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Research Article

MADM Problems with Correlation Coefficient of Trapezoidal Fuzzy Intuitionistic Fuzzy Sets

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This paper discusses on the notion of trapezoidal fuzzy intuitionistic fuzzy sets (TzFIFSs) and some of the arithmetic operations of the same. Correlation coefficient of TzFIFS is proposed based on the membership, nonmembership, and hesitation degrees. The weighted averaging (WA) operator and the weighted geometric (WG) operator are proposed for TzFIFSs. Based on these operators and the correlation coefficient defined for the TzFIFS, new multiattribute decision making (MADM) models are proposed and numerical illustration is given.

1. Introduction

Intuitionistic fuzzy sets (IFSs) proposed by Atanassov [1–3] are a generalization of the concept of fuzzy sets. Atanassov and Gargov [4] expanded the IFSs, using interval value to express membership and nonmembership function of IFSs. Liu and Yuan [5] introduced the concept of fuzzy number IFSs as a further generalization of IFSs. Among the works done in IFSs, Szmidt and Kacprzyk [6–8] can be mentioned. To the best of our knowledge, Burillo et al. [9] proposed the definition of intuitionistic fuzzy number (IFN) and studied the perturbations of IFN and the first properties of the correlation between these numbers. Many researchers have applied the IFS theory to the field of decision making. Recently some researches [5, 10–14] showed great interest in the fuzzy number IFSs and applied it to the field of decision making. Based on the arithmetic aggregation operators, Xu and Yager [15], Xu and Chen [16, 17], and Wang [13] developed some new geometric aggregation operators and intuitionistic fuzzy ordered weighted averaging (IFOWA) operator. Szmidt and Kacprzyk [7] proposed some solution concepts in group decision making with intuitionistic (individual and social) fuzzy preference relations. Szmidt and Kacprzyk [8] investigated the consensus-reaching process in group decision making based on individual intuitionistic fuzzy preference relations. Herrera et al. [18] developed an aggregation process for combining numerical, interval valued, and linguistic information and then proposed different extensions of this process to deal with contexts in which information such as IFSs or multigranular linguistic information can appear. Xu and Yager [15] developed some geometric aggregation operators for MADM problems. Li [19] investigated MADM problems with intuitionistic fuzzy information and constructed several linear programming models to generate optimal weights for attributes.

Multiattribute decision making (MADM) problems are of importance in most kinds of fields such as engineering, economics, and management. It is obvious that much knowledge in the real world is fuzzy rather than precise. Imprecision comes from a variety of sources such as unquantifiable information [20]. In many situations decision makers have imprecise/vague information about alternatives with respect to attributes. It is well known that the conventional decision making analysis using different techniques and tools has been found to be inadequate to handle uncertainty of fuzzy data. To overcome this problem, the concept of fuzzy approach has been used in the evaluation of decision making systems. For a long period of time, efforts have been made in designing various decision making systems suitable for the arising day-to-day problems. MADM problems are widespread in real-life decision making situations and the problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes, both quantitative and qualitative [21]. In order to choose a desirable solution, the decision maker often provides his/her preference information which takes the form of numerical values, such as exact values, interval number values, and fuzzy numbers. However, under many conditions, numerical values are inadequate or insufficient to model real-life decision problems. Indeed, human judgments including preference information may be stated in intuitionistic fuzzy information, especially in trapezoidal fuzzy intuitionistic fuzzy information. Hence, MADM problems under intuitionistic fuzzy or trapezoidal fuzzy intuitionistic fuzzy environment are an interesting area of study for researchers in the recent days.

It is well known that the conventional correlation analysis using probabilities and statistics has been found to be inadequate to handle uncertainty of failure data and modeling. The method to measure the correlation between two variables involving fuzziness is a challenge to classical statistical theory. Park et al. [22] proposed the correlation coefficient of interval valued intuitionistic fuzzy sets. Robinson and Amirtharaj [23–26] proposed correlation coefficient for vague sets, interval vague sets, triangular and trapezoidal IFSs, and a revised correlation coefficient for triangular and trapezoidal IFSs using graded mean integration representation. In this paper, a novel method of correlation coefficient of trapezoidal fuzzy intuitionistic fuzzy sets (TzFIFSs) is proposed and developed by taking into account the membership, nonmembership, and the hesitation degrees of TzFIFSs. The weighted averaging (WA) and weighted geometric (WG) operators for TzFIFSs are proposed for MADM problems. Based on these operators, an approach is suggested to solve uncertain multiple attribute group decision making problems, where the attribute values are trapezoidal fuzzy intuitionistic fuzzy numbers. A new algorithm is developed to solve the MADM problems in which the correlation coefficient of TzFIFSs is used for ranking alternatives.

2. Arithmetic Operations for TzFIFS

Arithmetic operations of TzFIFNs are based on the arithmetic operations of TzFNs. In TzFIFN the membership and nonmembership degrees take the form of trapezoidal fuzzy number. The basic concepts related to TzFIFNs are presented in the following.

