Research Article
Theoretical Study of Upper Critical Magnetic Field ($H_{C2}$) in Multiband Iron Based Superconductors

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This research work focuses on the theoretical investigation of the upper critical magnetic field, $H_{C2}$; Ginzburg-Landau coherence length, $\xi_{GL}(T)$; and Ginzburg-Landau penetration depth, $\lambda_{GL}(T)$, for the two-band iron based superconductors BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs. By employing the phenomenological Ginzburg-Landau (GL) equation for the two-band superconductors BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs, we obtained expressions for the upper critical magnetic field, $H_{C2}$; GL coherence length, $\xi_{GL}$; and GL penetration depth, $\lambda_{GL}$, as a function of temperature and the angular dependency of upper critical magnetic field. By using the experimental values in the obtained expressions, phase diagrams of the upper critical magnetic field parallel, $H_{C2}^\parallel$, and perpendicular, $H_{C2}^\perp$, to the symmetry axis (c-direction) versus temperature are drawn. We also plotted the phase diagrams of the upper critical magnetic field, $H_{C2}$, versus the angle $\theta$. Similarly, the phase diagrams of the GL coherence length, $\xi_{GL}$, and GL penetration depth, $\lambda_{GL}$, parallel and perpendicular to the symmetry axis versus temperature are drawn for the superconductors mentioned above. Our findings are in agreement with experimental observations.

1. Introduction

The discovery of superconductivity in iron based materials by Kamihara et al. [1] at $T_C = 26$ K in the iron-based superconductor LaFeAsO$_{1-x}$F$_x$ was welcomed surprise and this brought to an end the monopoly of cuprates as the only high temperature superconductors. The $T_C$ of F-doped LaFeAs was reported to increase with pressure and reach a maximum of about 43 K at 4 GPa, obtained from electrical resistance measurements under high pressure [2]. Up to now, the critical temperature for iron based superconductors has been raised to 57.3 K for Sm$_{0.25}$La$_{0.25}$O$_{0.85}$F$_{0.15}$FeAs which is the highest for noncuprate superconductors [3]. Among these new iron based superconductors, the iron based F-doped layered quaternary compound Nd(O$_{1-x}$F$_x$)FeAs is a good candidate for this study since it is the same family as LaFeAsO$_{1-x}$F$_x$ and has the same crystal structure. Previous extensive transport and heat capacity studies on Nd(O$_{1-x}$F$_x$)FeAs single crystal revealed that at zero external magnetic fields the compound was found to undergo a transition from the normal state to the superconducting state at superconducting critical temperature: $T_C = 51$ K at $x = 0.18$ at ambient pressure [4].

Superconductivity was also obtained in the 122-family iron based multiband superconductors and great efforts were made to raise $T_C$ to temperatures higher than 38 K as detected in (Ba$_{0.6}$K$_{0.4}$)Fe$_2$As$_2$ [5] and for optimally doped BaFe$_2$(As$_{1-x}$P$_x$)$_2$ at $T_C = 31$ K at $x = 0.32$ [6]. The isovalent doping of phosphorus for arsenic in BaFe$_2$(As$_{1-x}$P$_x$)$_2$ suppresses magnetic ordering without changing the number of Fe(3d) electrons and induces superconductivity with a maximum value of $T_C = 31$ K. The isovalently P-doped BaFe$_2$(As$_{1-x}$P$_x$)$_2$ system is particularly suitable to study the detailed evolution of the electronic properties. The isovalent doping does not add any charge carriers to the system and the dopant changes the electronic structure mainly because of differences in ion size.

The discovery of superconductivity in the III-family of iron based superconductor, LiFeAs, is reported by Wang et al. [7]. The compound crystallizes into a structure containing [FeAs] conducting layer that is interlaced with Li charge...
reservoir. Superconductivity was observed with $T_C$ up to 18 K [7] in this compound. The two-band nature of the order parameter in iron based compounds is confirmed by heat capacity measurements [8] and by ab initio calculations [9]. Several methods have been used to determine the temperature dependence and anisotropy of $H_{C2}$ for LiFeAs and the first results were obtained from resistivity measurements [10]. Among the iron based superconductors, LiFeAs shows a relatively small upper critical magnetic field.

