

Research Article

Nonstationary Superconductivity: Quantum Dissipation and Time-Dependent Ginzburg-Landau Equation

Anatoly A. Barybin

Electronics Department, Saint-Petersburg State Electrotechnical University, 197376 Saint-Petersburg, Russia

Correspondence should be addressed to Anatoly A. Barybin, barybin@mail.ru

Received 27 October 2010; Accepted 4 January 2011

Academic Editor: Victor V. Moshchalkov

Copyright © 2011 Anatoly A. Barybin. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Transport equations of the macroscopic superfluid dynamics are revised on the basis of a combination of the conventional (stationary) Ginzburg-Landau equation and Schrödinger's equation for the macroscopic wave function (often called the order parameter) by using the well-known Madelung-Feynman approach to representation of the quantum-mechanical equations in hydrodynamic form. Such an approach has given (a) three different contributions to the resulting chemical potential μ_s for the superfluid component, (b) a general hydrodynamic equation of superfluid motion, (c) the continuity equation for superfluid flow with a relaxation term involving the phenomenological parameters τ_{GL} and ξ_{GL} , (d) a new version of the time-dependent Ginzburg-Landau equation for the modulus of the order parameter which takes into account dissipation effects and reflects the charge conservation property for the superfluid component. The conventional Ginzburg-Landau equation also follows from our continuity equation as a particular case of stationarity. All the results obtained are mutually consistent within the scope of the chosen phenomenological description and, being model-neutral, applicable to both the low- T_c and high- T_c superconductors.

1. Introduction

In superconductivity, the present efforts of scientists are mainly concentrated on studying the high- T_c superconductors. Yet, there are some theoretical problems in the low- T_c superconductivity left unsolved up to now. One of them (relating to thermoelectricity and having remained mysterious for many years because of great discrepancy between theory and experiments) has recently been explained by using the Fermi liquid description and charge imbalance notion [1].

Another problem left unsolved from the past is associated with the nonequilibrium behavior of superfluid flow when time-dependent situations result in dissipative effects, which are non-existent in the stationary regime. In particular, many efforts were spent previously to convert the usual (stationary) Ginzburg-Landau equation into a time-dependent (non-stationary) form but without effect, unfortunately, because all such attempts resulted in an equation of charge non-conserving form.

Before pursuing this point in more detail, it is useful to consider the similar problem relating to elaboration of the

dissipative quantum theory for usual (non-superconductive) systems with damping.

On the classical level, a variational principle was first applied to dissipative systems by Bateman [2] on the basis of doubling the system degrees of freedom. Bateman's classical approach was later imported to quantum field theory [3] and this method has been since then applied to a number of problems. However, for our purposes it is more interesting to consider another approach set forth below.

Over sixty years ago, Caldirola [4] and Kanai [5] derived a quantum-mechanical equation for the wave function ψ . They have described the damped motion of a charged particle with charge e and mass m by introducing a phenomenological dissipation parameter λ . Another form of the nonlinear wave equation, known as the Schrödinger-Langevin equation, was obtained by Kostin [6] when examining the quantum motion of a Brownian particle.

Afterwards, Wagner [7] carried out a comparative study of the Caldirola-Kanai and Schrödinger-Langevin equations on the basis of the hydrodynamic treatment, which was first suggested by Madelung [8]. In accordance with Madelung's picture, the wave function is represented as

$\psi = \sqrt{\rho} \exp(iS/\hbar)$. In this case, the separation of Schrödinger's equation into the real and imaginary parts yields both the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

and the hydrodynamic equation of motion,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{m} \nabla(V + Q) - \lambda \mathbf{v}, \quad (2)$$

where

$$\mathbf{v} = \frac{1}{m} \nabla S, \quad Q = -\frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}}. \quad (3)$$

The quantum hydrodynamic equation (2) takes into account dissipative effects in the ordinary way by introducing a phenomenological relaxation term $-\lambda \mathbf{v}$. Besides the electric potential V , it contains the so-called *quantum potential* Q (both V and Q are measured in eV). In the classical limit $\hbar \rightarrow 0$, when the quantity S makes sense of the classical action, the quantum potential Q vanishes.

Expressions (3) are valid only for the Schrödinger-Langevin-Kostin formulation [6], while the Caldirola-Kanai equations [4, 5] produce some extra loss factors for \mathbf{v} and Q , $\exp(-\lambda t)$ and $\exp(-2\lambda t)$, respectively.

As is shown by Wagner [7], in the dynamic regime with taking into account both the scalar potential $\varphi = V/e$ and the vector potential \mathbf{A} , when (in the SI system)

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \varphi, \quad \mathbf{B} = \nabla \times \mathbf{A}, \quad (4)$$

expression (3) for the quantum potential Q remains unchanged. However, expression (3) for the hydrodynamic velocity \mathbf{v} assumes the following form:

$$\mathbf{v} = \frac{(\nabla S - e\mathbf{A})}{m}. \quad (5)$$

Moreover, with allowance for \mathbf{v} , the hydrodynamic equation (2) acquires some additional terms which, however, have no reasonable physical explanation [7].

It should be noted that the quantum potential concept appearing in Madelung's formulation of quantum mechanics [8] was applied by a number of authors to solve different physical problems (e.g., see [9–12]) including quantum dissipation effects in Schrödinger-Langevin-Kostin formulation [13, 14].

As applied to the superfluid dynamics in superconductors, the problem of quantum dissipation is treated in the form of the time-dependent Ginzburg-Landau (TDGL) equation (e.g., see [15]). Generalization of the usual Ginzburg-Landau (GL) stationary theory [16–18] to time-varying situations was the subject of much investigation (e.g., see reviews [19–21]). However, all known versions of the TDGL equation suffer from a general drawback: they have no charge conservation property, that is, do not provide the continuity equation for the superfluid flow (but *not* for the

total flow, superfluid and normal fluid together) (see the text after formula (38)).

As will be shown below, unlike the nonsuperconductive systems with losses subject to (1) and (2), insertion of the quantum dissipation effect in the TDGL equation for superfluid flow in the form with nonzero right-hand part, whereas the equation of supercarrier motion involves no relaxation term like $-\lambda \mathbf{v}$. This inference results from applying Madelung's treatment to our version of the TDGL equation which results from combining Schrödinger's equation with the conventional Ginzburg-Landau equation.

Section 2 deals with such a phenomenological treatment based on the quantum-mechanical Madelung-Feynman approach to obtain the transport equations for superfluid motion. Our modified version of the TDGL equation provides a basis for deriving the desired forms of the continuity equation, the dynamic equation of motion, and the general expression for the total chemical potential of the superfluid component in dynamic regime.

Section 3 converts the continuity equation for superfluid flow into two forms corresponding to the stationary and time-dependent Ginzburg-Landau equations written, for the modulus of the order parameter.

2. Transport Equations of Macroscopic Superfluid Dynamics

2.1. Madelung-Feynman's Approach to the Quantum-Mechanical Equations in Hydrodynamic Form. Madelung [8] was the first to suggest the hydrodynamic form of quantum theory by decomposing Schrödinger's equation into the real and imaginary parts. As is noted in Section 1, Madelung's approach was first applied [4–7] to the dissipative quantum-mechanical systems described by the *microscopic wave function* $\psi = \sqrt{\rho} \exp(iS/\hbar)$, which gives rise to the customary hydrodynamic equations (1) and (2).