Definition 1 (trapezoidal fuzzy number (TzFN)). $A = (a, b, c, d)$ is called a trapezoidal fuzzy number, if the membership function $\mu_A : R \to [0,1]$ is expressed as

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & \text{for } a \le x < b \\ 1 & \text{for } b \le x < c \\ \dfrac{c-x}{d-c} & \text{for } c \le x < d \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $x \in R$, $0 \le a \le b \le c \le d \le 1$.

Definition 2 (trapezoidal fuzzy intuitionistic fuzzy number (TzFIFN)). Let X be a nonempty set. Then $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$ is called a trapezoidal fuzzy intuitionistic fuzzy number if $\mu_A(x) = (\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \mu_{A_4}(x))$ and $\gamma_A(x) = (\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x), \gamma_{A_4}(x))$ are trapezoidal fuzzy numbers, which can express the membership degree and the nonmembership degree of x in X and fulfill $0 \le \mu_{A_4}(x) + \gamma_{A_4}(x) \le 1$, for all $x \in X$.

An intuitionistic fuzzy number expressed on the basis of trapezoidal fuzzy number is called trapezoidal fuzzy intuitionistic fuzzy number.

$\pi_A(x) = (\pi_{A_1}(x), \pi_{A_2}(x), \pi_{A_3}(x), \pi_{A_4}(x))$ is called the hesitation degree of the given trapezoidal fuzzy intuitionistic fuzzy set. Also we have $0 \le \pi_{A_1}(x), \pi_{A_2}(x), \pi_{A_3}(x), \pi_{A_4}(x) \le 1$, for all $x \in X$.

Suppose

$$\begin{aligned} A &= \left\{\left\langle x, \left[\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \mu_{A_4}(x)\right],\right.\right. \\ &\qquad \left.\left.\left[\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x), \gamma_{A_4}(x)\right]\right\rangle \times (x \in X)^{-1}\right\}, \\ B &= \left\{\left\langle x, \left[\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_3}(x), \mu_{B_4}(x)\right],\right.\right. \\ &\qquad \left.\left.\left[\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x), \gamma_{B_4}(x)\right]\right\rangle \times (x \in X)^{-1}\right\} \end{aligned} \quad (2)$$

are two TzFIFNs; then according to the above operation rules of intuitionistic fuzzy numbers and the operation rules of trapezoidal fuzzy numbers, the operational rules of TzFIFNs are as follows:

$$\begin{aligned} A + B &= \left\{\left\langle\left[\mu_{A_1}(x), \mu_{A_2}(x), \mu_{A_3}(x), \mu_{A_4}(x)\right],\right.\right. \\ &\qquad \left.\left[\gamma_{A_1}(x), \gamma_{A_2}(x), \gamma_{A_3}(x), \gamma_{A_4}(x)\right]\right\rangle \\ &\qquad + \left\langle\left[\mu_{B_1}(x), \mu_{B_2}(x), \mu_{B_4}(x)\right],\right. \\ &\qquad \left.\left.\left[\gamma_{B_1}(x), \gamma_{B_2}(x), \gamma_{B_3}(x), \gamma_{B_4}(x)\right]\right\rangle\right\} \\ &= \left\{\left\langle\left[\mu_{A_1}(x) + \mu_{B_1}(x) - \mu_{A_1}(x) \cdot \mu_{B_1}(x),\right.\right.\right. \\ &\qquad \mu_{A_2}(x) + \mu_{B_2}(x) - \mu_{A_2}(x) \cdot \mu_{B_2}(x), \\ &\qquad \mu_{A_3}(x) + \mu_{B_3}(x) - \mu_{A_3}(x) \cdot \mu_{B_3}(x), \\ &\qquad \left.\mu_{A_4}(x) + \mu_{B_4}(x) - \mu_{A_4}(x) \cdot \mu_{B_4}(x)\right], \\ &\qquad \left[\gamma_{A_1}(x) \cdot \gamma_{B_1}(x), \gamma_{A_2}(x) \cdot \gamma_{B_2}(x),\right. \\ &\qquad \left.\left.\left.\gamma_{A_3}(x) \cdot \gamma_{B_3}(x), \gamma_{A_4}(x) \cdot \gamma_{B_4}(x)\right]\right\rangle\right\}, \end{aligned} \quad (3)$$

