sigma = \pm 1$. This leads to a drag resistance much smaller than that of the drive layer, with an activation energy that is *symmetric* with respect to and increases with bias. These are the behaviors observed in experiment [35]. Figure 2 illustrates the expected activation energy from a microscopic model implementing this physics [36] as a function of relative density imbalance between the layers, $\Delta\nu/\nu_T$, where $\Delta\nu = \nu_T - \nu_B$ and $\nu_T = 1$, for different layer separations. For voltage drop measured in the drive layer, the results are asymmetric around $\Delta\nu = 0$, as shown explicitly for $d = 0.8\ell_0$. For the drive layer, the large circles represent the activation energy found from a voltage drop measurement when it has a higher density than the drag layer, and the small circles the activation energy when it has the smaller density. Thus a measurement on the drive layer while continuously adjusting the density imbalance from positive through zero to negative results in a line from the large circles to the small circles as zero is crossed, yielding an asymmetric result. For measurements in the drag layer, the result would follow the higher of the two activation energies, yielding a result symmetric in bias.

This result followed from the precise cancellation between the counterflow current force on the vorticity of merons of a particular polarization and the Lorentz force associated with meron charge and its associated effective flux. The experiments thus provide indirect evidence that the meron charges vary in precisely the way one expects from the spin-charge relation, (1), verifying this unique property of quantum Hall ferromagnetism.

5. Periodic Models of the Quantum Hall Bilayer

As explained in Section 3, disorder has a nonperturbative effect on the incompressible quantum Hall state. True disorder is extremely difficult to treat theoretically. In analogy with superfluid systems [38], and one-component quantum Hall systems near a plateau transition [39–42], one may hope that a periodic potential captures some of the nonperturbative effects of disorder. Once one has obtained a second-order phase transition, one then adds disorder or other perturbations and examines their relevance/irrelevance [43]. This approach has been very fruitful in the past.

FIGURE 3: (Color online) The ground state configuration for a 16×16 unit cell with the strength of the periodic potential (in units, where $J = 1$) being $V = 3.0$, and the Hubbard interaction is $U = 8.0$. The lengths of the spins denote their planar projection. Note a vortex/antivortex at the center of each puddle.

As a first attempt, one of us (G. Murthy) and Subir Sachdev [13] examined the Composite Boson theory [1] near a putative Superfluid/Mott Insulator transition [38]. The primary difference between the neutral system and this one is that the Composite Boson is charged and is minimally coupled to both the external gauge field and the statistical field which attaches flux. There are two natural phases: a Bose-condensed Higgs phase in which all excitations are massive and the system can be shown to have a quantized Hall conductivity and a Mott Insulating phase in which all conductivities vanish [13].

In the large- N approximation, one can integrate out the bosons, leaving behind an effective theory of gauge fluctuations. The phase transition turns out to be second-order in the large- N limit, with the critical point having the conductivities [13]

$$\sigma_{xx}^*(0, \omega) = \frac{e^2}{h} \frac{\pi/8}{1 + (\pi/8)^2}, \quad \sigma_{xy}^* = \frac{\pi}{8} \sigma_{xx}^*. \quad (12)$$

The XY-angle θ also acquires an imaginary part of the self-energy at the critical point which vanishes as ω^7 for zero interlayer tunneling and ω^5 when interlayer tunneling is nonzero [13].

While this model is fully quantum, it is overly simplistic in assuming that only a single phase transition exists between the uniform superfluid and a Mott Insulator.

Recently, the present authors in collaboration with Jianmin Sun and Noah Bray-Ali have taken some steps towards building a more realistic model [44]. In this model, fermions are assumed to be gapped out, and the real spin is assumed to be fully polarized, leaving dynamics only in the pseudospin. We put the system on a square lattice with

FIGURE 4: (Color online) The ground state configuration for $V = 7.0$ and $U = 8.0$.

