Research Article

The Study on Hybridized Two-Band Superconductor

T. Chanpoom,^{1,2} J. Seechumsang,^{2,3} S. Chantrapakajee,⁴ and P. Udomsamuthirun^{2,3}

¹ Program of Physics and General Science, Faculty of Science and Technology, Rajabhat Nakhon Ratchasima University, Thailand

² Prasarnmitr Physics Research Unit, Department of Physics, Faculty of Science, Srinakharinwirot University, Sukhumvit 23, Bangkok 10110, Thailand

³ Thailand Center of Excellence in Physics (ThEP), Si Ayutthaya Road, Bangkok 10400, Thailand

⁴ Rajamangala University of Technology Phra Nakhon, 399 Samsen Road, Dusit, Bangkok 10300, Thailand

Correspondence should be addressed to P. Udomsamuthirun; udomsamut55@yahoo.com

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The two-band hybridized superconductor which the pairing occurred by conduction electron band and other-electron band are considered within a mean-field approximation. The critical temperature, zero-temperature order parameter, gap-to- T_c ratio, and isotope effect coefficient are derived. We find that the hybridization coefficient shows a little effect on the superconductor that conduction electron band has the same energy as other-electron band but shows more effect on the superconductor that conduction electron band coexists with lower-energy other-electron band. The critical temperature is decreased as the hybridization coefficient increases. The higher value of hybridization coefficient, lower value of gap-to- T_c ratio, and higher value of isotope effect coefficient are found.

1. Introduction

Since Moskalenko [1, 2] Suhl et al. [3] introduced the two-band model that accounts for multiple energy bands in the vicinity of the Fermi energy contributing electron pairing in superconductor, the two-band model has been applied to high temperature superconductor in copper oxides [4–10], MgB_2 superconductor [11–13], and heavy Fermion superconductor [14, 15]. Dolgov et al. [16] studied the thermodynamic properties of the two-band superconductor: MgB_2 . The superconducting energy gap, free energy, the entropy, and heat capacity were calculated within the framework of two-band Eliashberg theory. Mazin et al. [17] studied the effect of interband impurity scattering on the critical temperature of two-band superconductor in MgB_2 . Askerzade and Tanatar [18] and Changjan and Udomsamuthirun [19] calculated the critical field of the two-band superconductor by Ginzburg-Landau approach and applied it to Fe-based superconductors. Golubov and Koshelev [20] investigated the two-band superconductor with strong intraband and weak interband electronic scattering rates in the framework of coupled Usadel equation.

The interplay of superconductivity and magnetism is one of the most interesting phenomena of superconductor. The cuprate superconductor exhibits the phase diagram having the magnetic ordered states in the vicinity of the superconducting phase. The antiferromagnetic and ferromagnetic phases are also found in the heavy Fermion superconductor. In ErRh_4B_6 [21] and HoMo_6S_8 [22] system, s -wave superconductivity shows ferromagnetism in the ground state at intermediate temperature.

The nesting properties of Fermi surface in low dimensional system arise the spin density wave (SDW) state and charge density wave (CDW) state in the interplay of superconductivity and magnetism system. The SDW and the CDW states occurred by Coulomb interaction between electron, and electron-phonon interaction, respectively. Nass et al. [23] used the BCS-type pairing to explain the antiferromagnetic superconductor. Suzumura and Nagi [24] investigated some properties of antiferromagnetic superconductor. They proposed the Hamiltonian of the superconductivity associated with conduction d -electron and the antiferromagnetism associated with f -electron of rare earth atoms that formed the BCS-type pairing. Ichimura et al. [25] investigated the effect