$$
\begin{aligned}
A\cdot B &= \left\{\left\langle \left[\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x)\right],\right.\right.\\
&\qquad \left.\left[\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x)\right]\right\rangle\\
&\qquad \cdot\left\langle \left[\mu_{B_1}(x),\mu_{B_2}(x),\ \mu_{B_4}(x)\right],\right.\\
&\qquad\quad \left.\left.\left[\gamma_{B_1}(x),\gamma_{B_2}(x),\gamma_{B_3}(x),\gamma_{B_4}(x)\right]\right\rangle\right\}\\
&= \left\{\left\langle \left[\mu_{A_1}(x)\cdot\mu_{B_1}(x),\mu_{A_2}(x)\cdot\mu_{B_2}(x),\right.\right.\right.\\
&\qquad \left.\mu_{A_3}(x)\cdot\mu_{B_3}(x),\mu_{A_4}(x)\cdot\mu_{B_4}(x)\right],\\
&\qquad \left[\gamma_{A_1}(x)+\gamma_{B_1}(x)-\gamma_{A_1}(x)\cdot\gamma_{B_1}(x),\right.\\
&\qquad\quad \gamma_{A_2}(x)+\gamma_{B_2}(x)-\gamma_{A_2}(x)\cdot\gamma_{B_2}(x),\\
&\qquad\quad \gamma_{A_3}(x)+\gamma_{B_3}(x)-\gamma_{A_3}(x)\cdot\gamma_{B_3}(x),\\
&\qquad\quad \left.\left.\left.\gamma_{A_4}(x)+\gamma_{B_4}(x)-\gamma_{A_4}(x)\cdot\gamma_{B_4}(x)\right]\right\rangle\right\},
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
\lambda A &= \lambda\left(\left[\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x)\right],\right.\\
&\qquad \left.\left[\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x)\right]\right)\\
&= \left\{\left\langle \left[1-\left(1-\mu_{A_1}(x)\right)^{\lambda},1-\left(1-\mu_{A_2}(x)\right)^{\lambda},\right.\right.\right.\\
&\qquad \left.1-\left(1-\mu_{A_3}(x)\right)^{\lambda},1-\left(1-\mu_{A_4}(x)\right)^{\lambda}\right],\\
&\qquad \left[\left(\gamma_{A_1}(x)\right)^{n},\left(\gamma_{A_2}(x)\right)^{n},\right.\\
&\qquad \left.\left.\left.\left(\gamma_{A_3}(x)\right)^{n},\left(\gamma_{A_4}(x)\right)^{n}\right]\right\rangle\right\},
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
A^{\lambda} &= \left(\left[\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x)\right],\right.\\
&\qquad \left.\left[\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x)\right]\right)^{\lambda}\\
&= \left\{\left\langle \left[\left(\mu_{A_1}(x)\right)^{n},\left(\mu_{A_2}(x)\right)^{n},\right.\right.\right.\\
&\qquad \left.\left(\mu_{A_3}(x)\right)^{n},\left(\mu_{A_4}(x)\right)^{n}\right],\\
&\qquad \left[1-\left(1-\gamma_{A_1}(x)\right)^{\lambda},1-\left(1-\gamma_{A_2}(x)\right)^{\lambda},\right.\\
&\qquad \left.\left.\left.1-\left(1-\gamma_{A_3}(x)\right)^{\lambda},1-\left(1-\gamma_{A_4}(x)\right)^{\lambda}\right]\right\rangle\right\},\\
&\qquad\qquad \lambda\ge 0.
\end{aligned}
\tag{6}
$$

For the above operation rules, the following are true:

(i) $A+B=B+A$,

(ii) $A\cdot B=B\cdot A$,

(iii) $\lambda(A+B)=\lambda A+\lambda B$, $\lambda\ge 0$,

(iv) $\lambda_1 A+\lambda_2 A=(\lambda_1+\lambda_2)A$, $\lambda_1,\lambda_2\ge 0$,

(v) $A^{\lambda_1}\cdot A^{\lambda_2}=(A)^{\lambda_1+\lambda_2}$, $\lambda_1,\lambda_2\ge 0$.

3. Correlation Coefficient of TzFIFS

Let $X=\{x_1,x_2,\ldots,x_n\}$ be the finite universal set and let A, $B\in \mathrm{TzFIFS}(X)$ be given by

$$
\begin{aligned}
A &= \left\{\left\langle x,\left[\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x)\right],\right.\right.\\
&\qquad \left.\left.\left[\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x)\right]\right\rangle\times(x\in X)^{-1}\right\},\\
B &= \left\{\left\langle x,\left[\mu_{B_1}(x),\mu_{B_2}(x),\mu_{B_3}(x),\mu_{B_4}(x)\right],\right.\right.\\
&\qquad \left.\left.\left[\gamma_{B_1}(x),\gamma_{B_2}(x),\gamma_{B_3}(x),\gamma_{B_4}(x)\right]\right\rangle\times(x\in X)^{-1}\right\},
\end{aligned}
\tag{7}
$$

which are two trapezoidal fuzzy intuitionistic fuzzy numbers.

Then the correlation of trapezoidal fuzzy intuitionistic fuzzy numbers (TzFIFNs) is defined as follows.