Soon, after the discovery of superconductivity in iron based superconductors, it was found that the iron based superconductors possess a large superconducting critical magnetic field ($H_{C2}$) and low anisotropy [11] when compared to the cuprates. The theory of upper critical magnetic field in superconductors was essentially developed right after Abrikosov proposed the idea of type II superconductors [12]. Determining the upper critical magnetic field $H_{C2}$, where superconductivity ceases in a type II superconductor, is one of the most important steps for gathering and understanding of unconventional superconductivity including the pairing mechanism, the pairing strength, and the coherence length. In an anisotropic type II superconductor, the magnetic field destroys superconductivity at the upper critical magnetic fields $H_{C2}^{ab}$ and $H_{C2}^c$ when the magnetic field is applied within the ab-plane ($H \parallel ab$) and c-axis ($H \parallel c$), respectively. It is well known that the Ginzburg-Landau theory remains a powerful instrument for the study of magnetic phase diagram of superconductors [13]. The free energy functional of two-band superconductors can be expressed by the power series of order parameters in the vicinity of the critical temperature and minimization of the free energy function gives the GL equations that can describe the field distribution in superconductors.

In conventional superconductors, the electron pairing is mediated by an electron-phonon interaction and can be well understood within the microscopic-model developed by Bardeen, Cooper, and Schrieffer (BCS) in the theory of superconductivity [14]. The electron-phonon coupling mechanism is not sufficient to explain the superconductivity in iron based superconductors and their role is rather speculated by spin-fluctuation coupling mechanism.

The main specialty of iron based superconductors is their multiband nature. In the multiband model of superconductivity, several intraband and interband interaction terms are present. The importance of interband pairing in multiband models has been emphasized by de Menezes [15]. The Fermi surface, which has contributions from all five possible Fe(3d) orbitals [16], is very similar to the different iron based compounds and all the five Fe(3d) orbitals pass through the Fermi surface. The first-principle calculations have shown that the energy bands near the Fermi surface are mainly contributed by the Fe(3d) $d_{xy}$ and $d_{yz}$ orbitals [17]. One of the main features of two-band superconductors is the presence of two energy gaps, $\Delta_1$ and $\Delta_2$, which vanish at the same critical temperature, $T_C$. According to the microscopic theory, the presence of the two gaps is explained by the fact that in each band $\Psi^*$ an intraband coupling constant $V_{ii}$ and the interband coupling constant $V_{ij}$ exist which on the one hand enhances pairing of electrons and on the other hand leads to the single critical temperature, $T_C$.

### 2. Theoretical Formulations

#### 2.1. Calculation of Upper Critical Magnetic Field

In the presence of two order parameters in a bulk isotropic s-wave superconductor, the phenomenological GL free energy density functional for two-band superconducting order parameters $\Psi_1$ and $\Psi_2$ can be written as [18]

$$F_{SC} = F_1 + F_2 + F_{12} + \frac{H^2}{8\pi},$$

where

$$F_1 = -\frac{\hbar^2}{2m_1^*} \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1 - \frac{\alpha_1}{2} |\Psi_1|^4 + \frac{\beta_1}{2} |\Psi_2|^4,$$

$$F_2 = -\frac{\hbar^2}{2m_2^*} \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2 - \frac{\alpha_2}{2} |\Psi_2|^4 + \frac{\beta_2}{2} |\Psi_1|^4,$$

$$F_{12} = \epsilon \left( \Psi_1 \Psi_2^* + \Psi_2 \Psi_1^* \right)$$

$$+ \epsilon_1 \left( \left[ \left( \nabla + \frac{2ie^*A}{\hbar c} \right) \Psi_1 \right]^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2 \right)$$

$$+ \left[ \left( \nabla + \frac{2ie^*A}{\hbar c} \right) \Psi_2 \right]^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1 \right),$$

where $F_1$ and $F_2$ are the free energies for each band. $F_{12}$ is the interaction free energy term between the two bands. $H^2/8\pi$ is the energy stored in the local magnetic fields. $\Psi_1$ and $\Psi_2$ are superconducting order parameters. The coefficients $\epsilon$ and $\epsilon_1$ describe the interband interaction of the two order parameters (proximity effect) and the interband mixing of gradients of two order parameters (drag effect) [18], respectively. $m_1^*$ and $m_2^*$ denote the effective mass of carriers for each band. $\alpha$ is the GL temperature-dependent parameter and $\beta_i$ is GL temperature-independent parameter. $H = \nabla \times A$ is the external magnetic field and $A$ is the vector potential.

