Hindawi Publishing Corporation
Journal of Applied Mathematics and Decision Sciences
Volume 2007, Article ID 56404, 10 pages
doi:10.1155/2007/56404

Research Article

A Decomposition-Based Pricing Method for Solving a Large-Scale MILP Model for an Integrated Fishery

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Received 6 September 2006; Revised 11 January 2007; Accepted 5 June 2007

Recommended by Stefanka Chukova

We study the integrated fishery planning problem (IFP). In this problem, a fishery manager must schedule fishing trawlers to determine when and where the trawlers should go fishing and when the trawlers should return the caught fish to the factory. The manager must then decide how to process the fish into products at the factory. The objective is to maximize profit. We have found that IFP is difficult to solve. The initial formulations for several planning horizons are solved using the AMPL modelling language and CPLEX with branch and bound. The IFP can be decomposed into a trawler-scheduling subproblem and a fish-processing subproblem in two different ways by relaxing different sets of constraints. We tried conventional decomposition techniques including subgradient optimization and Dantzig-Wolfe decomposition, both of which were unacceptably slow. We then developed a decomposition-based pricing method for solving the large fishery model, which gives excellent computation times. Numerical results for several planning horizon models are presented.

1. Introduction and literature review

Modern commercial fisheries are often vertically integrated, that is, a firm may own fishing trawlers and a processing factory. To maximise profit, a fishery manager must schedule the fishing trawlers to determine when and where the trawlers should go fishing and when the trawlers should return the caught fish to the factory. Given a trawler schedule, the manager must then decide how to process the fish into products at the factory. The objective is to maximise profit. The difficult part of this problem is coordinating trawler scheduling and fish-processing.

Wide-ranging research has been reported on fisheries. Many papers described biological models, and only a few discussed production-planning. Mikalsen and Vassdal [1] developed a multiperiod LP model for production-planning over a one month horizon for smoothing the seasonal fluctuations of fish supply. Their model was market-driven and incorporated the acquisition of raw material purchased, rather than acquired with their owned fishing fleet. Jensson [2] developed a product mix LP model to maximize profit of an Icelandic fish-processing firm over a five-period planning horizon. He addressed production-planning and labour allocation for that processing firm but did not address any fleet-specific issue or quota issue. Gunn et al. [3] developed a model for calculating the total profit of a Canadian company with integrated fishing and processing. Their model included a fleet of trawlers, a number of processing plants, and market requirements. However, their model ignored the trawler scheduling and labour allocation in the processing firm. Indeed, none of these papers discussed models that integrated both trawler scheduling and production.

We previously developed a model [4] for the integrated fishery problem (IFP). The IFP is designed to coordinate trawler scheduling and processing and to allocate labour. It can be updated and run periodically to aid in a manager's decision making. We experimented with real data from a New Zealand fishery. Unfortunately, for realistic planning horizons of 20 periods or more, the computational times for the IFP were quite long.

To find an effective solution method, in this paper we report on our work with subgradient optimisation (SO) and Dantzig-Wolfe decomposition (DWD). Despite experimenting with different alternatives for SO and DWD, we found that these algorithms are ineffective. We then developed an effective decomposition-based pricing (DBP) method.

The remainder of the paper is organized as follows. In Section 2, we briefly present the IFP in matrix notation and its LP relaxation. Section 3 describes SO with two different Lagrangean relaxations. Section 4 describes the DWD, also with two different relaxations. Neither SO nor DWD proved effective. Section 5 gives our decomposition-based pricing procedure. While technically a heuristic, we found decomposition-based pricing to be quite efficient. As DBP was developed for linear programs, we modified it for the fishery model. We conclude the paper in Section 6 with a discussion of decomposition-based pricing to other problems and future work.

2. The fishery model

In this section, we briefly describe our fishery model [4] in matrix notation.

Parameters.

(i) c^1, c^2, c^3 denote unit profit of trawler operation, raw fish inventory, and fish-processing, respectively,

(ii) A^0 denotes quantity of fish landed per trip in each period,

(iii) D^1 denotes mass balance coefficients on each trawler in each period,

(iv) D^2 denotes mass balance coefficients on fish within the processing factory,

(v) A^1, A^2 denote mass balance coefficients governing transformation of raw fish into a finished product.