Definition 3. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle/x\in X\}$ be a trapezoidal fuzzy intuitionistic fuzzy number, where $\mu_A(x)=(\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x))$ is the membership degree, $\gamma_A(x)=(\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x))$ is the nonmembership degree, and $\pi_A(x)=(\pi_{A_1}(x),\pi_{A_2}(x),\pi_{A_3}(x),\pi_{A_4}(x))$ is the hesitation degree. Then the trapezoidal fuzzy intuitionistic energy of the sets A and B is defined as

$$
\begin{aligned}
&E_{\mathrm{TzFIFS}}(A)\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\left[\mu_A^2(x_i)+\gamma_A^2(x_i)+\pi_A^2(x_i)\right]\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\left\{\left[\left(\mu_{A_1}^2(x_i)+\mu_{A_2}^2(x_i)+\mu_{A_3}^2(x_i)+\mu_{A_4}^2(x_i)\right)\right]\right.\\
&\qquad\quad+\left[\left(\gamma_{A_1}^2(x_i)+\gamma_{A_2}^2(x_i)+\gamma_{A_3}^2(x_i)+\gamma_{A_4}^2(x_i)\right)\right]\\
&\qquad\quad\left.+\left[\left(\pi_{A_1}^2(x_i)+\pi_{A_2}^2(x_i)+\pi_{A_3}^2(x_i)+\pi_{A_4}^2(x_i)\right)\right]\right\},
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&E_{\mathrm{TzFIFS}}(B)\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\left[\mu_B^2(x_i)+\gamma_B^2(x_i)+\pi_B^2(x_i)\right]\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\left\{\left[\left(\mu_{B_1}^2(x_i)+\mu_{B_2}^2(x_i)+\mu_{B_3}^2(x_i)+\mu_{B_4}^2(x_i)\right)\right]\right.\\
&\qquad\quad+\left[\left(\gamma_{B_1}^2(x_i)+\gamma_{B_2}^2(x_i)+\gamma_{B_3}^2(x_i)+\gamma_{B_4}^2(x_i)\right)\right]\\
&\qquad\quad\left.+\left[\left(\pi_{B_1}^2(x_i)+\pi_{B_2}^2(x_i)+\pi_{B_3}^2(x_i)+\pi_{B_4}^2(x_i)\right)\right]\right\}.
\end{aligned}
\tag{9}
$$

Definition 4. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle/x\in X\}$ be a trapezoidal fuzzy intuitionistic fuzzy number, where $\mu_A(x)=(\mu_{A_1}(x),\mu_{A_2}(x),\mu_{A_3}(x),\mu_{A_4}(x))$ is the membership degree, $\gamma_A(x)=(\gamma_{A_1}(x),\gamma_{A_2}(x),\gamma_{A_3}(x),\gamma_{A_4}(x))$ is the nonmembership degree, and $\pi_A(x)=(\pi_{A_1}(x),\pi_{A_2}(x),\pi_{A_3}(x),\pi_{A_4}(x))$ is the hesitation degree. Then the correlation between the sets A and B is defined as

$$\begin{aligned}
&C_{\text{TzFIFS}}(A,B)\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\left[\mu_A(x_i)\mu_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\pi_A(x_i)\pi_B(x_i)\right]\\
&\quad=\frac{1}{4}\sum_{i=1}^{n}\Big\{\Big[\Big(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)\\
&\qquad\qquad+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\mu_{A_4}(x_i)\mu_{B_4}(x_i)\Big)\Big]\\
&\qquad+\Big[\Big(\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\\
&\qquad\qquad+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)\Big)\Big]\\
&\qquad+\Big[\Big(\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\\
&\qquad\qquad+\pi_{A_3}(x_i)\pi_{B_3}(x_i)+\pi_{A_4}(x_i)\pi_{B_4}(x_i)\Big)\Big]\Big\}.
\end{aligned}\tag{10}$$

The correlation coefficient is given as

$$K_{\text{TzFIFS}}(A,B)=\frac{C_{\text{TzFIFS}}(A,B)}{\sqrt{E_{\text{TzFIFS}}(A)\cdot E_{\text{TzFIFS}}(B)}},\quad 0\le K_{\text{TzFIFS}}(A,B)\le 1.\tag{11}$$

Proposition 5. *For A, $B \in TzFIFS(X)$ the following are true:*

(1) $0 \le K_{TzFIFS}(A,B) \le 1$,

(2) $C_{TzFIFS}(A,B) = C_{TzFIFS}(B,A)$,

(3) $K_{TzFIFS}(A,B) = K_{TzFIFS}(B,A)$.

Theorem 6. *For A, $B \in TzFIFS(X)$, then $0 \le K_{TzFIFS}(A,B) \le 1$.*

Proof. Since $C_{\text{TzFIFS}}(A,B) \ge 0$, it can be proved that $K_{\text{TzFIFS}}(A,B) \le 1$.

