Spin Polarization Oscillations and Coherence Time in the Random Interaction Approach

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We study the time evolution of the survival probability and the spin polarization of a dissipative nondegenerate two-level system in the presence of a magnetic field in the Faraday configuration. We apply the Extended Gaussian Orthogonal Ensemble approach to model the stochastic system-environment interaction and calculate the survival and spin polarization to first and second order of the interaction picture. We present also the time evolution of the thermal average of these quantities as functions of the temperature, the magnetic field, and the energy-levels density, for $\rho(\epsilon) \propto \epsilon^s$, in the subohmic, ohmic, and superohmic dissipation forms. We show that the behavior of the spin polarization calculated here agrees rather well with the time evolution of spin polarization observed and calculated, recently, for the electron-nucleus dynamics of Ga centers in dilute (Ga,N)As semiconductors.

1. Introduction

The necessity to maintain the information for long periods of time and the efforts to control and manipulate the electronic spin degrees of freedom led to extensive research activities, such as, among others, the search for the best conditions to keep the spin polarization, as well as new mechanisms to enhance the spin coherence time [1–6]. In spite of abundant empirical knowledge of the spin depolarization rates and the spin coherence times, there is little knowledge of the explicit time evolution of these quantities, as functions of the relevant system-environment interaction parameters. In this paper, we focus on this problem and present a simple calculation of the time evolution of the spin polarization driven by the stochastic interaction of the system and its environment, with good results and few assumptions.

The physics of the two-state systems have been studied since the early days of the quantum theory, and various models and approaches have been proposed and published [7–18]. Attempts to solve completely the models for dissipative two-state systems are generally faced with mathematical complexities. Examples vary from entangled differential equations in master equation approaches [11, 19] to perturbative calculations in the ‘spin-boson’ [14] and the rotating-wave approximation [16]. Recently the master equation approach [19] was applied to describe the evolution of the electronic and nuclear spin polarizations of interstitial gallium defects, which behave as paramagnetic centers in dilute (Ga,N)As semiconductor that selectively capture electrons with opposite spin, and block the recombination of conduction band electrons with the same spin (which lead to an increase of the lifetime of conduction electrons and bound electrons from picoseconds to nanoseconds). The intricacy of this approach expressed through almost a hundred of coupled nonlinear differential equations (with assumptions on the dissipative interactions) reminds us of the ‘much too intractable (intermediate results) of the spin-boson model’ [15]. In the physics of complex systems, it has been frequently found that some processes are insensitive to the details of the interaction, being only a few “gross properties” relevant to describe them. This feature, which is not new to many body problems, has often been used to construct successful and enlightening approaches in terms of ensembles of stochastic interactions [17, 20–27] that make possible satisfying evaluations of ensemble averages for relevant quantities. We will present here a Gaussian stochastic spin-environment interaction approach that strengthens this idea.

In the master equation approach in [28, 29], the electronic and nuclear spin dynamics, obtained through rather detailed
with the continuous (black) curve on top of the gray-circles graph published in [15]. Another system considered three cases: random matrices that belong to finite-dimension GOE, to BEGOE, and to FEGOE, being the last two embedded ensembles generated by $k$-body interactions of spinless fermions and bosons, respectively. The dynamics and complexity of this system are different from the ones studied here and in [18]. For the purpose of this paper it is good enough to calculate the survival and spin polarization to second order of the interaction picture, and for times $t$ such that $\Delta_0t/\hbar$ is of the order of $1$. This means that when $t = 1-10\text{ns}$ the magnetic fields are of the order of $10\text{mT}$.

The Hamiltonian in (1) is similar to that of the spin–boson model for a two-state system in the fictitious spin 1/2 picture.

We will show here that the gross properties of the spin dynamics and spin polarization oscillations, observed in the above mentioned examples, result when a two-state system interacts stochastically with its environment. To take into account the magnetic field in the Faraday configuration, we will consider the Hamiltonian,

