Research Article

Electrostatic Potential in High-Temperature Superconducting Cuprates: Extended Ginzburg-Landau Theory

Madhuparna Karmakar

Department of Condensed Matter Physics and Material Sciences, S. N. Bose National Centre for Basic Sciences, JD Block, Sector-III, Saltlake, Kolkata 700098, India

Correspondence should be addressed to Madhuparna Karmakar, madhuparna@bose.res.in

Received 8 March 2011; Revised 12 May 2011; Accepted 13 July 2011

Academic Editor: P. Guptasarma

Copyright © 2011 Madhuparna Karmakar. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The electrostatic potential and the associated charge distribution in the vortices of high- T_c superconductors involving mixed symmetry state of the order parameters have been studied. The work is carried out in the framework of an extended Ginzburg-Landau (GL) theory involving the Gorter-Casimir two-fluid model and Bardeen's extension of GL theory applied to the high- T_c superconductors. The properties are calculated using the material parameters relevant for the high- T_c cuprate YBCO.

1. Introduction

A key feature which characterizes the vortices in any type-II superconductor is the ability of the vortex to support a magnetic flux, with the magnetic flux quantum being defined as $\Phi_0 = hc/2e$. However, what is less known and came into light only recently [1–3] is the fact that the vortices, along with supporting a magnetic flux quanta, can support an accumulation of finite electric charge in it. The accumulation of charge in the vortices is an artifact of the difference between the chemical potential inside the vortex core and of the region outside the core. The presence of diamagnetic electric current in any superconductor gives rise to inhomogeneity, as a result of which there arises an internal electric field in the superconductor, which maintain the neutrality of the charge distribution in the system and thus a constant electrochemical potential in the superconductor. The presence of an internal electric field in a superconductor with a stationary current was first discussed by Bopp [4] and such an electric field was later measured by a number of experiments [5, 6]. A uniform current in a superconductor results in an electric field analogous to the Bernoulli pressure variation associated with the nonuniform flow of a classical fluid [7]. The corresponding electric potential is thus termed as the Bernoulli potential [8] and is profoundly influenced

by the band structure of the superconducting material [9, 10]. The experimentally observed Hall anomaly in high-temperature superconductors has been attributed to the charge accumulation in the vortex core. In case of the high- T_c cuprates, it has been observed that there is a sign reversal of the flux flow Hall coefficient below T_c [11]. It has been suggested that this Hall anomaly of high-temperature superconducting cuprates is universal and is dependent on the doping of the material, with a reversal of sign of the Hall coefficient from the overdoped to the underdoped regime. Kumagai et al. [12] have for the first time experimentally studied the accumulation of charge in the vortex cores in high- T_c superconductor by using high-resolution measurements of the nuclear quadrupole frequency which is sensitive to the local charge density. The behavior of the vortex core and the charge accumulation in the core in high- T_c cuprate are different from that of the conventional type-II superconductors owing to the complicated Fermi surface architecture in these materials. A theoretical model is thus required which can be used to study the electrostatic potential and the associated charge distribution in the vortices of the high- T_c superconducting cuprates. In the present work, we attempt to do so in the framework of a phenomenological model. Such an approach has been used to study the electrostatic potential in

conventional type-II superconductors [13, 14], but for the high- T_c superconducting cuprates such a study is still lacking.

For developing the theoretical model, it is important to discuss the pairing state symmetry of the high- T_c superconducting cuprates. In spite of the conflicting evidence regarding the pairing state symmetry of high-temperature superconducting cuprates over the past decades, a consensus could now be reached that even in case of a strictly tetragonal system [15, 16], the high-temperature superconducting cuprates possess a mixed symmetry state of the order parameter components (for details see [17]). The mixed pairing state symmetry is characterized by the presence of a dominant d -wave order parameter component along with a subdominant s -wave order parameter component [18–20]. It has been found that the properties of such system can be theoretically explained by allowing for two or more order parameter components and their derivative mixing terms in the free energy density functional [17, 21–25]. In the present work, a similar two-order parameter Ginzburg-Landau (GL) theory involving mixed symmetry state of the order parameter components is used to study the electrostatic properties of the high- T_c superconducting cuprate. The other important aspect of the present work is the extension of the GL theory to the low temperature regime. The GL theory is known to be applicable to the temperature regions near the critical temperature (T_c); however, it has been observed that even at temperatures much below the T_c (i.e., $T \approx (2/3)T_c$) the GL theory provides very good result. At temperature lower than this, inaccuracies tend to set in the result obtained by employing the GL model. Extension of the present two-order parameter GL theory to the low temperature regime has been carried out in a manner parallel to the Bardeen's extension of the conventional GL theory [13, 14, 26]. The purpose of this paper is thus twofold, firstly it gives an extension of the two-order parameter GL theory corresponding to the high- T_c superconducting cuprates involving mixed symmetry state of the order parameter components to the low temperature regime, secondly it discusses about the electrostatic potential and charge distribution in the high- T_c superconducting system.

The rest of the paper is organized as follows: in Section 2 the theoretical formulation of the problem is discussed; Section 3 deals with the results obtained regarding the magnetic properties of the system and their analysis, while in Section 4 the details regarding the electrostatic properties have been discussed. Finally we conclude in Section 5 with the suggestion for future works.

2. Extended Ginzburg-Landau Model for High- T_c Cuprates

A second-order transition such as the normal-superconductor phase transition can be described by Gorter-Casimir two-fluid model [27]. In case of high-temperature superconductors involving mixed symmetry state of the order parameters, the d - and s -wave order parameter components can be expressed in terms of superconducting fractions $\bar{w}_d$

and $\bar{w}_s$, respectively. In presence of the superconducting electrons the normal-state free energy density is modified as

$$f_s = \left\langle U - \epsilon_{\text{cond}}\bar{w}_d + \epsilon_{\text{cons}}\bar{w}_s - \frac{1}{2}\gamma T^2\sqrt{1+\bar{w}_s} - \frac{1}{2}\gamma T^2\sqrt{1-\bar{w}_d} - \frac{1}{2}\beta_{sd}\gamma T^2\sqrt{1+\bar{w}_s}\sqrt{1-\bar{w}_d} \right\rangle. \quad (1)$$

The superconducting state is an ordered state. The transition of the electrons from the disordered normal state to the ordered superconducting state will give out a certain condensation energy which is expressed in (1) as $\epsilon_{\text{cond}}\bar{w}_d$ and $\epsilon_{\text{cons}}\bar{w}_s$ for the d - and s -wave type superconducting electrons, respectively. As can be seen from (1), in the present study the condition of stable d -wave configuration in the bulk has been considered, with a single transition temperature T_d . The d -wave pairing interaction is considered to be attractive, while for the s -wave a repulsive interaction is considered [28, 29]. In (1) U is the internal energy of the system, while γ is the linear coefficient of specific heat. The terms $(1/2)\gamma T^2\sqrt{1+\bar{w}_s}$ and $(1/2)\gamma T^2\sqrt{1-\bar{w}_d}$ correspond to the reduction in the entropy of the system due to the ordering of the electrons in the superconducting state. The term $(1/2)\beta_{sd}\gamma T^2\sqrt{1+\bar{w}_s}\sqrt{1-\bar{w}_d}$ corresponds to the contribution arising due to the interaction between the s - and d -wave order parameter components. The coefficient β_{sd} has been kept as a variable so as to understand the effect of the interaction term. At the critical temperature T_d , the ordering of the superconducting electron vanishes and the system return to the normal state, thus we can write

$$\epsilon_{\text{con}} = \epsilon_{\text{cons}} = \epsilon_{\text{cond}} = \frac{1}{4}\gamma T_d^2. \quad (2)$$

In terms of the total electron density n , the superconducting fractions are defined as $\bar{w}_s = 2|s|^2/n$ and $\bar{w}_d = 2|d|^2/n$, respectively, with $n = (2(|s|^2 + |d|^2) + n_n)$, where n_n is the normal state electron density. Equation (1) gives the condensation energy density of the system.