of the CDW on BCS superconductor within a mean-field approximation. In this model, the two order parameters of CDW and superconductor were introduced. The tight-binding band in 2D square lattice and nesting vector $Q = (\pi, \pi)$ was used in their calculations. Rout and Das [26] applied the periodic Anderson model (PAM) for calculating the nonmagnetic ground state of heavy Fermion superconductor. The Hamiltonian of the heavy Fermion systems composed of conduction electron band, f -electron band, the hybridization of conduction electron and f -electron band, BCS-like pairing band, and the intra-atomic Coulomb interaction of f -electron. The Hamiltonian was simplified by linearising the intra-atomic Coulomb interaction with the Hartree-Fock approximation then the f -electron band energy was $E_k = \varepsilon_f + Un_\sigma$, where ε_f was bare f -electron energy and Un_σ was the Coulomb energy of f -electron. Finally, their Hamiltonian were consisted of conduction and f -electron band including BCS-like pairing band and hybridization term. Panda and Rout [27] studied the interplay of CDW, SDW, and superconductivity in high temperature superconductor in low doping phase. The model of mean-field Hamiltonian including the CDW, SDW and superconductivity was introduced.

In this paper, we modified the hybridized Hamiltonian of Rout and Das [26] to be the two-band superconductor with hybridization. Some physical properties of two-band hybridized superconductor, that is, critical temperature, zero-temperature order parameter, gap-to- T_c ratio, and isotope effect coefficient, were investigated.

2. Model and Calculation

According to the hybridized Hamiltonian [26] that consisted of conduction electron and f -electron band, BCS-like pairing band, and hybridization term, we have

$$H = \sum_{k,\sigma} \varepsilon_k C_{k\sigma}^+ C_{k\sigma} + \varepsilon_f \sum_{k,\sigma} f_{k\sigma}^+ f_{k\sigma} + \gamma_0 \sum_{k,\sigma} (f_{k\sigma}^+ C_{k\sigma} + C_{k\sigma}^+ f_{k\sigma}) + \frac{U}{2} \sum_{i,\sigma} n_{i\sigma}^f n_{i,-\sigma}^f - \Delta \sum_k (C_{k\uparrow}^+ C_{-k\downarrow}^+ + C_{-k\downarrow} C_{k\uparrow}), \quad (1)$$

where $C_{k\sigma}^+ (C_{k\sigma})$ and $f_{k\sigma}^+ (f_{k\sigma})$ are the creation (annihilation) operator of conduction electron and f -electron. Δ is the superconducting order parameter that Cooper pairs involve only the conduction electron. γ_0 is the hybridization interaction coefficient of f -electron band and conduction band. $n_i^f = f_{i\sigma}^+ f_{i\sigma}$ is the intraatomic Coulomb interaction between f -electron. They [26] linearised the $(U/2) \sum_{i,\sigma} n_{i\sigma}^f n_{i,-\sigma}^f$ term by Hartree-Fock approximation that $(U/2) \sum_{i,\sigma} n_{i\sigma}^f n_{i,-\sigma}^f \approx U \sum_{i,\sigma} n_{-\sigma} f_{i\sigma}^+ f_{i\sigma}$; then the Hamiltonian became

$$H = \sum_{k,\sigma} \varepsilon_k C_{k\sigma}^+ C_{k\sigma} + \sum_{k,\sigma} E_0 f_{k\sigma}^+ f_{k\sigma} + \gamma_0 \sum_{k,\sigma} (f_{k\sigma}^+ C_{k\sigma} + C_{k\sigma}^+ f_{k\sigma}) - \Delta \sum_k (C_{k\uparrow}^+ C_{-k\downarrow}^+ + C_{-k\downarrow} C_{k\uparrow}), \quad (2)$$

where $E_0 = \varepsilon_f + Un_{-\sigma}$ that is the energy collected the non-interaction with a modified f -level.

In our model, the two-band superconductor comprise of conduction electron and other-electron band. The superconducting order parameters can occur by conduction electron and other-electron band. The conduction band makes the intra-atomic Coulomb interaction with other-electron band. We set

