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Research Article

Imperfect Production System under Reverse Logistics in Stock-Out Situation: EPQ Model

Himani Dem[1] and Leena Prasher[2]

[1] *Department of Science & Humanities, SRM University, Delhi-NCR Campus, Ghaziabad, Uttar Pradesh 201204, India*
[2] *Centre for Mathematical Sciences, Banasthali University, Banasthali, Rajasthan 304022, India*

Correspondence should be addressed to Himani Dem; himanidem@gmail.com

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This paper derives a reverse logistic inventory model with imperfect production, stock-dependent demand, flexible manufacturing, and shortages over infinite planning horizon. The objective is to determine the joint policy for optimal production, amount of remanufacturing, collection of reusable items, and collection as well as disposal of defective items which minimizes the total cost of the inventory system under consideration. To make the model more realistic, both of the cases of linear and nonlinear holding costs have been discussed. The results are discussed with a numerical example to illustrate the theory.

1. Introduction

The assumption of perfect production is not ideal for practical production system. Even the best production system may produce defective items. The governmental guidelines clearly state reduction of wastages, conservation of precious resources, protection of environment, and prevention of environmental degradation as a guiding principles for the business organizations. The manufacturing organizations may reuse the defective items after suitably repairing and removal of defects in order to avoid waste of resources. The defective items which are either irreparable or cannot be repaired easily and cost effectively are disposed off. They also prefer to reuse or recycle the items procured from the customers and reconvert through the appropriate process to appear as new and useful.

2. Literature Review

The classical production inventory model assumes that all the items produced are of perfect quality. Such an assumption appears impractical in real system. Therefore researchers have attracted towards model formation in which some parts of the items produced are of imperfect quality and they can be reworked and repaired. Rosenblatt and Lee [1], Lee and Rosenblatt [2], Cheng [3], Das and Sarkar [4], and Chung and Hou [5] worked on the issue of imperfect quality items and proved that production inventory cost is affected by rework or repair. Cheng [3] developed an economic order quantity model with demand-dependent unit production cost and imperfect production processes. Hayek and Salameh [6] assumed that all of the defective items produced are repairable and derived an optimal operating policy for EPQ model under the effect of reworking of imperfect quality items. Chung and Hou [5] investigated the production inventory model with imperfect production processes and allowable shortages. Chiu et al. [7] derived an economic production quantity (EPQ) model with scrap, rework, and stochastic machine breakdowns and assumed some portion of the defective items to be scrapped and the other parts to be repairable. S. R. Singh and C. Singh [8] developed an imperfect production process with exponential demand rate and Weibull distribution deterioration under inflation.

For the past few decades, reverse logistics have been receiving much attention. The problem of optimal lot sizes for production/procurement and recovery was first studied by Schrady [9]. For issues in the greening process, Nahmias and Rivera [10] studied an EPQ variant of Schrady's model [9] with a finite recovery rate. Richter [11, 12] and Richter and Dobos [13] investigated a waste disposal model considering the returned rate as a decision variable. They gave the optimal number of production and reproduction batches depended on the returned rate. Dobos and Richter [14, 15] investigated a production/remanufacturing system with constant demand that is satisfied by noninstantaneous production and remanufacturing for single and multiple remanufacturing and production cycle. Dobos and Richter [16] extended their previous model and assumed that the quality of collected returned items is not always suitable for further repairing. El Saadany and Jaber [17] extended the models developed in Dobos and Richter [14, 15] by assuming that the collection rate of returned items is dependent on the purchasing price and the acceptance quality level of these returns. That is, the flow of used/returned items increases as the purchasing price increases and decreases as the corresponding acceptance quality level increases. A general reverse logistics inventory model was developed by Alamri [18]. Chung and Wee [19] developed an inventory model on short life-cycle deteriorating product remanufacturing in a green supply chain model. Singh and Saxena [20] derived an optimal returned policy for a reverse logistics inventory model with backorders.

An increase in the shelf space can influence more customers. In this connection, the observations made by Levin et al. [21] and Silver and Peterson [22] may be noted. They observed that the presence of greater quantity of the same item tends to attract more customers. The reason behind this fact is a typical psychology of the customers. They may have the feeling of obtaining a wide range for selection when a large amount is stored/displayed. Gupta and Vrat [23] developed models for stock-dependent consumption rate. Mandal and Phaujdar [24] developed an inventory model for deteriorating items and stock-dependent consumption rate. Schweitzer and Seidmann [25] established optimizing processing rate for flexible manufacturing systems. Giri and Chaudhuri [26] developed deterministic model of perishable inventory with stock-dependent demand rate and nonlinear holding cost and proved that the nonlinear holding cost affected the total average cost. Sana et al. [27] established a production-inventory model for a deteriorating item with trended demand and shortages. Teng and Chang [28] proposed economic production model for deteriorating item with price and stock-dependent demand. Singh and Jain [29] worked on reserve money for an EOQ model in an inflationary environment under supplier credits. S. R. Singh and C. Singh [8] worked on supply chain model with stochastic lead time under imprecise partially backlogging for expiring items. Singh et al. [30] contributed on an inventory model for deteriorating items with shortages and stock-dependent demand under inflation for two shops under one management. Konstantaras and Skouri [31] presented a model by considering a general cycle pattern in which a variable number of reproduction lots of equal size were followed by a variable number of manufacturing lots of equal size. They also studied the case where shortages were allowed in each manufacturing and reproduction cycle and similar sufficient conditions, as the nonshortages case, were given. Yadav et al. [32] developed an inventory model of deteriorating items with stock-dependent demand using genetic algorithm in fuzzy environment. Dem and Singh [33] investigated an EPQ model for damageable items with multivariate demand and flexible manufacturing. Dem and Singh [34] developed a production model for imperfect production process under volume flexibility. Goyal et al. [35] explored an inventory system with variable demand as well as production under partially backordered shortages.

3. Assumptions and Notations

The following assumptions and notations are used throughout the model.

3.1. Assumptions

(1) Production rate is linear function of demand and demand is a nonlinear function of on-hand inventory. Thus production is a nonlinear function of on-hand inventory.