$$H = \frac{1}{2}\Delta_0\sigma_z + H_B + \sigma_zV = H_0 + \sigma_zV,$$

where $(1/2)\omega_0\sigma_z$ represents a particle of spin 1/2 in the magnetic field $B = (\Delta_0 g_\text{eff}B_0)\hat{x}$ and $H_B$ describes the environment (also referred as the bath), characterized by a level density $\rho(\epsilon)$. The potential $\sigma_zV$ represents the system–environment interaction where the operator $V$ represents the environment, with matrix elements modelled as statistically independent Gaussian variables. The Hamiltonians $H_B$ and $V$ belong to Gaussian orthogonal ensembles (GOEs) of random matrices with dimension $N \times N$, large enough that the order relations in (14) are fulfilled. A similar Hamiltonian with the terms $\sigma_x$ and $\sigma_z$ interchanged with each other was studied before [18]. In that case, it was possible to evaluate the whole series of the survival probability and the spin polarization, in the interaction representation, for times much larger than the collision time $t_{\text{coll}}$ and much smaller than the Poincaré recurrence time $t_p$. When the basis is chosen such that $|1\rangle$ and $|−1\rangle$ are the eigenstates of $\sigma_z$, the Hamiltonian describes the spin flip processes. If, instead, $|1\rangle$ and $|−1\rangle$ are the eigenstates of $\sigma_x$, the spin-flip processes are understood as tunneling process in the fictitious spin 1/2 picture [15]. Another system with a similar Hamiltonian $\sigma_x/2 + H_x + \lambda \sigma_xV$, and the two-state system in an eigenstate of $\sigma_z$, at $t = 0$, was numerically studied in [30], showing also that the decoherence depends greatly on the nature of the random environment $H_x$ and the interaction $V$. A result that agrees with the results obtained here and other previous papers where it was shown, analytically, that the survival probabilities depend explicitly on the density of levels $\rho(\epsilon)$ of the bath. The authors in [30] considered three cases: random matrices that belong to finite-dimension GOE, to BEGOE, and to FEGOE, being the last two embedded ensembles generated by $k$-body interactions of spinless fermions and bosons, respectively. The dynamics and complexity of this system are different from the ones studied here and in [18]. For the purpose of this paper it is good enough to calculate the survival and spin polarization to second order of the interaction picture, and for times $t$ such that $\Delta_0t/\hbar$ is of the order of $1$. This means that when $t = 1-10\text{ns}$ the magnetic fields are of the order of $10\text{mT}$.

We show the experimental results of the field effect, in the Faraday geometry, on the spin-dependent recombination ratio reported in [5], where the hyperfine interaction was used to explain this behavior when interstitial Ga atoms are present in dilute (Ga, N)As semiconductors. On top of these data we plot also our results (see blue curve) for the spin polarization, as function of the magnetic field, driven by the stochastic interaction. The oscillatory behavior of our results might be behind the large dispersion of data in the experimental results shown in Figure 2.

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In the spin-boson model the environment is modeled as a set of harmonic oscillators and the Hamiltonian describes effectively the tunneling between wells of a double well potential. Some quantities were calculated when appropriate approximations were introduced and the level density was assumed $\propto E^s$ (which, depending on whether the exponent $s$ is equal to 1, $< 1$ or $> 1$, corresponds to the so-called ohmic, subohmic and superohmic dissipation forms, respectively).

In the next section we will present the dissipative two-level model. We will present results for the first- and second-order terms of the interaction representation picture in Section 3. In Section 4 we will show some results for the survival probability and for the spin polarization. We discuss some conclusions at the end.

2. The Random Matrix Model and the Survival Probability

As mentioned before, we will consider the Hamiltonian in (1), where the interaction operator $V(\tau)$, with levels density $\rho(E)$, and $Z$ is the partition function. The states $|\alpha\rangle$, with $|\alpha\rangle$ an eigenstate of $\sigma_z$, form a complete set. If, at time $t = 0$, the interaction is switched on, we pose the problem of calculating the probability $P_{1 \rightarrow 1}(t)$ at time $t > 0$ the system remains in the state $|1\rangle$, regardless of the state of the bath, i.e., the problem of calculating the thermal average of the survival probability,

$$\langle P_{1 \rightarrow 1}(t) \rangle = \sum_{\alpha,\beta} p_{\alpha} \left\langle \left\langle 1 | e^{-i\hat{H}t/\hbar} | 1 \right\rangle \right\rangle, \quad (4)$$

where $\{\cdots\}$ stands for the ensemble average on the Gaussian variables and $\{\cdots\}_\beta$ for the thermal average. Concerning the Gaussian ensemble, it is worth mentioning that a wide research on random matrix ensembles underwent a rapid development leading to various modified versions, in particular the embedded ensembles (EE) generated by random $k$-body interactions, $k = 2$ being the most important; see [26, 27]. The statistical assumptions that we need in our calculations below are completely characterized by (2) and (12), which, according to [21], are compatible with the postulates of matrix elements given by the two-body random ensemble. Before we calculate these averages, we write the time evolution operator $e^{-i\hat{H}t/\hbar}$ in the interaction representation as

$$e^{-i\hat{H}t/\hbar} = e^{-i\hat{H}_{\text{d}}t/\hbar} \sum_{n=0}^{\infty} (-i)^n \sum_{b_{1,\ldots,b_n}} |b_n\rangle \int_0^t dt_n$$

$$\cdot \int_0^{t_n} V(t_{n-1}) dt_{n-1} \cdots \int_0^{t_2} V(t_2) dt_2 \cdot \int_0^{t_1} V(t_1) dt_1.$$ (5)