$$H = \sum_{k,\sigma} \varepsilon_k C_{k\sigma}^+ C_{k\sigma} + \sum_{k,\sigma} E_0 f_{k\sigma}^+ f_{k\sigma} + \gamma_0 \sum_{k,\sigma} (f_{k\sigma}^+ C_{k\sigma} + C_{k\sigma}^+ f_{k\sigma}) + \frac{U}{2} \sum_{i,\sigma} n_{i\sigma}^f n_{i,-\sigma}^f - \Delta \sum_k (C_{k\uparrow}^+ C_{-k\downarrow}^+ + C_{-k\downarrow} C_{k\uparrow}) - \Delta \sum_k (f_{k\uparrow}^+ f_{-k\downarrow}^+ + f_{-k\downarrow} f_{k\uparrow}), \quad (3)$$

where $C_{k\sigma}^+ (C_{k\sigma})$ and $f_{k\sigma}^+ (f_{k\sigma})$ are the creation (annihilation) operators of conduction electron and other-electron band. Δ is the superconducting order parameter. γ_0 is the hybridization interaction coefficient of other-electron band and conduction band. $n_i^f = f_{i\sigma}^+ f_{i\sigma}$ is the intra-atomic Coulomb interaction between other-electron. The Hamiltonian is linearised by [26]'s technique; then we get the simplified two-band-hybridized Hamiltonian. We can write the Hamiltonian as

$$H = H_1 + H_2 + H_{12}, \quad (4)$$

where

$$H_1 = \sum_{k\sigma} \varepsilon_k C_{k\sigma}^+ C_{k\sigma} - \Delta \sum_k (C_{k\uparrow}^+ C_{-k\downarrow}^+ + C_{-k\downarrow} C_{k\uparrow}), \quad (5a)$$

$$H_2 = \sum_{k\sigma} E_k f_{k\sigma}^+ f_{k\sigma} - \Delta \sum_k (f_{k\uparrow}^+ f_{-k\downarrow}^+ + f_{-k\downarrow} f_{k\uparrow}), \quad (5b)$$

$$H_{12} = \gamma_0 \sum_{k\sigma} (f_{k\sigma}^+ C_{k\sigma} + C_{k\sigma}^+ f_{k\sigma}). \quad (5c)$$

The first Hamiltonian describes the conduction electron Hamiltonian, the second Hamiltonian describes the Hamiltonian of other-electron band, and the third Hamiltonian describes the interact Hamiltonian. Where ε_k and E_k are the band energies of the conduction electron and other-electron band measured from the Fermi energy.

$E_k = E_0 + Un_{-\sigma}$ is the energy collected the non-interaction with a modified other-electron band. $C_{k\sigma}^+ (C_{k\sigma})$ and $f_{k\sigma}^+ (f_{k\sigma})$ are the creation (annihilation) operator of conduction electron and other-electron. γ_0 is the hybridization interaction coefficient of other-electron band and conduction band. Δ is the effective superconducting order parameter occurred by conduction electron and other-electron and assumed to be homogeneous in space. The effective superconducting order parameters is

$$\Delta = \frac{V}{2} \sum_k (\langle C_{k\uparrow}^+ C_{-k\downarrow}^+ \rangle + \langle f_{k\uparrow}^+ f_{-k\downarrow}^+ \rangle), \quad (6)$$

where the s -wave like BCS pairing interaction having the same coupling interaction potential is assumed. The effective

superconducting order parameters are the coupling equation of conduction electron and other-electron that can occur in magnetic superconductor [23–25]. However for simplicity of calculation, the same pairing strength is taken [28].

We introduce the finite-temperature Green function:

$$G(k, \tau) = -\langle T_\tau \psi_k(\tau) \psi_k^\dagger(0) \rangle, \quad (7)$$

where $\psi_k^\dagger = (C_{k\uparrow}^\dagger, C_{-k\downarrow}^\dagger, f_{k\uparrow}^\dagger, f_{-k\downarrow}^\dagger)$ and T_τ is the ordering operator for imaginary time, $\tau = it$.

