Hindawi
Advances in Condensed Matter Physics
Volume 2019, Article ID 7632852, 6 pages
https://doi.org/10.1155/2019/7632852

Research Article

Extreme Events in Lasers with Modulation of the Field Polarization

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Received 25 September 2018; Revised 13 December 2018; Accepted 6 January 2019; Published 4 February 2019

Academic Editor: Jan A. Jung

We develop a theoretical model for a unidirectional ring laser consisting of an isotropic active medium inside a cavity containing a birefringent Kerr cell. We analyze the dynamical behavior of the system as we modulate the voltage applied to the Kerr cell. We discuss the bifurcation diagram and we study the regions of control parameter space where it becomes possible to observe and predict extreme events.

1. Introduction

Lasers have been used as test benches for nonlinear dynamics in many different configurations, some of them requiring a complicated set up or involving a very large number of degrees of freedom. Lasers with optical feedback, laser with saturable absorbers, and lasers with large Fresnel number are typical examples appearing in recent literature. In particular, lasers with a modulated parameter are able to display a large variety of dynamical regimes [1–4]. Periodic behavior, period doubling transition to chaos [2–4], intermittency, crisis of chaotic attractors [5–7], and optical rogue waves [8–10] are among possible observed phenomena. Modulation of cavity losses [3], cavity length [4], and pump rate [11] have been reported as mechanisms generating chaotic behavior in Class B lasers.

Today there is an increasing interest on the study of the so called optical rogue waves. Optical rogue waves are high intensity pulses much larger than average and therefore rare events [8–10, 12–17]. Several optical systems have been reported [9, 13, 14, 18–20] as showing such type of pulses more frequently than what would be expected for a normal distribution probability of the light intensity. However the analysis of the physical mechanism at the origin of extreme events remains difficult in those experiments and models.

Here we analyze a theoretical model of a laser in which the modulation is applied to the relative phase between the two components of the linear polarizations of the field. Modulation of such parameter is usually achieved by introducing inside the cavity a birefringent material whose extraordinary refractive index is changed through a sinusoidal voltage. If we assume that the active medium and the cavity are isotropic, the laser may operate in principle at any polarization of the field. We identify the existence of generalized multistability, period doubling transition to chaos, and three types of crises of strange attractors. However the main objective of this work is to show the appearance of optical rogue waves and to identify the physical mechanism at their origin. Special interest is put also in establishing our ability to predict them [21–25]. We establish clearly the relevance of the existence of generalized multistability and the role played by an external crisis of the chaotic attractor in order to generate optical rogue waves. We construct also bifurcation diagrams taking the amplitude of the modulation as the main control parameter. All other parameter values like gain, losses, and dissipation are compatible with Class B lasers [26].

2. Theoretical Model

The theoretical model is based on a single mode, Class B unidirectional ring laser with an electro optic modulator (EOM) placed inside the cavity. After applying the rotating wave and slowly varying amplitude approximations and without taking into account diffraction, the set of Maxwell-Bloch equations describing the interaction between a single mode electromagnetic field and a two-level atom are

$$\begin{aligned} \partial_\tau \mathbf{E} &= -\kappa \mathbf{E} + \mu \mathbf{P} \\ \partial_\tau \mathbf{P} &= -(1+i\delta)\mathbf{P} - \mathbf{E}D \\ \partial_\tau D &= -\gamma_\parallel \left\{ \frac{1}{2}\left(\mathbf{E}^*\mathbf{P} + \mathbf{E}\mathbf{P}^*\right) + D - D_0 \right\} \end{aligned} \tag{1}$$

where $\mathbf{E}$ and $\mathbf{P}$ are the vectorial form of the electromagnetic field and the atomic polarization respectively; D is the atomic population inversion, κ is the cavity loss rate, δ is the detuning between the atomic frequency and the cavity resonance, D_0 is the population inversion given by the pump, and $\gamma_\parallel$ is the relaxation rate of the population inversion. It is worthwhile noting that time and all relaxation rates in (1) are normalized to the atomic linewidth $\gamma_\perp$. A birefringent material (EOM) placed inside the cavity changes the relative phase of one component of the linear polarization with respect to the other without changing their amplitudes. Writing the electromagnetic field at the input of the EOM as

$$\mathbf{E_{in}} = E_x \mathbf{x} + E_y e^{i\psi_y} \mathbf{y} \tag{2}$$

it becomes at the output:

$$\mathbf{E_{out}} = E_x \mathbf{x} + E_y e^{i(\psi + \Delta\phi)} \mathbf{y} \tag{3}$$

where E_x and E_y are the components of the electric field in the direction of the axis corresponding to the ordinary and extraordinary indexes of refraction of the EOM crystal, respectively, and $\Delta\phi$ for the EOM is given by