Here

$$V(t_j) = e^{i\hat{H}_{\text{d}} t_j /\hbar} \sigma_x V e^{-i\hat{H}_{\text{d}} t_j /\hbar}$$

$$\left( \sigma_z \cos \frac{\Delta t_j}{\hbar} + \sigma_y \sin \frac{\Delta t_j}{\hbar} \right) e^{i\hat{H}_{\text{d}} t_j /\hbar} V e^{-i\hat{H}_{\text{d}} t_j /\hbar}$$

that can be written as

$$V(t_j) = \left( \sigma_z \cos \frac{\Delta t_j}{\hbar} + \sigma_y \sin \frac{\Delta t_j}{\hbar} \right) e^{i\hat{H}_{\text{d}} t_j /\hbar} V e^{-i\hat{H}_{\text{d}} t_j /\hbar}$$

$$\left( \sigma_z \cos \frac{\Delta t_j}{\hbar} + \sigma_y \sin \frac{\Delta t_j}{\hbar} \right) e^{i\hat{H}_{\text{d}} t_j /\hbar} V e^{-i\hat{H}_{\text{d}} t_j /\hbar}$$

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$$\left( \sigma_z \cos \frac{\Delta t_j}{\hbar} + \sigma_y \sin \frac{\Delta t_j}{\hbar} \right) e^{i\hat{H}_{\text{d}} t_j /\hbar} V e^{-i\hat{H}_{\text{d}} t_j /\hbar}$$

To simplify the notation, from here onwards we will define $\varepsilon = E / \hbar$ and $\omega_0 = \Delta / \hbar$. Using these quantities in (5), the amplitude of the survival probability becomes

$$\langle 1 \alpha | e^{-i\hat{H}t/\hbar} | 1 \alpha \rangle = \langle 1 \alpha | e^{-i\hat{H}_{\text{d}} t_j /\hbar} \sum_{n=0}^{\infty} (-i)^n \sum_{b_{1,\ldots,b_n}} |b_n\rangle \int_0^t dt_n$$

$$\cdot \left( \sigma_z \cos \omega_0 t_n + \sigma_y \sin \omega_0 t_n \right)$$

$$\cdot V_{b_{n+1}} e^{i(\varepsilon_0 - \varepsilon_b) t_n} \cdots \times \int_0^{t_1} dt_2 \cdot \left( \sigma_z \cos \omega_0 t_1 + \sigma_y \sin \omega_0 t_1 \right)$$

$$\cdot V_{b_2} e^{i(\varepsilon_0 - \varepsilon_2) t_1} \cdots \times \int_0^{t_1} dt_1.$$ (8)
This amplitude can straightforwardly be written as
\[
\langle a | e^{-iHt} | 1a \rangle = e^{-iE_a t} \sum_{n=0}^{\infty} (-i)^n \cdot \sum_{b_1,\ldots,b_n} V_{ab_{1-n}} \cdots V_{b_n} V_{b_1 a} \int_0^t dt_\alpha e^{i(\varepsilon_{a-b_{1-n}}) t_\alpha} \cdots \int_0^t dt_\beta e^{i(\varepsilon_{b_{n-1}-b_{n}}) t_\beta} (1)
\]
\[
\cdot e^{-i(\varepsilon_{a-b}) t} \langle 1 | \cos \omega_0 (t_n - t_{n-1} + \cdots - (-1)^n t_1) + (-1)^n \sin \omega_0 (t_n - t_{n-1} + \cdots - (-1)^n t_1) \rangle.
\]

Therefore,
\[
\langle | e^{-iHt} | a \rangle \rangle^2 = P_{a \rightarrow \bar{a}} (t) = \sum_{m,n=0}^{\infty} (-i)^m (i)^n \cdot \sum_{a,b_1,\ldots,b_m,b_{m+1},\ldots,b_{m+n}} V_{ab_1} \cdots V_{b_m a} V_{b_{m+1} b_1} \cdots V_{b_m b_{m+n}} \cdot \int_0^t dt_\alpha \cdots \int_0^t dt_\beta \int_0^t dt_\gamma \cdots \int_0^t dt_\delta \cdot e^{i[(\varepsilon_{a-b_1} + t_{\alpha-\beta}) t_\alpha + \cdots + (\varepsilon_{b_m-b_{m+n}}) t_\delta]} \cdot \cos \omega_0 \left( \frac{t}{2} - t_n + \cdots + (-1)^n t_1 \right) \cos \omega_0 \ \cdot \left( \frac{t}{2} - t_m + \cdots + (-1)^n t_1 \right).
\]