Afterwards, Feynman [22, 23] extended this approach to superconductivity assuming the *macroscopic wave function* (often called the *order parameter*) written in the following form

$$\Psi(\mathbf{r}, t) = a(\mathbf{r}, t) \exp[i\Theta(\mathbf{r}, t)], \quad (6)$$

satisfies the ordinary Schrödinger equation to describe the superconducting condensate. The reasoning for such an assumption is that the macroscopic wave function describes a large number of zero-spin particles (Cooper's pairs), all doing precisely the same thing (strongly correlated motion). So the time rate of change in this function must be exactly the same as the rate for any single particle. This wave function, being complex-valued, is normalized so that $|\Psi|^2 \equiv a^2 = N_s/2$, where N_s is the superconducting electron density. The phase $\Theta \equiv S/\hbar$ is associated with the canonical (generalized) momentum of Cooper's pairs

$$\mathbf{P} = 2(m\mathbf{v}_s + e\mathbf{A}), \quad (7)$$

by the known relation $\mathbf{P} = \hbar \nabla \Theta \equiv \nabla S$ [17, 18].

Relation (7) is a generalization of expression (5) to Cooper's pairs with charge $q_s = 2e$ and mass $m_s = 2m$, having velocity \mathbf{v}_s and concentration $N_s/2$.

On these grounds, following Feynman [22, 23] we write Schrödinger's equation for the macroscopic wave function (6) in the conventional form (using the SI system)

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m_s} (-i\hbar \nabla - q_s \mathbf{A})^2 + U \right] \Psi. \quad (8)$$

Here the potential energy, written as

$$U = q_s \varphi + 2\mu_{s0} = 2(e\varphi + \mu_{s0}), \quad (9)$$

is determined not only by the electric potential φ but also by the chemical potential $2\mu_{s0}$ for Cooper's pairs in the condensate. A physical meaning of the chemical potential μ_{s0} will be discussed later (see formula (49)). Its insertion into (9) is an *essential feature* of our treatment. This point distinguishes between our form (8) of Schrödinger's equation for the *macroscopic* wave function Ψ and the similar one for the *microscopic* wave function ψ (with no insertion of the chemical potential into the potential energy (9), as it is usually done in the conventional quantum-mechanical theory).

Substituting the expression (6) for Ψ in Schrödinger's equation (8) and separating the result into the real and imaginary parts, following Madelung [8], yield:

(i) from the imaginary part

$$\frac{\partial N_s}{\partial t} + \nabla \cdot (N_s \mathbf{v}_s) = 0, \quad (10)$$

(ii) from the real part

$$\mu_{s0} + \mu_{s1} + e\varphi + \frac{mv_s^2}{2} + \frac{\hbar}{2} \frac{\partial \Theta}{\partial t} = 0. \quad (11)$$

Here, in addition to μ_{s0} , there is an extra contribution to the total chemical potential

$$\mu_{s1} = -\frac{\hbar^2}{8m} \frac{\nabla^2 N_s^{1/2}}{N_s^{1/2}}. \quad (12)$$

As is seen from expression (12), the additional chemical potential μ_{s1} is due to a spatial nonuniformity of the superfluid density N_s , which takes place, for example, in the vicinity of any vortex line or the NS-interface.

Relation (11) includes also the term $mv_s^2/2$ caused by superfluid flow with velocity \mathbf{v}_s . Let us apply the gradient operation to (11) by using the known identity [24]

$$\nabla \frac{mv_s^2}{2} = m\mathbf{v}_s \cdot \nabla \mathbf{v}_s + m\mathbf{v}_s \times (\nabla \times \mathbf{v}_s) \quad (13)$$

and the total derivative of \mathbf{v}_s with respect to time

$$\frac{d\mathbf{v}_s}{dt} \equiv \frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{v}_s \cdot \nabla \mathbf{v}_s = \frac{\partial \mathbf{v}_s}{\partial t} + \nabla \frac{v_s^2}{2} - \mathbf{v}_s \times (\nabla \times \mathbf{v}_s). \quad (14)$$

Employing the following expression for \mathbf{v}_s resulted from (7):

$$\mathbf{v}_s = \frac{\mathbf{P}}{2m} - \left(\frac{e}{m} \right) \mathbf{A} \equiv \left(\frac{\hbar}{2m} \right) \nabla \Theta - \left(\frac{e}{m} \right) \mathbf{A} \quad (15)$$

and expression (4) for \mathbf{B} , we obtain

$$\nabla \times \mathbf{v}_s = -\frac{e}{m} \nabla \times \mathbf{A} = -\frac{e}{m} \mathbf{B}. \quad (16)$$

After the above transformations applied to (11), we arrive at the following equation of superfluid motion:

$$m \frac{d\mathbf{v}_s}{dt} = e(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \nabla(\mu_{s0} + \mu_{s1}). \quad (17)$$

A comparison of (2) and (4) with (17) shows that the chemical potential (12), being doubled, can be interpreted, by analogy with formula (3) for Q , as the quantum potential of Cooper's pairs ($\rho_s = q_s N_s/2 = eN_s$)

$$Q_s \equiv 2\mu_{s1} = -\frac{\hbar^2}{2m_s} \frac{\nabla^2 \rho_s^{1/2}}{\rho_s^{1/2}}. \quad (18)$$

The continuity equation (10) obtained at this stage of our examination does not take into account the dynamic processes of breaking and creating for Cooper's pairs. This point should not be surprising because such processes are necessarily beyond the ordinary Schrödinger equation (8). This equation has been used above as an initial one to obtain (10) and (11) and it describes only the dynamic processes of *nondissipative* character. In the phenomenological theory of nonstationary superconductivity, the dissipation effects are usually attained by using the time-dependent Ginzburg-Landau (TDGL) equation [18–21]. The TDGL equation is a generalization of the ordinary GL equation (originally obtained [16] for the steady-state nonuniform superconductors) to time-varying situations.

2.2. Combined Approach to the Schrödinger and Ginzburg-Landau Equations. In their pioneer work [16], Ginzburg and Landau began searching phenomenologically a proper stationary form for the free energy functional,

$$\mathcal{F}_s = \int f_s(\Psi, \Psi^*, \mathbf{A}) dV, \quad (19)$$

where its density f_s depends on the vector potential \mathbf{A} , as well as on the complex order parameter Ψ given in the form of (6). To find $\Psi(\mathbf{r})$, (19) was derived by requiring \mathcal{F}_s to be minimum with respect to variation of Ψ^* , that is, $\delta \mathcal{F}_s / \delta \Psi^* = 0$. Such an operation has led to the usual GL equation [16, 18]

$$\frac{\partial f_s}{\partial \Psi^*} \equiv \left[\frac{1}{2m_s} (-i\hbar \nabla - q_s \mathbf{A})^2 + \alpha + \beta |\Psi|^2 \right] \Psi = 0, \quad (20)$$

where the zero equality is required to ensure stationary conditions. The parameters α and β introduced phenomenologically are often associated with two other measurable temperature-dependent quantities: the *equilibrium* superfluid density $N_s^\circ(T) = 2|\Psi^\circ|^2$ and the thermodynamic

critical magnetic field $H_c(T)$ by means of the following relations (written in the SI system of units) [18, 25]:

$$\alpha(T) = -2\mu_0 \frac{H_c^2(T)}{N_s^\circ(T)}, \quad \beta(T) = 2 \frac{|\alpha(T)|}{N_s^\circ(T)}. \quad (21)$$

Here (and also below) the superscript \circ corresponds to the equilibrium state and μ_0 is the vacuum permeability.