(2) Demand rate remains stock-dependent for a certain period after which a uniform demand rate follows as the stock comes down to zero level. The functional relationship between the demand rate $f(q)$ and the inventory level $q(t)$ is given by the following expression:

$$f(q) = Dq^{\beta}, \quad D > 0,\ 0 < \beta < 1,\ q \geq 0, \tag{1}$$

where β denotes the shape parameter and is a measure of responsiveness of the demand to changes in the level of on hand inventory and D denotes the scale parameter.

(3) The repair work of defective items starts when the regular production work stops. After repair, they are as good as the new ones.

(4) Deterioration rate is constant.

(5) Items are returnable and are remanufactured. Remanufactured items are as good as new ones and they are used during the shortage period of forward manufacturing.

(6) The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered, and all remaining cycles are identical.

(7) Shortages are allowed and completely backlogged.

(8) The production time interval including repair time during forward production coincides with the collection time interval for reverse manufacturing. (This assumption is not applicable during the period of shortages.)

3.2. Notations

$q(t)$: On hand inventory level during regular production uptime

$f(q)$: Demand rate, $f(q) = Dq^{\beta}$, $D > 0$, $0 < \beta < 1$

P: Production rate during regular production uptime, $P = lf(q)$, where l is a scale parameter, $P > f(q)$, $l > 1$

P_1: Production rate of defective items repaired, $P_1 = l_1 f(q)$, where l_1 is a scale parameter, $l_1 > 1$

K: Setup cost

h_1: Holding cost per unit per unit time during regular production uptime, repair time, and reverse manufacturing

h_2: Holding cost per unit per unit time during the collecting and consuming process of defective items

h_3: Holding cost per unit per unit time during the collecting and consuming process for the reverse manufacturing

$q_c(t_c)$: Inventory level during collecting and consuming process of defective items

$q_{rc}(t_{rc})$: Inventory level during the collecting process of reverse manufacturing

$q_{rm}(t_{rm})$: Inventory level during the remanufacturing process for reverse manufacturing

ξ: Fraction of the production lot size in the interval $[0, t_{p_1}]$ where $0 < \xi < 1$

Q: Maximum inventory level during regular production uptime

Q_c: Maximum inventory level of defective items collected

Q_h: Inventory level when repair work stops

t_p: Time when regular production stops and repair starts. It also represents the time when collection process of defective items stops

t_{p_1}: Time when repair work of defective items stops and remanufacturing starts. At this point of time collection process of reusable items also stops

t_s: Time when remanufacturing stops and also the time when accumulated inventory of forward manufacturing vanishes

t_{s_1}: Time when accumulated remanufactured inventory vanishes and shortages start

t_m: Time when production starts again during the period of shortage. At this time collection of defective items start again

t_{m_1}: Time when collection process of defective items stops and their repair work starts during the period of shortage

T: Length of a complete cycle

C_p: Production cost per unit of regular good quality items produced

C_r: Repair cost per unit of defective items repaired

C_s: Shortage cost per unit per unit time

s_r: Scrap cost per unit

S': Maximum shortages

S_1: Maximum inventory level of remanufactured items

S_c: Collection of defective items during period of shortage

S_h: Shortage level when repair work starts

R_c: Rate of collection of reusable items

R_m: Rate of remanufacturing of reusable items

θ_1: Rate of deterioration

HC: Total holding cost

SC: Total shortage cost

SR: Total scrap cost

RC: Total repair cost

PC: Total production cost.

4. Formulation of Model

This model consists of two systems: forward manufacturing and reverse manufacturing. The manufacturing process is flexible but imperfect, and hence it can produce as per the demand rate, but it produces perfect as well as imperfect items. At the beginning of each cycle, the inventory is zero. The production starts at the very beginning of the cycle and system produces perfect as well as imperfect items. As production progresses, the inventory of perfect items piles up even after meeting the market demand and imperfect items are collected in a separate store. During the regular production uptime, x portion of produced items is assumed to be defective and is generated at a production rate $d = Px$ where the regular production rate P is a function of demand. Among these defective items, θ portion is considered to be scrap and the other portions can be reworked and repaired. The rework or repair work starts when the regular production work stops. Along with the two stores of perfect quality items and imperfect quality items from the forward production system, there is one more store in which reusable items are collected from the customers, and therefore at the beginning of each cycle, the process of collecting returnable items in a separate store also begins and is continued till the forward production process stops. More precisely, at a point where the repair work during forward manufacturing system stops; the collection process of returnable items also stops at the same point (for simplicity, we assume that there is no collection of used items, once the remanufacturing of collected items starts). At this very point, the remanufacturing of reusable items begin at a constant rate. The accumulated inventory produced from the advanced manufacturing system in the meanwhile starts getting consumed and ultimately becomes nil. The accumulated inventory of remanufacturing products (which are assumed to be as good as the newly produced products) is consumed when the shortages from the forward manufacturing system begin to surface. Also at this stage, there is no production and inventory of remanufactured items is consumed till it becomes nil. When the inventory of remanufactured items is also nil, inventory shortages begin to accumulate for some time. Thereafter, production starts and collection of defective items also starts. After certain time, repair work starts and after clearing the shortages, the cycle

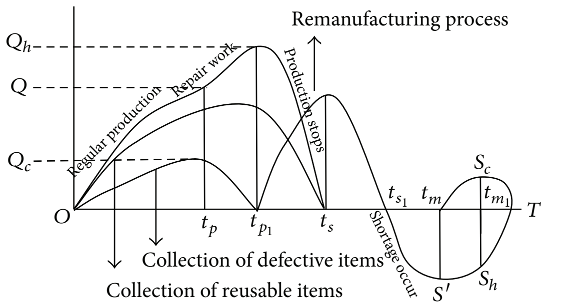

Figure 1: Production inventory model with imperfect production and reverse manufacturing.

ends with zero inventory. Graphical description is shown in Figure 1.