Now, substituting (2) into (1), we get

$$F_{SC} = -\frac{\hbar^2}{2m_1^*} \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1 - \frac{\alpha_1}{2} |\Psi_1|^4 + \frac{\beta_1}{2} |\Psi_1|^4$$

$$- \frac{\hbar^2}{2m_2^*} \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2 - \frac{\alpha_2}{2} |\Psi_2|^4 + \frac{\beta_2}{2} |\Psi_2|^4$$

$$+ \epsilon \left( \Psi_1 \Psi_2^* + \Psi_2 \Psi_1^* \right)$$

$$+ \epsilon_1 \left( \left[ \left( \nabla + \frac{2ie^*A}{\hbar c} \right) \Psi_1 \right]^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_2 \right)$$

$$+ \left[ \left( \nabla + \frac{2ie^*A}{\hbar c} \right) \Psi_2 \right]^* \left( \nabla - \frac{2ie^*A}{\hbar c} \right) \Psi_1 \right) + \frac{H^2}{8\pi}.$$
respect to variations in the complex conjugate of the order
parameters $\Psi_1^*, \Psi_2^*$, such that

$$\frac{\partial F_{SC}}{\partial \Psi_1^*} = 0,$$
$$\frac{\partial F_{SC}}{\partial \Psi_2^*} = 0. \tag{4}$$

Near the critical temperature, $T_c$, we have $\Psi_1^* \rightarrow 0$. Thus, using (3) in (4), the first GL equations for the two bands can be obtained as

$$-\frac{\hbar^2}{2m_1^*} \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 \Psi_1 - \alpha_1 \Psi_1 + e \Psi_2$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = \begin{pmatrix} E_{11} & -\frac{\hbar^2}{2m_1^*} \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 - \alpha_1 \\ e + e_1 \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 & E_{22} - \alpha_2 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0. \tag{6}$$

We can compare (6) to the free-particle time independent Schrödinger equation with particle of mass $m_1$ and charge of $e^*$ moving in a magnetic field which is the same as eigenvalues of the harmonic oscillator. For the field in the direction of the $c$-axis, we can linearize the GL equations as follows:

$$H_{11} \Psi_1 = -\frac{\hbar^2}{2m_1^*} \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 \Psi_1 = E_{11} \Psi_1,$$
$$H_{22} \Psi_2 = -\frac{\hbar^2}{2m_2^*} \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 \Psi_2 = E_{22} \Psi_2. \tag{7}$$

Hence, (6) becomes

$$\begin{pmatrix} E_{11} - \alpha_1 & e + e_1 \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 \\ e + e_1 \left( \nabla - \frac{2ie^* A}{\hbar c} \right)^2 & E_{22} - \alpha_2 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0. \tag{8}$$

The energy eigenvalues of the quantum harmonic oscillator are given by $E_n = (n + 1/2)\hbar \omega$. In order to determine $H_{C2}$, we must choose $n = 0$ which gives the upper critical magnetic field. The energy of the two bands for the $n = 0$ state becomes $E_{11} = (1/2)\hbar \omega_c$ and $E_{22} = (1/2)\hbar \omega_c$, where the frequency of the oscillator is given by $\omega_c = 2e^* H_{C2}/m_1^* c$. For a displacement $x$ in one direction, the vector potential is given by $A = H_{C2} x$.

Thus, (8) can be written as

$$\begin{pmatrix} \frac{e^* H_{C2}}{m_1^* c} & -\alpha_1 \\ e - \frac{2e^* H_{C2}}{\hbar c} & \frac{e^* H_{C2}}{m_1^* c} - \alpha_2 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0. \tag{9}$$

By considering $m_1^* = m_2^* = m^*$, the upper critical magnetic field $H_{C2}$ is obtained by setting the determinant of the matrix in (9) to zero; that is,

$$\left| \frac{e^* H_{C2}}{m^* c} - \alpha_1 \\ e - \frac{2e^* H_{C2}}{\hbar c} & \frac{e^* H_{C2}}{m^* c} - \alpha_2 \right| = 0, \tag{10}$$

from which we get

$$\left[ \frac{e^* \hbar}{m^* c} \left( \alpha_1 + \alpha_2 \right) - \frac{4e^* \epsilon_1}{\hbar^2 c^2} \right] H_{C2}^2 - \left[ \frac{e^* \hbar}{m^* c} \left( \alpha_1 + \alpha_2 \right) - \frac{4e^* \epsilon_1}{\hbar^2 c^2} \right] H_{C2} + \alpha_1 \alpha_2 - \epsilon^2 = 0. \tag{11}$$