In the literature, there are different versions of the time-dependent generalization of the stationary GL equation (20). A common feature of all the known phenomenological approaches is the assumption that the function $\Psi(\mathbf{r}, t)$ tends to its stationary value (corresponding to minimum of the free energy functional \mathcal{F}_s) at a rate governed by the slope of the density $f_s(\Psi, \Psi^*, \mathbf{A})$ in the functional space, that is, by the value $-\partial f_s / \partial \Psi^*$ which is now other than zero, unlike (20). This way has led to the following general form of the TDGL equation (e.g., see [15, 19, 20, 26]):

$$\begin{aligned} \left(\frac{\partial}{\partial t} - \frac{2\mu}{i\hbar} \right) \Psi &= -\frac{1}{\hbar\gamma} \frac{\partial f_s}{\partial \Psi^*} \\ &\equiv -\frac{1}{\hbar\gamma} \left[\frac{1}{2m_s} (-i\hbar\nabla - q_s \mathbf{A})^2 + \alpha + \beta |\Psi|^2 \right] \Psi. \end{aligned} \quad (22)$$

The right-hand part of (22) includes the relaxation parameter γ introduced phenomenologically and the functional derivative $\partial f_s / \partial \Psi^*$ in the form of (20) but not being equal to zero under nonequilibrium conditions. Second term on the left reduces to the following time dependence $\Psi^\circ(t) = |\Psi^\circ| \exp(-2i\mu t/\hbar)$ in equilibrium when $\gamma \rightarrow \infty$.

Microscopic theory of the time-dependent superconductivity meets with serious complications arising from existence of the energy gap and singularity in the density of states at the gap edges. For this reason, the TDGL equation was derived by Gor'kov and Eliashberg [27] only for the gapless regime with predominant effect of paramagnetic impurities. Schmid [28] has obtained the analogous equation valid only near the transition temperature T_c and proposed that μ appearing in (22) should be considered as the electrochemical potential. A detailed discussion of the time-dependent problems can be found in a review [20].

All versions of the TDGL equation have the left-hand side similar to that of (22). As is shown below, such a structure of the TDGL equation is not of charge-conservation character because it cannot provide the continuity equation for the superfluid motion (but *not* for the total, normal and superfluid, motion which always conserves the charge).

Our idea of obtaining another version of the time-dependent GL equation is based on an analogy with the conventional way in deriving the kinetic equation for the distribution function to describe the nonstationary behavior of carriers in semiconductors and normal metals.

As is well-known (e.g., see books [29, 30]), in kinetics theory the time rate of change in the distribution function $f(\mathbf{r}, t)$ is taken to be divided into two parts: (i) the *field contribution* $(\partial f / \partial t)_{\text{field}}$ caused by the coherent action of electromagnetic fields and (ii) the *dissipation contribution* $(\partial f / \partial t)_{\text{diss}}$ caused by the incoherent relaxation processes of

electron scattering and known as the collision integral, so that

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_{\text{field}} + \left(\frac{\partial f}{\partial t} \right)_{\text{diss}}, \quad (23)$$

discarding the actual expressions for the two terms in (23), which can be found in the literature (see [29, 30]).

In accordance with the line of our reasoning, the similar approach can be phenomenologically applied to the macroscopic wave function $\Psi(\mathbf{r}, t)$ of the superconducting state, so that its time rate of change is represented, by analogy with expression (23), as a sum of the similar two contributions:

$$\frac{\partial \Psi}{\partial t} = \left(\frac{\partial \Psi}{\partial t} \right)_{\text{field}} + \left(\frac{\partial \Psi}{\partial t} \right)_{\text{diss}}. \quad (24)$$

The *field contribution* is assumed to obey Schrödinger's equation (8) rewritten in the following form:

$$\left(\frac{\partial \Psi}{\partial t} \right)_{\text{field}} = \frac{1}{i\hbar} \hat{H} \Psi, \quad (25)$$

where, in accordance with (8) and (9), the Hamiltonian operator for Cooper's pairs is

$$\hat{H} = \frac{1}{2m_s} (-i\hbar\nabla - q_s \mathbf{A})^2 + q_s \varphi + 2\mu_{s0}. \quad (26)$$

The *dissipation contribution* is taken to satisfy the following equation:

$$\left(\frac{\partial \Psi}{\partial t} \right)_{\text{diss}} = -\frac{1}{\hbar\gamma} \frac{\partial f_s}{\partial \Psi^*}, \quad (27)$$

whose right-hand part coincides with that of (22), where the functional derivative

$$\frac{\partial f_s}{\partial \Psi^*} = \left[\frac{1}{2m_s} (-i\hbar\nabla - q_s \mathbf{A})^2 + \alpha + \beta |\Psi|^2 \right] \Psi \quad (28)$$

has the form of (20) theoretically substantiated by Ginzburg and Landau [16] but different from zero under nonequilibrium conditions.

Substitution of relations (25)–(28) into (24) gives a desired novel version of the nonstationary equation, giving rise to both the usual GL equation (see (62)–(64)) and the TDGL equation (see (67)–(69)):

$$\begin{aligned} \frac{\partial \Psi}{\partial t} - \frac{1}{i\hbar} \left[\frac{1}{2m_s} (-i\hbar\nabla - q_s \mathbf{A})^2 + q_s \varphi + 2\mu_{s0} \right] \Psi \\ = -\frac{1}{\hbar\gamma} \left[\frac{1}{2m_s} (-i\hbar\nabla - q_s \mathbf{A})^2 + \alpha + \beta |\Psi|^2 \right] \Psi. \end{aligned} \quad (29)$$

This equation, being time-dependent and space-nonuniform, allows for dissipation processes characterized by the phenomenological parameter γ , as well as by α and β . In comparison with the conventional equation (22) existing now in the literature [15, 19, 20, 26], (29) contains in the left-hand side the additional terms involving the space operator ∇ and the electromagnetic potentials \mathbf{A} and φ ,

provided the quantity μ in (22) means μ_{s0} . It is these terms that turn out to be of crucial importance in obtaining below the unambiguous and consistent results, including: (i) the continuity equation (36) with nonzero relaxation term instead of (10) and (ii) the superfluid motion equation (52) instead of (17).

Phenomenological parameters α , β , and γ are often replaced by others [15, 25]:

$$\begin{aligned}\tau_{\text{GL}}(T) &= \frac{\hbar\gamma(T)}{|\alpha(T)|} \equiv \frac{\hbar\gamma(T)}{2} \frac{N_s^\circ(T)}{\mu_0 H_c^2(T)}, \\ \xi_{\text{GL}}(T) &= \frac{\hbar^2}{2m_s |\alpha(T)|} \equiv \frac{\hbar^2}{8m} \frac{N_s^\circ(T)}{\mu_0 H_c^2(T)}, \\ \eta_{\text{GL}}(T) &= \frac{\beta(T)}{|\alpha(T)|} \equiv \frac{2}{N_s^\circ(T)}\end{aligned}\quad (30)$$

where the last equalities are written by applying formula (21). The quantities $\tau_{\text{GL}}(T)$ and $\xi_{\text{GL}}(T)$ are known as the temperature-dependent relaxation time and coherence length of the GL theory [17–21, 27].