FIGURE 5: The spin wave velocity of the G -mode as a function of V . Note that it never vanishes, but that it can vary by a factor of three, and have significant discontinuities at transitions.

lattice constant $a = l_0 \sqrt{2\pi}$, so that there is one electron per site. There is a periodic potential with period $L = Na$ in each direction coupling to the charge associated with pseudospin textures, which assumes the following form for a triplet of spins on the lattice [45]:

$$\delta Q_{123} = \frac{e}{2\pi} \tan^{-1} \left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2 \times \mathbf{n}_3}{1 + \mathbf{n}_1 \cdot \mathbf{n}_2 + \mathbf{n}_2 \cdot \mathbf{n}_3 + \mathbf{n}_3 \cdot \mathbf{n}_1} \right). \quad (13)$$

The usual spin-stiffness, planar anisotropy, and interlayer tunneling terms are present. Finally, to model the Coulomb interaction between pseudospin textures, we introduce a Hubbard interaction for the charge on each plaquette $\square$.

FIGURE 6: The lowest-lying modes for $V = 4.3$ and $U = 18$. The system is very close to a ground state transition. Note the almost gapless Q -mode.

Thus, the Hamiltonian is

$$\begin{aligned} \mathcal{H} = & -J \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \left(n_x(\mathbf{r})n_x(\mathbf{r}') + n_y(\mathbf{r})n_y(\mathbf{r}') \right) + \frac{\Gamma}{2} \sum_{\mathbf{r}} (n_z(\mathbf{r}))^2 \\ & - h \sum_{\mathbf{r}} n_x(\mathbf{r}) - V \sum_{\square} f(X, Y) \delta Q_{\square} + H_U [\delta Q^2]. \end{aligned} \quad (14)$$

Here V is the strength of the periodic potential living on the dual lattice, and $f(X, Y)$ its functional form. We choose the simple form $f = \sin(2\pi X/L) \sin(2\pi Y/L)$, resulting in each unit cell having four puddles, two each being positive and negative. We show results for $J = \Gamma$ and measure all energies in units of J . We find the ground states (really saddle point configurations of the spins) numerically by simulated annealing. For small V the uniform ferromagnetic state is the ground state, but as V increases the ground state nucleates more and more merons/antimerons in each of the puddles. An example is shown Figure 3.

As the strength of the periodic potential increases, more merons/antimerons are nucleated to screen the potential, as exemplified by Figure 4.

The phase transitions between these ground states are generically first order, though occasionally they can become weakly first order or even second order. Various physical quantities, such as the spin stiffness (without vortex corrections) and spin wave velocity, also exhibit jumps at these transitions, as shown in Figure 5.

We note an important qualitative difference between the models with and without the periodic potential as a function of interlayer coupling. In the clean model, with no potential, the interlayer tunneling strength h is strongly relevant, and thus systems with weak and strong h at the microscopic scale are expected to behave similarly at large length scales. However, in the model with a periodic potential, as h increases, the system undergoes transitions in which merons/antimerons are lost from the puddles, ending at very large h in the uniform ferromagnetic state. Thus, the

FIGURE 7: T_{KT} in units of J/k_B as a function of V for a 16×16 unit cell at $U = 8$. Dramatic decreases in the transition temperature T_{KT} separating the low-temperature ferromagnetic phase from high-temperature paramagnetic phase occur due to changes in the topological density of the ground-state at critical potential strengths.

weak and strong h systems are in different phases, and there is no reason to expect similar behavior from them. Indeed, experimentally one sees the puzzling features only in systems with a tiny h .

Secondly, we examined the collective mode spectrum, and found that when the transition is weakly first order there is a new, quadratically dispersing, mode which becomes low in energy and can be even lower in energy than the linearly dispersing Goldstone mode (or G -mode). We call this new mode the Q -mode. Examining the wave functions of the Q -mode reveals that it represents vortex motion within the puddle. An example of the low-energy part of the collective mode dispersions is shown in Figure 6.