Decision variables.

(i) w denotes binary variables indicating whether a trawler takes a given trip,

(ii) f denotes raw fish inventory, indicating the current quantity of each type of raw fish in each period,

(iii) x denotes fish-processing variables, indicating that a given type of raw fish is converted into a given product.

$$\begin{array}{rll} \text{Model IFP :} \quad \text{maximize} & c^1 w + c^2 f + c^3 x & \\ \text{subject to} & & \\ \text{Inventory supply constraints} & A^0 w + f = 0, & (2.1) \\ \text{Trawler scheduling constraints} & D^1 w = b^1, & (2.2) \\ \text{Processing constraints} & D^2 x = b^2, & (2.3) \\ \text{Inventory demand constraints} & A^1 f + A^2 x = b^0, & (2.4) \\ & w \in \{0,1\}, & (2.5) \\ & f, x \geq 0. & (2.6) \end{array}$$

Equation (2.1) represents the relationship of the trawler-scheduling variables w to landed fish f, as a mass balance in movement of fish from trawlers to the factory. Equation (2.2) expresses the constraints involving only trawler scheduling, indicating, for example, that a trawler may be in only one place at a time. Equation (2.3) expresses fish-processing constraints, modelling the flow of fish through the factory as raw fish is made into various products. Equation (2.4) represents the mass balance constraints, representing the flow of raw landed fish inventory into the fish-processing factory.

When the integer constraints (2.5) are relaxed, the model is the usual linear programming relaxation. However, other relaxations are possible. Observe that IFP decomposes into a trawler-scheduling problem and a fish-processing problem if either constraint set (2.1) or constraint set (2.4) were relaxed. In the next section, we use both decompositions with SO.

3. Subgradient optimisation for the fishery model

Lagrangean relaxation is based on the existence of complicating constraints. When these complicating constraints are relaxed, the resulting model is often easier to solve. Geoffrion [5] introduced the term "Lagrangean relaxation," developed relevant theories, and explored its usefulness for IP branch and bound. Fisher [6] reviewed Lagrangean relaxation and documented a number of successful applications of this method. To obtain the Lagrangean relaxation of IFP, we attach multipliers θ to complicating constraints of IFP, and bring this term into the objective function. SO is a commonly used method of finding the optimal multipliers θ (Held et al. [7], Held and Karp [8], and Shepardson and Marsten [9]). This approach yields θ directly. In this section, we describe our attempts to solve IFP with SO, with two different decompositions.

Table 3.1. Numerical results for SO, relaxing constraint set (2.4).

	5 periods	10 periods	30 periods
IP optimum	\$522 764	\$1 065 775	\$2 300 871
LP optimum	\$522 764	\$1 066 350	\$2 331 036
SO optimum	\$522 764	\$1 065 991	\$2 325 650
SO solution time (s)	718	1120	3625

3.1. Relaxation of inventory balance constraints. In this section, we use SO to solve the fishery model by relaxing the complicating inventory balance constraints (2.4)

$$\begin{aligned}\mathrm{PR1}(\theta):\operatorname*{Max}_{f,w,x}\{&c^1w+c^2f+c^3x-\theta(A^1f+A^2x-b^0)\mid A^0w+f=0,\\&D^1w\le b^1,\ D^2x\le b^2\}.\end{aligned}\tag{3.1}$$

The SO algorithm for IFP can be stated as follows. Denote θ^k as the Lagrangean multiplier at iteration k.

Step 1. Initialize iteration $k=0$ and set jump size t^k. θ^0 was taken from the dual values of constraint set (2.4) from the LP relaxation.

Step 2. Solve $\mathrm{PR1}(\theta)$ for θ^k.

Step 3. Let $\theta^{k+1}=\theta^k+t^k(A^1f+A^2x-b^0)$.

Step 4. Set $t^{k+1}=t^k(0.9998)*$ (Lagrangean value − LP value)/slack, where slack = slack + $(A^1f+A^2x-b^0)^2$.

Step 5. For convergence:
if $|\theta^{k+1}-\theta^k|<\varepsilon$, then stop,
else if the maximum number of iterations was reached, then stop,
else let $k=k+1$ and go back to Step 1.

Table 3.1 shows numerical results for models with various different planning horizons.