In the case of employing parameters (30), our initial equation (29) takes the following form:

$$i\hbar \frac{\partial \Psi}{\partial t} - \hat{H}\Psi = \frac{i\hbar}{\tau_{\text{GL}}} \hat{R}\Psi. \quad (31)$$

Here the Hamiltonian operator \hat{H} (nondissipative) is given by (26) and the relaxation operator \hat{R} (dissipative) on the right is now defined as

$$\hat{R} = \xi_{\text{GL}}^2 \left(\nabla + \frac{2e}{i\hbar} \mathbf{A} \right)^2 - \eta_{\text{GL}} |\Psi|^2 + 1. \quad (32)$$

It is easy to see that (31) is converted either into the ordinary GL equation (20) when $\tau_{\text{GL}} \rightarrow 0$ (relaxation processes are dominant) or into Schrödinger's equation (8) when $\tau_{\text{GL}}^{-1} \rightarrow 0$ (relaxation processes are negligible). Something close to our result was remarked by Abrahams and Tsuneto [26].

It is very important to note that the right-hand side of (31) can exactly be reduced to the form obtained on the basis of microscopic theory for gapless superconductors by Gor'kov and Eliashberg [27] (see also works [18–20]) only if the quantity $D_{\text{GL}} \equiv \xi_{\text{GL}}^2/\tau_{\text{GL}} = \hbar/4my$ is identified with the normal-state diffusion coefficient $D = v_F^2 \tau/3$.

The left-hand and right-hand sides of (31) are, respectively, transformed by using the macroscopic wave function in the form of $\Psi = \sqrt{N_s/2} \exp(i\Theta)$ (see (6)) into the following:

$$\begin{aligned}i\hbar \frac{\partial \Psi}{\partial t} - \hat{H}\Psi &= - \left[2 \left(\mu_{s0} + \mu_{s1} + e\varphi + \frac{mv_s^2}{2} + \frac{\hbar}{2} \frac{\partial \Theta}{\partial t} \right) \right. \\ &\quad \left. - \frac{i\hbar}{2N_s} \left(\frac{\partial N_s}{\partial t} + \nabla \cdot (N_s \mathbf{v}_s) \right) \right] \Psi,\end{aligned}\quad (33)$$

$$\frac{i\hbar}{\tau_{\text{GL}}} \hat{R}\Psi = - \frac{i\hbar}{\tau_{\text{GL}}} \left(G + \xi_{\text{GL}}^2 \frac{2m}{i\hbar} \frac{\nabla \cdot (N_s \mathbf{v}_s)}{N_s} \right) \Psi. \quad (34)$$

Here we have used formula (7) the relations $(\mathbf{v}_s \cdot \nabla a)/a = (\nabla \cdot N_s \mathbf{v}_s)/2N_s$ and $\nabla \cdot \mathbf{v}_s = 0$ (see (44)), and also introduced a quantity

$$G = \xi_{\text{GL}}^2 \frac{8m}{\hbar^2} \left(\frac{mv_s^2}{2} + \mu_{s1} \right) + \frac{N_s}{N_s^\circ} - 1. \quad (35)$$

After substituting (33) and (34) into (31) and separating out the imaginary and real parts, we obtain:

(i) the continuity equation for superfluid motion

$$\frac{\partial N_s}{\partial t} + \nabla \cdot (N_s \mathbf{v}_s) = - \frac{2G}{\tau_{\text{GL}}} N_s, \quad (36)$$

(ii) the relation for chemical potentials

$$\mu_{s0} + \mu_{s1} + \mu_{s2} + e\varphi + \frac{mv_s^2}{2} + \frac{\hbar}{2} \frac{\partial \Theta}{\partial t} = 0. \quad (37)$$

Here, in addition to the chemical potentials μ_{s0} and μ_{s1} we have introduced another contribution

$$\mu_{s2} = - \frac{\hbar}{4\gamma N_s} \nabla \cdot (N_s \mathbf{v}_s) \equiv - \frac{\xi_{\text{GL}}^2}{\tau_{\text{GL}}} \frac{m}{e} \frac{\nabla \cdot \mathbf{J}_s}{N_s}. \quad (38)$$

The chemical potential μ_{s2} , just like μ_{s1} , is due to the nonuniform spatial distribution of supercarriers but, unlike μ_{s1} , is solely of dissipative nature. It is interesting to notice that μ_{s2} fully coincides with the quantity $2(\mu_p - \mu)$ introduced by Rieger et al. [31] to describe mutual conversion of the superfluid and normal fluid components in the vicinity of NS-interfaces.

It is pertinent to stress that if we employed the conventional TDGL equation (22) instead of our equations (31)–(32) as a basis for the transformations done above, it would lead us to the different result: there would be neither the term $\nabla \cdot (N_s \mathbf{v}_s)$ in (36) nor the terms μ_{s1} , $e\varphi$ and $mv_s^2/2$ in (37). It is the former fact that allows us to draw the conclusion about the charge-nonconservation character of all the existing versions of the TDGL equation, which have the left-hand side similar to that of (22). The above-mentioned arguments count in favor of our version of the TDGL equation given by (29) or (31)–(32).

If we define the total chemical potential of the superfluid component in a dynamic regime as the sum of the four contributions (see (12) and (38)):

$$\begin{aligned}\mu_\Sigma &= \mu_{s0} + \mu_{s1} + \mu_{s2} + \frac{mv_s^2}{2} = \mu_s + \frac{mv_s^2}{2} \\ &\equiv \left(\mu_{s0} - \frac{\hbar^2}{8m} \frac{\nabla^2 N_s^{1/2}}{N_s^{1/2}} - \frac{\xi_{\text{GL}}^2 m}{\tau_{\text{GL}} e} \frac{\nabla \cdot \mathbf{J}_s}{N_s} \right) + \frac{mv_s^2}{2},\end{aligned}\quad (39)$$

then formula (37) may be rewritten in two ways

$$\Phi \equiv e\varphi + \frac{\hbar}{2} \frac{\partial \Theta}{\partial t} = - \left(\mu_s + \frac{mv_s^2}{2} \right) \equiv -\mu_\Sigma, \quad (40)$$

$$\zeta_\Sigma \equiv \zeta_s + \frac{mv_s^2}{2} \equiv \mu_\Sigma + e\varphi = - \frac{\hbar}{2} \frac{\partial \Theta}{\partial t}, \quad (41)$$

where $\zeta_s = \mu_s + e\varphi$ is the electrochemical potential of the superfluid component.