We next write the contribution of the kinetic energy of the superconducting condensate towards the total free energy density of the system. In presence of the mixed symmetry state of the order parameter components, the kinetic energy density contribution can be written as

$$f_{\text{kin}} = \left\langle \left[\gamma_s |\Pi s|^2 + \gamma_d |\Pi d|^2 + \gamma_v \left[(\Pi_y s)^* (\Pi_y d) - (\Pi_x s)^* (\Pi_x d) + c.c. \right] \right] \right\rangle, \quad (3)$$

where $\Pi = (-i\nabla - 2e\mathbf{A}/\hbar c)$ and γ_i is related to the effective electronic masses as $\gamma_i = \hbar^2/2m_i^*$ with $i = s, d$, and v . The above expression consists of three terms. The first term with the coefficient γ_s corresponds to the contribution of the s -wave order parameter component to the kinetic energy of the system, the second term with the coefficient γ_d gives the contribution of the d -wave order parameter component. Finally, the third term with the coefficient γ_v is the mixed gradient coupling term and combines the gradient

FIGURE 1: Variation of relative magnitude of d - and s -wave order parameter components $\langle \omega_d \rangle / \langle \omega_s \rangle$ with the coupling parameter β_{sd} at different magnetic field inductions b . Other relevant parameters are $t = T/T_d = 0.5$, $\kappa = 72$, $\gamma_v/\gamma_d = \epsilon_v = 0.1$.

contributions of the d - and s -wave type order parameter components. The mixed gradient coupling term is the most important term regarding the generation of s -wave order parameter component in the system. Previous theoretical studies of high- T_c superconductors carried out in the framework of two-order parameter GL theory have shown that properties of these materials are significantly affected by the admixture of s -wave order parameter component and the decisive role regarding the contribution of the s -wave order parameter component in the system is played by the mixed gradient coupling term.

It has been observed through the linear stability analysis that with the vanishing of the mixed gradient coupling term, the bulk d -wave solution is stable against the admixture of the s -wave order parameter component [16]. Thus, it can be said that in case of the standard two-order parameter GL model, the higher order coupling terms, namely $|s|^2|d|^2$ does not give rise to any significant contribution towards the admixture of the s -wave order parameter component. It must be noted that in case of the extended GL theory the model deals with temperatures much below the critical temperature where the density of the superconducting electrons gets enhanced. In order to verify the relative contribution of the d - and s -wave type superconducting electrons, the average value of the order parameter components has been calculated at different temperatures and magnetic field inductions. Irrespective of presence or absence of the interaction term with coefficient β_{sd} , the magnitude of the s -wave order parameter component is found to be significantly smaller than that of the d -wave order parameter component. Even at a low temperature of $t = 0.5$ and very low magnetic field, it has been observed that the relative magnitude of the d - and s -wave contribution does not change significantly with the presence or absence of the coupling term. This indicates the fact that even at low temperatures the principal contribution

towards the s -wave order parameter component arises from the mixed gradient coupling term only. The observation is presented in Figure 1 for a significant value of the coupling term $\beta_{sd} = 0.5$, and it can be seen that over the wide range of magnetic field induction and specially at lower magnetic fields (where the superconducting electron density is enhanced), the relative magnitude of the d - and s -wave order parameter components remains unchanged for the presence and absence of the interaction term. The observation justifies the approximation that for studying the properties of the high- T_c superconductors in the framework of extended two-order parameter GL theory, it is sufficient to consider the contribution of the s -wave order parameter component arising from the mixed gradient coupling term only and thus for the remaining part of the present study one can set $\beta_{sd} = 0$.

Next the contribution of the electrostatic potential towards the free energy density of the system is discussed. The corresponding Coulomb energy as a part of the free energy density is expressed as

$$f_c = \left\langle \phi \rho - \frac{1}{2} \epsilon_0 |\nabla \phi|^2 \right\rangle, \quad (4)$$

where the total charge density is defined as $\rho = en + \rho_{\text{lattice}}$. In this expression, ρ_{lattice} represents the lattice charge density. The electrostatic potential is determined from the Coulomb interaction and can be given as

$$\phi(\mathbf{r}) = \int d\mathbf{r}' \frac{1}{4\pi\epsilon} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \rho(\mathbf{r}') \quad (5)$$

or in its differential form by the Poisson equation

$$-\epsilon \nabla^2 \phi = \rho. \quad (6)$$

Finally, we take into account the contribution of the magnetic energy towards the free energy density of the system. For the applied magnetic field B_a the resulting magnetic energy density is given as

$$f_M = \left\langle \frac{1}{2\mu_0} (\mathbf{B} - \mathbf{B}_a)^2 \right\rangle = \left\langle \frac{1}{8\pi} (\mathbf{B} - \mathbf{B}_a)^2 \right\rangle, \quad (7)$$

where $\mu_0 = 4\pi$. Using (1)–(7), the resulting free energy density can be expressed as

$$\begin{aligned} f = & \left\langle U + \frac{1}{4} \gamma T_d^2 \frac{2|s|^2}{n} - \frac{1}{4} \gamma T_d^2 \frac{2|d|^2}{n} - \frac{1}{2} \gamma T^2 \sqrt{1 + \frac{2|s|^2}{n}} \right. \\ & + \gamma_s |\Pi s|^2 + \gamma_d |\Pi d|^2 - \frac{1}{2} \gamma T^2 \sqrt{1 - \frac{2|d|^2}{n}} \\ & - \frac{1}{2} \gamma T^2 \beta_{sd} \sqrt{1 + \frac{2|s|^2}{n}} \sqrt{1 - \frac{2|d|^2}{n}} \\ & + \gamma_v \left[(\Pi_y s)^* (\Pi_y d) - (\Pi_x s)^* (\Pi_x d) + c.c \right] \\ & \left. + \phi(en + \rho_{\text{lattice}}) - \frac{1}{2} \epsilon_0 |\nabla \phi|^2 + \frac{1}{8\pi} (\mathbf{B} - \mathbf{B}_a)^2 \right\rangle, \quad (8) \end{aligned}$$

with $n = (2|s|^2 + 2|d|^2 + n_n)$ being the total electron density. Equation (8) gives the extended two-order parameter Ginzburg-Landau (GL) free energy density for the high- T_c superconductors involving mixed symmetry state of the order parameters. Near the critical temperature (T_d) the equation reduces to the standard two-order parameter Ginzburg-Landau free energy density functional, consisting of higher-order interaction terms of the order parameter components. The total free energy is a function of the order parameter components s and d , the vector potential $\mathbf{A}$, and the normal state electron density n_n . The other physical quantities, namely, $\mathbf{B}$, ϕ , n , ρ , and so forth are to be understood in terms of s , d , $\mathbf{A}$, and n_n . The variables are dependent on the material parameter U , T_d , γ , $m_s^* = m_d^*$, ρ_{lattice} , ϵ_0 , and n . Since the density dependence of the material parameters lead to significant corrections, we assume that U , T_d , and γ are dependent on the density n . However, ϵ_0 , $m_s^* = m_d^*$, ρ_{lattice} , and so forth are taken as constants.