As the production system is not perfect, the items produced during regular production are a mixture of perfect and imperfect quality. The production cycle begins with zero inventory and starts at $t = 0$. In the interval $(0, t_p)$, the inventory of perfect quality items piles up after meeting demand and removing imperfect items and imperfect quality items are collected in a separate store as shown by (2) and (5). Feasibility of this assumption implies that $l - 1 - lx$ must be greater than zero. Regular production is stopped at $t = t_p$, and at this very point repair work starts. The repairable portion of imperfect items collected in the interval $(0, t_p)$ is repaired at a repair rate $P_1 = l_1 D(q)$ during the interval (t_p, t_{p_1}). After repairing, these items are assumed to be as good to serve the demand as the perfect quality items and contribute in rising inventory of perfect quality items after meeting demand. As the inventory of perfect quality items rises, the inventory of defective items falls as shown by (3) and (6). The forward production process stops at $t = t_{p_1}$. The inventory of perfect quality items falls to zero level at $t = t_s$.

4.1. Forward Manufacturing Process. The following equations depict the inventory level of perfect quality items in various intervals:

$$\frac{dq}{dt} = P - Dq^{\beta} - Px, \quad q(0) = 0, \ 0 \le t \le t_p, \tag{2}$$

$$\frac{dq}{dt} = P_1 - Dq^{\beta}, \quad q(t_p) = Q, \ t_p \le t \le t_{p_1}, \tag{3}$$

$$\frac{dq}{dt} = -Dq^{\beta}, \quad q(t_s) = 0, \ t_{p_1} \le t \le t_s. \tag{4}$$

The following equations depict the inventory level of imperfect quality items (collection and consumption process):

$$\frac{dq_c}{dt_c} = Px(1-\theta), \quad q_c(0) = 0, \ 0 \le t_c \le t_p, \tag{5}$$

$$\frac{dq_c}{dt_c} = -P_1, \quad q_c(t_{p_1}) = 0, \ t_p \le t_c \le t_{p_1}. \tag{6}$$

Along with the two stores of perfect quality items and imperfect quality items from the forward production system, there is one more store in which reusable items are collected from the customers. At the beginning of each cycle, the process of collecting reusable items also starts at a rate of collection R_c. At time $t = t_{p_1}$, the collection process is stopped and remanufacturing of collected items starts at a rate R_m. At time $t = t_s$, the inventory of reusable collected items falls to zero and the inventory of remanufactured items rises to S_1. Also at this point, there is no inventory to serve the demand from the forward production system, so that the remanufactured items are used to serve the demand till its falling to zero.

4.2. Reverse Collection and Consumption Process. One has

$$\begin{aligned} \frac{dq_{rc}}{dt_{rc}} &= R_c - \theta_1 q_{rc}, \quad q_{rc}(0) = 0, \ 0 \le t_{rc} \le t_{p_1}, \\ \frac{dq_{rc}}{dt_{rc}} &= -R_m - \theta_1 q_{rc}, \quad q_{rc}(t_{p_1}) = B\xi, \ t_{p_1} \le t_{rc} \le t_s, \end{aligned} \tag{7}$$

where $B = (l/(l-1-lx))Q + (l_1/(l_1-1))(Q_h - Q)$ is the production lot size in the interval $[0, t_{p_1}]$ (see Appendix A).

4.3. Reverse Manufacturing Process.

$$\begin{aligned} \frac{dq_{rm}}{dt_{rm}} &= R_m - \theta_1 q_{rm}, \quad q_{rm}(t_{p_1}) = 0, \ t_{p_1} \le t_{rm} \le t_s, \\ \frac{dq}{dt} &= -Dq^{\beta} - \theta_1 q, \quad q(t_s) = S_1, \ t_s \le t \le t_{s_1}. \end{aligned} \tag{8}$$

4.4. Shortage Phase. At time $t = t_{s_1}$, the inventory of remanufactured items is also nil and shortages accumulate in the interval (t_{s_1}, t_m) and rise to S' at $t = t_m$. At time $t = t_m$ production restarts and shortages start clearing gradually. As the production starts, imperfect quality items also start to accumulate. The imperfect quality items collected in the interval (t_m, t_{m_1}) are repaired and used to clear the demand in the interval (t_{m_1}, T).

The following are the equations depicting the behavior of inventory during shortages as described above:

$$\begin{aligned} \frac{dq}{dt} &= -D_1, \quad q(t_{s_1}) = 0, \ t_{s_1} \le t \le t_m, \\ \frac{dq}{dt} &= (l-1-lx) D_1, \quad q(t_m) = S', \ t_m \le t \le t_{m_1}, \\ \frac{dq}{dt} &= (l_1 - 1) D_1, \quad q(t_{m_1}) = S_h, \ t_{m_1} \le t \le T. \end{aligned} \tag{9}$$

The following are the equations depicting the collection and consumption of imperfect quality items during shortage period:

$$\begin{aligned} \frac{dq_c}{dt_c} &= Px(1-\theta), \quad q_c(t_m) = 0, \ t_m \le t_c \le t_{m_1}, \\ \frac{dq_c}{dt_c} &= -P_1, \quad q_c(T) = 0, \ t_{m_1} \le t_c \le T. \end{aligned} \tag{10}$$

Solving (2), (3), and (4) and using the associated boundary conditions, we get the inventory level of perfect quality items at various stages during forward manufacturing:

$$q^{\alpha} = (l - 1 - lx) D\alpha t, \quad 0 \le t \le t_p, \tag{11}$$

$$q^{\alpha} = Q^{\alpha} + (l_1 - 1) D\alpha (t - t_p), \quad t_p \le t \le t_{p_1}, \tag{12}$$

$$q^{\alpha} = -\alpha D (t - t_s), \quad t_{p_1} \le t \le t_s. \tag{13}$$

Using $q(t_p) = Q$, $q(t_{p_1}) = Q_h$, and $q(t_{p_1}) = Q_h$ in (11), (12), and (13), respectively, we get the following relations:

$$t_p = \frac{Q^{\alpha}}{(l - 1 - lx) D\alpha}, \qquad t_{p_1} = t_p + \frac{Q_h^{\alpha} - Q^{\alpha}}{(l_1 - 1) D\alpha},$$
$$t_s = \frac{1}{\alpha D}\left(Q_h^{\alpha} + \alpha D t_{p_1}\right). \tag{14}$$

For feasibility of the model t_{p_1} must be greater than t_p; otherwise the remanufacturing process will start during the regular production time. Therefore $t_{p_1} > t_p$, give rise to the constraint $(Q_h^{\alpha} - Q^{\alpha})/(l_1 - 1)D\alpha > 0$ must be satisfied which holds true (see Appendix B). The above constraint also implies $Q_h^{\alpha} > Q^{\alpha}$ with means that inventory level at t_{p_1} is higher than the inventory level at t_p which is shown in Figure 1. Also, $t_s > t_{p_1}$ as the interval (t_{p_1}, t_s) represents the interval of remanufacturing which implies that $Q_h^{\alpha}/\alpha D > 0$ and which holds true.