Finally, the solution of the quadratic equation becomes

$$H_{C2} = \frac{(e^* \hbar/m^* c)(\alpha_1 + \alpha_2) - 4e^* \epsilon_1}{4e^* \epsilon_1 - \alpha_1 \alpha_2 - (2e^* \hbar/m^* c^2) \alpha_1 \alpha_2 - (8e^* \epsilon_1/m^* c^2) \alpha_1 + (16e^* \epsilon_1^2/m^* c^2) \alpha_1 + 4e^* \hbar^2 \epsilon_1/m^* c^2}}{2 \left( e^* \hbar/m^* c^2 - 4e^* \epsilon_1/\hbar^2 c^2 \right)}. \tag{12}$$
Now let $\alpha_1 = \hbar^2/2m^*\xi_{\text{GL1}}^2$, $\alpha_2 = \hbar^2/2m^*\xi_{\text{GL2}}^2$, and $\epsilon = \hbar^2/2m^*\xi_{\text{GL12}}^2$, where $\xi_{\text{GL1}}$, $\xi_{\text{GL2}}$, and $\xi_{\text{GL12}}$ are the first band, second band, and interband effective coherence lengths, respectively, $\epsilon_i = v\hbar^2/2m^*$, where $v$ is the gradients of interband mixing of two order parameters in energy unit and $\Phi_0 = 2\pi\hbar/e$ and is the quantum flux.

Hence, (12) becomes

$$H_{C2} = \frac{\Phi_0}{2\pi(1 - \nu^2)} \left[ \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - \nu \frac{1}{\xi_{\text{GL12}}^2} \right)^2 \right] \pm \frac{\Phi_0}{2\pi(1 - \nu^2)} \left[ \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - \nu \frac{1}{\xi_{\text{GL12}}^2} \right)^2 \right]^{1/2} - (1 - \nu^2) \left( \frac{1}{\xi_{\text{GL1}}^2} \frac{1}{\xi_{\text{GL2}}^2} - \frac{1}{\xi_{\text{GL12}}^2} \right)$$

For the positive value of the square root in (13) we get

$$H_{C2}^+ = \frac{\Phi_0}{2\pi(1 - \nu^2)} \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - \nu \frac{1}{\xi_{\text{GL12}}^2} \right)$$

$$+ \frac{\Phi_0}{2\pi} \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - v\xi_{\text{GL12}}^2 \right)$$

(17)

Due to the drag effect, the GL theory for a two-band superconductor can be reduced to the GL theory for an effective single-band superconductor. Two-band model can be reduced to one band model if we set $\alpha_2 = \epsilon = \epsilon_1 = 0$. We observe that (16) cannot be reduced to one band model. So it does not have a physical meaning. But (17) can be reduced to one band model and is the analytical equation of upper critical magnetic field and thus we have

$$H_{C2} = \frac{\Phi_0}{2\pi\xi_{\text{GL12}}^2} \left( \frac{1}{2\xi_{\text{GL12}}^2} + \frac{1}{2\xi_{\text{GL12}}^2} - 2\nu \left( \frac{1}{2\xi_{\text{GL12}}^2} + \frac{1}{2\xi_{\text{GL12}}^2} - v\xi_{\text{GL12}}^2 \right) \right)$$

(18)

The effect of anisotropy mass tensor on the upper critical magnetic field is included by replacing $\xi$ in (18) with coherence length tensor, $\xi$.

Finally we have

$$H_{C2} = \frac{\Phi_0}{2\pi\xi_{\text{GL12}}^2} \left( \frac{1}{2\xi_{\text{GL12}}^2} + \frac{1}{2\xi_{\text{GL12}}^2} - 2\nu \left( \frac{1}{2\xi_{\text{GL12}}^2} + \frac{1}{2\xi_{\text{GL12}}^2} - v\xi_{\text{GL12}}^2 \right) \right)$$

(19)

The interband mixing of gradients of two order parameters in an isotropic bulk two-band superconductor plays important role. The two-band superconductor is characterized with a single upper critical magnetic field, $H_{C2}$; GL coherence length, $\xi_{\text{GL}}$; GL penetration depth, $\lambda_{\text{GL}}$; and GL characteristic parameter, $\kappa$. 

Case 1. Consider

$$\left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - \nu \frac{1}{\xi_{\text{GL12}}^2} \right)$$

$$\ll (1 - \nu^2) \left( \frac{1}{\xi_{\text{GL1}}^2} \frac{1}{\xi_{\text{GL2}}^2} - \frac{1}{\xi_{\text{GL12}}^2} \right)$$

In this case the term in the square root in (13) becomes complex. Therefore, we can conclude that the first case is not valid since magnetic field is a real quantity.