After some calculations, the Green function in Nambu representation is obtained:

$$G(\omega_n, k) = \left(i\omega_n - \left(\frac{\varepsilon_k - E_k}{2} \right) \rho_3 \sigma_3 - \left(\frac{\varepsilon_k + E_k}{2} \right) \sigma_3 + \Delta \sigma_1 - \gamma_0 \rho_1 \sigma_3 \right)^{-1}, \quad (8)$$

where $\omega_n = \pi T(2n+1)$, T is temperature, n is an integer, and ρ_i and σ_i ($i = 1, 2, 3$) are the Pauli matrices. Our Green function obtained shows the same form as [25] that investigated the effect of the CDW on BCS superconductor within a mean-field approximation, $G^{-1}(\omega_n, k) = i\omega_n - \gamma_k \rho_3 \sigma_3 - \delta_k \rho_0 \sigma_3 + \Delta \rho_0 \sigma_1 + w \rho_1 \sigma_3$. All parameters detailed can be found in [25]. We find that $(\varepsilon_k - E_k)/2 \equiv \gamma_k$ and $(\varepsilon_k + E_k)/2 \equiv \delta_k$, where γ_k and δ_k are the band structure energies in 2D square lattice of nearest-neighbor and next-nearest-neighbor transfer, respectively. And $-\gamma_0 \equiv w$ which w is the order parameter of CDW. This result means that the CDW consideration gives the same result as the hybridization consideration within a mean-field approximation.

From (6) and (8), the superconducting gap equation is

$$\frac{1}{V} = \frac{1}{4} \sum_k \left(\frac{\tanh\left(\frac{\sqrt{\Delta^2 + \varepsilon_-^2}}{2T}\right)}{\sqrt{\Delta^2 + \varepsilon_-^2}} + \frac{\tanh\left(\frac{\sqrt{\Delta^2 + \varepsilon_+^2}}{2T}\right)}{\sqrt{\Delta^2 + \varepsilon_+^2}} \right), \quad (9)$$

where $\varepsilon_+ = (\varepsilon_k + E_k)/2 + \sqrt{((\varepsilon_k - E_k)/2)^2 + \gamma_0^2}$ and $\varepsilon_- = (\varepsilon_k + E_k)/2 - \sqrt{((\varepsilon_k - E_k)/2)^2 + \gamma_0^2}$. The ε_+ and ε_- represent the upper and lower bands of quasiparticle energy spectra of the hybridization system. We can determine the superconducting critical temperature T_c by putting $\Delta \rightarrow 0$; then

$$\frac{1}{V} = \frac{1}{4} \sum_k \left(\frac{\tanh(\varepsilon_-/2T_c)}{\varepsilon_-} + \frac{\tanh(\varepsilon_+/2T_c)}{\varepsilon_+} \right). \quad (10)$$

In the absence of the hybridization interaction, $\gamma_0 = 0$; that is,

$$\frac{1}{V} = \frac{1}{2} \sum_k \left(\frac{\tanh(\varepsilon/2T_{c0})}{\varepsilon} \right) \quad \text{or} \quad \frac{1}{\lambda} = \ln \left(\frac{2\gamma\omega_D}{\pi T_{c0}} \right), \quad (11)$$

where $\gamma = 1.78$. $\lambda = N(0)V$, λ is the coupling constant, and $N(0)$ is the constant density of state at the Fermi surface, and

ω_D is the Debye cutoff energy. T_{c0} is the critical temperature of superconductor without hybridization that the BCS's result.

Because of the complicated quasi-particle energy spectra obtained, we introduce the two approximated conditions to calculate analytically; the superconductor with conduction electron band having the same energy as other-electron band ($\varepsilon_k \approx E_k$) and the superconductor with conduction electron band coexisting with lower-energy other-electron band ($\varepsilon_k \gg E_k, E_k \approx 0$).

Case 1. The superconductor that the conduction electron band having the same energy as other-electron band.

In this case, the approximation is $\varepsilon_k \approx E_k$. Then, we get $\varepsilon_- \approx \varepsilon_k - \gamma_0$ and $\varepsilon_+ \approx \varepsilon_k + \gamma_0$. The difference of the lower and upper energy spectra is equal to $2\gamma_0$. If $\gamma_0 = 0$, the BCS's superconductor is obtained. In this case, the effect of the hybridization interaction on the BCS superconductor is considered.