Equation (40) shows that the total chemical potential μ_Σ always varies oppositely in phase with respect to the so-called *gauge-invariant potential* Φ , being equal to it in magnitude. Besides Φ , the superconductivity theory deals with another gauge-invariant quantity, namely, the superfluid velocity \mathbf{v}_s [17, 18]. The use of them allows us, following Stephen [32], to introduce the gauge-invariant electromagnetic potentials (marked with subscript g)

$$\mathbf{A}_g = -\frac{mv_s}{e}, \quad \varphi_g = \frac{\Phi}{e}. \quad (42)$$

In this case, the electric field (4) can be rewritten in the gauge-invariant form:

$$\mathbf{E} = -\frac{\partial \mathbf{A}_g}{\partial t} - \nabla \varphi_g. \quad (43)$$

As is known from the classical electrodynamics [33], in order for the gradient term in (43) to be actually the potential field $\mathbf{E}_p = -\nabla \varphi_g$ with the scalar potential φ_g obeying Poisson's equation, it is necessary to employ the Coulomb gauge ($\nabla \cdot \mathbf{A}_g = 0$). Hence, using formulae (16) and (42), we arrive at the following requirements imposed on the superfluid velocity:

$$\nabla \cdot \mathbf{v}_s = 0, \quad \nabla \times \mathbf{v}_s = -\left(\frac{e}{m}\right)\mathbf{B} \neq 0. \quad (44)$$

Formula (44) expresses the property of *solenoidality* inherent in the superfluid flow. The universal condition $\nabla \cdot \mathbf{v}_s = 0$ ensures that the component of superfluid velocity normal to any boundary vanishes ($\mathbf{n} \cdot \mathbf{v}_s = 0$). In the vicinity of NS-interface there is a conversion of the normal current with $\mathbf{n} \cdot \mathbf{v}_n \neq 0$ into the supercurrent with $\mathbf{n} \cdot \mathbf{v}_s = 0$ and nonuniform distribution of N_s .

According to (41), the electrochemical potential $\zeta_s = \mu_s + e\varphi$ is determined by the rate of change in the phase Θ of the order parameter Ψ . One should recall that all the energy values are measured with respect to the unperturbed bottom of the energy band ($\mathcal{E}_b^\circ = 0$). Then, by definition, in equilibrium $\mu_s^\circ \equiv \zeta_s^\circ - \mathcal{E}_b^\circ = \zeta_s^\circ$. There being the potential electric field $\mathbf{E}_p = -\nabla \varphi$ with the London gauge $\nabla \cdot \mathbf{A} = 0$, the band bottom suffers an offset from $\mathcal{E}_b^\circ = 0$, so that $\mathcal{E}_b = e\varphi$ and $\mu_s \equiv \zeta_s - \mathcal{E}_b = \zeta_s - e\varphi$.

Let us clarify a sense of the initial chemical potential μ_{s0} appearing in (9) and find its relationship to the Fermi energy \mathcal{E}_F . First consider the equilibrium superconducting state with no superfluid flow ($\mathbf{v}_s = 0$) when the chemical potentials μ_s^{eq} and μ_n^{eq} are both equal to \mathcal{E}_F . Then the Gibbs free energy per unit volume is

$$\mathcal{G}(\mathbf{v}_s = 0) = \mu_s^{\text{eq}} N_s^{\text{eq}} + \mu_n^{\text{eq}} N_n^{\text{eq}} = \mathcal{E}_F N. \quad (45)$$

In the presence of the stationary superfluid flow ($\mathbf{v}_s \neq 0$), the equilibrium is not disturbed, yet both the chemical potentials and electron densities become altered so that $\mu_s^\circ = \mu_n^\circ \neq \mathcal{E}_F$ and $N_s^\circ + N_n^\circ = N_s^{\text{eq}} + N_n^{\text{eq}} = N$. Then

$$\mathcal{G}(\mathbf{v}_s \neq 0) = \mu_s^\circ N_s^\circ + \mu_n^\circ N_n^\circ = \mu_s^\circ N. \quad (46)$$

It is evident that the alteration in \mathcal{G} is brought about by an increase in the kinetic energy of supercarriers

$$\mathcal{G}(\mathbf{v}_s \neq 0) - \mathcal{G}(\mathbf{v}_s = 0) = \frac{mv_s^2}{2} N_s^\circ. \quad (47)$$

From the above reasoning it follows that in the case of stationary superfluid flow,

$$\mu_s^\circ = \mathcal{E}_F + \frac{mv_s^2}{2} \frac{N_s^\circ}{N} = \mu_n^\circ. \quad (48)$$

It is this quantity taking account of the superfluid motion that can be identified with the above introduced value $\mu_{s0} + mv_s^2/2$, then

$$\mu_{s0} = \mu_s^\circ - \frac{mv_s^2}{2} = \mathcal{E}_F - \frac{mv_s^2}{2} \frac{N_n^\circ}{N}. \quad (49)$$

Equations (48) and (49) are similar to those given by Ginzburg and Zharkov [34, formula (9.8) and (9.9)], the addition to \mathcal{E}_F in (48) being associated with the Bernoulli potential, which was observed experimentally [35, 36].

Thus, we can rewrite the total chemical potential (39) in the following form:

$$\mu_\Sigma = \mu_s^\circ + \delta\mu_s, \quad (50)$$

where the stationary value μ_s° and its perturbation $\delta\mu_s$ are defined as follows (see (12), (38), and (49)):

$$\begin{aligned} \mu_s^\circ &= \mu_{s0} + \frac{mv_s^2}{2}, \\ \delta\mu_s &= \mu_{s1} + \mu_{s2} \equiv -\frac{\hbar^2}{8m} \frac{\nabla^2 N_s^{1/2}}{N_s^{1/2}} - \frac{\xi_{\text{GL}}^2}{\tau_{\text{GL}}} \frac{m}{e} \frac{\nabla \cdot \mathbf{J}_s}{N_s}. \end{aligned} \quad (51)$$

The dynamic addition $\delta\mu_s$ to the equilibrium chemical potential $\mu_s^\circ = \mu_n^\circ$ reflects an appearance of the normal current $\mathbf{J}_n = eN_n\mathbf{v}_n$ together with the nonuniform superfluid density $\delta N_s = N_s - N_s^\circ$ as a result of the time-dependent and space-dependent external actions.

Applying the gradient operation to (37) and carrying out the same procedure, as applied to formula (11), we finally obtain instead of (17) the required hydrodynamic equation of superfluid motion:

$$m \frac{d\mathbf{v}_s}{dt} = e(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \nabla \mu_s. \quad (52)$$

Here, the resulting chemical potential μ_s of the superfluid component consists of three contributions,

$$\mu_s = \mu_{s0} + \mu_{s1} + \mu_{s2}, \quad (53)$$

defined by (49), (12), and (38), respectively.

In comparison with (17), (52) contains, besides μ_{s0} and μ_{s1} , the chemical potential μ_{s2} , (38). The latter reflects influence of peculiar dissipative effects specified by the parameter γ entering originally into the right-hand side of the time-dependent equation (29). This influence differs from that expressed by the dissipative term $-\lambda\mathbf{v}$ in (2) and, being proportional to the gradient ∇N_s (since $\nabla \cdot (N_s \mathbf{v}_s) = \mathbf{v}_s \cdot \nabla N_s$ owing to $\nabla \cdot \mathbf{v}_s = 0$), it is of specific diffusion character.

3. Continuity Equation and Time-Dependent Ginzburg-Landau Equation

Let us return to our result in the form of (36) giving the continuity equation for the superfluid flow, which can be rewritten as

$$\frac{dN_s}{dt} + \nabla \cdot (N_s \mathbf{v}_s) = -\mathcal{R}_{\text{diss}}. \quad (54)$$

Here the relaxation (dissipation) term, in accordance with (35), is equal to

$$\mathcal{R}_{\text{diss}} = \frac{2N_s}{\tau_{\text{GL}}} \left[\xi_{\text{GL}}^2 \frac{8m}{\hbar^2} \left(\frac{mv_s^2}{2} + \mu_{s1} \right) + \frac{N_s}{N_s^\circ} - 1 \right], \quad (55)$$

with τ_{GL} and ξ_{GL} being phenomenological parameters of the GL theory [17–21, 27] (see formulae (30)).