Case 2. Consider

$$\left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - \nu \frac{1}{\xi_{\text{GL12}}^2} \right)$$

$$\gg (1 - \nu^2) \left( \frac{1}{\xi_{\text{GL1}}^2} \frac{1}{\xi_{\text{GL2}}^2} - \frac{1}{\xi_{\text{GL12}}^2} \right)$$

Using Taylor’s binomial series expansions, for the negative value of the term in the square root in (13) we get

$$H_{C2}^- = \frac{\Phi_0}{2\pi} \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - v\xi_{\text{GL12}}^2 \right)$$

$$+ \frac{1}{8\pi} \left( \frac{1}{2\xi_{\text{GL1}}^2} + \frac{1}{2\xi_{\text{GL2}}^2} - v\xi_{\text{GL12}}^2 \right)$$

(16)
Thus, the GL effective coherence length, $\xi_{\text{GL eff}}$, can be expressed as

\[
\frac{1}{\xi_{\text{GL eff}}^2} = \frac{1}{\xi_{\text{G}}^2} + \frac{1}{\xi_{\text{L}}^2} + \frac{1}{\xi_{\text{GL}}^2} - 2(1 - \nu^2) \left[ \frac{1}{\xi_{\text{GL}}^2} - \frac{1}{\xi_{\text{G}}^2} \right] (\frac{\xi_{\text{GL}}^2}{\xi_{\text{G}}^2} + \frac{\xi_{\text{GL}}^2}{\xi_{\text{L}}^2} - 2(1 - \nu^2) \left[ \frac{1}{\xi_{\text{GL}}^2} - \frac{1}{\xi_{\text{L}}^2} \right] + \frac{1}{\xi_{\text{GL}}^2} (1 - \nu^2) \left( \frac{\xi_{\text{L}}^2}{\xi_{\text{GL}}^2} - 1 \right)^2 \right] .
\]

Therefore, (19) becomes

\[
H_{C_2}^c(T) = \frac{\Phi_0}{2\pi\xi_{\text{GL eff}}^2(T)} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] .
\]

If the direction of the applied magnetic field is parallel to the $c$-axis, then the expression for the upper critical magnetic field, $H_{C_2}^c$, as a function of temperature in (21) is given by

\[
H_{C_2}^c(T) = \frac{\Phi_0}{2\pi\xi_{\text{GL}}^2(0)} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] ,
\]

where $H_{C_2}^c(0) = \Phi_0/2\pi\xi_{\text{GL}}^2(0)$ is the zero temperature upper critical field parallel to the $c$-axis.

If the direction of the applied magnetic field is perpendicular to the $c$-axis, then the expression for the upper critical magnetic field, $H_{C_2}^c(T)$, in (21) is given by

\[
H_{C_2}^c(T) = \frac{\Phi_0}{2\pi\xi_{\text{GL}}^2(0)} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] ,
\]

where $H_{C_2}^c(0) = \Phi_0/2\pi\xi_{\text{GL}}^2(0)$ is the zero temperature upper critical field perpendicular to the $c$-axis.

Using the experimental values $H_{C_2}^c(0) = 36$ T and $H_{C_2}^c(0) = 77$ T [19] at $T_C = 311$ K, the mathematical expressions of the temperature dependence of the upper critical magnetic field parallel to the symmetry axis (in the $c$-direction) $H_{C_2}^c(T)$ and perpendicular to the symmetry axis $H_{C_2}^c(T)$ in BaFe$_2$(As$_{1-x}$P$_x$)$_2$ become

\[
H_{C_2}^c(T) = 36 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] ,
\]

\[
H_{C_2}^c(T) = 36 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] .
\]

Finally, the angular dependence of the upper critical magnetic field for BaFe$_2$(As$_{1-x}$P$_x$)$_2$ at an angle $\theta$ from the $c$-axis can be expressed as

\[
H_{C_2}^{\text{GL}}(\theta) = \frac{H_{C_2}^c}{\sqrt{\cos^2 \theta + \gamma^2 \sin^2 \theta}} .
\]

It has been shown that a single band anisotropic model can properly describe the angular dependence of $H_{C_2}^c(\theta)$ in a multiband system near $T_C$ [21].

Similarly, the current model/theory can be applied to IIII-family of iron based two-band superconductor Nd(O$_{1-x}$F$_x$)FeAs by considering its experimental values. The expressions of the temperature dependence of the upper critical magnetic field parallel and perpendicular to the symmetry axis (in the $c$-axis) in Nd(O$_{1-x}$F$_x$)FeAs for $H_{C_2}^c(0) = 70$ T and $H_{C_2}^c(0) = 304$ T [7] at $T_C = 51$ K become

\[
H_{C_2}^c(T) = 70 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] ,
\]

\[
H_{C_2}^c(T) = 304 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] .
\]

The angular dependence of the upper critical magnetic field at an angle $\theta$ from the $c$-axis can be expressed as

\[
H_{C_2}^{\text{GL}}(\theta) = \frac{70}{\sqrt{\cos^2 \theta + 0.053 \sin^2 \theta}} .
\]