For any arbitrary real number ξ, the following relation is true:

$$\begin{aligned}
0&\le\sum_{i=1}^{n}\Big\{\left(\mu_{A_1}(x_i)-\xi\mu_{B_1}(x_i)\right)^2+\left(\mu_{A_2}(x_i)-\xi\mu_{B_2}(x_i)\right)^2\\
&\quad+\left(\mu_{A_3}(x_i)-\xi\mu_{B_3}(x_i)\right)^2+\left(\mu_{A_4}(x_i)-\xi\mu_{B_4}(x_i)\right)^2\\
&\quad+\left(\gamma_{A_1}(x_i)-\xi\gamma_{B_1}(x_i)\right)^2+\left(\gamma_{A_2}(x_i)-\xi\gamma_{B_2}(x_i)\right)^2\\
&\quad+\left(\gamma_{A_3}(x_i)-\xi\gamma_{B_3}(x_i)\right)^2+\left(\gamma_{A_4}(x_i)-\xi\gamma_{B_4}(x_i)\right)^2\\
&\quad+\left(\pi_{A_1}(x_i)-\xi\pi_{B_1}(x_i)\right)^2+\left(\pi_{A_2}(x_i)-\xi\pi_{B_2}(x_i)\right)^2\\
&\quad+\left(\pi_{A_3}(x_i)-\xi\pi_{B_3}(x_i)\right)^2+\left(\pi_{A_4}(x_i)-\xi\pi_{B_4}(x_i)\right)^2\Big\}\\
&=\sum_{i=1}^{n}\Big\{\left(\mu^2_{A_1}(x_i)+\gamma^2_{A_1}(x_i)+\pi^2_{A_1}(x_i)\right)\\
&\quad+\left(\mu^2_{A_2}(x_i)+\gamma^2_{A_2}(x_i)+\pi^2_{A_2}(x_i)\right)\\
&\quad+\left(\mu^2_{A_3}(x_i)+\gamma^2_{A_3}(x_i)+\pi^2_{A_3}(x_i)\right)\\
&\quad+\left(\mu^2_{A_4}(x_i)+\gamma^2_{A_4}(x_i)+\pi^2_{A_4}(x_i)\right)\\
&\quad-2\xi\left(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\pi_{A_1}(x_i)\pi_{B_1}(x_i)\right)\\
&\quad-2\xi\left(\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\right)\\
&\quad-2\xi\left(\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\pi_{A_3}(x_i)\pi_{B_3}(x_i)\right)\\
&\quad-2\xi\left(\mu_{A_4}(x_i)\mu_{B_4}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)+\pi_{A_4}(x_i)\pi_{B_4}(x_i)\right)\\
&\quad+\xi^2\left(\mu^2_{B_1}(x_i)+\gamma^2_{B_1}(x_i)+\pi^2_{B_1}(x_i)\right)\\
&\quad+\xi^2\left(\mu^2_{B_2}(x_i)+\gamma^2_{B_2}(x_i)+\pi^2_{B_2}(x_i)\right)\\
&\quad+\xi^2\left(\mu^2_{B_3}(x_i)+\gamma^2_{B_3}(x_i)+\pi^2_{B_3}(x_i)\right)\\
&\quad+\xi^2\left(\mu^2_{B_4}(x_i)+\gamma^2_{B_4}(x_i)+\pi^2_{B_4}(x_i)\right)\Big\}.
\end{aligned}\tag{12}$$

Hence,

$$\begin{aligned}
&\Big\{\sum_{i=1}^{n}\Big[\Big(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)\\
&\qquad+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\mu_{A_4}(x_i)\mu_{B_4}(x_i)\\
&\qquad+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\\
&\qquad+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)\\
&\qquad+\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\\
&\qquad+\pi_{A_3}(x_i)\pi_{B_3}(x_i)+\pi_{A_4}(x_i)\pi_{B_4}(x_i)\Big)\Big]\Big\}^2\\
&\le\Big(\sum_{i=1}^{n}\Big\{\left(\mu^2_{A_1}(x_i)+\gamma^2_{A_1}(x_i)+\pi^2_{A_1}(x_i)\right)\\
&\qquad+\left(\mu^2_{A_2}(x_i)+\gamma^2_{A_2}(x_i)+\pi^2_{A_2}(x_i)\right)\\
&\qquad+\left(\mu^2_{A_3}(x_i)+\gamma^2_{A_3}(x_i)+\pi^2_{A_3}(x_i)\right)\\
&\qquad+\left(\mu^2_{A_4}(x_i)+\gamma^2_{A_4}(x_i)+\pi^2_{A_4}(x_i)\right)\Big\}
\end{aligned}$$

$$\times \sum_{i=1}^{n} \{ (\mu_{B_1}^2(x_i) + \gamma_{B_1}^2(x_i) + \pi_{B_1}^2(x_i)) + (\mu_{B_2}^2(x_i) + \gamma_{B_2}^2(x_i) + \pi_{B_2}^2(x_i)) + (\mu_{B_3}^2(x_i) + \gamma_{B_3}^2(x_i) + \pi_{B_3}^2(x_i)) \} + (\mu_{B_4}^2(x_i) + \gamma_{B_4}^2(x_i) + \pi_{B_4}^2(x_i)) \Bigg). \tag{13}$$