3.2. Relaxation of landed fish constraints. We next attempted SO by relaxing constraint set (2.1) as follows:

$$\mathrm{PR2}(\theta):\operatorname*{Max}_{f,w,x}\{c^1w+c^2f+c^3x-\theta(A^0w+f)\mid D^1w\le b^1,\ D^2x\le b^2,\ A^1f+A^2x=b^0\}.\tag{3.2}$$

Numerical results of various planning horizon models are shown in Table 3.2.

SO was ineffective in both of these decompositions, taking far too long to converge. Subgradient optimization has been reported to result in unpredictable convergence behaviour (Guignard and kim [10]) and such was the case with this model. We experimented with modifications to update the Lagrangean multipliers, but this decreased computational time only slightly. We therefore turned to Dantzig-Wolfe decomposition.

Table 3.2. Comparison between LP and LR relaxation solutions and true optimum (IP).

	5 periods	10 periods
IP optimum	\$522 764	\$1 065 775
LP optimum	\$522 764	\$1 066 350
LR optimum	\$522 764	\$1 070 450
SO solution time (s)	952	1360

4. Dantzig-Wolfe decomposition (DWD) for the IFP

In this section, we apply DWD (Dantzig [11]). DWD yields θ as the dual variable associated with the relaxed constraints. This decomposition may be interpreted in the following way: the fishery manager uses a master model to generate prices for raw fish. These prices are passed to the fishing-trawler captains who propose trawler schedules, and to the factory manager who proposes a production schedule. Their proposals are passed to the fishery manager, who uses the master model to find the best mix of proposals and new prices for raw fish. The procedure terminates when no new proposals come from the subproblems.

Algebraically, we express the feasible region of the trawler-scheduling subproblem as a convex combination of the extreme points for constraint sets (2.1) and (2.2). Since the trawler-scheduling variables are bounded, this set is bounded. Similarly, we can express the feasible region of the production-planning subproblem and constraint set (2.3) as a convex combination of its extreme points. Without loss of generality, these variables are bounded, so their convex set is bounded. Let λ^1 and λ^2 be variables associated, respectively, with the subproblems for trawler scheduling and fish-processing, with extreme points numbered $1,\dots,K1$ and $1,\dots,K2$. We can then write the DWD master problem as follows:

$$\text{Maximize} \sum_{k=1}^{K1} \lambda^{1k}\left(c^1 w + c^2 f\right) + \lambda^{2k} \sum_{k=1}^{K2} c^3 x, \quad \text{subject to}$$

$$\text{Inventory balance rows} \quad \sum_{k=1}^{K1} \lambda^{1k}\left(A^1 f\right) - \sum_{k=1}^{K2} \lambda^{2k}\left(A^2 x\right) = 0, \tag{4.1}$$

$$\begin{aligned} &\text{Trawler scheduling} && \sum_{k=1}^{K1} \lambda^{1k} = 1, \\ &\text{Fish-processing} && \sum_{k=1}^{K2} \lambda^{2k} = 1, \quad \lambda^{1k}, \lambda^{2k} \geq 0. \end{aligned} \tag{4.2}$$

Note that λ^{1k} is continuous, so this model will only provide an upper bound.

Let θ be the dual prices associated with the inventory balance constraint (4.1). The subproblems are as follows.

(1) Trawler-scheduling subproblem S_1^k,

$$\begin{aligned}&\text{maximize } c^1 w + c^2 f - \theta(A^1 f), \quad \text{subject to}\\&\text{constraint sets (2.1) and (2.2)},\\&f \geq 0,\ w \in \{0,1\}.\end{aligned} \tag{4.3}$$

(2) Processing subproblem S_2^k,

$$\begin{aligned}&\text{maximize } c^3 x - \theta(A^2 x), \quad \text{subject to}\\&\text{constraint set (2.3)},\\&x \geq 0.\end{aligned} \tag{4.4}$$

We used AMPL [12] to solve IFP with DWD on a Pentium3 computer. The results were really quite disastrous. Giving the master a good initial feasible solution, a small five-period model required 1168 iterations and 4 hours, 54 minutes. Giving the master an initial feasible solution of the zero vector, the five-period model required 1068 iterations and 3 hours, 59 minutes, but a direct solution with CPLEX takes only four seconds to solve a five-period model directly. We further attempted to use DWD to solve models with more periods, but these took a very long time to solve.