Furthermore, the model/theory can be applied to III-family of iron based superconductor LiFeAs by considering its experimental values. The expressions of the temperature dependence of the upper critical magnetic field parallel and perpendicular to the symmetry axis (in the $c$-axis) in LiFeAs for $H_{C_2}^c(0) = 15$ T and $H_{C_2}^c(0) = 24.2$ T [22] at $T_C = 18$ K become

\[
H_{C_2}^c(T) = 15 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] ,
\]

\[
H_{C_2}^c(T) = 24.2 \text{ Tesla} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right] .
\]

The angular dependence of the upper critical magnetic field at an angle $\theta$ from the $c$-axis can be expressed as

\[
H_{C_2}^{\text{GL}}(\theta) = \frac{15}{\sqrt{\cos^2 \theta + 0.38 \sin^2 \theta}} .
\]

The expression for the anisotropy parameter ($\gamma$) is given by

\[
\gamma = H_{C_2}^{\text{LC}}/H_{C_2}^c .
\]

Thus, we calculate the values of $\gamma$ to be 2.14, 4.34, and 1.6 for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs, respectively.
2.2. Calculation of Ginzburg-Landau Coherence Length and Penetration Depth. There are two important temperature dependent material parameters which arise from the GL model that characterize the phenomenological properties of a superconductor. The coherence length, $\xi_{\text{GL}}$, is defined as

$$\xi_{\text{GL}}(T) = \left( \frac{\hbar^2}{2m^*|\alpha(T)|} \right)^{1/2}. \quad (32)$$

This parameter specifies the spatial width of the transition layer of the order parameter, $\Psi$, in the neighborhood of the boundary between a normal region and a superconducting region.

Using the assumption of Changjan and Udomsamuthirun [23], the GL temperature-dependent parameter, $\alpha$, can be written as a function of temperature in the vicinity of the critical temperature, $T_C$, as

$$\alpha(T) \approx \alpha_0 \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]. \quad (33)$$

Here, $\alpha_0$ is the temperature-independent positive constant.

Hence, the expression for the temperature dependent GL coherence length, $\xi_{\text{GL}}(T)$, becomes

$$\xi_{\text{GL}}(T) = \xi_{\text{GL}}(0) \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}, \quad (34)$$

where $\xi_{\text{GL}}(0)$ is the zero temperature coherence length. Using the experimental values of the zero temperature coherence lengths $\xi_{\text{GL}}^{ab}(0) = 3.2 \text{ nm}$ and $\xi_{\text{GL}}^{c}(0) = 1.3 \text{ nm}$ [19], the temperature dependent coherence length in the $ab$-plane, $\xi_{\text{GL}}^{ab}$, and along the $c$-axis, $\xi_{\text{GL}}^{c}$, for BaFe$_2$(As$_{1-x}$P$_x$)$_2$ becomes

$$\xi_{\text{GL}}^{ab}(T) = 3.2 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}, \quad (35)$$

$$\xi_{\text{GL}}^{c}(T) = 1.3 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}. \quad (36)$$

Similarly, using the experimental values of the zero temperature coherence lengths $\xi_{\text{GL}}^{ab}(0) = 3.7 \text{ nm}$ and $\xi_{\text{GL}}^{c}(0) = 0.9 \text{ nm}$ [24], the temperature dependent coherence length in the $ab$-plane, $\xi_{\text{GL}}^{ab}$, and along the $c$-axis, $\xi_{\text{GL}}^{c}$, for Nd(O$_{1-x}$F$_x$)$_2$FeAs becomes

$$\xi_{\text{GL}}^{ab}(T) = 3.7 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}, \quad (37)$$

$$\xi_{\text{GL}}^{c}(T) = 0.9 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}. \quad (38)$$

Furthermore, using the experimental values of the zero temperature coherence lengths $\xi_{\text{GL}}^{ab}(0) = 4.8 \text{ nm}$ and $\xi_{\text{GL}}^{c}(0) = 1.7 \text{ nm}$ [22], the temperature dependent coherence length in the $ab$-plane, $\xi_{\text{GL}}^{ab}$, and along the $c$-axis, $\xi_{\text{GL}}^{c}$, for LiFeAs becomes

$$\xi_{\text{GL}}^{ab}(T) = 4.8 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}, \quad (39)$$

$$\xi_{\text{GL}}^{c}(T) = 1.7 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^2 \right]^{-1/2}. \quad (40)$$