We substitute above approximations into (10); then the gap equation becomes

$$\begin{aligned} \frac{1}{\lambda} &= \frac{1}{4} \left(\int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_-/2T_c)}{\varepsilon_-} d\varepsilon_k + \int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_+/2T_c)}{\varepsilon_+} d\varepsilon_k \right) \\ &\approx \ln \left(\frac{2\gamma\sqrt{\omega_D^2 - \gamma_0^2}}{\pi T_c} \right). \end{aligned} \quad (12)$$

The critical temperature is

$$T_c = 1.13 \sqrt{\omega_D^2 - \gamma_0^2} e^{-1/\lambda}. \quad (13)$$

And the zero-temperature energy gap can be found as

$$\begin{aligned} \frac{1}{\lambda} &= \frac{1}{4} \int_{-\omega_D}^{\omega_D} \left(\frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_k - \gamma_0)^2}} + \frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_k + \gamma_0)^2}} \right) d\varepsilon_k \\ &= \frac{1}{2} \left(\sinh^{-1} \left(\frac{\omega_D - \gamma_0}{\Delta(0)} \right) + \sinh^{-1} \left(\frac{\omega_D + \gamma_0}{\Delta(0)} \right) \right). \end{aligned} \quad (14)$$

For $\omega_D \gg \Delta(0)$, we can get

$$\Delta(0) = 2\sqrt{\omega_D^2 - \gamma_0^2} e^{-1/\lambda}. \quad (15)$$

From (13) and (15), the gap-to- T_c ratio is obtained:

$$R = \frac{2\Delta(0)}{T_c} = 3.53. \quad (16)$$

In this case, we find that the hybridization interaction coefficient decreases the critical temperature and zero-temperature energy gap but has no effect on gap-to- T_c ratio.

To investigate the effect of hybridization and the Debye cutoff on gap-to- T_c ratio, we rewrite the gap equation at critical temperature and at zero-temperature into the form [29]

$$\int_{(-\omega_D+\gamma_0)/(2T_c)}^{(\omega_D+\gamma_0)/(2T_c)} \frac{\tanh x}{x} dx = \int_{-2(\omega_D-\gamma_0)/T_c}^{2(\omega_D+\gamma_0)/T_c} \frac{1}{\sqrt{R^2+x^2}} dx. \quad (17)$$

The numerical calculation of (17) is shown in Figure 2.

Within the definition of isotope effect coefficient in harmonic approximation; $\alpha = (1/2)(\omega_D/T_c)(dT_c/d\omega_D)$ and equation (9), we

$$\alpha = \frac{\omega_D}{2} \left(\frac{\tanh(\omega_D^-/2T_c)/\omega_D^- + \tanh(\omega_D^+/2T_c)/\omega_D^+}{\tanh(\omega_D^-/2T_c) + \tanh(\omega_D^+/2T_c)} \right), \quad (18)$$

where $\omega_D^+ = \omega_D + \gamma_0$ and $\omega_D^- = \omega_D - \gamma_0$.

Consider the limiting cases that the hybridization is so small with respect to Debye cutoff energy, $\omega_D \gg \gamma_0$; for $\omega_D > 2T_c$, we can get that $\alpha \approx (\omega_D/4)(1/\omega_D^- + 1/\omega_D^+) \approx 1/2$, and for $\omega_D < 2T_c$, we get $\alpha \approx (\omega_D/2)((1/T_c)/(\omega_D/T_c)) \approx 1/2$ that the BCS' result.

Case 2. The superconductor that the conduction electron band coexisting with lower-energy other-electron band.