We now proceed to determine the equations of motion from the extended GL-free energy density functional. On minimizing (8) with respect to the electrostatic potential ϕ , the order parameters (s and d), the vector potential $\mathbf{A}$, and the normal state electron density n_n , we get a complete set of equations. Equations (9)–(13) comprising of the Poisson equation, GL equations corresponding to the d - and s -wave order parameter components, Ampere's law and Bernoulli potential, respectively, give the complete set of equations corresponding to the high- T_c superconductors involving mixed symmetry state of order parameters in presence of electrostatic potential,

$$-\epsilon_0 \nabla^2 \phi = e(n_n + 2|s|^2 + 2|d|^2) + \rho_{\text{lattice}}, \quad (9)$$

$$0 = \gamma_d \Pi^2 d + \gamma_v (\Pi_y^2 - \Pi_x^2) s + \left(\frac{-\gamma T_d^2}{2n} + \frac{1}{\sqrt{1 - 2|d|^2/n}} \frac{\gamma T^2}{2n} + \frac{\gamma T^2}{2n} \beta_{sd} \right) \quad (10)$$

$$\times \frac{\sqrt{1 + 2|s|^2/n}}{\sqrt{1 - 2|d|^2/n}} d,$$

$$0 = \gamma_s \Pi^2 s + \gamma_v (\Pi_y^2 - \Pi_x^2) d + \left(\frac{\gamma T_d^2}{2n} - \frac{1}{\sqrt{1 + 2|s|^2/n}} \frac{\gamma T^2}{2n} - \frac{\gamma T^2}{2n} \beta_{sd} \frac{\sqrt{1 - 2|d|^2/n}}{\sqrt{1 + 2|s|^2/n}} \right) s, \quad (11)$$

$$\nabla^2 \mathbf{A} = 4\pi \left\{ \left(-\frac{\hbar e^*}{m_s^*} \right) [s^* (\Pi s) + s (\Pi s)^*] + \left(-\frac{\hbar e^*}{m_d^*} \right) [d^* (\Pi d) + d (\Pi d)^*] \right\}$$

$$+ \left(-\frac{\hbar e^*}{m_v^*} \right) \left(\hat{y} [s^* (\Pi_y d) + d (\Pi_y s)^* + c.c.] - \hat{x} [s^* (\Pi_x d) + d (\Pi_x s)^* + c.c.] \right), \quad (12)$$

$$e\phi = \lambda_{\text{TF}}^2 \nabla^2 e\phi - \left(\frac{1}{n} \right) \{ \gamma_d d^* \Pi^2 d + \gamma_v d^* (\Pi_y^2 - \Pi_x^2) s \} - \left(\frac{1}{n} \right) \{ \gamma_s s^* \Pi^2 s + \gamma_v s^* (\Pi_y^2 - \Pi_x^2) d \} - \left(\frac{|s|^2}{2n} \right) T_d^2 \left(\frac{\partial \gamma}{\partial n} \right) + \left(\frac{|d|^2}{2n} \right) T_d^2 \left(\frac{\partial \gamma}{\partial n} \right) - \left(\frac{|s|^2}{n} \right) \gamma T_d \left(\frac{\partial T_d}{\partial n} \right) + \left(\frac{|d|^2}{n} \right) \gamma T_d \left(\frac{\partial T_d}{\partial n} \right) + \left(\frac{T^2}{2} \right) \left(\frac{\partial \gamma}{\partial n} \right) \sqrt{\left(1 + \frac{2|s|^2}{n} \right)} + \left(\frac{T^2}{2} \right) \left(\frac{\partial \gamma}{\partial n} \right) \sqrt{\left(1 - \frac{2|d|^2}{n} \right)} + \left(\frac{T^2}{2} \right) \left(\frac{\partial \gamma}{\partial n} \right) \times \beta_{sd} \sqrt{1 + \frac{2|s|^2}{n}} \sqrt{1 - \frac{2|d|^2}{n}}, \quad (13)$$

where the contribution $\lambda_{\text{TF}}^2 \nabla^2 e\phi$ arises due to the Thomas-Fermi screening. In the above equations, we have used the effective potential $\chi = \chi_d + \chi_s$ acting on the s - and d -wave type electrons. The effective potentials corresponding to d - and s -wave type electrons are defined as

$$\chi_d = -\frac{2}{n} \epsilon_{\text{con}} + \frac{\gamma T^2}{2n} \frac{1}{\sqrt{1 - 2|d|^2/n}} + \frac{\gamma T^2}{2n} \beta_{sd} \frac{\sqrt{1 + 2|s|^2/n}}{\sqrt{1 - 2|d|^2/n}}, \quad (14)$$

$$\chi_s = \frac{2}{n} \epsilon_{\text{con}} - \frac{\gamma T^2}{2n} \frac{1}{\sqrt{1 + 2|s|^2/n}} - \frac{\gamma T^2}{2n} \beta_{sd} \frac{\sqrt{1 - 2|d|^2/n}}{\sqrt{1 + 2|s|^2/n}}.$$

In the above set of equations, the basic material parameters are γ , T_d , $m_s^* = m_d^*$, $\partial T_d / \partial n$, and $\partial \gamma / \partial n$.

3. Magnetic Properties of the System

We begin the discussion by studying the magnetic properties of high-temperature superconductors involving mixed

symmetry state of the order parameters. In terms of reduced unit, the quantities used in the calculation are expressed as

$$\begin{aligned}
 t &= \frac{T}{T_d}, \\
 \mathbf{B}' &= \frac{\lambda_{\text{Lon}} \mathbf{B}}{\lambda_0 \sqrt{B_c B_0}}, \\
 \mathbf{A}' &= \frac{\mathbf{A}}{\lambda_0 \sqrt{B_c B_0}}, \\
 \mathbf{r}' &= \frac{\mathbf{r}}{\lambda_{\text{Lon}}}, \\
 \lambda_{\text{Lon}} &= \frac{\lambda_0}{\sqrt{1 - t^4}}, \\
 \kappa(t) &= \kappa_0 \sqrt{\frac{2}{1 + t^2}}, \\
 B_c(t) &= B_0(1 - t^2), \\
 B_{c2}(t) &= \sqrt{2} \kappa B_c,
 \end{aligned} \tag{15}$$

where $B_0 = T_d \sqrt{\mu_0 \gamma / 2}$, $\lambda_0 = \sqrt{m_d / e^2 n \mu_0}$, and $\kappa_0 = (m_d T_d / n e \hbar) \sqrt{\gamma / \mu_0}$. The extended GL free energy density functional is then expressed in terms of the gauge invariant real quantities $\omega_d = 2|d|^2 / n(1 - t^4)$, $\omega_s = 2|s|^2 / n(1 - t^4)$, and $\mathbf{Q}$, where $s(x', y') = \sqrt{\omega_s(x', y')} e^{i\phi_s(x', y')}$ and $d(x', y') = \sqrt{\omega_d(x', y')} e^{i\phi_d(x', y')}$ correspond to the order parameter components, while $\mathbf{Q}'(x', y') = \mathbf{A}'(x', y') - \nabla' \phi(x', y') / \kappa$ is the velocity of the superconducting electrons. The corresponding two-dimensional free energy density in terms of dimensionless unit can be thus written as

$$\begin{aligned}
 f' &= \left\langle -\frac{(-\omega_s + \omega_d)}{(1 - t^2)} - \frac{2t^2 \sqrt{1 + \omega_s(1 - t^4)}}{(1 - t^2)(1 - t^4)} - 2t^2 \right. \\
 &\quad \times \frac{\sqrt{1 - \omega_d(1 - t^4)}}{(1 - t^2)(1 - t^4)} - 2t^2 \beta_{sd} \sqrt{1 + \omega_s(1 - t^4)} \\
 &\quad \times \frac{\sqrt{1 - \omega_d(1 - t^4)}}{(1 - t^2)(1 - t^4)} \\
 &\quad + g_s + \omega_s Q'^2 + g_d + \omega_d Q'^2 \\
 &\quad + \epsilon_v \left[2 \cos(\phi) \left\{ \frac{(\nabla'_y \omega_s)(\nabla'_y \omega_d)}{4\kappa^2 (\omega_s \omega_d)^{1/2}} \right. \right. \\
 &\quad \left. \left. - \frac{(\nabla'_x \omega_s)(\nabla'_x \omega_d)}{4\kappa^2 (\omega_s \omega_d)^{1/2}} \right. \right. \\
 &\quad \left. \left. + (Q_y'^2 - Q_x'^2)(\omega_d \omega_s)^{1/2} \right\} \right. \\
 &\quad \left. + 2 \sin(\phi) \left\{ \frac{Q'_y (\nabla'_y \omega_s)}{2\kappa} - \frac{Q'_x (\nabla'_x \omega_s)}{2\kappa} \right\} \left(\frac{\omega_d}{\omega_s} \right)^{1/2} \right.
 \end{aligned}$$

$$\begin{aligned}
 &- 2 \sin(\phi) \left\{ \frac{Q'_y (\nabla'_y \omega_d)}{2\kappa} - \frac{Q'_x (\nabla'_x \omega_d)}{2\kappa} \right\} \\
 &\quad \times \left(\frac{\omega_s}{\omega_d} \right)^{1/2} \left. \right] + (\nabla' \times \mathbf{Q}' - B'_a)^2 \Bigg\rangle', \tag{16}
 \end{aligned}$$