According to the Meissner effect, an external magnetic field is expelled completely from the interior of a superconductor. But there was experimental evidence that magnetic fields penetrate a superconductor and a surface current flows in a very thin layer of thickness which is called the GL penetration depth, $\lambda_{\text{GL}}$, and can be expressed in terms of the superconducting electron density, $n_s$, as

$$\lambda_{\text{GL}}(T) = \left( \frac{m^* c^2}{16\pi e^2 n_s} \right)^{1/2}. \quad (41)$$

For $T \rightarrow 0$, $n_s \rightarrow n$ (total electron density), thus, we have

$$\lambda_{\text{GL}}(0) = \left( \frac{m^* c^2}{16\pi e^2 n} \right)^{1/2}. \quad (42)$$

In the two-fluid theory of Gorter and Casimir [25] we can have

$$\frac{n_s}{n} = \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]. \quad (43)$$

Now, combining (41), (42), and (43) we have

$$\lambda_{\text{GL}}(T) = \lambda_{\text{GL}}(0) \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{-1/2}. \quad (44)$$

Using the experimental value $\lambda_{\text{GL}}^{ab}(0) = 101 \text{ nm}$ [26] for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, the expressions for the temperature dependent penetration depth in the $ab$-plane, $\lambda_{\text{GL}}^{ab}$, and along the $c$-axis, $\lambda_{\text{GL}}^{c}$, become

$$\lambda_{\text{GL}}^{ab}(T) = 101 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{-1/2}, \quad (45)$$

$$\lambda_{\text{GL}}^{c}(T) = 216 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{-1/2}. \quad (46)$$

Similarly, using the experimental value $\lambda_{\text{GL}}^{ab}(0) = 200 \text{ nm}$ [24], the expressions for the temperature dependent penetration depth in the $ab$-plane, $\lambda_{\text{GL}}^{ab}$, and along the $c$-axis, $\lambda_{\text{GL}}^{c}$, for Nd(O$_{1-x}$F$_x$)$_2$FeAs become

$$\lambda_{\text{GL}}^{ab}(T) = 200 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{-1/2}, \quad (47)$$

$$\lambda_{\text{GL}}^{c}(T) = 868.6 \text{ nm} \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{-1/2}. \quad (48)$$
Using the experimental values $\lambda_{\text{GL}}^{ab}(0) = 198.4 \text{ nm}$ and $\lambda_{\text{GL}}^c(0) = 250 \text{ nm}$ \cite{27}, the expressions for the temperature dependent penetration depth in the $ab$-plane, $\lambda_{\text{GL}}^{ab}$, and along the $c$-axis, $\lambda_{\text{GL}}^c$, for LiFeAs become

$$\lambda_{\text{GL}}^{ab}(T) = 198.4 \text{ nm} \left[1 - \left(\frac{T}{T_C}\right)^4\right]^{-1/2},$$

$$\lambda_{\text{GL}}^c(T) = 250 \text{ nm} \left[1 - \left(\frac{T}{T_C}\right)^4\right]^{-1/2}.$$  \hspace{1cm} (47)

The GL parameters in the $c$-axis and $ab$-plane are given by

$$\kappa_{\text{GL}}^{ab} = \frac{\lambda_{\text{GL}}^{ab}}{\lambda_{\text{GL}}^c}, \quad \kappa_{\text{GL}}^c = \frac{\lambda_{\text{GL}}^c}{\lambda_{\text{GL}}^{ab}},$$

Using the experimental values at $T = 0 \text{ K}$ for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, $\kappa_{\text{GL}}$ becomes

$$\kappa_{\text{GL}}^{ab} = 31.6,$$

$$\kappa_{\text{GL}}^c = 72.4.$$  \hspace{1cm} (48)

Using the experimental values at $T = 0 \text{ K}$ for Nd(O$_{1-x}$F$_x$)FeAs, $\kappa_{\text{GL}}$ becomes

$$\kappa_{\text{GL}}^{ab} = 54,$$

$$\kappa_{\text{GL}}^c = 228.4.$$  \hspace{1cm} (49)

Similarly, at $T = 0 \text{ K}$ for LiFeAs, $\kappa_{\text{GL}}$ becomes

$$\kappa_{\text{GL}}^{ab} = 41.3,$$

$$\kappa_{\text{GL}}^c = 78.$$  \hspace{1cm} (50)

As is well known, Abrikosov showed that the breakdown point between type I and type II superconductors is $\kappa = 1/\sqrt{2}$ \cite{12}. If the value of $\kappa < 1/\sqrt{2}$, then the superconductor is said to be type I and is type II if $\kappa > 1/\sqrt{2}$. Therefore, from (48) to (50), we conclude that the iron based superconductors BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs are type II superconductors.