Consider

$$\begin{aligned}
&\text{Holding cost in } [0, t_s] \\
&\quad = \text{holding cost in } [0, t_p] + \text{holding cost in } [t_p, t_{p_1}] \\
&\qquad + \text{holding cost in } [t_{p_1}, t_s] \\
&\quad = h_1 \left(\int_O^Q q^n dt + \int_Q^{Q_h} q^n dt + \int_{Q_h}^0 q^n dt \right) \\
&\quad = h_1 \left[\frac{Q^{n+\alpha}}{(l - 1 - lx) D (n + \alpha)} + \frac{Q_h^{n+\alpha} l_1}{(l_1 - 1) D (n + \alpha)} \right. \\
&\qquad \left. - \frac{Q^{n+\alpha}}{(l_1 - 1) D (n + \alpha)} \right].
\end{aligned} \tag{15}$$

Solving (5) and (6) and using boundary conditions, we get inventory level of imperfect quality items during forward manufacturing process:

$$q_c^{\alpha} = lx (1 - \theta) D\alpha t_c, \quad 0 \le t_c \le t_p,$$
$$q_c^{\alpha} = -D\alpha (t_c - t_{p_1}) l_1, \quad t_p \le t_c \le t_{p_1}. \tag{16}$$

Using $q_c(t_p) = Q_c$ in (16) and also using the values of t_p and t_{p_1} (already derived), we have

$$Q_c^{\alpha} = \frac{lx (1 - \theta)}{l - 1 - lx} Q^{\alpha}, \qquad Q_h^{\alpha} - Q^{\alpha} = (l_1 - 1) D\alpha \left(\frac{Q_c^{\alpha}}{l_1 D\alpha} \right). \tag{17}$$

As $l - 1 - lx > 0$, $l_1 - 1 > 0 \Rightarrow Q_c^{\alpha} \ge 0$ and $Q_h^{\alpha} > Q^{\alpha}$ which shows the existence of inventory levels at various stages.

Consider

$$\begin{aligned}
&\text{Holding cost of imperfect quality items during interval} \\
&[0, t_{p_1}] = \text{holding cost in } [0, t_p] \\
&\qquad + \text{holding cost in } [t_p, t_{p_1}] \\
&\qquad = h_2 \left[\frac{Q_c^{n+\alpha}}{lx (1 - \theta) D (n + \alpha)} + \frac{Q_c^{n+\alpha}}{l_1 D (n + \alpha)} \right].
\end{aligned} \tag{18}$$

Solving (7) and using boundary conditions, the inventory level of reusable items in various intervals is calculated as follows:

$$q_{rc} = \frac{R_c}{\theta_1}\left(1 - e^{-\theta_1 t_{rc}}\right), \quad 0 \le t_{rc} \le t_{p_1}, \tag{19}$$

$$q_{rc} = \frac{-R_m}{\theta_1} + \left(B\xi + \frac{R_m}{\theta_1}\right) e^{\theta_1 (t_{p_1} - t_{rc})}, \quad t_{p_1} \le t_{rc} \le t_s. \tag{20}$$

Using $q_{rc}(t_{p_1}) = B\xi$ in (19), we have $B\xi = R_c(t_{p_1} - \theta_1 t_{p_1}^2/2)$ which can be used to find the rate of collection of reusable items.

Using $q_{rc}(t_s) = 0$, in (20), we have

$$\frac{B\xi}{R_m} - \frac{B^2 \xi^2 \theta_1}{2R_m^2} = (t_s - t_{p_1}). \tag{21}$$

The above relation gives the rate of remanufacturing.

Consider

$$\begin{aligned}
&\text{Holding cost of reverse collection in interval } [0, t_s] \\
&\quad = \text{holding cost in } [0, t_{p_1}] + \text{holding cost in } [t_{p_1}, t_s] \\
&\quad = h_2 \left[\int_0^{B\xi} q_{rc}^n dt + \int_{t_{p_1}}^{t_s} q_{rc}^n dt_{rc} \right] \\
&\quad = h_2 \left[\frac{(B\xi)^{n+1}}{R_c (n + 1)} + \frac{(B\xi)^{n+2} \theta_1}{R_c^2 (n + 2)} + \frac{(B\xi)^{n+1}}{R_m (n + 1)} \right. \\
&\qquad \left. - \frac{(B\xi)^{n+2} \theta_1}{(n + 2) R_m^2} \right], \\
&\text{Deterioration cost} \\
&\quad = \theta_1 C_p \left[\frac{(B\xi)^{n+1}}{R_c (n + 1)} + \frac{(B\xi)^{n+2} \theta_1}{R_c^2 (n + 2)} + \frac{(B\xi)^{n+1}}{R_m (n + 1)} \right. \\
&\qquad \left. - \frac{(B\xi)^{n+2} \theta_1}{(n + 2) R_m^2} \right].
\end{aligned} \tag{22}$$

For reverse manufacturing process, after solving (8) and using boundary conditions, the inventory level of remanufactured items can be obtained as

$$q_{rm} = \frac{R_m}{\theta_1}\left(1 - e^{\theta_1(t_{p_1} - t_{rm})}\right), \quad t_{p_1} \le t_{rm} \le t_s, \qquad (23)$$

$$q^{\alpha} = \frac{-D}{\theta_1} + \left(S_1^{\alpha} + \frac{D}{\theta_1}\right) e^{\theta_1 \alpha (t_s - t)}, \quad t_s \le t \le t_{s_1}, \qquad (24)$$