For the calculation of the ensemble average of the survival probability \( P_{1 \rightarrow \bar{1}} (t) \), we follow the procedure and the assumptions explained in detail in [17, 18, 21]. In the calculation of the average
\[
\langle V_{a,b_1} V_{b_2} \cdots V_{b_{m-1},a} V_{b_m} V_{b_{m+1} b_1} \cdots V_{b_m b_{m+n}} \rangle,
\]
using the statistical assumptions given in (2), one has to take into account that only the covariance of \( V_{ab} \) with itself or with \( V_{ba} \) is nonzero. This property implies that the average of a product factorizes into products of averages of pairs of matrix elements, more specifically into a number of configurations, such as \( \langle V_{a,b_1} V_{b_2} \cdots V_{b_{m-1},a} V_{b_m} V_{b_{m+1} b_1} \cdots V_{b_m b_{m+n}} \rangle \). The Lorentzian weight factor \( W_{\Delta_a} (\varepsilon_a - \varepsilon_b) \) in (2) is assumed to have the property
\[
W_{\Delta_a} (\varepsilon_a - \varepsilon_b) = \begin{cases} 1, & |\varepsilon_a - \varepsilon_b| \leq \Delta (\varepsilon_a) \\ 0, & |\varepsilon_a - \varepsilon_b| > \Delta (\varepsilon_a) \end{cases}
\]
which, as mentioned above, restricts the scope of the interaction \( V \) to connect eigenstates of \( H_{E_1} \) within the energy interval \( \Delta_a = \Delta (\varepsilon_a) = \Delta (\varepsilon_{ab}) \). Here \( \varepsilon_{ab} \equiv \varepsilon_a = (\varepsilon_a + \varepsilon_b)/2 \). The quantity
\[
f_{\text{coll}} \sim \frac{1}{\Delta_a}
\]
has time dimensions and as mentioned before is associated with the collision time, the duration of one application of the interaction \( V \). We assume that \( \Delta_a \) contains many band levels. In the following we will write \( \Delta \) for \( \Delta_a \). If \( D \) is the mean level spacing of the energy eigenvalues of \( H_{E_1} \), we will also assume that the characteristic tunneling frequencies, which are of the order of \( \omega_0 \), are much larger than \( D \). Therefore, the times involved in the calculation satisfy the inequalities
\[
\frac{1}{\Delta} \sim f_{\text{coll}} \ll t \ll t_p \sim \frac{1}{D}
\]
where \( t_p \) is the Poincaré recurrence time. Notice that, in the same way as \( \varepsilon \) and \( \omega_0 \), the energies \( \Delta \), \( \Delta_{st} \), and \( D \) are in units of \( h \). The assumptions in (14) are taken into account in the calculation of the ensemble average of (10). In this calculation, we meet with quantities like
\[
\sum_{b_1,b_{m+1}} \langle V_{b_1 b_{m+1}} \rangle e^{i(\varepsilon_{\alpha+1} - \varepsilon_\beta) t_{\alpha+1} + (\varepsilon_{\beta+1} - \varepsilon_\gamma) t_{\beta+1}}
\]
which become
\[
\int_0^\infty \! d\varepsilon_j \rho (\varepsilon_j) \nu^2 (\varepsilon_j) W_{\Delta} e^{i(\varepsilon_{\alpha+1} - \varepsilon_\beta) t_{\alpha+1} + (\varepsilon_{\beta+1} - \varepsilon_\gamma) t_{\beta+1}}
\]
\[
\nu^2 (\varepsilon_j) \rho (\varepsilon_j) e^{-i\Delta t_{\beta+1} t_j}
\]
where \( \nu^2 (\varepsilon_j) \), \( \rho (\varepsilon_j) \), and \( \Delta \) are assumed to vary slowly with the energy. Since the product of \( \nu^2 (\varepsilon_j) \) and \( \rho (\varepsilon_j) \) appear systematically and the energies \( \varepsilon_j \) and \( \varepsilon_{j+1} \) vary almost continuously along the bath spectrum, we define the density \( \rho_\varepsilon (\varepsilon_j) = \nu^2 (\varepsilon_j) \rho (\varepsilon_j) \), in units of \( (E^2 / \hbar^2) (h/E) = 1/s \). The values of \( m \) and \( n \) that determine the terms \( (m, n) \) of the sum in (10) determine also the order of the contribution to the survival probability, which is given by \( \gamma = (m+n)/2 \). For easy reference we write the survival probability as
\[
\langle P_{1 \rightarrow \bar{1}} (t) \rangle = \sum_{m,n=0}^{\infty} \langle P_{1 \rightarrow \bar{1}}^{m,n} (t) \rangle = \sum_{m=0}^{\infty} \langle P_{1 \rightarrow \bar{1}}^{(m)} (t) \rangle
\]
with
\[
\langle P_{1 \rightarrow \bar{1}}^{(m,n)} (t) \rangle = (-i)^n (i)^m \sum_{a,b_1,\ldots,b_m, b_{m+1}, \ldots, b_{m+n}} \langle V_{a,b_1} V_{b_2} \cdots V_{b_{m-1},a} V_{b_m} V_{b_{m+1} b_1} \cdots V_{b_m b_{m+n}} \rangle \cdot \int_0^t dt_\alpha \cdots \int_0^t dt_\beta \cdots \int_0^t dt_\delta \cdot e^{i[(\varepsilon_a - \varepsilon_{b_1}) + \varepsilon_{b_{m-1} - b_{m+n}}] t_\delta} \cdot \cos \omega_0 \left( \frac{t}{2} - t_n + \cdots + (-1)^n t_1 \right) \cos \omega_0 \left( \frac{t}{2} - t_m + \cdots + (-1)^n t_1 \right) \cdots (-1)^m t_1 \right)
\]
3. Time Evolution of the Survival Probability and the Spin Polarization