where B'_a is the applied magnetic field and $f' = f / ((1/4) \gamma T_d^2 (1 - t^2)(1 - t^4))$. For studying the magnetic properties of the system, the contribution of the Coulomb energy and the internal energy to the free energy density functional is neglected. In the above equation, ϵ_v gives the strength of the s -wave order parameter component in the system and is defined as $\epsilon_v = \gamma_v / \gamma_d$. We now discuss about the selection of the parameters in this work. Theoretical studies carried out on high- T_c superconductors involving mixed pairing state symmetry of order parameters suggested the value of mixed gradient coupling parameter to be $\epsilon_v = \gamma_v / \gamma_d \approx 0.1 - 0.4$ [30]. Further, it was also observed that with $\epsilon_v = 0.1$ the theoretical results obtained in the framework of two-order parameter GL theory were in excellent agreement with the experimental data corresponding to $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ [17, 22, 24, 25]. Thus, for the present study the mixed gradient coupling parameter ϵ_v has been chosen to be $\epsilon_v = 0.1$. The effects of the higher values of the coupling parameter ϵ_v have also been verified, so as to understand the influence of the admixture of s -wave order parameter component in the system at various temperatures. The results obtained for different values of ϵ_v have been found to be qualitatively the same, with difference in magnitude depending upon the amount of admixture of the s -wave order parameter component in the system. The effects of the admixture of s -wave order parameter component on the various properties of the high- T_c cuprates, namely, the vortex lattice structure, local spatial behavior of the order parameter and magnetic field profiles, reversible magnetization of the system, and shear modulus of the flux line lattice and so forth, studied in the framework of the standard two-order parameter GL model, have already been reported in the literature [17, 22, 24, 25].

The three GL equations obtained by minimizing the free energy density functional equation (16) with respect to the order parameter components ω_s , ω_d and supervelocity $\mathbf{Q}'$ as, $\delta f'(\mathbf{r}') / \delta \omega_s(\mathbf{r}') = \delta f'(\mathbf{r}') / \delta \omega_d(\mathbf{r}') = 0 = \delta f'(\mathbf{r}') / \delta \mathbf{Q}'(\mathbf{r}')$ can be written as

$$\begin{aligned}
 -\nabla'^2 \omega_d &= 2\kappa^2 \left[\omega_d - s_d - g_d - \omega_d Q'^2 \right. \\
 &\quad \left. - \epsilon_v \left\{ \cos(\phi) \right. \right. \\
 &\quad \times \left[\left(-\frac{(\nabla_y'^2 \omega_s)}{2\kappa^2} + \frac{(\nabla_x'^2 \omega_s)}{2\kappa^2} + g_{sy} - g_{sx} \right) \right. \right. \\
 &\quad \left. \left. \times \left(\frac{\omega_d}{\omega_s} \right)^{1/2} + (Q_y'^2 - Q_x'^2)(\omega_s \omega_d)^{1/2} \right] \right.
 \end{aligned}$$

$$+ 2 \left(\frac{\sin(\phi)}{2\kappa} \right) \left(\frac{\omega_d}{\omega_s} \right)^{1/2} \times \left(Q'_y(\nabla'_y \omega_s) - Q'_x(\nabla'_x \omega_s) \right) \Bigg], \quad (17)$$

$$\begin{aligned} -\nabla'^2 \omega_s = 2\kappa^2 \Bigg[& -\omega_s - s_s - g_s - \omega_s Q'^2 \\ & - \epsilon_v \left\{ \cos(\phi) \right. \\ & \times \left[\left(-\frac{(\nabla_y'^2 \omega_d)}{2\kappa^2} + \frac{(\nabla_x'^2 \omega_d)}{2\kappa^2} + g_{dy} - g_{dx} \right) \right. \\ & \times \left(\frac{\omega_s}{\omega_d} \right)^{1/2} + (Q_y'^2 - Q_x'^2)(\omega_s \omega_d)^{1/2} \Bigg] \\ & \left. + 2 \left(\frac{\sin(\phi)}{2\kappa} \right) \left(\frac{\omega_s}{\omega_d} \right)^{1/2} \right. \\ & \left. \times (Q'_y(\nabla'_y \omega_d) - Q'_x(\nabla'_x \omega_d)) \right\} \Bigg], \quad (18) \end{aligned}$$

$$\begin{aligned} -\nabla'^2 \mathbf{Q}' = & -(\omega_s + \omega_d) \mathbf{Q}' \\ & - \epsilon_v \left[2 \cos(\phi) (\omega_s \omega_d)^{1/2} (\hat{y} Q'_y - \hat{x} Q'_x) + \left(\frac{\sin(\phi)}{2\kappa} \right) \right. \\ & \times (\hat{y}(\nabla'_y \omega_s) - \hat{x}(\nabla'_x \omega_s)) \left(\frac{\omega_s}{\omega_d} \right)^{1/2} \\ & - \left(\frac{\sin(\phi)}{2\kappa} \right) (\hat{y}(\nabla'_y \omega_d) - \hat{x}(\nabla'_x \omega_d)) \\ & \left. \times \left(\frac{\omega_s}{\omega_d} \right)^{1/2} \right], \quad (19) \end{aligned}$$

where $g_i = (\nabla'_i \omega_i)^2 / 4\kappa^2 \omega_i$ and $g_{ij} = (\nabla'_j \omega_i)^2 / 4\kappa^2 \omega_i$, with $i = s, d$; $j = x, y$; and

$$\begin{aligned} s_s &= \frac{t^2}{(1-t^2)} \left(1 - \frac{1}{\sqrt{1+\omega_s(1-t^4)}} - \beta_{sd} \frac{\sqrt{1-\omega_d(1-t^4)}}{\sqrt{1+\omega_s(1-t^4)}} \right) \omega_s, \\ s_d &= \frac{t^2}{(1-t^2)} \left(\frac{1}{\sqrt{1-\omega_d(1-t^4)}} - 1 + \beta_{sd} \frac{\sqrt{1+\omega_s(1-t^4)}}{\sqrt{1-\omega_d(1-t^4)}} \right) \omega_d. \end{aligned} \quad (20)$$

The order parameters and the magnetic field are now expressed in terms of Fourier series [22], and the positions of the vortices are defined in terms of the reciprocal lattice vectors as $\mathbf{K} = \mathbf{K}_{mn} = (2\pi/x_1 y_2)(m y_2, n x_1 + m x_2)$, where m and n are integers and x_1, x_2 , and y_2 are lattice parameters.

The order parameter components and the magnetic field are determined by solving a set of iterative equations using a numerical iteration technique [17, 22–25]. The iteration process is continued till the solution remains constant upto 15 digits. The high precision solutions of the GL equations are thus obtained and these solutions can be used to study the various magnetic and electrostatic properties of the high- T_c superconductors involving mixed symmetry state of the order parameter components at different temperatures.

3.1. Single Vortex and Vortex Lattice Structure. The first step to study the magnetic properties of the high- T_c superconducting cuprates is to study the structure of the flux line lattice. Small angle neutron scattering (SANS) [31] and Scanning tunneling microscopy (STM) [32] experiments on high- T_c superconductors have shown that unlike the case of the conventional type-II superconductors, which are characterized by a triangular vortex lattice, the high- T_c superconductors exhibit an oblique vortex lattice configuration. The experimental observations have been substantiated by the previous theoretical works where an oblique vortex lattice structure have been observed [17]. Another important feature of the high- T_c superconductors involving mixed symmetry state of the order parameters is the fourfold symmetric structure of the s -wave order parameter component [15–17]. Figure 2 shows the vortex lattice structure of the s -wave and d -wave order parameter components corresponding to different temperatures at a particular magnetic field induction and mixed gradient coupling parameter ϵ_v mentioned in the figure caption. It can be seen from the figure that at different temperatures the structure of the vortex lattice is essentially oblique and the s -wave order parameter component possesses a fourfold symmetric structure. Similar oblique flux line lattice structure is observed for the magnetic field distribution also as can be seen from Figure 3.