If the right-hand side of (54) was absent, the universal solenoidality condition $\nabla \cdot \mathbf{v}_s = 0$ for the superfluid flow would imply that the superfluid behaves as an incompressible ideal fluid. However, in effect this is not the case because from (54) it follows that

$$\frac{dN_s}{dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) N_s = -\mathcal{R}_{\text{diss}} \neq 0, \quad (56)$$

where the right-hand side is not equal to zero in the general case. This refutes the conclusion about the superfluid incompressibility, which can incorrectly be inferred from the condition $\nabla \cdot \mathbf{v}_s = 0$, as it was done before [26].

It is convenient to represent $\mathcal{R}_{\text{diss}}$ in the following form:

$$\mathcal{R}_{\text{diss}} = \Gamma_R N_s \quad \text{with} \quad \Gamma_R \equiv \frac{1}{\tau_R} = \frac{2G}{\tau_{\text{GL}}}, \quad (57)$$

where, according to formula (35), we have

$$\begin{aligned} G &= \xi_{\text{GL}}^2 \frac{8m}{\hbar^2} \left(\frac{mv_s^2}{2} + \mu_{s1} \right) + \frac{N_s}{N_s^\circ} - 1 \\ &\equiv \xi_{\text{GL}}^2 \left(\frac{2m\mathbf{v}_s}{\hbar} \right)^2 - \xi_{\text{GL}}^2 \frac{\nabla^2 N_s^{1/2}}{N_s^{1/2}} + \frac{N_s}{N_s^\circ} - 1. \end{aligned} \quad (58)$$

The last equality in (58) has been obtained by employing expression (12) for the chemical potential μ_{s1} .

The quantity $\Gamma_R \equiv \tau_R^{-1} = 2G/\tau_{\text{GL}}$, appearing in (57) and playing a role of the effective pair-breaking rate, consists of two terms: $\Gamma_R = \Gamma_1 + \Gamma_2$, which correspond to the contributions from two terms in (58).

The first term Γ_1 is related to the contribution of $mv_s^2/2$ to the quantity G and realized in the particular case of spatial uniformity, when $N_s \simeq N_s^\circ$ and the quantity $\mu_{s1} \propto \nabla^2 N_s^{1/2}$ is vanishingly small, so that

$$\Gamma_1 = \frac{\xi_{\text{GL}}^2}{\tau_{\text{GL}}} \frac{8m}{\hbar^2} mv_s^2 = 2D_{\text{GL}} \left(\frac{2mv_s}{\hbar} \right)^2, \quad (59)$$

where $D_{\text{GL}} = \xi_{\text{GL}}^2/\tau_{\text{GL}}$.

In the literature (e.g., see [21, 37, 38]), there is a similar quantity $\Gamma = 2D(mv_s/\hbar)^2$ involving the normal-state

diffusion constant $D = v_F^2 \tau/3$. If the diffusion constants D and D_{GL} are identified with each other, the quantity Γ becomes four times smaller than Γ_1 because it includes the electron mass m instead of the effective mass $m_s = 2m$ for Cooper's pairs, as it occurs in the real case. It is known that $\Gamma = \tau_E^{-1}/2 + \tau_s^{-1}$ near T_c [21, 37], whence

$$\Gamma_1 = 4\Gamma = 2 \left(\frac{1}{\tau_E} + \frac{2}{\tau_s} \right). \quad (60)$$

Here τ_E is the energy relaxation time due to the inelastic electron-phonon collisions and τ_s is the elastic spin-flip scattering time due to magnetic impurities (if any), with both being taken for the normal state of a superconductor.

The second term Γ_2 is related to the contributions from μ_{s1} and the two last terms in (58) to the quantity G and caused by spatial nonuniformity of $N_s \neq N_s^\circ$. This is also due to the pair-breaking processes with participating phonons, whose relaxation time ($\tau_{\text{ph}} \approx \tau_E$) is usually much more than the GL relaxation time τ_{GL} [18, 21, 38].

So in order to ensure $\tau_E \approx \tau_R \equiv \tau_{\text{GL}}/2G \gg \tau_{\text{GL}}$, it is necessary that values of G should be sufficiently small. The above inequality provides sufficiently fast formation of the quasilocal equilibrium in the condensate (characterized by τ_{GL}) as compared with the relatively slow pair-breaking processes (characterized by $\tau_E \approx \tau_R$). Such an assertion should be true at any temperature near T_c when $\tau_{\text{GL}} \simeq \pi\hbar/8k_B(T_c - T)$ [18, 21, 38]. Consequently, the closer temperature T is to T_c , the smaller should be the quantity G ($G \rightarrow 0$ in the limiting case $T \rightarrow T_c$) to ensure the inequality $\tau_{\text{GL}} \ll \tau_E \approx \tau_R \equiv \tau_{\text{GL}}/2G$. This provides fast formation of the quasi-local equilibrium in the condensate under nonstationary physical conditions.

Before rearranging the continuity equation (54), let us consider the extremely limiting case of $G = 0$ and show that it can be realized, irrespective of temperature, only in the particular case of stationary situations described by the usual (time-independent) GL equation (20).

3.1. Stationary Ginzburg-Landau Equation. A comparison of equations (20) and (29) displays a condition for realizing the GL equation in the form of $\gamma = 0$ imposed on (29), which reflects the dominant effect of relaxation processes. It is clearly evident from (31) that the condition $\gamma = 0$ (or $\tau_{\text{GL}} = 0$) is equivalent to the equation $\hat{R}\Psi = 0$, where the relaxation operator \hat{R} is defined as (32). From expression (34), it follows that the stationary GL equation (20) assumes the form

$$\hat{R}\Psi \equiv - \left(G - i\xi_{\text{GL}}^2 \frac{2m}{\hbar} \frac{\nabla \cdot \mathbf{J}_s}{\rho_s} \right) \Psi = 0. \quad (61)$$

The quantity (61) must be equal to zero for any arbitrary function Ψ , which is fulfilled if and only if both the real and imaginary terms enclosed in parentheses vanish.

Vanishing the real part of \hat{R} in (61) means $G = 0$. Formula (58) for G , being written in terms of the normalized modulus $\psi = |\Psi|/|\Psi^\circ| \equiv (N_s/N_s^\circ)^{1/2}$ of the complex order

parameter Ψ , converts the equality $G = 0$ into the following equation:

$$\xi_{\text{GL}}^2 \nabla^2 \psi + \left[1 - \xi_{\text{GL}}^2 \left(\frac{2mv_s}{\hbar} \right)^2 \right] \psi - \psi^3 = 0 \quad (62)$$

(ψ here and below should not be confused with the microscopic wave function ψ mentioned in Section 1).