In this section we present, in detail, results to zeroth and first order of the interaction picture (IP) of the survival probability described in previous section for a spin 1/2 system interacting with a magnetic field in the Faraday configuration and randomly with its environment. The second-order terms of the survival probability are calculated, in detail, in the appendix.

3.1. Survival and Spin Polarization to Zeroth Order of the IP.

The zeroth order contribution describes the time evolution of the isolated spin 1/2 system in the Faraday geometry. This contribution is given by the term \( \langle P_{1\rightarrow 1}^{(0)} (t) \rangle \) in (18). The survival probability to zeroth order is then the well-known quantity

\[
\langle P_{1\rightarrow 1}^{(0)} (t) \rangle = \frac{1}{2} + \frac{1}{2} \cos \omega_o t. \tag{19}
\]

This means that the probability to find the isolated spin 1/2 system in the eigenstate \( | -1 \rangle \) of \( \sigma_z \), in the Faraday geometry, is

\[
\langle P_{1\rightarrow -1}^{(0)} (t) \rangle = 1 - \langle P_{1\rightarrow -1}^{(0)} (t) \rangle = \frac{1}{2} - \frac{1}{2} \cos \omega_o t. \tag{20}
\]

Therefore, as expected, the spin polarization of the isolated spin 1/2 system, is

\[
\langle \Pi_z (t) \rangle = \langle P_{1\rightarrow -1}^{(0)} (t) \rangle - \langle P_{1\rightarrow 1}^{(0)} (t) \rangle = \cos \omega_o t. \tag{21}
\]

It is clear that, in this case, the thermal average is

\[
\langle \Pi_z (t) \rangle_\beta = \frac{1}{Z} \sum_{n_i} \rho_{n_i} \langle \Pi_z (t) \rangle = \cos \omega_o t. \tag{22}
\]

3.2. Survival and Spin Polarization to First Order of the IP.

We calculate now the survival probability and spin polarization of the spin 1/2 system in the Faraday geometry, to first order of the interaction picture. In this calculation, we use the property

\[
\langle P_{1\rightarrow 1}^{(m,n)} (t) \rangle = \langle P_{1\rightarrow 1}^{(n,m)} (t) \rangle^*. \tag{23}
\]

For the first-order contribution, we need to evaluate \( \langle P_{1\rightarrow 1}^{(1)} (t) \rangle, \langle P_{1\rightarrow -1}^{(1)} (t) \rangle, \) and \( \langle P_{1\rightarrow -1}^{(2)} (t) \rangle \). The term \( P_{1\rightarrow 1}^{(1)} (t) \) is given by

\[
\langle P_{1\rightarrow 1}^{(1)} (t) \rangle = (i)^2 \int_0^t dt_2 \int_0^{t_2} dt_1 \sum_{a,b} \langle V_{a,b} V_{b,a} \rangle \\
\cdot e^{i(\epsilon_n - \epsilon_a t_1 + (\epsilon_n - \epsilon_a) t_2)} \cos \omega_o \left( \frac{t}{2} - t_2 + t_1 \right) \cos \omega_o t_2. \tag{24}
\]