3.2. Reversible Magnetization. An important and experimentally determinable quantity for any superconducting material is the reversible magnetization of the system. The reversible magnetization is defined as $M = \bar{B} - B_a$, where B_a is the equilibrium applied magnetic field and is expressed as $B_a = 4\pi(\partial f / \partial \bar{B})$. Thus, the determination of the equilibrium applied magnetic field of the system involves the computation of the numerical derivative of the free energy density functional which is a complex function of two-order parameter components and magnetic field. An alternative approach is to determine the equilibrium applied magnetic field by making use of the virial theorem applicable to the two-order parameter model [22]. In the framework of the two-order parameter extended GL theory, the virial theorem has been formulated and the resulting equilibrium applied magnetic field can be expressed as

$$B_a = \frac{\langle 2B^2 + \omega_d - \omega_s - s_d - s_s \rangle}{2\bar{B}}. \quad (21)$$

Using this expression for the equilibrium-applied magnetic field the corresponding reversible magnetization of the system can be calculated as per the relation $M = \bar{B} - B_a$.

FIGURE 2: Variation of vortex lattice structure corresponding to the s - and d -wave order parameter components $\omega_s(x, y)$ ((a)–(c)) and $\omega_d(x, y)$ ((d)–(f)) for different temperatures ($t = 0.9, 0.7, 0.5$), respectively, calculated by the extended two-order parameter GL theory. Other parameters used for the calculation are GL parameter $\kappa = 72$, magnetic field induction $b = 0.8$, and mixed gradient coupling parameter $\epsilon_v = 0.1, \beta_{sd} = 0$.

FIGURE 3: Variation of magnetic field distribution $B(x, y)$ for different temperatures ($t = 0.9, 0.7, 0.5$) ((a)–(c)) respectively, calculated by the extended two-order parameter GL theory. Other parameters used for the calculation are same as in Figure 2.

The reversible magnetization of the system calculated by this expression at different temperatures is plotted in Figure 4. It can be seen from the figure that for higher values of t , that is, at temperatures close to T_d , the reversible magnetization shows the behavior well known for the GL theory. However, as one move to lower temperatures an anomalous behavior of the reversible magnetization is observed, characterized by an s -shape of the curve whose curvature increases with the decrease in temperature. In case of conventional type-II superconductors also the curvature has been found to increase with the decrease in the temperature and below a certain temperature termed as T_a the reversible magnetization shows an anomalous behavior and the system undergoes a first-order transition [13, 14]. At temperatures below T_a , a finite magnetization is observed for an applied magnetic field above B_{c2} . For the conventional type-II superconductors with $\kappa \approx 1/\sqrt{2}$, a first-order transition has been predicted near the H_{c1} and H_{c2} , depending upon the ratio of the mean free path l to the coherence length ξ_0 . For the conventional low temperature type-II superconductors, the anomalous behavior of the reversible magnetization of the system has been attributed to the presence of impurities in the superconducting system [33, 34].

In case of the high-temperature superconductors involving mixed symmetry state of the order parameters, such a first-order transition has not been observed for the temperatures plotted in Figure 4. The absence of such a transition can be attributed to the large value of the GL parameter κ in case of the high- T_c superconductors. Before coming to this point, it will be useful to determine the temperature T_a in case of the high- T_c superconductors. Near the upper critical field B_{c2} the density of superconducting condensate is small and the coefficients of the GL free energy functional can be defined as

$$\alpha = \frac{\gamma}{2n} (T^2 - T_d^2), \quad (22)$$

$$\beta = \frac{\gamma T^2}{2n^2}.$$

We now define an asymptotic GL parameter κ_{as} as [35]

$$\kappa_{as} = \sqrt{\frac{m_d^2 \beta}{2\mu_0 \hbar^2 e^2}}. \quad (23)$$

Thus, the asymptotic GL parameter κ_{as} is related to temperature as per the relation

$$\kappa_{as} = \frac{\kappa_0 t}{2}. \quad (24)$$

The first-order transition should be observed for $\kappa_{as} = 1/\sqrt{2}$ and with $\kappa_0 = 72$ corresponding to high- T_c superconductor $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$, the transition temperature (T_a) amounts to

$$T_a = \frac{\sqrt{2}}{\kappa_0} T_d = 0.01964. \quad (25)$$

Thus, in case of high- T_c superconductors (particularly $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$) such a transition is likely to be observed at a very low temperature and the range of validity of the extended GL theory for high- T_c superconductors is larger as compared to that for the conventional type-II superconductors. This justifies the observation that in Figure 4 a finite reversible magnetization is not observed at applied magnetic fields higher than B_{c2} even at a low temperature of $t = 0.4$. One may possibly define as in the case of the applied magnetic field ($H_{c1} < H < H_{c2}$), a range of temperature as $T_a < T < T_c$, over which the extended GL theory is valid for a type-II superconducting material. Below this temperature T_a , one should be careful regarding the validity of the extended GL theory and the accuracy of the results obtained.

4. Electrostatic Potential and Charge Distribution in Flux Line Lattice

We next concentrate on the electrostatic potential and the associated charge distribution in the vortices of the high- T_c superconductors involving mixed symmetry state of the order parameter components.

4.1. Electrostatic Potential. The electrostatic potential of the system is expressed as per equation (13). For the sake of simplicity, in the present calculation of the electrostatic potential the Thomas-Fermi screening is neglected, that is, $\lambda_{\text{TF}}^2 \nabla^2 e\phi = 0$. Such an approximation is justified since the Thomas-Fermi screening length is small as has been shown below. In terms of dimensionless units, we define the electrostatic potential as

$$\Phi = \frac{en}{(1/4)\gamma T_d^2(1-t^2)(1-t^4)} \phi. \quad (26)$$

For the high- T_c superconductors involving mixed symmetry state of the order parameter components, the resulting electrostatic potential is

$$\begin{aligned} \Phi = & (\omega_s - \omega_d) + (s_s + s_d) + C_1(\omega_d - \omega_s) \\ & + C_2 \left(\sqrt{1 + \omega_s(1-t^4)} + \sqrt{1 - \omega_d(1-t^4)} \right. \\ & \left. + \beta_{sd} \sqrt{1 + \omega_s(1-t^4)} \sqrt{1 - \omega_d(1-t^4)} \right), \end{aligned} \quad (27)$$

FIGURE 4: Applied magnetic field B_a (measured in units of upper critical field B_{c2}) dependence of reversible magnetization calculated by the extended two-order parameter GL theory for different temperatures. Other parameters used in the calculation are GL parameter $\kappa = 72$ and mixed gradient coupling parameters $\epsilon_v = 0.1$ and $\beta_{sd} = 0$.

where the temperature-dependent coefficients C_1 and C_2 are given as

$$\begin{aligned} C_1 &= \frac{1}{(1-t^2)} \frac{\partial \ln \epsilon_{\text{con}}}{\partial \ln n}, \\ C_2 &= \frac{2t^2}{(1-t^2)(1-t^4)} \frac{\partial \ln \gamma}{\partial \ln n}. \end{aligned} \quad (28)$$

The coefficients C_1 and C_2 depend upon the material parameter of the system under consideration, in this case YBCO. Details regarding the determination of the coefficients C_1 and C_2 for YBCO are discussed in the appendix at the end of the paper.

The electrostatic potential expressed by (27) consists of three components as $\Phi_1 = C_1(\omega_d - \omega_s)$, which arises due to the dependence of the condensation energy on the superconducting electron density, $\Phi_2 = C_2(\sqrt{1 + \omega_s(1-t^4)} + \sqrt{1 - \omega_d(1-t^4)} + \beta_{sd}\sqrt{1 + \omega_s(1-t^4)}\sqrt{1 - \omega_d(1-t^4)})$ arising from the reduced normal-state thermoelectric potential and $\Phi_B = (\omega_s - \omega_d) + (s_s + s_d)$ corresponding to the Bernoulli potential. The individual potential components are shown in Figure 5 for the temperature t and magnetic field induction b mentioned in the figure caption. A characteristic feature that can be observed from the figure is the flatness of the curves corresponding to the different components of potential, this is, unlike the case observed for conventional type-II superconductor niobium [13, 14]. It is worth mentioning that Kumagai et al. [12] in their NQR measurement of vortex charge accumulation for $\text{YBa}_2\text{Cu}_3\text{O}_7$ and $\text{YBa}_2\text{Cu}_4\text{O}_8$ have observed a flat response of the vortex charge distribution. One can thus predict that the corresponding electrostatic potential should also show similar flatness in their profiles.