We next tried to solve IFP by relaxing the landed-fish constraint set (2.1). This proved even worse computationally. A trivial three-period model required 1367 iterations with a naive initial solution, and 1787 iterations with an initial solution of the zero vector. A five-period model required 3923 iterations, and a ten-period model was abandoned after 4536 iterations.

We conclude that Dantzig-Wolfe decomposition is not effective for IFP.

5. Decomposition-based pricing (DBP)

In this section, we apply decomposition-based pricing (DBP) for the efficient solution of IFP. Mamer and McBride [13] developed DBP for multicommodity flow problems. As with DWD, subproblems are created by dualizing some constraints, and these subproblems are identical to S_1^k and S_2^k from the DWD. Instead of using the subproblem to produce an extreme point of the relaxed polytope for inclusion in a master problem, the optimal basic columns of the subproblem are included in a restricted master. The DWD master is replaced by a version of the original problem with all of the original rows and a subset of original columns. This restricted master problem is solved to obtain an improved primal solution and new dual prices. The restricted master is not the same as the DWD master. It has a full IFP formulation with restricted rows. The procedure terminates when no positive variables entered into the restricted master or when the objective value of the subproblems and that of the restricted master are equal. Dual prices from the inventory balance constraints (2.4) are passed to the subproblems.

The fishery problem is an MILP model, so we cannot guarantee strong duality (outside of a custom branch and bound algorithm). Hence this is a heuristic method. However,

Table 5.1. LP relaxation solution and IP solution of different planning horizon models.

Planning horizon	Variables in original problem	LP objective value	IP objective value
5	2193	\$522 764	\$522 764
10	4423	\$1 066 350	\$1 065 775
15	6803	\$1 607 944	\$1 582 008
20	9333	\$1 898 411	\$1 880 196
25	11 989	\$2 141 757	\$2 121 887
30	16 139	\$2 331 037	\$2 300 871

through a careful choice of initial feasible solution and stopping criteria, we obtain excellent bounds, and the solution times obtained are much faster than the direct solutions with CPLEX.

5.1. DBP procedure for the fishery model. We use Lagrangean relaxation to relax the inventory balance constraints (2.4), as in Section 3. Let θ be the simplex multipliers for the restricted master where θ is associated with the inventory balance constraints (2.4). We define the restricted master as the original problem for IFP, but restricted to a smaller set of variables I^k. Set I^k is the set of positive variables in the master at iteration k. Set I^k increases in size with each iteration because each iteration of the subproblems adds new variables to I^k. Computationally, we found (as did Mamer and McBride [13]) that the number of variables in I^k at any iteration is much less than the number of variables in the original problem:

$$\begin{aligned}(M^k)\,\text{maximize } c^1 w + c^2 f + c^3 x, &\quad \text{subject to} \\ \text{constraint sets (2.1) to (2.4)}&, \\ f \geq 0, \quad w \in \{0,1\}, &\quad x \geq 0,\end{aligned} \tag{5.1}$$

with $f, w, x \in I^k$, here I^k is the index set of all positive variables $f, w, x \geq 0$.

Our decomposition-based pricing procedure is summarised as follows.

Step 1. Initialize. Set iteration $k = 1$. We used three alternate methods to pick an initial set of prices θ^1.

(I1) Start with $\theta^1 = 0$.

(I2) Start with θ^1 as the dual prices from the relaxed constraints of the IFP LP relaxation.

(I3) Start with heuristic dual prices, $\theta^1_{ilt} = -\sum_{j:Fij>0} P_{ij}/(2.5 \cdot F_{ij})$, where $F_{i,j}$ is the fillet percentage of raw material and $P_{i,j,l}$ is the profit of processing product j of quality l from raw materials i.

Step 2. Solve subproblems S^k_1 and S^k_2. For each f_i, w_i, or $x_i > 0$, add the variable to I^k. Thus, $I^k = \{f_i, w_i, x_i > 0$ in S_1 or S_2 for any iteration $1, 2, \ldots, k\}$.

Step 3. Solve the restricted master M^k and get dual prices θ^k.