Neglecting terms of order \( 1/\Delta \) and smaller, we have

\[
\langle P_{1\rightarrow 1}^{(0,2)} (t) \rangle = -\rho_o \langle e^{\Delta^2 t} \cos \omega_o t \rangle. \tag{25}
\]

The term (1, 1) is given by

\[
\langle P_{1\rightarrow 1}^{(1,1)} (t) \rangle = (-i) \int_0^t dt_1 \int_0^{t_1} dt_2 \sum_a \langle V_{a,a} V_{a,a} \rangle e^{i(\epsilon_n - \epsilon_a) t_1} e^{-i(\epsilon_n - \epsilon_a) t_2} \cos \omega_o \left( \frac{t}{2} - t_2 + t_1 \right). \tag{26}
\]

To order \( 1/\Delta \) we have

\[
\langle P_{1\rightarrow 1}^{(1)} (t) \rangle = \rho_o \left( \frac{t}{2} - t_2 + t_1 \right) \cos \omega_o t. \tag{27}
\]

Taking into account these contributions and the zeroth-order term \( \langle P_{1\rightarrow 1}^{(0)} (t) \rangle \), we have, to first order of the interaction picture and for \( \Delta \gg \omega_o \), the survival probability

\[
\langle P_{1\rightarrow 1}^{(1)} (t) \rangle = \frac{1}{2} + \frac{1}{2} (1 - 2\rho_o) \cos \omega_o t \tag{28}
\]

and the spin polarization

\[
\langle \Pi_z (t) \rangle = (1 - 2\rho_o) \cos \omega_o t + \frac{2\rho_o}{\omega_o} \sin t \omega_o. \tag{29}
\]

These quantities depend on the level density \( \rho_o \) and, through the frequency \( \omega_o \), on the magnetic field. We show in Figure 3 the time evolution of the survival probability (upper panel) and of the spin polarization (lower panel), for two values of the density of levels \( \rho_o \) each. For the blue curves we considered \( \rho_o = 2 \times 10^4 \) s\(^{-1}\) while for the red curves we have \( \rho_o = 5 \times 10^4 \) s\(^{-1}\). In all of these graphs, we consider the frequency \( \omega_o = 2.73556 \times 10^5 \) s\(^{-1}\). In Figure 4 we plot the thermal averages

\[
\langle P_{1\rightarrow 1}^{(1)} (t) \rangle_\beta = \frac{1}{Z} \sum_{n_i} \rho_{n_i} \langle P_{1\rightarrow 1}^{(1)} (t) \rangle \tag{30}
\]

and

\[
\langle \Pi_z (t) \rangle_\beta = \frac{1}{Z} \sum_{n_i} \rho_{n_i} \langle \Pi_z (t) \rangle, \tag{31}
\]

assuming that the density of levels \( \rho_o \) is proportional to \( e^s \). The graphs are plotted for \( T = 300 \) K and for \( s = 0.2 \) and \( s = 1 \) and \( s = 1.5 \), which correspond to the subohmic, ohmic, and superohmic dissipation forms, in the spin-boson model [15].

In Figure 5, we have the thermal average of the spin polarization calculated in [28, 29], and on top of it the thermal
Figure 3: The survival probability (upper panel) and the spin polarization (lower panel) for two values of \( \rho_0(\epsilon) \), in arbitrary units. The blue curve corresponds to \( \rho_0(\epsilon) = 2 \times 10^8 \text{s}^{-1} \) and the red curve to \( \rho_0(\epsilon) = 5 \times 10^8 \text{s}^{-1} \). The frequency in both cases is \( \omega_0 = 2.73556 \times 10^{10} \text{s}^{-1} \).

Figure 4: The thermal averages of the survival probability (upper panel) and the spin polarization (lower panel) assuming that the density of levels \( \rho_0(\epsilon) \) is \( \propto \epsilon^s \). The plots shown here are for the subohmic \( s = 0.2 \), ohmic \( s = 1 \) and superohmic \( s = 1.5 \) dissipation forms and for \( T = 300 \text{K} \).