Holding cost in $\left[t_{p_1}, t_{s_1}\right]$

$$\begin{aligned}
&= \text{holding cost in } \left[t_{p_1}, t_s\right] + \text{holding cost in } \left[t_s, t_{s_1}\right] \\
&= h_1 \left[\int_0^{S_1} q_{rm}^n \left(\frac{1}{R_m} + \frac{\theta_1 q_{rm}}{R_m^2}\right) dq_{rm} + \int_{S_1}^{0} q^n \left(\frac{-q^{\alpha-1}}{D} + \frac{\theta_1 q^{2\alpha-1}}{D^2}\right) dq\right] \\
&= h_1 \left[\frac{S_1^{n+1}}{(n+1) R_m} + \frac{\theta_1 S_1^{(n+2)}}{R_m^2 (n+2)} + \left(\frac{S_1^{\alpha+n}}{D(\alpha+n)} - \frac{\theta_1 S_1^{n+2\alpha}}{D^2 (n+2\alpha)}\right)\right],
\end{aligned}$$

Deterioration cost

$$= \theta_1 C_p \left[\frac{S_1^{n+1}}{(n+1) R_m} + \frac{\theta_1 S_1^{(n+2)}}{R_m^2 (n+2)} + \left(\frac{S_1^{\alpha+n}}{D(\alpha+n)} - \frac{\theta_1 S_1^{n+2\alpha}}{D^2 (n+2\alpha)}\right)\right]. \qquad (25)$$

Using $q(t_{s_1}) = 0$ in (24), we have $t_{s_1} = t_s + S_1^{\alpha}/D\alpha - \theta_1 S_1^{2\alpha}/2D^2\alpha$, where $t_s = Q^{\alpha} l(1 - x\theta)/\alpha D(l - 1 - lx)$.

Solving (9), we have the shortage level in various intervals as given as follows:

$$q = D_1 \left(t_{s_1} - t\right), \quad t_{s_1} \le t \le t_m, \qquad (26)$$

$$q = (l - 1 - lx) D_1 (t - t_m) + S', \quad t_m \le t \le t_{m_1}, \qquad (27)$$

$$q = (l_1 - 1) D_1 \left(t - t_{m_1}\right) + S_h', \quad t_{m_1} \le t \le T. \qquad (28)$$

Using $q(t_m) = S'$, $q(t_{m_1}) = S_h$, and $q(T) = 0$ in (26), (27), and (28), respectively, we have the following relations:

$$t_m = t_{s_1} - \frac{S'}{D_1}, \qquad t_{m_1} = t_m + \frac{S_h - S'}{(l - 1 - lx) D_1}, \qquad \frac{-S_h}{(l_1 - 1) D_1} + t_{m_1} = T. \qquad (29)$$

For feasibility of the practical situation, t_{m_1} must be greater than or equal to t_m as there cannot be any collection of imperfect items before production. Therefore $(S_h - S')/(l - 1 - lx)D_1 \ge 0$ which implies $S_h > S'$ which holds true.

Substituting all the relevant values, the cycle time is given as

$$T = \left[\frac{Q^{\alpha} l (1 - x\theta)}{\alpha D (l - 1 - lx)} + \frac{S_1^{\alpha}}{D\alpha} - \frac{\theta_1 S_1^{2\alpha}}{2D^2\alpha} - \frac{S'}{D_1}\left(\frac{l(1-x)}{l - 1 - lx}\right) + \frac{S_h}{D_1}\left(\frac{l_1 - l + lx}{(l - 1 - lx)(l_1 - 1)}\right)\right], \qquad (30)$$

where $S_h = ((l_1 - 1)lx(1 - \theta)/(lx\theta(1 - l_1) - lx + l_1(l - 1)))S'$.

Consider

Total shortage cost in $\left[t_{s_1}, T\right]$

$$= \text{SC} = C_s \left[\frac{S'^2}{2D_1} - \frac{1}{D_1 (l - 1 - lx)} \times \left(\frac{S_h^2}{2} - \frac{S'^2}{2}\right) + \frac{S_h^2}{2(l_1 - 1) D_1}\right]. \qquad (31)$$

Solving (10), the inventory levels of imperfect quality items during period of shortage are

$$\begin{aligned}
q_c &= lx(1 - \theta) D_1 (t_c - t_m), \qquad t_m \le t_c \le t_{m_1}, \\
q_c &= -l_1 D_1 (t_c - T), \qquad t_{m_1} \le t_c \le T.
\end{aligned} \qquad (32)$$

Using $q_c(t_{m_1}) = S_c$ in (32), we get

$$\frac{S'}{l - 1 - lx} = S_h \left(\frac{1}{l - 1 - lx} + \frac{l_1}{(l_1 - 1) lx (1 - \theta)}\right). \qquad (33)$$

Holding cost of imperfect quality items during period of shortage

$$= h_2 \left[\frac{S_c^{(n+1)}}{lx(1-\theta) D_1 (n+1)} - \frac{S_c^{(n+1)}}{l_1 D_1}\right]. \qquad (34)$$

Total holding cost in complete cycle $[0, T]$ is as follows:

$$\begin{aligned}
\text{HC} = {} & h_1 \left[\frac{Q^{n+\alpha}}{(l - 1 - lx) D (n+\alpha)} + \frac{Q_h^{n+\alpha}}{(l_1 - 1) D (n+\alpha)} - \frac{Q^{n+\alpha}}{(l_1 - 1) D (n+\alpha)}\right] \\
& + h_2 \left[\frac{Q_c^{n+\alpha}}{lx(1-\theta) D (n+\alpha)} + \frac{Q_c^{n+\alpha}}{l_1 D (n+\alpha)}\right]
\end{aligned}$$

$$+ h_3 \left[\frac{(B\xi)^{n+1}}{R_c(n+1)} + \frac{(B\xi)^{n+2}\theta_1}{R_c^2(n+2)} + \frac{(B\xi)^{n+1}}{R_m(n+1)} - \frac{(B\xi)^{n+2}\theta_1}{(n+2)R_m^2} \right]$$
$$+ h_1 \left[\frac{S_1^{n+1}}{(n+1)R_m} + \frac{\theta_1 S_1^{(n+2)}}{R_m^2(n+2)} + \left(\frac{S_1^{\alpha+n}}{D(\alpha+n)} - \frac{\theta_1 S_1^{n+2\alpha}}{D^2(n+2\alpha)} \right) \right]$$
$$+ h_2 \left[\frac{S_c^{(n+1)}}{lx(1-\theta)D_1(n+1)} - \frac{S_c^{(n+1)}}{l_1 D_1} \right]. \tag{35}$$