Because of the hybridization Hamiltonian having the same Green's function as the charge density wave model, we can apply this model to the superconducting state found in heavy Fermion superconductor. The heavy Fermion superconductor has its origin in the interplay of strong Coulomb repulsion in 4f- and 5f-shells and their hybridizations with the conduction band. The f -electron is associated with the magnetic ordering having lower energy than conduction electron. We can make the assumption that the f -electron band is at the Fermi level which can be taken as $E_k \approx 0$; then

$$\begin{aligned} \varepsilon_- &\approx \frac{\varepsilon_k}{2} - \gamma_0, & \varepsilon_+ &\approx \frac{\varepsilon_k}{2} + \gamma_0, & \text{for } \frac{\varepsilon_k}{2} < \gamma_0, \\ \varepsilon_- &\approx 0, & \varepsilon_+ &\approx \varepsilon_k, & \text{for } \frac{\varepsilon_k}{2} > \gamma_0. \end{aligned} \quad (19)$$

Substituting above approximation into (10), we can get

$$\begin{aligned} \frac{1}{\lambda} &= \frac{1}{4} \left(\int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_-/2T_c)}{\varepsilon_-} d\varepsilon_k + \int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_+/2T_c)}{\varepsilon_+} d\varepsilon_k \right) \\ &\approx \ln \left(\frac{2\gamma}{\pi T_c} \sqrt{2\omega_D\gamma_0} \right) + \frac{1}{2} \left(\frac{\omega_D}{2T_c} - \frac{\gamma_0}{T_c} \right), \end{aligned} \quad (20)$$

where,

$$\begin{aligned} \int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_-/2T_c)}{\varepsilon_-} d\varepsilon_k &\approx 2 \left(\frac{\omega_D}{2T_c} - \frac{2\gamma_0}{2T_c} \right) + 2 \ln \left(\frac{4\gamma\gamma_0}{\pi T_c} \right), \\ \int_{-\omega_D}^{\omega_D} \frac{\tanh(\varepsilon_+/2T_c)}{\varepsilon_+} d\varepsilon_k &\approx 2 \ln \left(\frac{2\gamma\omega_D}{\pi T_c} \right). \end{aligned} \quad (21)$$

The critical temperature is

$$T_c = 1.13 \sqrt{2\gamma_0\omega_D} e^{-1/\lambda+(1/2)(\omega_D/2T_c-\gamma_0/T_c)}. \quad (22)$$

The gap equation as zero-temperature is

$$\frac{1}{\lambda} = \frac{1}{4} \int_{-\omega_D}^{\omega_D} \left(\frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_-)^2}} + \frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_+)^2}} \right) d\varepsilon_k, \quad (23)$$

where,

$$\begin{aligned} \int_{-\omega_D}^{\omega_D} \frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_-)^2}} d\varepsilon_k \\ \approx \left(\frac{2}{\Delta(0)} \right) (\omega_D - 2\gamma_0) + 2 \sinh^{-1} \left(\frac{2\gamma_0}{\Delta(0)} \right), \end{aligned} \quad (24)$$

$$\int_{-\omega_D}^{\omega_D} \frac{1}{\sqrt{\Delta^2(0) + (\varepsilon_+)^2}} d\varepsilon_k \approx 2 \sinh^{-1} \left(\frac{\omega_D}{\Delta(0)} \right).$$

Then, we get

$$\Delta(0) = 2\sqrt{2\gamma_0\omega_D} e^{-1/\lambda+(1/2)((\omega_D-2\gamma_0)/\Delta(0))}. \quad (25)$$

According to (22) and (25), the gap-to- T_c ratio is

$$R = \frac{2\Delta(0)}{T_c} = 3.53 e^{((\omega_D-2\gamma_0)/T_c)(1/R-1/4)}. \quad (26)$$

To investigate the effect of hybridization and the Debye cutoff on gap-to- T_c ratio, we rewrite the gap equation at critical temperature and at zero-temperature into the form [29]

$$\int_{-\omega_D/T_c}^{\omega_D/T_c} \frac{\tanh(y/4 - \sqrt{(y/4)})}{x} dx = \int_{-2(\omega_D-\gamma_0)/T_c}^{2(\omega_D+\gamma_0)/T_c} \frac{1}{\sqrt{R^2+x^2}} dx. \quad (27)$$

The numerical calculation of this equation is shown in Figure 2.