\section{3. Results and Discussion}

In this study we obtained expressions for the angular dependence of upper critical magnetic field, temperature dependence of the upper critical magnetic field parallel and perpendicular to the symmetry axis, the temperature dependence of $\xi_{\text{GL}}$ and $\lambda_{\text{GL}}$. Parallel and perpendicular to the symmetry axis by using the GL approach in two-band iron based superconductors BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs.

Firstly, using (27), (29), and (31) the phase diagram of $H_{C2}$ versus $\theta$ is plotted for iron based superconductors BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs, respectively, as shown in Figure 1. As can be seen from Figure 1, the upper critical magnetic field is maximum in the $ab$-plane ($\pm90^\circ$) and minimum in the $c$-axis ($0^\circ$) for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs.

Secondly, using (24), (28), and (30) and by taking some available experimental data, the phase diagrams of $H_{C2}^{Fe}$ and $H_{C2}^c$ versus temperature are plotted for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs, respectively, as shown in Figure 2. As can be seen from Figure 2, the upper critical magnetic field decreases with increasing temperature and has nonlinear dependence for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs.

Thirdly, the expressions for the temperature dependent $\xi_{\text{GL}}$ and $\lambda_{\text{GL}}$ for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs are given in (35) and (36); (37) and (38); (39) and (40), respectively. Using those equations and by considering some
available experimental data, the phase diagrams of $\xi_{ab}^{GL}$ and $\xi_{c}^{GL}$ versus temperature are plotted as shown in Figure 3. As can be seen in Figure 3, the GL coherence length increases with temperature which subsequently diverges at the critical temperature: $\xi_{GL}(T \to T_C) \to \infty$, for BaFe$_2$(As$_{1-x}$P$_x$)$_2$, Nd(O$_{1-x}$F$_x$)FeAs, and LiFeAs.

Finally, using (45), (46), and (47) and taking the experimental values, we plotted the phase diagrams of $\lambda_{GL}^{ab}$ and
\( \lambda_{GL} \) versus temperature as shown in Figure 4. As can be seen in Figure 4, the GL penetration depth increases with temperature and diverges at the critical temperature, \( \lambda_{GL}(T \rightarrow T_C) \rightarrow \infty \) for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs. According to the mathematical computations, the coherence length is strongly temperature dependent, so that it can cause temperature dependency and anisotropy nature on the upper critical magnetic field of BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs. This indicates that the upper critical magnetic field along \( ab\)-plane, \( H_{C2}^{ab} \), is quite different from the upper critical magnetic field along \( c\)-axis, \( H_{C2}^{c} \). We plotted upper critical magnetic field versus temperature phase diagrams and the critical magnetic field decays with increasing temperature as shown in Figure 2. We also plotted GL coherence length and GL penetration depth versus temperature for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs and we obtained that the characteristic length increases with temperature and each diverges to infinite at its critical temperature as shown in Figures 3 and 4. Furthermore, we plotted the angular dependence of the upper critical magnetic field for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs as shown in Figure 1. Finally, we conclude that the model we used works for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\) (122), Nd(O\(_{1-x}\)F\(_x\))FeAs (111), and LiFeAs (111) and the results we obtained are in agreement with experimental findings [4, 19, 22, 27, 29–32].

### 4. Conclusion

In this research work, we used two-band GL approach in order to determine the upper critical magnetic field, GL coherence length, and GL penetration depth for the two-band iron based superconductors BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs. According to the mathematical computations, the coherence length is strongly temperature dependent, so that it can cause temperature dependency and anisotropy nature on the upper critical magnetic field of BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs. This indicates that the upper critical magnetic field along \( ab\)-plane, \( H_{C2}^{ab} \), is quite different from the upper critical magnetic field along \( c\)-axis, \( H_{C2}^{c} \). We plotted upper critical magnetic field versus temperature phase diagrams and the critical magnetic field decays with increasing temperature as shown in Figure 2. We also plotted GL coherence length and GL penetration depth versus temperature for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs and we obtained that the characteristic length increases with temperature and each diverges to infinite at its critical temperature as shown in Figures 3 and 4. Furthermore, we plotted the angular dependence of the upper critical magnetic field for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\), Nd(O\(_{1-x}\)F\(_x\))FeAs, and LiFeAs as shown in Figure 1. Finally, we conclude that the model we used works for BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\) (122), Nd(O\(_{1-x}\)F\(_x\))FeAs (111), and LiFeAs (111) and the results we obtained are in agreement with experimental findings [4, 19, 22, 27, 29–32].

### Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

### References


10 Advances in Condensed Matter Physics