The general expression $v_s = -(e/m)\mathbf{A}$ for the simply connected and vortex-free superconductors [17, 18, 25] allows one to write the following relation:

$$\frac{2mv_s}{\hbar} = \frac{2eA}{\hbar} \equiv \frac{2\pi A}{\phi_0}, \quad (63)$$

with $\phi_0 = \pi\hbar/e$ being the quantum of magnetic flux (written in the SI system of units). Then equation (62) assumes the following form:

$$\xi_{\text{GL}}^2 \nabla^2 \psi + \left[1 - \xi_{\text{GL}}^2 \left(\frac{2eA}{\hbar} \right)^2 \right] \psi - \psi^3 = 0. \quad (64)$$

For simply-connected and vortex-free superconductors the order parameter Ψ is always real-valued, then $\psi = \Psi/\Psi^\circ \equiv (N_s/N_s^\circ)^{1/2}$. Consequently, (64) is the ordinary GL equation and it fully coincides with the similar ones existing now in the literature (e.g., see books [17, 18, 25]).

Vanishing the *imaginary part* of \hat{R} in (61) establishes the following requirement for stationary situations:

$$\nabla \cdot \mathbf{J}_s = 0. \quad (65)$$

The requirement (65) removes the chemical potential μ_{s2} of relaxation nature from both the chemical potential relation (37) and the force equation (52) because $\mu_{s2} \propto \nabla \cdot \mathbf{J}_s = 0$, in accordance with (38).

Therefore, in the stationary situation with steady-state superfluid flow described by the GL equation (62) or (64), the superfluid motion is governed by the simplified equation (17), which is due to $\mu_{s2} = 0$, rather than the general equation (52). In (17), there are the uniform chemical potential μ_{s0} given by (49) and the nonuniform chemical potential $\delta\mu_s = \mu_{s1}$ determined by the quantum potential Q_s of Cooper's pairs in the form of (18).

Since the superfluid flow is always solenoidal, the requirement (65) by virtue of the solenoidality condition $\nabla \cdot \mathbf{v}_s = 0$ (see formula (44)) yields

$$\mathbf{v}_s \cdot \nabla N_s = 0. \quad (66)$$

So under stationary conditions we have $\nabla N_s \perp \mathbf{v}_s$, that is, any spatial nonuniformity in N_s arises only in the plane perpendicular to the supervelocity vector \mathbf{v}_s . Thus, the longitudinal component of the gradient ∇N_s ($\nabla N_s \parallel \mathbf{v}_s$) can appear solely in nonstationary situations when the requirement (65) and its consequence (66) fail to occur for solenoidal motion of the superfluid component.

3.2. Time-Dependent Ginzburg-Landau Equation. In the case of nonstationary situations, the general continuity equation

(54) for the superfluid flow can be rearranged with the help of (57) and (58) and reduced to the following form:

$$D_{\text{GL}} \nabla^2 \psi - \frac{d\psi}{dt} = - \left[(1 - \psi^2) - \xi_{\text{GL}}^2 \left(\frac{2mv_s}{\hbar} \right)^2 \right] \frac{\psi}{\tau_{\text{GL}}}, \quad (67)$$

where $D_{\text{GL}} = \xi_{\text{GL}}^2 / \tau_{\text{GL}}$ and $d\psi/dt = \partial\psi/\partial t + \mathbf{v}_s \cdot \nabla \psi$.

Equation (67) is the desired version of the time-dependent Ginzburg-Landau equation which describes the time evolution of the normalized modulus $\psi = |\Psi|/|\Psi^\circ| \equiv (N_s/N_s^\circ)^{1/2}$ of the nonequilibrium order parameter $\Psi(\mathbf{r}, t)$.

Expressions (59) and (60) with $\tau_s^{-1} = 0$ (no magnetic impurities) yield a ratio of the relaxation times

$$\frac{\tau_{\text{GL}}}{\tau_E} = \xi_{\text{GL}}^2 \left(\frac{2mv_s}{\hbar} \right)^2 = \xi_{\text{GL}}^2 \left(\frac{2eA}{\hbar} \right)^2 \ll 1. \quad (68)$$

Here the second equality conforms to simply connected and vortex-free superconductors, whereas the inequality physically means relatively fast formation of the quasi-local equilibrium (as compared with the pair-breaking processes) in the condensate under nonstationary conditions.

Application of formula (68) reduces the general TDGL equation (67) to the simplified form

$$D_{\text{GL}} \nabla^2 \psi - \frac{d\psi}{dt} = -(1 - \psi^2) \frac{\psi}{\tau_{\text{GL}}}, \quad (69)$$

where we have used the inequality $\tau_{\text{GL}} \ll \tau_E$.

It is evident that the general form (67) of the TDGL equation assumes the special form (62) of the GL equation with $\tau_{\text{GL}} \ll \tau_E$ if $d\psi/dt = 0$. This is indeed realized under the stationary conditions when (i) $\partial\psi/\partial t = 0$ and (ii) $\mathbf{v}_s \cdot \nabla \psi \equiv (4N_s N_s^\circ)^{-1/2} (\mathbf{v}_s \cdot \nabla N_s) = 0$ as a result of the stationary requirement (66). In this case, (69) also changes its form into the known one [17, 18, 25]

$$\xi_{\text{GL}}^2 \nabla^2 \psi + \psi - \psi^3 = 0, \quad (70)$$

which fully coincides with the stationary GL equations (62) and (64) when employing the inequality (68).

Thus, any time-dependent and space-nonuniform physical situation is governed by the equation of superfluid motion in hydrodynamic form of (52) and the TDGL equation (67) for the modulus of the order parameter. The superfluid motion equation (52) follows from the chemical potential relation (37) and is fully equivalent to this relation. Our version (67) of the TDGL equation, being a consequence of the continuity equation (54), is of charge conservation character. In general, this equation has the form of a nonhomogeneous diffusion equation with nonlinear terms allowing for the superfluid motion and dissipation processes, which assumes the conventional form (62) of the GL equation under stationary conditions.

4. Conclusion

The macroscopic equations of superfluid transport have been derived on the basis of a phenomenological combination of the usual Ginzburg-Landau equation and Schrödinger's

equation for the macroscopic wave function (see (24)–(28)), which has resulted in the desired form of (31) to describe nonstationary superconductivity. Application of the Madelung-Feynman approach [8, 22, 23] to the newly derived equation (31) by separating it into the real and imaginary parts has finally provided us with the required equation (52) to describe the superfluid motion in hydrodynamic form. Besides, such an approach has given: (i) four different contributions to the total chemical potential μ_s for superfluid component in the form of (39) and (ii) the continuity equation (36) or (54) for the superfluid flow with the relaxation term in the form of (55) involving the phenomenological Ginzburg-Landau parameters τ_{GL} and ξ_{GL} .

The hydrodynamic equation (52) of superfluid motion and the novel version (67) of the TDGL equation for the modulus of the order parameter have turned out to be a consequence, respectively, of the chemical potential relation (37) and of the continuity equation (54). This result explains why, unlike semiconductor electronics, in stationary superconductor theory nobody applies the equation of superfluid motion and the continuity equation. Instead of them, the chemical potential relation (37) and the GL equation (62) are commonly used.

Taking into account quantum dissipation effects with the help of the relaxation term $-\lambda \mathbf{v}$ entering into the hydrodynamic equation of motion in the form of (2), as is commonly used for nonsuperconductive media, is not the case for the theory of time-dependent superconductivity. For the latter, the hydrodynamic equation (52) of superfluid motion has taken into account the relaxation processes in terms of μ_{s2} given by (38). Again, the dissipative effects have proved to be inserted into the continuity equation (54) or into its output in the form of the time-dependent Ginzburg-Landau equation (67) for the modulus of the order parameter.