Within the definition of isotope effect coefficient in harmonic approximation, (10) and $E_k \approx 0$, we can get

$$\begin{aligned} \alpha &= \left(\frac{\omega_D}{2} \right) \\ &\times \left(\frac{1 + (2T_c/\omega_D) \tanh(\omega_D/2T_c)}{\omega_D - 2\gamma_0 + 2T_c (\tanh(\omega_D/2T_c) + \tanh(\gamma_0/T_c))} \right). \end{aligned} \quad (28)$$

3. Results and Discussions

We use the hybridized two-band Hamiltonian to investigate the critical temperature, zero-temperature order parameter, gap-to- T_c ratio, and isotope effect coefficient of superconductor. The Green function and gap equation are derived analytically. However, the quasi-particle energy spectra obtained

FIGURE 1: The T_c versus hybridization coefficients of Cases 1 and 2.

are complicated; then we introduce two approximated cases as, the superconductor that conduction electron band has the same energy as other-electron band ($\epsilon_k \approx E_k$) and the superconductor that conduction electron band coexists with lower-energy other-electron band ($\epsilon_k \gg E_k$, $E_k \approx 0$). The critical temperature, zero-temperature order parameter, gap-to- T_c ratio, and isotope effect coefficient are shown in the exact forms.

We use the integration forms of involved equations for more accuracy in numerical calculation. After the numerical calculations of T_c , (Figure 1), we find that the weak-coupling limit ($\lambda < 0.4$) can be found in range of $\omega_D/T_c > 10$. Then, we investigate the effect of hybridization in weak-coupling limit with $\omega_D/T_c > 10$ and $\gamma_0/T_c = 0.2, 4.0$. We find that T_c is decreased when the hybridization coefficient increased in Case 2 and no effect in Case 1. In Figure 2, the gap-to- T_c ratios (R) of Cases 1 and 2 with varied hybridization coefficients are shown. In Case 1, the R is tended to BCS value, 3.53, as $\omega_D/T_c \rightarrow \infty$. Case 2, $R \approx 3.3$ – 3.9 can be found to depend strongly on the value of γ_0/T_c . The higher value of γ_0/T_c and the lower value of R are found to agree with superconducting gap's behavior of HF system of Rout et al. [30].

The isotope effect coefficient is also investigated and shown in Figure 3. In Case 2, we find that the isotope effect coefficient can be more than the BCS ($\alpha > 0.5$) and less than BCS value ($\alpha < 0.5$), but in Case 1 we can find only for $\alpha > 0.5$. However, α in both cases is converse to 0.5 as $\omega_D/T_c \rightarrow \infty$.

The Fe-based superconductors are mutiband system which comprises at least two bands which propose s -wave pairing state. The $R = 3.68$ [31] which shows a consistent manner with the BCS prediction is found. The value of isotope effect coefficient were found to be $\alpha \approx 0.35$ – 0.4 [32] and $\alpha_{Fe} = 0.81$ [33]. These results indicate that electron-phonon interaction plays some role in the superconducting mechanism by affecting the magnetic properties [31]. According to our model, the effective superconducting order parameters

FIGURE 2: The gap-to- T_c ratio of Cases 1 and 2 with varied hybridization coefficients.

FIGURE 3: The isotope effect coefficient of Cases 1 and 2 with varied hybridization coefficients.

are the coupling equation of conduction electron and other-electron and the magnetic order are included in our calculation as in Case 2. We can get the experimental data of Fe-based superconductor from Figures 2 and 3.

4. Conclusions

The two-band hybridized superconductor that the pairing occurred by conduction electron band and other-electron band is studied in weak-coupling limit. The formula of critical temperature, zero-temperature order parameter, gap-to- T_c ratio and isotope effect coefficient are calculated. For the superconductor that conduction electron band has the same energy as other-electron band, the hybridization coefficient

shows a little effect. The numerical results do not differ much from the BCS's results. For the superconductor that conduction electron band coexists with lower-energy other-electron band, the hybridization coefficient show more effect. T_c is decreased when the hybridization coefficient increases. We can get higher and lower value of R and α than BCS's results depending on the hybridization coefficient. Higher value of hybridization coefficient, lower value of R , and higher value of α are found.

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