FIGURE 5: The components of the electrostatic potential Φ plotted along the x -direction. The parameter values used for the figure are $t = 0.5$, magnetic induction parameter $b = 0.5$, and GL parameters $\kappa = 72$ and $\beta_{sd} = 0$.

The Bernoulli potential ϕ_B reaches zero at the center of the vortex. The potential is repulsive inside the vortex core, while outside the core the Bernoulli potential is attractive. Similar to the Bernoulli potential ϕ_B , the ϕ_1 component of electrostatic potential minimizes at the center of the vortex. This is an expected observation, since ϕ_1 arises from the condensation energy and is thus proportional to the density of superconducting electrons. The density of superconducting electron density vanishes at the center of the vortex and so is the corresponding component of the electrostatic potential. The third component of the electrostatic potential ϕ_2 is the only component which gives a nonzero contribution at the center of the vortex.

4.2. Electrostatic Charge Distribution. We next calculate the distribution of electrostatic charge in the flux line lattice. As per the Poisson's equation, the charge accumulation in the vortex core can be calculated from the electrostatic potential of the system as per the relation $\rho = -\epsilon \nabla^2 \phi$. We define charge in terms of dimensionless units as

$$\rho' = \frac{\rho}{en}. \quad (29)$$

The corresponding Poisson's equation in terms of the dimensionless unit is given as

$$\rho' = -C_3 \frac{\lambda_{TF}^2}{\lambda_{Lon}^2} \nabla'^2 \Phi, \quad (30)$$

where the coefficient C_3 is dependent upon the material parameters of the superconducting system under consideration, in this case YBCO, and is expressed as

$$C_3 = \frac{2D\epsilon_{con}}{n^2} (1 - t^2)(1 - t^4) \quad (31)$$

with D being the density of states. As in case of the electrostatic potential of the vortex lattice, the net electrostatic charge in the vortex lattice also consists of three parts as ρ'_1 ,

FIGURE 6: Different components of the charge distribution profile calculated by two-order parameter extended GL theory. The parameter values are the same as in Figure 5.

ρ'_2 , and ρ'_B corresponding to the electrostatic potential components ϕ_1 , ϕ_2 , and ϕ_B , respectively.

The different components of charge are plotted in Figure 6, and the particular shape of the vortex charge profiles are strongly dependent on the material parameters of YBCO through the coefficients C_1 , C_2 , and C_3 . At the center of the vortex, the components of charge ρ_B and ρ_2 give a positive contribution while the magnitude of the component ρ_1 is negative at the center of the vortex. The net charge at the center of the vortex has a finite negative value due to the large negative contribution arising from the component ρ_1 . The total charge density (ρ) shows a flat response outside the vortex. The behavior can be considered to be in agreement with the experimental observation reported by Kumagai et al. for YBCO [12].

For the calculation of charge distribution in the vortex lattice, the effect of Thomas-Fermi screening has been taken into account. It must however be noted that the magnitude of screening is very small. In case of YBCO, this screening amounts to

$$\frac{\lambda_{TF}^2}{\lambda_{Lon}^2} = 1.4646 \times 10^{-7} \quad (32)$$

with $\lambda_{TF}^2 = \epsilon/2De^2$ and $\lambda_{Lon} = 1.4 \times 10^{-6}$ m. Consequently, for YBCO the coefficient

$$C_3 \frac{\lambda_{TF}^2}{\lambda_{Lon}^2} = 1.54 \times 10^{-9} (1 - t^2)(1 - t^4). \quad (33)$$

In Figure 7, the magnetic field dependence of peak amplitude of charge calculated at different temperatures is presented. A higher applied magnetic field corresponds to a lower amplitude of charge. At higher magnetic field inductions, the peak amplitude of charge almost saturates. A possible reason for the observation can be the greater overlap between the vortices at higher magnetic field. At high magnetic field, the charge accumulation in the vortex decreases along the line joining the neighboring vortices. A lower temperature corresponds to a greater accumulation of charges in the vortex core.

FIGURE 7: Magnetic field dependence of peak amplitude of charge $\rho(x, 0)$ calculated along the x -direction at different temperatures. Other relevant parameters are $\kappa = 72$, $\epsilon_v = 0.1$, and $\beta_{sd} = 0$.

5. Conclusions

The study carried out in the present work consists of two parts, the first part is the extension of the GL theory to the low temperature regime in case of the high- T_c superconducting cuprates involving mixed symmetry state of the order parameter components. The second part of the work involves the study of the electrostatic potential and the associated charge distribution in the vortices. In order to study the electrostatic potential and charge accumulation in the vortices, a set of equations are derived which consist of the Poisson's equation for electrostatic charge, Bernoulli potential for the electrostatic potential, Ginzburg-Landau equation for s - and d -wave order parameter components, and Ampere's law for magnetic vector potential. The resulting equations are solved by using a numerical iteration technique for arbitrary magnetic field induction and wide range of temperature. At any temperature, the flux line lattice shows an oblique structure characteristic to the high- T_c cuprates.

The equilibrium applied magnetic field and the resulting reversible magnetization is calculated by using the virial theorem developed for the two-order parameter system. The reversible magnetization could thus be obtained without taking the numerical derivative of the free energy density functional. In the applied magnetic field versus reversible magnetization plot, an interesting S-shaped feature has been observed. The curvature of the S-shaped curve has been found to get enhanced at the lower values of temperature; however, a first-order transition, as has been found in case of conventional type-II superconductors, is absent in case of the high- T_c superconducting cuprates for the temperature region studied. It can be attributed to the value of the temperature T_a , which determines the lower limit of

applicability of the extended GL theory. In case of high- T_c superconducting cuprates, the value of T_a is much lower than that in case of the conventional type-II superconductors and thus any anomalous feature of the reversible magnetization curve can be expected to be observed only below this temperature. The observation also signifies that as in case of the applied magnetic field ($H_{c1} < H < H_{c2}$), for temperature also it is possible to define a range ($T_a < T < T_c$) over which the extended GL theory is applicable. In case of high- T_c superconducting cuprates, the extended GL theory is valid over a wider temperature regime as compared to the conventional low-temperature type-II superconductors.

The work further deals with the determination of the electrostatic potential and the associated charge distribution in the vortices. The net electrostatic potential and charge distribution in the vortex lattice consists of three different components and their magnitudes are strongly dependent on the material parameters. In case of the high- T_c cuprate YBCO, the spatial distribution of the electrostatic potential and charge shows a flat behavior unlike the case of the conventional low-temperature type-II superconductors. Here, it is worth mentioning that Kumagai et al. [12] through their experimental study have indeed suggested a flatness in the vortex charge distribution profile in high- T_c superconducting cuprates.

A correspondence between the theoretical results and experimental observations can be achieved by utilizing the present theoretical model to explain the result obtained from the NMR and NQR studies carried out on the high- T_c superconducting cuprates. The present model can further be generalized to take into account the various important features associated with the high- T_c cuprates such as, the presence of anisotropy or orthorhombic distortion. Apart from the high- T_c cuprates, the method can be used to analyze the electrostatic potential and charge distribution in other materials involving complex order parameters. These issues will be addressed in future.

Appendix

A. Calculation of Material Parameters for YBCO

In this section, we determine the various material parameters of high- T_c cuprate YBCO that has been used to calculate the coefficients C_1 and C_2 .

A.1. Calculation of $\partial\gamma/\partial n$. The linear coefficient of specific heat γ is related to the density of states D as per the relation

$$\gamma = \frac{2}{3} \pi^2 k_B^2 D. \quad (\text{A.1})$$

Here, the linear coefficient of specific heat associated with the superconducting CuO_2 -plane of YBCO has been taken into account. The effect of the chains has not been considered. For the high- T_c cuprate $\text{YBa}_2\text{Cu}_3\text{O}_7$, the linear coefficient of specific heat is chosen to be $302 \text{ J K}^{-2} \text{ m}^{-3}$ [36]. Thus, the

TABLE 1: Material parameters for YBCO.