The above inequality can be written as

$$\begin{aligned}\Bigg(\Bigg\{ \sum_{i=1}^{n} [& \mu_{A_1}(x_i)\mu_{B_1}(x_i) + \mu_{A_2}(x_i)\mu_{B_2}(x_i) \\ & + \mu_{A_3}(x_i)\mu_{B_3}(x_i) + \mu_{A_4}(x_i)\mu_{B_4}(x_i) \\ & + \gamma_{A_1}(x_i)\gamma_{B_1}(x_i) + \gamma_{A_2}(x_i)\gamma_{B_2}(x_i) \\ & + \gamma_{A_3}(x_i)\gamma_{B_3}(x_i) + \gamma_{A_4}(x_i)\gamma_{B_4}(x_i) \\ & + \pi_{A_1}(x_i)\pi_{B_1}(x_i) + \pi_{A_2}(x_i)\pi_{B_2}(x_i) \\ & + \pi_{A_3}(x_i)\pi_{B_3}(x_i) + \pi_{A_4}(x_i)\pi_{B_4}(x_i)]^2 \Bigg\} \\ \times \Bigg(\Bigg(\sum_{i=1}^{n} \{ & (\mu_{A_1}^2(x_i) + \gamma_{A_1}^2(x_i) + \pi_{A_1}^2(x_i)) \\ & + (\mu_{A_2}^2(x_i) + \gamma_{A_2}^2(x_i) + \pi_{A_2}^2(x_i)) \\ & + (\mu_{A_3}^2(x_i) + \gamma_{A_3}^2(x_i) + \pi_{A_3}^2(x_i)) \\ & + (\mu_{A_4}^2(x_i) + \gamma_{A_4}^2(x_i) + \pi_{A_4}^2(x_i)) \} \\ \times \sum_{i=1}^{n} \{ & (\mu_{B_1}^2(x_i) + \gamma_{B_1}^2(x_i) + \pi_{B_1}^2(x_i)) \\ & + (\mu_{B_2}^2(x_i) + \gamma_{B_2}^2(x_i) + \pi_{B_2}^2(x_i)) \\ & + (\mu_{B_3}^2(x_i) + \gamma_{B_3}^2(x_i) + \pi_{B_3}^2(x_i)) \\ & + (\mu_{B_4}^2(x_i) + \gamma_{B_4}^2(x_i) + \pi_{B_4}^2(x_i)) \} \Bigg)\Bigg)^{-1} \Bigg). \\ & \leq 1. \end{aligned} \tag{14}$$

Therefore $[C_{\mathrm{TzFIFS}}(A, B)]^2 / E_{\mathrm{TzFIFS}}(A) \cdot E_{\mathrm{TzFIFS}}(B) \leq 1$; hence $K_{\mathrm{TzFIFS}}(A, B) = C_{\mathrm{TzFIFS}}(A, B) / \sqrt{E_{\mathrm{TzFIFS}}(A) \cdot E_{\mathrm{TzFIFS}}(B)} \leq 1$. □

Theorem 7. *Consider $K_{TzFIFS}(A, B) = 1 \Leftrightarrow A = B$.*

Proof. Considering the inequality in the proof of Theorem 6, then the equality holds if and only if the following are satisfied

(i) $\mu_{A_1}(x_i) = \xi\mu_{B_1}(x_i)$, $\mu_{A_2}(x_i) = \xi\mu_{B_2}(x_i)$, $\mu_{A_3}(x_i) = \xi\mu_{B_3}(x_i)$, $\mu_{A_4}(x_i) = \xi\mu_{B_4}(x_i)$,

(ii) $\gamma_{A_1}(x_i) = \xi\gamma_{B_1}(x_i)$, $\gamma_{A_2}(x_i) = \xi\gamma_{B_2}(x_i)$, $\gamma_{A_3}(x_i) = \xi\gamma_{B_3}(x_i)$, $\gamma_{A_4}(x_i) = \xi\gamma_{B_4}(x_i)$,

(iii) $\pi_{A_1}(x_i) = \xi\pi_{B_1}(x_i)$, $\pi_{A_2}(x_i) = \xi\pi_{B_2}(x_i)$, $\pi_{A_3}(x_i) = \xi\pi_{B_3}(x_i)$, $\pi_{A_4}(x_i) = \xi\pi_{B_4}(x_i)$,

for some positive real ξ.