Production cost is as follows:

$$\begin{aligned} \text{PC} &= C_p \left[\int_0^{t_p} P dt + \int_{t_m}^{t_{m_1}} P dt \right] \\ &= \frac{C_p l}{l-1-lx} \left[Q + \left(S_h - S' \right) \right]. \end{aligned} \tag{36}$$

Repair cost is as follows:

$$\begin{aligned} \text{RC} &= C_r \left[\int_{t_p}^{t_{p_1}} P_1 dt + \int_{t_{m_1}}^{T} P_1 dt \right] \\ &= C_r \left[\frac{l_1(Q_h - Q)}{l_1 - 1} - \frac{l_1 S_h}{l_1 - 1} \right]. \end{aligned} \tag{37}$$

Scrap cost is as follows:

$$\text{SR} = S_r \theta x \left[\frac{lQ}{l-1-lx} + \frac{l\left(S_h - S'\right)}{l-1-lx} \right]. \tag{38}$$

Total deterioration cost is as follows:

$$\text{DC} = \theta_1 C_p \times \left[\frac{(B\xi)^{n+1}}{R_c(n+1)} + \frac{(B\xi)^{n+2}\theta_1}{R_c^2(n+2)} + \frac{(B\xi)^{n+1}}{R_m(n+1)} - \frac{(B\xi)^{n+2}\theta_1}{(n+2)R_m^2} + \frac{S_1^{n+1}}{(n+1)R_m} + \frac{\theta_1 S_1^{(n+2)}}{R_m^2(n+2)} \right]. \tag{39}$$

The total inventory cost per unit time is given by

$$\text{TAC}\left(Q, S'\right) = \frac{K + \text{HC} + \text{DC} + \text{SC} + \text{PC} + \text{SC} + \text{RC}}{T}. \tag{40}$$

Our problem is to find the time to stop the production when q takes optimum value Q and the time to again start the production when maximum shortages accumulate. For minimum value of total cost, we must have

$$\frac{\partial}{\partial Q}(\text{TAC}) = 0, \qquad \frac{\partial}{\partial S'}(\text{TAC}) = 0,$$

$$T \left[\frac{\partial \text{HC}}{\partial S'} + \frac{\partial \text{SC}}{\partial S'} + \frac{\partial \text{PC}}{\partial S'} + \frac{\partial \text{DC}}{\partial S'} + \frac{\partial \text{RC}}{\partial S'} + \frac{\partial \text{SR}}{\partial S'} \right] = (K + \text{HC} + \text{DC} + \text{SC} + \text{PC} + \text{RC} + \text{SR}) \frac{\partial T}{\partial S'}, \tag{C1}$$

$$T \left[\frac{\partial \text{HC}}{\partial Q'} + \frac{\partial \text{SC}}{\partial Q'} + \frac{\partial \text{PC}}{\partial Q'} + \frac{\partial \text{DC}}{\partial Q'} + \frac{\partial \text{RC}}{\partial Q'} + \frac{\partial \text{SR}}{\partial Q'} \right] = (K + \text{HC} + \text{DC} + \text{SC} + \text{PC} + \text{RC} + \text{SR}) \frac{\partial T}{\partial Q'}.$$

Provided the following condition of Hessian matrix, H is satisfied, where

$$H = \begin{bmatrix} \dfrac{\partial^2 \text{TAC}}{\partial Q^2} & \dfrac{\partial^2 \text{TAC}}{\partial Q \partial S} \\ \dfrac{\partial^2 \text{TAC}}{\partial S \partial Q} & \dfrac{\partial^2 \text{TAC}}{\partial S^2} \end{bmatrix}. \tag{41}$$

The first principle minor determinant of H, $|H_{11}| > 0$ and the second principle minor determinant of H, $|H_{22}| > 0$ (see Appendix C).

Using (C1). We get the relation

$$\left(\frac{\partial \text{HC}}{\partial S'} + \frac{\partial \text{SC}}{\partial S'} + \frac{\partial \text{PC}}{\partial S'} + \frac{\partial \text{DC}}{\partial S'} + \frac{\partial \text{RC}}{\partial S'} + \frac{\partial \text{SR}}{\partial S'} \right) \times \left(\frac{\partial \text{HC}}{\partial Q} + \frac{\partial \text{SC}}{\partial Q} + \frac{\partial \text{PC}}{\partial Q} + \frac{\partial \text{DC}}{\partial Q} + \frac{\partial \text{RC}}{\partial Q} + \frac{\partial \text{SR}}{\partial Q} \right)^{-1} = \frac{\partial T / \partial S'}{\partial T / \partial Q}. \tag{C2}$$

4.5. Special Case. If we relax some conditions so as to approach the basic EOQ model with shortages.

As $x \to 0$, $\xi \to 0$, $\theta_1 \to 0$, $\alpha \to 1$, $n \to 1$, and $D = D_1$, we have the following results: DC $= 0$, RC $= 0$, SR $= 0$, PC $= C_p(l/(l-1))(Q - S')$, HC $= h_1 l Q^2 / 2(l-1)D$, SC $= C_s S'^2 l / 2(l-1)D_1$, and $T = Ql/(l-1)D + (l/(1-l)D_1)S'$.