All the above results are mutually consistent within the scope of the chosen macroscopic description and, being phenomenological and model-neutral, can be applied to both the low- T_c and high- T_c superconductors.

Acknowledgment

The author wishes to acknowledge the Russian Foundation for Basic Research for financial support (Grant no. 10-03-00845-a).

References

- [1] A. A. Barybin, “A Fermi liquid approach to an explanation of the thermoelectric effects experimentally observed in superconductors,” *Superconductor Science and Technology*, vol. 21, no. 10, Article ID 105005, 2008.
- [2] H. Bateman, “On dissipative systems and related variational principles,” *Physical Review*, vol. 38, no. 4, pp. 815–819, 1931.
- [3] E. Celeghini, M. Rasetti, and G. Vitiello, “Quantum dissipation,” *Annals of Physics*, vol. 215, no. 1, pp. 156–170, 1992.
- [4] P. Caldirola, “Forze non conservative nella meccanica quantistica,” *Il Nuovo Cimento*, vol. 18, no. 9, pp. 393–400, 1941.
- [5] E. Kanai, “On the quantization of the dissipative systems,” *Progress of Theoretical Physics*, vol. 3, no. 4, pp. 440–442, 1948.
- [6] M. D. Kostin, “On the schrödinger-langevin equation,” *The Journal of Chemical Physics*, vol. 57, no. 9, pp. 3589–3591, 1972.
- [7] H.-J. Wagner, “Schrödinger quantization and variational principles in dissipative quantum theory,” *Zeitschrift für Physik B*, vol. 95, no. 2, pp. 261–273, 1994.
- [8] E. Madelung, “Quantentheorie in hydrodynamischer form,” *Zeitschrift für Physik*, vol. 40, no. 3-4, pp. 322–326, 1926.
- [9] D. Bohm and B. J. Hiley, “Non-locality and locality in the stochastic interpretation of quantum mechanics,” *Physics Reports*, vol. 172, no. 3, pp. 93–122, 1989.
- [10] P. K. Chattaraj and S. Sengupta, “Quantum fluid dynamics of a classically chaotic oscillator,” *Physics Letters A*, vol. 181, no. 3, pp. 225–231, 1993.
- [11] T. C. Wallstrom, “On the initial-value problem for the Madelung hydrodynamic equations,” *Physics Letters A*, vol. 184, no. 3, pp. 229–233, 1994.
- [12] H. Wu and D. W. L. Sprung, “Ballistic transport: a view from the quantum theory of motion,” *Physics Letters A*, vol. 196, no. 1-2, pp. 229–236, 1994.
- [13] A. L. Sanin and A. A. Smirnovsky, “Oscillatory motion in confined potential systems with dissipation in the context of the Schrödinger-Langevin-Kostin equation,” *Physics Letters A*, vol. 372, no. 1, pp. 21–27, 2007.
- [14] A. A. Smirnovsky and A. L. Sanin, “Influence of dissipation on quantum wave dynamics in confined potential systems,” in *Nanodesign, Technology, and Computer Simulations*, vol. 6597 of *Proceedings of SPIE*, June 2006.
- [15] T. J. Rieger, D. J. Scalapino, and J. E. Mercereau, “Time-dependent superconductivity and quantum dissipation,” *Physical Review B*, vol. 6, no. 5, pp. 1734–1743, 1972.
- [16] V. L. Ginzburg and L. D. Landau, “On superconductivity theory,” *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, vol. 20, p. 1064, 1950.
- [17] A. A. Abrikosov, *Fundamentals of the Theory of Metals*, North-Holland, Amsterdam, The Netherlands, 1988.
- [18] M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill, New York, NY, USA, 1975.
- [19] N. R. Werthamer, “The Ginzburg-Landau equations and their extensions,” in *Superconductivity*, R. D. Parks, Ed., vol. 1, pp. 321–370, Marcel Dekker, New York, NY, USA, 1969.
- [20] M. Cyrot, “Ginzburg-Landau theory for superconductors,” *Reports on Progress in Physics*, vol. 36, no. 2, pp. 103–158, 1973.
- [21] J. A. Pals, K. Weiss, P. M. T. M. van Attekum, R. E. Horstman, and J. Wolter, “Non-equilibrium superconductivity in homogeneous thin films,” *Physics Reports*, vol. 89, no. 4, pp. 323–390, 1982.
- [22] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics. Vol. 3: Quantum Mechanics*, Addison-Wesley, Reading, Mass, USA, 1965.
- [23] R. P. Feynman, *Statistical Mechanics*, chapter 10, Benjamin, Reading, Mass, USA, 1972.
- [24] G. A. Korn and T. M. Korn, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill, New York, NY, USA, 1961.
- [25] T. Van Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits*, Elsevier, Amsterdam, The Netherlands, 1981.
- [26] E. Abrahams and T. Tsuneto, “Time variation of the ginzburg-landau order parameter,” *Physical Review*, vol. 152, no. 1, pp. 416–432, 1966.

- [27] L. P. Gor'kov and G. M. Eliashberg, "An extension of the Ginzburg-Landau equations to nonstationary problems for alloys with paramagnetic impurities," *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, vol. 54, p. 612, 1968, [Soviet Physics JETP, 27, 328 (1968)].
- [28] A. Schmid, "A time dependent Ginzburg-Landau equation and its application to the problem of resistivity in the mixed state," *Physik der Kondensierten Materie*, vol. 5, no. 4, pp. 302–317, 1966.
- [29] I. P. Shkarofsky, T. W. Johnston, and M. P. Bachynsky, *The Particle Kinetics of Plasmas*, Addison-Wesley, Reading, Mass, USA, 1966.
- [30] A. A. Barybin, *Waves in Thin-film Semiconductor Structures with Hot Electrons*, Nauka, Moscow, Russia, 1986.
- [31] T. J. Rieger, D. J. Scalapino, and J. E. Mercereau, "Charge conservation and chemical potentials in time-dependent ginzburg-landau theory," *Physical Review Letters*, vol. 27, no. 26, pp. 1787–1790, 1971.
- [32] M. J. Stephen, "Transport equations for superconductors," *Physical Review*, vol. 139, pp. A197–A205, 1965.
- [33] J. D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, New York, NY, USA, 1962.
- [34] V. L. Ginzburg and G. F. Zharkov, "Thermoelectric effects in superconductors," *Uspekhi Fizicheskikh Nauk*, vol. 125, p. 19, 1978, [Soviet Physics Uspekhi, 21, 381 (1979)].
- [35] J. Bok and J. Klein, "Electric fields in superconductors," *Physical Review Letters*, vol. 20, no. 13, pp. 660–662, 1968.
- [36] T. D. Morris and J. B. Brown, "The temperature dependence of the Bernoulli effect in a type-I superconductor," *Physica*, vol. 55, pp. 760–764, 1971.
- [37] A. Schmid and G. Schön, "Linearized kinetic equations and relaxation processes of a superconductor near T," *Journal of Low Temperature Physics*, vol. 20, no. 1-2, pp. 207–227, 1975.
- [38] A. M. Kadin and A. M. Goldman, "Dynamical effects in nonequilibrium superconductors," in *Nonequilibrium Superconductivity*, D. N. Lan-genberg and A. I. Larkin, Eds., pp. 253–324, North-Holland, Amsterdam, The Netherlands, 1986.