As

$$\begin{aligned} & \mu_{A_1}(x_i) + \gamma_{A_1}(x_i) + \pi_{A_1}(x_i) \\ & \quad = \mu_{B_1}(x_i) + \gamma_{B_1}(x_i) + \pi_{B_1}(x_i) = 1, \\ & \mu_{A_2}(x_i) + \gamma_{A_2}(x_i) + \pi_{A_2}(x_i) \\ & \quad = \mu_{B_2}(x_i) + \gamma_{B_2}(x_i) + \pi_{B_2}(x_i) = 1, \\ & \mu_{A_3}(x_i) + \gamma_{A_3}(x_i) + \pi_{A_3}(x_i) \\ & \quad = \mu_{B_3}(x_i) + \gamma_{B_3}(x_i) + \pi_{B_3}(x_i) = 1, \\ & \mu_{A_4}(x_i) + \gamma_{A_4}(x_i) + \pi_{A_4}(x_i) \\ & \quad = \mu_{B_4}(x_i) + \gamma_{B_4}(x_i) + \pi_{B_4}(x_i) = 1, \end{aligned} \tag{15}$$

then it means $\xi = 1$, and therefore $A = B$. □

Theorem 8. *$C_{TrFIFS}(A, B) = 0 \Leftrightarrow A$ and B are nonfuzzy sets and satisfy the condition $\mu_A(x_i) + \mu_B(x_i) = 1$ or $\gamma_A(x_i) + \gamma_B(x_i) = 1$ or $\pi_A(x_i) + \pi_B(x_i) = 1$, for all $x_i \in X$.*

Proof. For all $x_i \in X$, the following are true:

$$\begin{aligned} & (\mu_{A_1}(x_i)\mu_{B_1}(x_i) + \gamma_{A_1}(x_i)\gamma_{B_1}(x_i) \\ & \quad + \pi_{A_1}(x_i)\pi_{B_1}(x_i)) \geq 0, \\ & (\mu_{A_2}(x_i)\mu_{B_2}(x_i) + \gamma_{A_2}(x_i)\gamma_{B_2}(x_i) \\ & \quad + \pi_{A_2}(x_i)\pi_{B_2}(x_i)) \geq 0, \\ & (\mu_{A_3}(x_i)\mu_{B_3}(x_i) + \gamma_{A_3}(x_i)\gamma_{B_3}(x_i) \\ & \quad + \pi_{A_3}(x_i)\pi_{B_3}(x_i)) \geq 0, \\ & (\mu_{A_4}(x_i)\mu_{B_4}(x_i) + \gamma_{A_4}(x_i)\gamma_{B_4}(x_i) \\ & \quad + \pi_{A_4}(x_i)\pi_{B_4}(x_i)) \geq 0. \end{aligned} \tag{16}$$

Hence,

$$\begin{aligned}
&\{(\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)\\
&\quad+\pi_{A_1}(x_i)\pi_{B_1}(x_i))\\
&\quad+(\mu_{A_2}(x_i)\mu_{B_2}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\\
&\qquad+\pi_{A_2}(x_i)\pi_{B_2}(x_i))\\
&\quad+(\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)\\
&\qquad+\pi_{A_3}(x_i)\pi_{B_3}(x_i))\\
&\quad+(\mu_{A_4}(x_i)\mu_{B_4}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)\\
&\qquad+\pi_{A_4}(x_i)\pi_{B_4}(x_i))\}\ge 0,\\
&\{\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)\\
&\quad+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\mu_{A_4}(x_i)\mu_{B_4}(x_i)\\
&\quad+\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\\
&\quad+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)\\
&\quad+\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\\
&\quad+\pi_{A_3}(x_i)\pi_{B_3}(x_i)+\pi_{A_4}(x_i)\pi_{B_4}(x_i)\}\ge 0.
\end{aligned}\tag{17}$$

If $C_{\text{TzFIFS}}(A,B)=0$ for all $x_i\in X$, then the following should be true:

$$\begin{aligned}
&\mu_{A_1}(x_i)\mu_{B_1}(x_i)+\mu_{A_2}(x_i)\mu_{B_2}(x_i)\\
&\quad+\mu_{A_3}(x_i)\mu_{B_3}(x_i)+\mu_{A_4}(x_i)\mu_{B_4}(x_i)=0,\\
&\gamma_{A_1}(x_i)\gamma_{B_1}(x_i)+\gamma_{A_2}(x_i)\gamma_{B_2}(x_i)\\
&\quad+\gamma_{A_3}(x_i)\gamma_{B_3}(x_i)+\gamma_{A_4}(x_i)\gamma_{B_4}(x_i)=0,\\
&\pi_{A_1}(x_i)\pi_{B_1}(x_i)+\pi_{A_2}(x_i)\pi_{B_2}(x_i)\\
&\quad+\pi_{A_3}(x_i)\pi_{B_3}(x_i)+\pi_{A_4}(x_i)\pi_{B_4}(x_i)=0.
\end{aligned}\tag{18}$$

(i) If $\mu_{A_1}(x_i)=1$ then $\mu_{B_1}(x_i)=0$ and $\gamma_{A_1}(x_i)=\pi_{A_1}(x_i)=0$,

(ii) if $\mu_{A_2}(x_i)=1$ then $\mu_{B_2}(x_i)=0$ and $\gamma_{A_2}(x_i)=\pi_{A_2}(x_i)=0$,

(iii) if $\mu_{A_3}(x_i)=1$ then $\mu_{B_3}(x_i)=0$ and $\gamma_{A_3}(x_i)=\pi_{A_3}(x_i)=0$,

(iv) if $\mu_{A_4}(x_i)=1$ then $\mu_{B_4}(x_i)=0$ and $\gamma_{A_4}(x_i)=\pi_{A_4}(x_i)=0$.