$$\Delta\phi = \Delta\phi_0 - \pi\frac{V}{V_\pi} = \left[2\pi\left(n_e - n_0\right)\frac{L}{\lambda}\right] - \pi\frac{V}{V_\pi} \tag{4}$$

where V_π is the voltage making a variation on the birefringence equivalent to a phase shift of π, $\Delta\phi_0$ is the medium birefringency at $V = 0$, n_e and n_0 are the extraordinary and ordinary indexes of refraction, respectively, L is the length of the EOM crystal, and λ is the wavelength. If the voltage V applied to the EOM changes sinusoidally with frequency Ω, (4) produces a periodic modulation of $\Delta\phi$. The relative phase of the y-component with respect to the x-component of the field is then changed by

$$\Delta\phi = \Delta\phi_0 + m\cos\Omega\tau \tag{5}$$

where m is the modulation amplitude normalized to the free spectral range of the cavity. Thus $m = 1$ implies a change in frequency corresponding to the intermode spacing. Introducing this effect into the boundary conditions for the field, we obtain, after some algebra, the following set of equations:

$$\begin{aligned} \partial_\tau E_x &= -\kappa E_x + \mu P_x \\ \partial_\tau E_y &= -\kappa E_y + \mu P_y + i\left(\Delta\phi_0 + m\cos(\Omega\tau)\right)E_y \\ \partial_\tau P_x &= -(1+i\delta)P_x - E_x D \\ \partial_\tau P_y &= -(1+i\delta)P_y - E_y D \\ \partial_\tau D & \\ &= -\gamma_\parallel\left(D - D_0 + \frac{1}{2}\left(E_x^* P_x + E_x P_x^* + E_y^* P_y + E_y P_y^*\right)\right) \end{aligned} \tag{6}$$

It is worthwhile noting that an adiabatic elimination of the atomic polarization is not easy to perform even if it decays much faster than the other variables because the phase is modulated and therefore the frequency of the electromagnetic field and the atomic polarization depends on time. We notice also that the modulation appears only in the equation of the y-component of the field polarization because we choose x and y as the axes corresponding to the ordinary and extraordinary indexes of refraction of the electrooptic device.

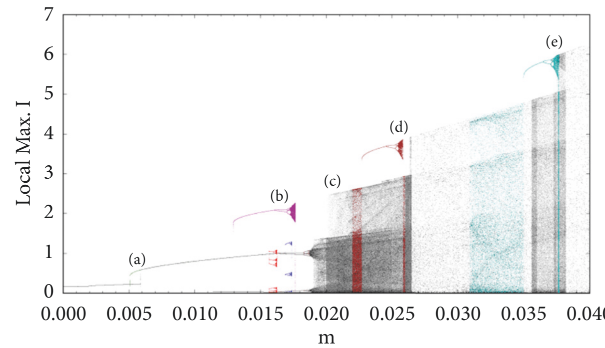

FIGURE 1: Bifurcation diagram showing the Intensity local maxima, I, as a function of the modulation amplitude, m. The parameters corresponding to the figure are $\Omega = 3.79\ 10^{-2}$, $\delta = 1$, $\kappa = 9.0\ 10^{-2}$, $\mu = 2.5\ 10^{-5}$, $\gamma_\parallel = 2.5\ 10^{-5}$, $D_0 = 7200$, and $\Delta\phi_0 = 0$. Note that $D_0 = 3600$ corresponds to the laser threshold. We use parameters corresponding approximately to solid state lasers. We recognize five interesting regions which are marked as (a), (b), (c), (d), and (e) in the figure.

3. Results

Figure 1 shows a typical bifurcation diagram corresponding to the dynamical system described above. We plot the maxima of the intensity, I, as a function of the amplitude of the modulation, m, for a fixed modulation frequency and detuning.

Several facts appear from a direct observation of the bifurcation diagram:

(1) In the region marked by (a) there is a bistable cycle between two periodic solutions. The lower branch is period 1 (T1) and the upper solution is a period

two (T2) with respect to the modulation frequency. It is clear that there is a subcritical bifurcation of T1 leading to T2.

(2) In (b) there is coexistence of several attractors of different periods. We identify the previous T2 and then three other branches of period 3, period 5, and period 6. Each of them gives rise to a period doubling bifurcation leading to chaos. Only the chaotic behavior resulting from T2 survives as the amplitude of the modulation is increased. As described in [6] the other branches disappear in boundary crisis of the strange attractors.