Table 5.2. Numerical results for dbp under different initial dual prices and stopping criteria.

Solution method	Planning horizon	Iterations	Solution time(s)	Variables in final master	DBP solution	Solution gap (%)
I1-SC1	5	26	156	1308	\$522 764	0.00%
I1-SC1	10	29	257	2815	\$1 065 775	0.00%
I1-SC1	15	32	341	4272	\$1 579 440	0.16%
I1-SC1	20	29	365	5691	\$1 874 097	0.32%
I1-SC1	25	29	414	7026	\$2 119 938	0.09%
I1-SC1	30	25	544	8115	\$2 293 803	0.31%
I1-SC2	5	29	211	1252	\$522 764	0.00%
I1-SC2	10	30	258	2576	\$1 065 538	0.02%
I1-SC2	15	32	335	3881	\$1 579 309	0.17%
I1-SC2	20	27	348	5065	\$1 870 047	0.54%
I1-SC2	25	29	557	6253	\$2 118 528	0.16%
I1-SC2	30	31	1,737	7324	\$2 288 997	0.52%
I2-SC1	5	27	192	1356	\$522 764	0.00%
I2-SC1	10	33	292	2873	\$1 065 531	0.02%
I2-SC1	15	30	322	4378	\$1 579 321	0.17%
I2-SC1	20	28	496	5874	\$1 864 368	0.84%
I2-SC1	25	27	433	7135	\$2 117 990	0.18%
I2-SC1	30	32	1,042	8277	\$2 266 274	1.50%
I2-SC2	5	28	208	1282	\$522 764	0.00%
I2-SC2	10	28	252	2724	\$1 065 712	0.01%
I2-SC2	15	35	373	4092	\$1 579 466	0.16%
I2-SC2	20	29	359	5420	\$1 875 597	0.24%
I2-SC2	25	35	534	6540	\$2 111 616	0.48%
I2-SC2	30	30	650	7623	\$2 292 894	0.35%
I3-SC1	5	26	178	1325	\$522 764	0.00%
I3-SC1	10	32	275	2784	\$1 065 775	0.00%
I3-SC1	15	30	312	4130	\$1 579 447	0.16%
I3-SC1	20	31	351	5524	\$1 876 023	0.22%
I3-SC1	25	32	487	7135	\$2 120 282	0.08%
I3-SC1	30	27	613	8052	\$2 295 376	0.23%

Step 4. For stopping criterion, we used two alternate methods.

(SC1) Stop when the objective values of the subproblem and restricted master are equal, $v(S_1^k + S_2^k) = v(M^{k+1})$. Here, we solve the trawler-scheduling subproblem as an LP. By solving this subproblem as LP, we find good variables to add to the restricted master, with fast computation time.

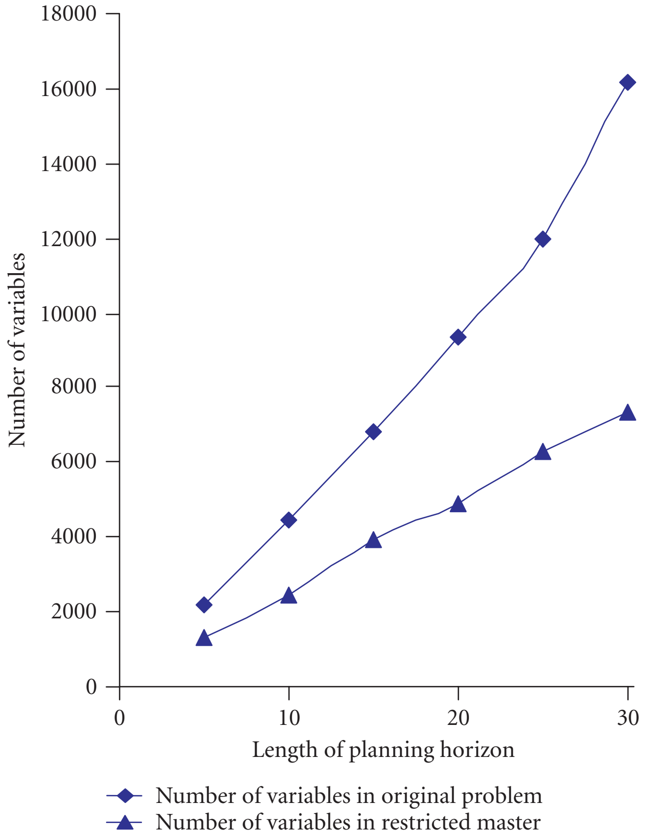

Figure 5.1. Comparison of the number of decision variables in DBP and that of IP.