The average of the spin polarization in (31). For this graph we considered \( \rho = 10^9 \times \epsilon^s \text{s}^{-1} \), with \( s = 0.36 \) and \( \omega_o = 2.58 \times 10^{10} \text{s}^{-1} \). The agreement is good up to times of the order 500ps. In the appendix we obtain the polarization whose thermal average

\[
\langle \Pi_z^{(2)}(t) \rangle_\beta = \frac{1}{Z} \sum_n \rho_n \langle \Pi_z^{(2)}(t) \rangle,
\]

was plotted in Figure 1 on top of the spin polarization calculated in [19]. Our calculation of the spin polarization in Figure 1 was for \( T = 300 \text{K}, \rho = 10^9 \times \epsilon^s \text{s}^{-1} \), with \( s = 0.28 \), and \( \omega_o = 2.58 \times 10^{10} \text{s}^{-1} \). As mentioned before, the agreement is rather good and strengthens the idea that in complex systems some processes are insensitive to the
Figure 5: The spin polarization calculated in Ref. [28] (gray circles) and our results (black curve), to first order of the interaction picture, for level density \( \rho(\epsilon) = 10^9 \epsilon^s \), with \( s = 0.36 \), and frequency \( \omega_0 = 2.73556 \times 10^{10} \text{s}^{-1} \). The gray circles graph published with author’s permission.

Figure 6: The survival probabilities \( \langle P^{(1)}_{1\rightarrow 1}(t) \rangle_\beta \) and \( \langle P^{(3)}_{1\rightarrow 1}(t) \rangle_\beta \) at \( t = 1.2 \times 10^{-9} \), \( T = 300 \text{K} \) and \( \rho = \epsilon^s \times 10^5 \text{s}^{-1} \), with \( s = 0.2 \).

Figure 7: The spin polarization \( \langle \Pi^{(1)}_z(t) \rangle_\beta \) as function of the magnetic field and time for \( T = 300 \text{K} \) and \( \rho = \epsilon^s \times 10^5 \text{s}^{-1} \), with \( s = 0.2 \). The oscillating behavior shown in this graph is compatible with the previous results, not only as function of time but also as function of the magnetic field.

4. The Field Effect on the Spin Polarization

A great amount of experimental research has been published to show the behavior of the spin polarization as function of the magnetic field. Nevertheless, we shall present here the behavior of the thermal averages of the survival probability \( \langle P^{(m)}_{1\rightarrow 1}(t) \rangle_\beta \) and the spin polarization \( \langle \Pi^{(m)}_z(t) \rangle_\beta \), as functions of the magnetic field and time.

In Figure 6 we plot the magnetic field behavior of the survival probabilities \( \langle P^{(1)}_{1\rightarrow 1}(t) \rangle_\beta \) and \( \langle P^{(3)}_{1\rightarrow 1}(t) \rangle_\beta \), as functions of the magnetic field at \( t = 1.210^{-9} \) and for level density \( \rho(\epsilon) = 10^9 \epsilon^s \), with \( s = 0.2 \). As expected, the survival probability tends to a probability of 1/2 as the magnetic field increases.

In Figure 7 we show the spin polarization \( \langle \Pi^{(1)}_z(t) \rangle_\beta \) as function of time and of the magnetic field for a level density \( \rho(\epsilon) = 10^9 \epsilon^s \), with \( s = 0.2 \). The oscillating behavior shown in this graph is compatible with the previous results, not only as function of time but also as function of the magnetic field.

5. Conclusions

We presented here a simple model to study the behavior of a two-level system interacting stochastically with its environment in the presence of a magnetic field in the Faraday configuration. We calculated the survival and spin polarization to first and second order of the interaction picture. We have shown the oscillating evolution of the thermal average of these quantities as function of time and the magnetic field, for different values of the temperature and for the level density \( \rho(\epsilon) \propto \epsilon^s \), in the subohmic, ohmic, and superohmic dissipation forms. We have shown that the spin polarization behavior agrees rather well with details of the interaction, and only few “gross properties” are relevant and can be described by suitable statistical models.

The analytical expressions of the survival probabilities and spin polarizations reported here allow also exploring the behavior of these quantities as functions of the magnetic field. This is the purpose in the next section.
the time evolution of the spin polarization observed and calculated, recently, for the electron-nucleus dynamics of Ga centers in dilute (Ga,N)As semiconductors. The calculation of the higher order terms, in the interaction picture, is ongoing and we hope to obtain more accurate results and a better understanding of the oscillating behaviors reported here.