(3) In (c) we observe an abrupt expansion of the chaotic attractor. This expansion is produced by a bifurcation called "external crisis" in which the chaotic attractor collides with an unstable orbit generated in the saddle bifurcation of a different preexisting branch. Each collision generates a sudden expansion of the attractor in phase space. This process generates the appearance of high intensity pulses. Close to the bifurcation they are rare but then they become more and more frequent causing a significative increase in the average intensity of the maxima. The first region where such optical rogue waves appear corresponds to the interval between $m = 0.02$ and $m = 0.021$. Thus the interesting interval for the appearance of rogue waves is $\delta m = .001$. Thus, if a voltage of 100 volts on the electrooptic crystal produces a phase shift in π, the interval of m where optical rogue waves appear is $\Delta V = 0.1 volts$.

(4) In region (d) it appears a new branch of period 4. The chaos originated in such branch disappears at a boundary crisis. The unstable period 4 orbit collides with the existing chaotic attractor generating a new expansion in phase space. It is important to note that the bifurcation producing the crisis of the chaotic attractors were in all cases supercritical, and there is no bistable behavior.

(5) As the modulation amplitude increases, the strange attractor expands gradually until region (e). It appears then another periodic solution, of period 5 (T5), in a saddle node bifurcation. The stable T5 solution shows period doubling bifurcations, ending in chaotic behavior. The unstable T5 solution collides with the preexisting strange attractor and this external crisis produces a chaos merging [27].

(6) Inside every chaotic region there are periodic windows. The transition from the strange attractor to a periodic solution can be done through a process of intermittency and therefore it may alternate high intensity peaks with regular periods of low intensity.

The bifurcation diagram of Figure 1 has been obtained for a detuning value of the order of the linewidth $\delta = 1$ and modulation frequency Ω of the order of the relaxation oscillation frequency. In that region of the control parameter space only one linear polarization survives for modulation

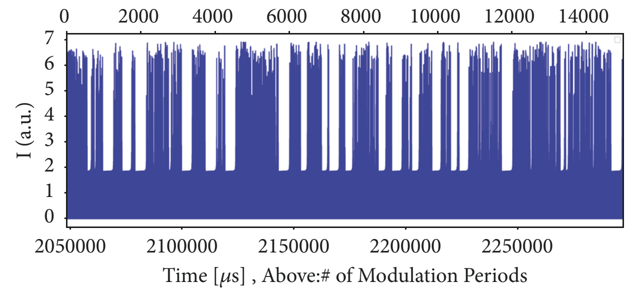

Figure 2: Intensity, I, as a function of time measured in units of the modulation period for a modulation amplitude $m = 3.8874\ 10^{-2}$, and $\Delta\phi_0 = -6\ 10^{-2}$. All other control parameter values are the same to those of Figure 1.

amplitude m different from zero. The symmetry between both linear polarizations has been broken by the modulation. If the detuning vanishes, there is no effect of the modulation and there is no possibility to observe chaotic behavior. A complete analysis of the symmetry breaking effects and the prevalence of one or the other linear polarization under modulation of their relative phases will be presented elsewhere. Several studies of lasers selecting polarization of the field in isotropic cavities can be found in [28–30] and references therein. High intensity peaks surrounded by small ones may appear in two cases: the passage from chaos to a periodic window and the sudden expansions of strange attractors. Previous works [31] show that some processes of intermittency may generate Levy's probability distribution and therefore they make probable the appearance of "rogue waves." Figure 2 shows a typical trace of the intensity as a function of time in a region of intermittency in which there is a strong asymmetry between the low maxima of the periodic solution of the laminar period and the high intensity pulses in the chaotic domains. The probability distribution of the maxima of intensity corresponding to the signal of Figure 2 is shown in Figure 3. If we define rogue wave as intensity pulses overcoming the average intensity plus four times the standard deviation, then there are no rogue waves in this region because of the large average and dispersion. The peak of the distribution at the intensity maxima of the periodic solution appears clearly.

In fact we are thinking that the process we observe here corresponds to the dynamical behavior described in reference [32] and, in this case, it does not produce optical rogue waves. Similar tests inside the chaotic regions far away in control parameter space from an abrupt expansion of the strange attractor give the same result. Thus there are no pulses satisfying the definition of "rogue wave" for most of the control parameter space. The chaotic attractor explores almost uniformly all possible values of the intensity in a large but limited region of phase space. The average of the intensity plus four times the variance is much higher than the maximum accessible for the laser intensity. There is a clear cut at the extreme value even if the probability of finding such extreme values is greater than finding smaller ones. Such extreme events which are typical in a deterministic nonlinear system have been called "Dragon Kings" [33]. However those

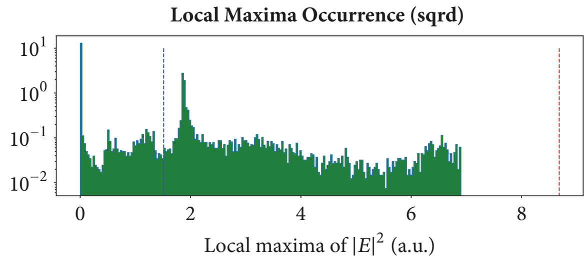

FIGURE 3: Histogram of the number of intensity peaks as a function of the intensity maxima for the temporal sequence of Figure 2. The dashed lines indicate the position of the average of the maxima intensity and the intensity corresponding to the average value plus four times the standard deviation which is the limit corresponding to the definition of rogue wave.