Critical temperature	T_c	90 K
Debye temperature	Θ_D	440 K
Coupling parameter	λ	3
Coulomb pseudo potential	μ^*	0.2
Coefficient of specific heat	γ	$302 \text{ JK}^{-2} \text{ m}^{-3}$
Hole density	n	$5 \times 10^{27} \text{ m}^{-3}$
Logarithmic derivative	$\partial D_0 / \partial E_F$	$7.5 \times 10^{19} \text{ J}^{-1}$
Density of states	D	$2.41 \times 10^{47} \text{ J}^{-1} \text{ m}^{-3}$
Bare density	D_0	$0.602 \times 10^{47} \text{ J}^{-1} \text{ m}^{-3}$
BCS interaction	V	$4.98 \times 10^{-47} \text{ Jm}^3$
Condensation energy	ϵ_{con}	$5.45 \times 10^5 \text{ Jm}^{-3}$
Coefficient (A6)	$\partial \gamma / \partial n$	$8.23 \times 10^{-26} \text{ J}$
Coefficient (A11)	$\partial \epsilon_{\text{con}} / \partial n$	$16.02 \times 10^{-23} \text{ J}$
Coefficient of C_1	$\partial \ln \epsilon_{\text{con}} / \partial \ln n$	1.47
Coefficient of C_2	$\partial \ln \gamma / \partial \ln n$	1.4

corresponding density of states can be determined by the relation

$$D = \frac{3\gamma}{2\pi^2 k_B^2}. \quad (\text{A.2})$$

The density of states D , which includes the mass renormalization due to the electron-phonon coupling, is related to the bare density of states D_0 as

$$D = D_0(1 + \lambda), \quad (\text{A.3})$$

where λ is the electron-phonon coupling constant. As per the Kresin-Wolf two-gap theory [37–40], in case of the high- T_c superconductors involving mixed symmetry state of order parameters, λ is a mixture of the coupling constants corresponding to superconductivity in the CuO_2 -planes, in the CuO -chains and the interaction between the plane and the chain. The plane exhibits strong-coupling superconductivity characterized by the coupling constant $\lambda = 3$. The contribution to λ arising from the other two sources mentioned above ranges from 0.5 to 0.9. In the present study, a two-dimensional approach to the problem is implemented wherein the superconducting property of the high- T_c cuprates is considered to be originating from the CuO_2 -plane. Both the dominant d -wave and the subdominant s -wave order parameter components are considered to be residing in the CuO_2 -plane, as has been interpreted from the recent experimental study carried out on YBCO [18, 19]. Consequent to the two-dimensional approach to the problem, in the present study the electron-phonon coupling constant is considered to be $\lambda = 3$.

From the bare density of states, one can determine the corresponding BCS interaction (V) as per the expression

$$\lambda = D_0 V. \quad (\text{A.4})$$

Using (A.1) the density derivative of the linear coefficient of specific heat can be expressed as

$$\frac{\partial \gamma}{\partial n} = \frac{1}{3} \pi^2 k_B^2 \frac{\partial \ln D}{\partial E_F}, \quad (\text{A.5})$$

where we have used $\partial E_F / \partial n = 1/2D$ and $n = 5 \times 10^{27} \text{ m}^{-3}$ is the density of holes [36]. Using (A.2)–(A.4), the density derivative of the linear coefficient of specific heat can be expressed as

$$\frac{\partial \gamma}{\partial n} = \frac{1}{3} \pi^2 k_B^2 \frac{1 + 2\lambda}{1 + \lambda} \frac{\partial \ln D_0}{\partial E_F}, \quad (\text{A.6})$$

where the BCS interaction V has been taken to be constant, that is, $\partial V / \partial n = 0$ or $\partial V / \partial E_F = 0$.

Using (A.6), the coefficient $\partial \gamma / \partial n$ can be determined. The energy derivative of the bare density of state $\partial D_0 / \partial E_F$ is obtained from the experimental data [41].

A.2. Calculation of $\partial \epsilon_{\text{con}} / \partial n$. For determining $\partial \epsilon_{\text{con}} / \partial n$, we begin by expressing the critical temperature (in the present case T_d) in terms of McMillan formula [42]

$$T_d = \frac{\Theta_D}{1.45} \exp \left[-\frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right], \quad (\text{A.7})$$

where $\Theta_D = 440 \text{ K}$ [43] is the Debye temperature and $\mu^* = 0.2$ is the Coulomb pseudopotential. The corresponding condensation energy can thus be expressed as

$$\epsilon_{\text{con}} = \frac{\pi^2}{12.6} k_B^2 (1 + \lambda) D_0 \Theta_D^2 \exp \left[-2 \frac{1.04(1 + \lambda)}{\lambda - \mu^*(1 + 0.62\lambda)} \right], \quad (\text{A.8})$$

where we have used the relation $\epsilon_{\text{con}} = (1/4) \gamma T_d^2$. The density derivative of the condensation energy can thus be given as

$$\frac{\partial \epsilon_{\text{con}}}{\partial n} = \epsilon_{\text{con}} \frac{\partial \lambda}{\partial n} \left[\frac{2.08(1 + 0.38\mu^*)}{[\lambda - \mu^*(1 + 0.62\lambda)]^2} + \frac{1}{1 + \lambda} \right], \quad (\text{A.9})$$

where we have considered the product $D_0 \Theta_D^2$ and μ^* to be constant with respect to the variation in the density of hole n .

Using the relations between λ and D_0 given by (A.2)–(A.4) one can write

$$\frac{\partial \lambda}{\partial n} = \frac{V}{2(1 + \lambda)} \frac{\partial \ln D_0}{\partial E_F}, \quad (\text{A.10})$$

where the energy derivative of the bare density of state is again determined from experimental data for YBCO [41]. The resulting expression for the density derivative of the condensation energy is thus given as

$$\frac{\partial \epsilon_{\text{con}}}{\partial n} = \frac{\epsilon_{\text{con}} V}{(1 + \lambda)^2} \frac{\partial \ln D_0}{\partial E_F} \left(\frac{1.04(1 + 0.38\mu^*)(1 + \lambda)}{[\lambda - \mu^*(1 + 0.62\lambda)]^2} + \frac{1}{2} \right). \quad (\text{A.11})$$

Using (A.11) one can calculate the coefficient $\partial \epsilon_{\text{con}} / \partial n$. In Table 1, we have tabulated the important material parameters for YBCO that has been used for the present study.

Acknowledgments

The author gratefully acknowledges E. H. Brandt and Pavel Lipavský for fruitful discussions. The work is financially supported by S. N. Bose National Centre for Basic Sciences, India.