(SC2) Stop when no new variables come into the restricted master problem. Here, we solve the trawler-scheduling subproblem as an IP.

Else go to Step 2.

Step 5. After the LP optimum is found, solve the final restricted master problem as an IP.

Table 5.1 shows the optimal LP and IP objective values. Depending on the initial feasible solution and stopping criterion, we ran the DBP algorithm in five different ways: I1-SC1, I1-SC2, I2-SC1, I2-SC2, and I3-SC1, as shown in Table 5.2.

The objective values obtained from our DBP procedure are very close to the optimal solutions. The best method, I3-SC1, had an average percentage solution gap of only 0.12%. Thus DBP takes far fewer iterations and much less time than DWD.

Figure 5.1 shows that the number of variables in the final DBP restricted master is typically fewer than half the number of original variables.

6. Conclusion

In this paper, we described our work with relaxation and decomposition techniques for the IFP. We found that both subgradient optimization and Dantzig-Wolfe decomposition were ineffective, under either of two different decompositions. Finally, we applied decomposition-based pricing to IFP. Our decomposition-based pricing procedure was the most effective method by far. We used real data from a commercial fishery, but the work has not been implemented in the fishery operation as yet. Using DBP, we see no impediment to implementation. More importantly, we believe that DBP can be adapted for other integer programs. Future work includes continuing to explore ways to improve the efficiency of DBP for integer programs.

References

[1] B. Mikalsen and T. Vassdal, "A short term production planning model in fish processing," in *Applied Operations Research in Fishing*, K. B. Haley, Ed., pp. 223–233, Plenum Press, New York, NY, USA, 1981.

[2] P. Jensson, "Daily production planning in fish processing firms," *European Journal of Operational Research*, vol. 36, no. 3, pp. 410–415, 1988.

[3] E. A. Gunn, H. H. Millar, and S. M. Newbold, "Planning harvesting and marketing activities for integrated fishing firms under an enterprise allocation scheme," *European Journal of Operational Research*, vol. 55, no. 2, pp. 243–259, 1991.

[4] M. B. Hasan and J. F. Raffensperger, "A mixed integer linear program for an integrated fishery," *ORiON*, vol. 22, no. 1, pp. 19–34, 2006.

[5] A. M. Geoffrion, "Lagrangean relaxation for integer programming," *Mathematical Programming Study*, no. 2, pp. 82–114, 1974.

[6] M. L. Fisher, "The Lagrangian relaxation method for solving integer programming problems," *Management Science*, vol. 27, no. 1, pp. 1–18, 1981.

[7] M. Held, P. Wolfe, and H. P. Crowder, "Validation of subgradient optimization," *Mathematical Programming*, vol. 6, no. 1, pp. 62–88, 1974.

[8] M. Held and R. M. Karp, "The traveling-salesman problem and minimum spanning trees," *Operations Research*, vol. 18, no. 6, pp. 1138–1162, 1970.

[9] F. Shepardson and R. E. Marsten, "A Lagrangean relaxation algorithm for the two duty period scheduling problem," *Management Science*, vol. 26, no. 3, pp. 274–281, 1980.

[10] M. Guignard and S. Kim, "Lagrangean decomposition: a model yielding stronger Lagrangean bounds," *Mathematical Programming*, vol. 39, no. 2, pp. 215–228, 1987.

[11] G. B. Dantzig, *Linear Programming and Extensions*, Princeton University Press, Princeton, NJ, USA, 1963.

[12] R. Fourer, D. M. Gay, and B. W. Kernighan, *AMPL: A Modelling Language for Mathematical Programming*, Curt Hinrichs, Pacific Grove, Calif, USA, 1993.

[13] J. W. Mamer and R. D. McBride, "A decomposition-based pricing procedure for large-scale linear programs: an application to the linear multicommodity flow problem," *Management Science*, vol. 46, no. 5, pp. 693–709, 2000.

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