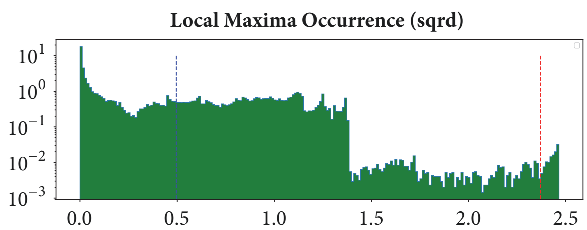

FIGURE 4: Histogram of the maxima of intensity at $m = 0.02025$. All other control parameter values are the same as in Figure 1. The dashed lines indicate the average intensity and the threshold intensity for optical rogue waves as in Figure 3.

extreme events do not overcome the threshold corresponding to the definition of rogue wave at the control parameter values when the system is far away from an external crisis.

Figure 4 shows instead the probability distribution of the intensity maxima for values of m just after an abrupt expansion of the chaotic attractor. Those regions could be described as those in which there is a modification of the structure of the chaotic attractor. In fact, the probability distribution of the maxima intensity shows two plateaus of different density. The second plateau extends over very high values of the intensity. Thus attractor does not cover uniformly all possible values. There are some intensity peaks overcoming the average peak intensity plus four times the standard deviation; therefore those extreme events can be called optical rogue waves.

Predictability of extreme events in deterministic systems was studied in [21–25]. Figure 5 shows a superposition of traces of the intensity as a function of time all of them triggered at the center on extreme events. We show that the evolution of the intensity follows always the same trajectory in each case since almost ten periods of the modulation before the extreme event while it loses correlation much faster after the pulse. Thus the extreme event may, in principle using a good protocol, is predicted with anticipation time much larger than the inverse of the greatest positive Lyapunov exponent. The maximum Lyapunov exponent Λ measured at $m = 0.02$ is $\Lambda = 1.921$ representing 3.14 modulation periods. At $m = 0.03$ the value of the maximum Lyapunov

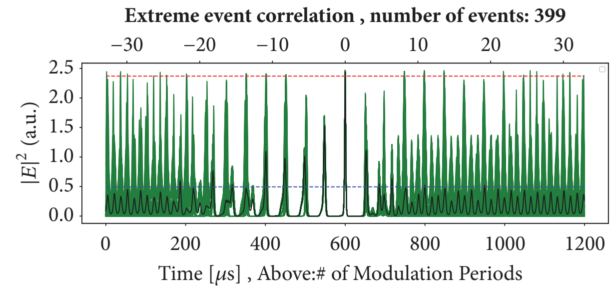

FIGURE 5: Intensity as a function of time in units of the modulation period using an extreme event as trigger and superposing 322 signals.

exponent increases and it represents 2.71 modulation periods. It is worthwhile noticing that Figure 5 shows only one of the variables of the system, the total intensity. It is clear that knowledge of the value of several dynamical variables will increase the predictive time because we are able to identify more precisely an initial condition for the corresponding trajectory in phase space. However the total intensity is one of the most relevant variables in determining the dynamical evolution allowing the identification of an extreme event to predict the rogue waves with an anticipation of 15 modulation periods. This time is at least four times larger than the inverse of the maximum Lyapunov exponent for this system.

4. Conclusions

It is worthwhile mentioning that optical rogue waves in vector lasers have been observed in Er doped fiber lasers [34, 35] and VECSELs [36]. In none of those cases did a modulated parameter existed. By including a controllable birefringent material inside a laser cavity in order to perform a phase modulation of the electromagnetic field, it is possible to observe several nonlinear phenomena. Multistability, crisis of strange attractors, chaos merging, and intermittency are among them. Furthermore external crises are able to produce large expansion of the strange attractors giving raise to the appearance of very high intensity pulses fulfilling the definition of optical rogue waves. Such phenomena are rare and dangerous but their appearance can be predicted by just following the trajectory of a single variable in phase space. Further work is necessary to find a protocol able to indicate their appearance without giving false alarms.

Data Availability

The [DATA TYPE] data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

The actual address of Alexis Gomel is Universite de Geneve, Suisse, and Jorge R. Tredicce's is also at Departamento de

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Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The authors wish to acknowledge support from the Project ECOS-Sud A14E03 "Evenements extremes en dynamique non lineaire."

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