Appendix

Survival Probability and Spin Polarization to Second Order

For the second-order contribution, one has to evaluate the following terms $\langle P_{1,1}^{0,2}(t) \rangle$, $\langle P_{1,1}^{1,3}(t) \rangle$, and $\langle P_{1,1}^{2,2}(t) \rangle$. For the term $\langle P_{1,1}^{0,4}(t) \rangle$ we have to evaluate

$$\langle P_{1,1}^{0,4}(t) \rangle = (i)^4 \int_0^t dt_4 \int_0^{t_4} dt_3 \int_0^{t_3} dt_2 \int_0^{t_2} dt_1$$

$$\cdot \sum_{a,b_1,b_2,b_3} \langle V_{a,b_1} V_{b_1,b_2} V_{b_2,b_3} V_{b_3,a} \rangle$$

$$\cdot e^{i \left( (\varepsilon_2 - \varepsilon_3)t_1 + (\varepsilon_3 - \varepsilon_1)t_2 + (\varepsilon_1 - \varepsilon_2)t_3 \right)} \cos \omega_o t \frac{t}{2}$$

with ensemble average

$$\langle P_{1,1}^{0,4}(t) \rangle = \int_0^t dt_4 \int_0^{t_4} dt_3 \int_0^{t_3} dt_2 \int_0^{t_2} dt_1 \rho_o^2 (e)$$

$$\cdot e^{-\Delta(t_2-t_1)} e^{-\Delta(t_3-t_2)} \cos \omega_o t \frac{t}{2} \cdot \left( \frac{t}{2} - t_4 + t_3 - t_2 + t_1 \right) \cos \omega_o t \frac{t}{2}.$$ (A.1)

Similarly, for the term $\langle P_{1,1}^{1,3}(t) \rangle$, the pairing of the matrix elements $V_{ab}$, that contribute to the same order of the survival probability are shown in Figure 9, in configurations A and B. The corresponding averages are

$$\langle P_{1,1}^{1,3}(t) \rangle_A = \int_0^t dt_2 \int_0^{t_2} dt_1 \int_0^{t_2} dt_1' \rho_o^2 (e)$$

$$\cdot e^{-\Delta(t_2-t_1)} e^{-\Delta(t_1-t_2')} \cos \omega_o \left( \frac{t}{2} - t_2 + t_1 \right) \cos \omega_o$$ (A.2)

For the term $\langle P_{1,1}^{2,2}(t) \rangle$, there are two possible configurations for the pairing of the matrix elements $V_{ab}$ that contribute to the same order in the leading order result. In Figure 8, we show graphically these pairing in configurations A and B. The corresponding averages are

$$\langle P_{1,1}^{2,2}(t) \rangle_A = \int_0^t dt_2 \int_0^{t_2} dt_1 \int_0^{t_2} dt_1' \rho_o^2 (e)$$

$$\cdot e^{-\Delta(t_2-t_1)} e^{-\Delta(t_1-t_2')} \cos \omega_o \left( \frac{t}{2} - t_2 + t_1 \right) \cos \omega_o$$ (A.5)

$$\langle P_{1,1}^{2,2}(t) \rangle_B = \int_0^t dt_2 \int_0^{t_2} dt_1 \int_0^{t_2} dt_1' \rho_o^2 (e)$$

$$\cdot e^{-\Delta(t_2-t_1)} e^{-\Delta(t_1-t_2')} \cos \omega_o \left( \frac{t}{2} - t_2 + t_1 \right) \cos \omega_o$$ (A.6)

$$\cdot \left( \frac{t}{2} - t_2 + t_1' \right).$$
Neglecting terms of order $1/\Delta$ and higher, adding the contributions of all these terms and taking into account also the first-order term $\langle P_{1\rightarrow1}^{(1)}(t) \rangle$, we have

$$
\langle P_{1\rightarrow1}^{(2)}(t) \rangle = \frac{1}{2} \frac{t^2 \rho_0^2}{2} + \frac{1}{2} \left( 1 - 2t\rho_0 + t^2 \rho_0^2 \right) \cos t\omega_o \tag{A.7}
$$

$$
+ \rho_o \omega_o \sin t\omega_o.
$$

Hence the probability that at time $t$ the particle is in the spin state $| -1 \rangle$ is

$$
\langle P_{1\rightarrow-1}^{(2)}(t) \rangle = \frac{1}{2} \frac{t^2 \rho_0^2}{2} - \frac{1}{2} \left( 1 - 2t\rho_0 + t^2 \rho_0^2 \right) \cos t\omega_o \tag{A.8}
$$

$$
- \frac{\rho_o}{\omega_o} \left( 1 - t\rho_o \right) \sin t\omega_o.
$$

Therefore the polarization, to second order of the interaction picture, is given by

$$
\langle \Pi_z^{(2)}(t) \rangle = -t^2 \rho_0^2 + \left( 1 - 2t\rho_0 + t^2 \rho_0^2 \right) \cos t\omega_o + \frac{2\rho_o}{\omega_o} \left( 1 - t\rho_o \right) \sin t\omega_o. \tag{A.9}
$$

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The author declares that there are no conflicts of interest regarding the publication of this paper.

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**References**