Substituting all the above values in (C1) and (C2) and solving for Q, we have

$$Q^* = \sqrt{\frac{2k(l-1)Dc_S}{lh_1(c_s + h_1)}}. \tag{42}$$

As l increases, production occurs at a more rapid rate. Hence for large l, the model should approach the instantaneous delivery situation of the EOQ model. For large l, $1 - 1/l \to 1$. Thus as l increases towards infinity, the optimal run size for the model approaches the EOQ when shortages are allowed.

Table 1: Effect of ξ on optimal values of Q, T, and TAC when holding cost is linear.

		$\alpha = 1$				
	ξ	0.2	0.4	0.6	0.8	1
	Q	11.103	8.67645	7.12382	6.04315	5.24999
	T	42.9282	42.02219	41.44897	37.75955	36.47535
	HC	90.85782	87.58654	85.42628	83.89338	82.77261
$n = 1$	SC	109.1283	112.4023	114.562	116.0942	117.2156
	PC	40.43753	37.15183	35.05367	33.59508	32.52352
	SR	0.202188	0.185759	0.175268	0.167975	0.162618
	RC	1.950454	1.686772	1.61653	1.586091	1.569654
	TAC	5.650744	5.700709	5.725624	6.243906	6.432465

Table 2: Effect of ξ on optimal values of Q, T, and TAC when holding cost is nonlinear.

		$\alpha = 1$				
	ξ	0.2	0.4	0.6	0.8	1
	Q	3.98022	2.86567	2.06824	1.55932	1.22518
	T	31.06676	30.12545	29.4805	29.07657	28.81484
	HC	50.06126	46.94452	44.62028	43.07698	42.03811
$n = 2$	SC	100.0975	106.1109	110.7606	113.8455	115.9239
	PC	28.74804	27.75043	27.06259	26.63059	26.35012
	SR	0.14374	0.138752	0.135313	0.133153	0.131751
	RC	1.890104	1.645637	1.592055	1.57176	1.561479
	TAC	5.824253	6.060996	6.247208	6.371384	6.455194

5. Numerical Example

In this part, we have presented computational results obtained by using Mathematica 7.0 which give insight about the behavior of optimal run size Q^*, production cycle time T, and the effects of reverse manufacturing on the total average cost TAC. The parametric values in the models are taken as

$$\begin{gathered} D = 2, \quad h_1 = h_2 = h_3 = 0.5, \quad K = 200, \\ l = 2, \quad l_1 = 1.5, \quad C_s = 0.5, \quad \theta = 0.1, \\ C_p = 0.6, \quad C_r = 0.4, \quad \theta_1 = 0.0001, \\ D_1 = 1.5, \quad x = 0.1, \quad S_r = 0.3. \end{gathered} \tag{43}$$

The following observations are made from Table 1.

Case 1 (when holding cost is linear). Consider the following.

(i) As ξ increases, production cost, scrap cost, and repair cost decrease. For higher values of ξ, the reusable items are produced more that result in less production from the forward manufacturing as some of the demand is adjusted by the reusable items. Consequently, less production leads to less production cost. The lower production also leads to lower defectives and hence decrease in scrap cost and repair cost.

(ii) As ξ increases, holding cost decreases and the cycle length also decreases. It is reasonable that lower holding cost and lower cycle length lead to more shortages and hence higher shortage cost. Consequently, there is slight increase in the total average cost.

The following observations are made from Table 2.

Case 2 (when holding cost is nonlinear). In view of the reasons discussed in Case 1, similar types of changes are observed in Case 2 also.

(i) For higher values of ξ, inventory level Q decreases and decrease in inventory level leads to lower inventory cost.

(ii) As ξ increases, inventory level of regular production falls and lower inventory level causes more shortages which results in higher shortage cost. While balancing between the inventory cost and shortage cost, there is slight increase in the total average cost.

(iii) As ξ increases, production cost, scrap cost, and repair cost decrease.

The following are the comparative observations from Tables 1 and 2.

(i) The optimal value of Q is much higher in case of linear holding cost as compared to nonlinear holding cost. Generally, nonlinear holding cost leads to very high inventory cost as compared to linear holding cost. It is natural that higher inventory cost decreases the production rate to reduce the inventory cost. So it is very reasonable to have lower inventory level in case of nonlinear holding cost as compared to inventory level in case of linear holding cost.

(ii) The total average cost incurred is slightly less in case of linear holding cost as compared to nonlinear holding cost. Nonlinear holding cost causes high increase in the inventory cost. To avoid high increase, there is much decrease in the inventory level of Case 2 as compared to Case 1. In spite of the lower inventory level in Case 2 as compared to Case 1, there is slight increase in the total average cost which shows the existence of nonlinear factor.

The following observations are made from Table 3.

Table 3: Effect of α on optimal solution.

	$\alpha = 1 - \beta$	0.2	0.4	0.6	0.8
	Q	3.55002	6.61844	8.76254	10.101
	T	35.8966	36.316	37.6392	39.6777
	HC	24.0557	33.5122	45.9702	64.2293
$n = 1$	SC	98.64717	120.8368	123.672	119.2628
	PC	27.93712	34.95404	38.46209	40.01435
	SR	0.139686	0.17477	0.19231	0.200072
	RC	1.880162	1.740152	1.673579	1.606051
	TAC	4.252766	5.265391	5.578497	5.678569

(i) As β decreases, inventory level Q rises and holding cost also increases. Higher inventory leads to higher inventory cost.

(ii) As β decreases, scrap cost and production cost increase but repair cost slightly decreases.

(iii) As β decreases, total average cost slightly increases and cycle length T also increases.

6. Conclusion

This model addresses the various expected realistic features that usually arise while working on the optimal production policy that minimizes the associated inventory cost. It is very important to take the production system imperfect as no real system can be perfect in production. In view of the highly competitive situations in the real business problems, the production system cannot afford to be inflexible, therefore we have taken the production system flexible also. Along with the issues raised above, the issue of environmental protection by undertaking reverse manufacturing has also been incorporated and optimal production policy is derived which can reduce the inventory cost as much as possible and also can take care of the requirement of environmental protection. When remanufacturing is undertaken along with imperfect production, as the ratio ξ increases, there is slight increase in the total average cost consisting of holding cost, shortage cost, deterioration cost, set-up cost, scrap cost, repair cost, and production cost. The total average cost incurred is slightly more in case of nonlinear holding cost as compared to linear holding cost. Optimal solution is derived for different values of the shape parameter. Further research can be extended to consider more realistic assumptions into the proposed model, for example, multiple production, stochastic nature of demand, machine breakdown, collection of used items during reverse manufacturing period, and so forth.