The presence of first-order ground state phase transitions results in the strong suppression of the Berezinskii-Kosterlitz-Thouless transition temperature T_{KT} near the V_0 corresponding to the transition. This can be seen in a simple way as follows. The ground state transition occurs between states having different numbers of merons/antimerons in each puddle. The simplest picture is when the ground state on one side of the transition has no merons/antimerons while the ground state on the other side has a checkerboard pattern of merons/antimerons. Integrating out the spin waves, the difference in ground state energy can be modelled as arising from a Coulomb gas energy of the form

$$E[\{m(\mathbf{R})\}] = \frac{1}{2} \sum_{\mathbf{R} \neq \mathbf{R}'} \mathcal{K}_s m(\mathbf{R}) m(\mathbf{R}') \log \left(\frac{|\mathbf{R} - \mathbf{R}'|}{\xi} \right) - E_c(V) \sum_{\mathbf{R}} (m(\mathbf{R}))^2, \quad (15)$$

where $E_c(V)$ is a core energy that depends on the potential strength, $m(\mathbf{R})$ is the vorticity at the dual lattice site $\mathbf{R}$, and ξ is a cutoff which can be tuned so that the groundstate goes through the transition seen to occur at V_c in the simulation. Near the transition the difference in ground state energies

behaves as $\alpha(V - V_c)$. Attributing this difference to the difference in vortex/antivortex core energies, we can extract $E_c \approx (\alpha/4)|V - V_c|$. Note that the vortex and the absence of a vortex interchange roles as the excitation as one goes through the transition. As the core energy of a vortex/antivortex vanishes, they proliferate and disorder the system. One can solve the renormalization group equations to find T_{KT} as one varies V . We find that T_{KT} is typically suppressed by an order of magnitude compared to Hartree-Fock estimates, as shown in Figure 7.

Now consider the suppression of interlayer tunneling near a ground state transition with a nearly gapless Q -mode. It is important to note that the XY angle θ couples to both the G - and Q -modes, and for small deviations, can be written as a linear combination of them. To see the effect of the Q -mode, one decomposes the tunneling term as

$$h \cos(\theta) \simeq h(e^{i\theta_G} e^{i\theta_Q} + c.c) \quad (16)$$

and integrates out the Q -mode at nonzero temperature T . Assuming a quadratic dispersion $\omega_Q(\mathbf{k}) = E_{Q0} + \gamma \mathbf{k}^2$, after integrating the Q -mode one obtains a renormalized h

$$h_R \approx h e^{-T \log(\Lambda/E_{Q0})/\gamma}. \quad (17)$$

Here Λ is a cutoff, and it can be seen that as the gap E_{Q0} of the Q -mode vanishes, $h_R \rightarrow 0$.

One of the most important open questions is whether, and how much of, the phenomenology of the system with the periodic potential survives in the system with true disorder. Our expectation is that the qualitative difference between weak and strong tunneling will survive, as will the fact that the ground state has topological content. One can expect the disorder to smooth the first-order transitions into second-order ones with a gapless mode at the transition [46–48]. In such a case, near a transition one expects the suppression of interlayer tunneling to survive as well. Furthermore, an important effect of disorder is to create large rare regions in which the system is close to critical (the Griffiths phase [49]), and thus even for a *generic* disorder strength one expects low-energy modes to exist in the system. However, whether interlayer tunneling is thus suppressed for generic disorder strength is not clear.

6. Conclusion

Quantum Hall bilayers have natural descriptions in the languages of ferromagnetism and thin film superconductivity. Both these descriptions suggest that the system should have vortex-like excitations, called merons, which can play a key role in the dissipative properties of the system. Real bilayer systems are inevitably subject to disorder, which is likely to be strong, and to induce such vortices in the groundstate, forming a “coherence network” which surrounds puddles of merons. We have discussed how the motion of merons accounts for the deviation of the system from perfect superfluid behavior and how the model naturally explains dissipation at finite temperatures in in counterflow, tunneling, and drag geometries. To better

understand quantum effects in this system, one can consider a simpler model in which the puddles are periodically arranged and study the low energy collective modes of the system. In this case we found a series of first-order zero temperature transitions separating states of different meron occupations in the puddles and argued that the Kosterlitz-Thouless temperature should drop to zero at these transitions. Approaching these transitions, the collective mode spectrum may develop a low energy mode associated with vortex motion which can greatly suppress the effect of tunneling on the system.

There is much yet to understand about this rich and fascinating system, including how quantum effects impact the zero temperature state and its transport properties, whether there is any attainable limit in which true counterflow superfluidity could be observed, and whether the competing effects of tunneling and disorder can lead to further exotic states. We anticipate that this system will remain a subject of keen interest for years to come.

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