References

- [1] D. I. Khomskii and A. Freimuth, “Charged vortices in high temperature superconductors,” *Physical Review Letters*, vol. 75, no. 7, pp. 1384–1386, 1995.
- [2] G. Blatter, M. Feigel’man, V. Geshkenbein, A. Larkin, and A. van Otterlo, “Electrostatics of vortices in type-II superconductors,” *Physical Review Letters*, vol. 77, no. 3, pp. 566–569, 1996.
- [3] H. Hayashi, M. Ichioka, and K. Machida, “Relation between Vortex core charge and Vortex Bound States,” *Journal of the Physical Society of Japan*, vol. 67, no. 10, pp. 3368–3371, 1998.
- [4] F. Bopp, “über die beziehungen der Londonschen gleichungen zur beschleunigungstheorie der supraleitung,” *Zeitschrift für Physik*, vol. 107, no. 9–10, pp. 623–632, 1937.
- [5] J. Bok and J. Klein, “‘Electric Fields’ in superconductors,” *Physical Review Letters*, vol. 20, no. 13, pp. 660–662, 1968.
- [6] J. B. Brown and T. D. Morris, in *Proceedings of 11th International Conference on Low Temperature Physics*, J. F. Allen, D. M. Finlayson, and D. M. McCall, Eds., vol. 2, p. 768, St. Andrews, Scotland, August 1968.
- [7] K. M. Hong, “Bernoulli potential in superconductors,” *Physical Review B*, vol. 12, no. 5, pp. 1766–1771, 1975.
- [8] F. London, *Superfluids*, vol. I, Sec. 8, Wiley, New York, NY, USA, 1950.
- [9] C. J. Adkins and J. R. Waldram, “Bernoulli potentials in superconductors,” *Physical Review Letters*, vol. 21, no. 2, pp. 76–78, 1968.
- [10] G. Rickayzen, “Bernoulli potentials in superconductors,” *Journal of Physics C*, vol. 2, no. 7, pp. 1334–1337, 1969.
- [11] T. Nagaoka, Y. Matsuda, H. Obara et al., “Hall anomaly in the superconducting state of high- T_c cuprates: universality in doping dependence,” *Physical Review Letters*, vol. 80, no. 16, pp. 3594–3597, 1998.
- [12] K. Kumagai, K. Nozaki, and Y. Matsuda, “Charged vortices in high-temperature superconductors probed by NMR,” *Physical Review B*, vol. 63, no. 14, Article ID 144502, 5 pages, 2001.
- [13] P. Lipavský, J. Koláček, K. Morawetz, and E. H. Brandt, “Electrostatic potential in a superconductor,” *Physical Review B*, vol. 65, no. 14, Article ID 144511, 18 pages, 2002.
- [14] P. Lipavský, J. Koláček, K. Morawetz, E. H. Brandt, and T.-J. Yang, *Bernoulli Potential in Superconductors*, vol. 733 of *Lecture Notes in Physics*, Springer, Berlin, Germany, 2008.
- [15] M. Franz, C. Kallin, P. I. Soininen, A. J. Berlinsky, and A. L. Fetter, “Vortex state in a d -wave superconductor,” *Physical Review B*, vol. 53, no. 9, pp. 5795–5814, 1996.
- [16] R. Heeb, A. van Otterlo, M. Sigrist, and G. Blatter, “Vortices in d -wave superconductors,” *Physical Review B*, vol. 54, no. 13, pp. 9385–9398, 1996.
- [17] M. Karmakar and B. Dey, “Properties of high temperature superconductors in states of mixed symmetry,” *Journal of Physics*, vol. 20, no. 25, Article ID 255218, 2008.
- [18] R. Khasanov, S. Strössl, D. Di Castro et al., “Multiple gap symmetries for the order parameter of cuprate superconductors from penetration depth measurements,” *Physical Review Letters*, vol. 99, no. 23, Article ID 237601, 4 pages, 2007.
- [19] R. Khasanov, A. Shengelaya, A. Maisuradze et al., “Experimental evidence for two gaps in the high-temperature $\text{La}_{1.83}\text{Sr}_{0.17}\text{CuO}_4$ superconductor,” *Physical Review Letters*, vol. 98, no. 5, Article ID 057007, 4 pages, 2007.
- [20] H. J. H. Smilde, A. A. Golubov, Ariando et al., “Admixtures to d -wave gap symmetry in untwinned $\text{YBa}_2\text{Cu}_3\text{O}_7$ superconducting films measured by angle-resolved electron tunneling,” *Physical Review Letters*, vol. 95, no. 25, Article ID 257001, 4 pages, 2005.
- [21] I. Affleck, M. Franz, and M. H. Sharifzadeh, “Generalized London free energy for high- T_c vortex lattices,” *Physical Review B*, vol. 55, no. 2, pp. R704–R707, 1997.
- [22] M. Karmakar and B. Dey, “Reversible magnetization of the layered high- T_c superconductors,” *Physical Review B*, vol. 74, no. 17, Article ID 172508, 4 pages, 2006.
- [23] M. Karmakar, “Pairing state symmetries of high- T_c superconductors: a comparative study using two Ginzburg-Landau models,” *Journal of Physics*, vol. 21, no. 17, Article ID 175702, 2009.
- [24] M. Karmakar and B. Dey, “The effect of in-plane mass anisotropy on the properties of high temperature superconductors in states of mixed symmetry,” *Journal of Physics*, vol. 21, no. 40, Article ID 405702, 2009.
- [25] M. Karmakar, “Anomalous magnetic field dependence of γ $\text{Ba}_2\text{Cu}_3\text{O}_7 - \delta$: effect of orthorhombic distortion: effect of orthorhombic distortion,” *Journal of Physics*, vol. 22, no. 25, Article ID 255701, 2010.
- [26] J. Bardeen, “Theory of boundary effects of superconductors,” *Physical Review*, vol. 94, no. 3, pp. 554–563, 1954.
- [27] C. J. Gorter and H. B. G. Casimir, *Z. Phys.*, vol. 35, p. 963, 1934.
- [28] Y. Ren, J.-H. Xu, and C. S. Ting, “Ginzburg-Landau equations and vortex structure of a $d_{x^2-y^2}$ superconductor,” *Physical Review Letters*, vol. 74, no. 18, pp. 3680–3683, 1995.
- [29] J. -H. Xu, Y. Ren, and C. S. Ting, “Ginzburg-Landau equations for a d -wave superconductor with applications to vortex structure and surface problems,” *Physical Review B*, vol. 52, no. 10, pp. 7663–7674, 1995.
- [30] D. L. Feder and C. Kallin, “Microscopic derivation of the Ginzburg-Landau equations for a d -wave superconductor,” *Physical Review B*, vol. 55, no. 1, pp. 559–574, 1997.
- [31] B. Keimer, W. Y. Shih, R. W. Erwin, J. W. Lynn, F. Dogan, and I. A. Aksay, “Vortex lattice symmetry and electronic structure in $\text{YBa}_2\text{Cu}_3\text{O}_7$,” *Physical Review Letters*, vol. 73, no. 25, pp. 3459–3462, 1994.
- [32] I. Maggio-Aprile, C. Renner, A. Erb, E. Walker, and O. Fischer, “Direct vortex lattice imaging and tunneling spectroscopy of flux lines on $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$,” *Physical Review Letters*, vol. 75, no. 14, pp. 2754–2757, 1995.
- [33] A. E. Jacobs, “First-order transitions at h_{c1} and h_{c2} in type-II superconductors,” *Physical Review Letters*, vol. 26, no. 11, pp. 629–631, 1971.
- [34] A. E. Jacobs, “First-order transitions at h_{c1} and h_{c2} in type-II superconductors,” *Physical Review B*, vol. 4, no. 9, pp. 3022–3029, 1971.
- [35] J. R. Waldram, *Superconductivity of Metals and Cuprates*, Arrowsmith, Bristol, UK, 1996.
- [36] N. M. Plakida, *High-Temperature Superconductivity*, Springer, Berlin, Germany, 1995.
- [37] V. Z. Kresin and S. A. Wolf, “Multigap structure in the cuprates,” *Physica C*, vol. 169, no. 5–6, pp. 476–484, 1990.
- [38] V. Z. Kresin and S. A. Wolf, “Major normal and superconducting parameters of high- T_c oxides,” *Physical Review B*, vol. 41, no. 7, pp. 4278–4285, 1990.
- [39] V. Z. Kresin and S. A. Wolf, “Induced superconducting state and two-gap structure: application to cuprate superconductors and conventional multilayers,” *Physical Review B*, vol. 46, no. 10, pp. 6458–6471, 1992.

- [40] V. Z. Kresin, "On the critical temperature for any strength of the electron-phonon coupling," *Physics Letters A*, vol. 122, no. 8, pp. 434–438, 1987.
- [41] U. Schwingenschlögl and C. Schuster, "Intrinsic doping at YBCO-metal interfaces: quantitative results," *European Physics Letters*, vol. 77, no. 3, Article ID 37007, 2007.
- [42] C. C. Koch, J. O. Scarbrough, and D. M. Kroeger, "Effects of interstitial oxygen on the superconductivity of niobium," *Physical Review B*, vol. 9, no. 3, pp. 888–897, 1974.
- [43] S. Inderhees, M. Salamon, T. Friedmann, and D. Ginsberg, "Measurement of the specific-heat anomaly at the superconducting transition of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$," *Physical Review B*, vol. 36, no. 4, pp. 2401–2403, 1987.