Appendices

A. Total Production

B is production lot size during forward manufacturing in interval $[0, t_{p_1}]$

$$\int_0^{t_p} P dt + \int_{t_p}^{t_{p_1}} P_1 dt. \tag{A.1}$$

Using (11) and (12)

$$\begin{aligned} &= \int_0^{Q} \frac{l q^{\beta+\alpha-1} dq}{l-1-lx} + \int_Q^{Q_h} \frac{l_1 q^{\alpha+\beta-1} dq}{l_1 - 1} \\ &= \frac{lQ}{l-1-lx} + \frac{l_1 (Q_h - Q)}{l_1 - 1}. \end{aligned} \tag{A.2}$$

B. Comparison between Maximum and after Repair Inventory Levels

Consider

$$Q_h^{\alpha} - Q^{\alpha} = (l_1 - 1) D\alpha \left(\frac{Q_c^{\alpha}}{l_1 D\alpha}\right) = \frac{lx(1-\theta) Q^{\alpha}}{l-1-lx}. \tag{B.1}$$

As $l - 1 - lx > 0$, $l, x > 0$, $0 < \theta < 1 \Rightarrow Q_h^{\alpha} - Q^{\alpha} > 0$.

C. Derivative Analysis

The first principle minor determinant of H, $|H_{11}| > 0$ and the second principle minor determinant of H, $|H_{22}| > 0$.

First we check $\partial^2 \mathrm{TAC}/\partial S^2 > 0$

$$\frac{\partial^2 \mathrm{TAC}}{\partial S^2} = \frac{\partial^2 (\mathrm{TC}/T)}{\partial S^2} = \frac{T\left(\partial^2 \mathrm{TC}/\partial S^2\right) - \mathrm{TC}\left(\partial^2 T/\partial S^2\right)}{T^2}. \tag{C.1}$$

Now $\partial^2 \mathrm{TAC}/\partial S^2 > 0$ if $T(\partial^2 \mathrm{TC}/\partial S^2) - \mathrm{TC}(\partial^2 T/\partial S^2) > 0$.

Using the value of T, we have $\partial^2 T/\partial S^2 = 0$.

Therefore $\partial^2 \mathrm{TAC}/\partial S^2 > 0$ if $T(\partial^2 \mathrm{TC}/\partial S^2) > 0$,

$$\frac{\partial^2 \mathrm{TC}}{\partial S^2} = \frac{h_1 n S_c^{n-1}}{D_1} \left(\frac{\partial S_c}{\partial S}\right)^2 \left(\frac{l_1 - lx(1-\theta)}{l_1 lx(1-\theta)}\right). \tag{C.2}$$

As $S_c > 0$, $l_1 > 1$, $l > 1$, $0 < x(1-\theta) < 1$.

So $l_1 - lx(1-\theta) > 0$.

Hence $\partial^2 \mathrm{TC}/\partial S^2 > 0$.

Similarly we check

$$\frac{\partial^2 \mathrm{TC}}{\partial S^2} > 0 \quad \text{if } T\frac{\partial^2 \mathrm{TC}}{\partial Q^2} - \mathrm{TC}\frac{\partial^2 T}{\partial Q^2} > 0. \tag{C3}$$

Using the value of T, we have $\partial^2 T/\partial Q^2 < 0$ (as $\alpha - 1 < 0$).

Also $\partial^2 \mathrm{TC}/\partial Q^2 > 0$ (as $l - 1 - lx > 0$, $D > 0$, $l_1 - 1 > 0$, $lx(1-\theta) > 0$, $n + \alpha > 1$).

Hence condition (C3) is satisfied.

Now to check $(\partial^2 \mathrm{TAC}/\partial S^2) \cdot (\partial^2 \mathrm{TAC}/\partial Q^2) - (\partial^2 \mathrm{TAC}/\partial Q \partial S)^2 > 0$.

If

$$\begin{aligned} &T^2 \frac{\partial^2 \mathrm{TC}}{\partial S^2}\frac{\partial^2 \mathrm{TC}}{\partial Q^2} - T \cdot \mathrm{TC}\frac{\partial^2 \mathrm{TC}}{\partial S^2}\frac{\partial^2 T}{\partial Q^2} - T \cdot \mathrm{TC}\frac{\partial^2 T}{\partial S^2}\frac{\partial^2 \mathrm{TC}}{\partial Q^2} \\ &\quad + (\mathrm{TC})^2 \frac{\partial^2 T}{\partial S^2}\frac{\partial^2 T}{\partial Q^2} > T^2 \left(\frac{\partial^2 \mathrm{TC}}{\partial Q \partial S}\right)^2. \end{aligned} \tag{C.3}$$

Using

$$\frac{\partial^2 T}{\partial S^2} = 0, \qquad \frac{\partial^2 T}{\partial Q \partial S} = 0, \qquad \frac{\partial^2 \mathrm{TC}}{\partial Q \partial S} = 0. \tag{C.4}$$

If

$$T^2 \frac{\partial^2 \mathrm{TC}}{\partial S^2}\frac{\partial^2 \mathrm{TC}}{\partial Q^2} - T \cdot \mathrm{TC}\frac{\partial^2 \mathrm{TC}}{\partial S^2}\frac{\partial^2 T}{\partial Q^2} > 0. \tag{C.5}$$

If

$$T\frac{\partial^2 \mathrm{TC}}{\partial Q^2} > \mathrm{TC}\frac{\partial^2 T}{\partial Q^2}. \tag{C.6}$$

Which is true as

$$\frac{\partial^2 T}{\partial Q^2} < 0, \qquad \frac{\partial^2 \mathrm{TC}}{\partial Q^2} > 0, \quad T > 0,\ \mathrm{TC} > 0. \tag{C.7}$$

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