Also,

(i) if $\mu_{B_1}(x_i)=1$ then $\mu_{A_1}(x_i)=0$ and $\gamma_{B_1}(x_i)=\pi_{B_1}(x_i)=0$,

(ii) if $\mu_{B_2}(x_i)=1$ then $\mu_{A_2}(x_i)=0$ and $\gamma_{B_2}(x_i)=\pi_{B_2}(x_i)=0$,

(iii) if $\mu_{B_3}(x_i)=1$ then $\mu_{A_3}(x_i)=0$ and $\gamma_{B_3}(x_i)=\pi_{B_3}(x_i)=0$,

(iv) if $\mu_{B_4}(x_i)=1$ then $\mu_{A_4}(x_i)=0$ and $\gamma_{B_4}(x_i)=\pi_{B_4}(x_i)=0$.

Hence $\mu_{A_1}(x_i)+\mu_{B_1}(x_i)=1$, $\mu_{A_2}(x_i)+\mu_{B_2}(x_i)=1$, $\mu_{A_3}(x_i)+\mu_{B_3}(x_i)=1$, and $\mu_{A_4}(x_i)+\mu_{B_4}(x_i)=1$.

Conversely, when A and B are nonfuzzy sets and $\mu_{A_1}(x_i)+\mu_{B_1}(x_i)=1$, $\mu_{A_2}(x_i)+\mu_{B_2}(x_i)=1$, $\mu_{A_3}(x_i)+\mu_{B_3}(x_i)=1$, and $\mu_{A_4}(x_i)+\mu_{B_4}(x_i)=1$.

If $\mu_{A_1}(x_i)=1$ then $\mu_{B_1}(x_i)=0$ and $\gamma_{A_1}(x_i)=\pi_{A_1}(x_i)=0$.

This property can be observed similarly for all other entries. Therefore $C_{\text{TrFIFS}}(A,B)=0$.

The cases $\gamma_A(x_i)+\gamma_B(x_i)=1$ and $\pi_A(x_i)+\pi_B(x_i)=1$ can be proved similarly. □

Theorem 9. *$C_{TzFIFS}(A,A)=1\Leftrightarrow A$ is a nonfuzzy set.*

Proof. If A is a nonfuzzy set, then $C_{\text{TzFIFS}}(A,A)=1$ is obvious.

Conversely, it can be proved by the method of contradiction.

Assume A is not a nonfuzzy set.

Then $0\le\mu_A(x_i)<1$, $0\le\gamma_A(x_i)<1$, and $0\le\pi_A(x_i)<1$, for some x_i.

Hence $\mu_A^2(x_i)+\gamma_A^2(x_i)+\pi_A^2(x_i)<1$.

That is,

$$\begin{aligned}
&\mu_{A_1}^2(x_i)+\gamma_{A_1}^2(x_i)+\pi_{A_1}^2(x_i)<1,\\
&\mu_{A_2}^2(x_i)+\gamma_{A_2}^2(x_i)+\pi_{A_2}^2(x_i)<1,\\
&\mu_{A_3}^2(x_i)+\gamma_{A_3}^2(x_i)+\pi_{A_3}^2(x_i)<1,\\
&\mu_{A_4}^2(x_i)+\gamma_{A_4}^2(x_i)+\pi_{A_4}^2(x_i)<1.
\end{aligned}\tag{19}$$

Also,

$$\begin{aligned}
&\{\mu_{A_1}^2(x_i)+\mu_{A_2}^2(x_i)+\mu_{A_3}^2(x_i)+\mu_{A_4}^2(x_i)\\
&\quad+\gamma_{A_1}^2(x_i)+\gamma_{A_2}^2(x_i)+\gamma_{A_3}^2(x_i)+\gamma_{A_4}^2(x_i)\\
&\quad+\pi_{A_1}^2(x_i)+\pi_{A_2}^2(x_i)+\pi_{A_3}^2(x_i)+\pi_{A_4}^2(x_i)\}<1.
\end{aligned}\tag{20}$$

Then

$$\begin{aligned}
C_{\text{TzFIFS}}(A,A)=\frac{1}{3}\sum_{i=1}^{n}&\{\mu_{A_1}^2(x_i)+\mu_{A_2}^2(x_i)+\mu_{A_3}^2(x_i)\\
&+\mu_{A_4}^2(x_i)+\gamma_{A_1}^2(x_i)+\gamma_{A_2}^2(x_i)\\
&+\gamma_{A_3}^2(x_i)+\gamma_{A_4}^2(x_i)\\
&+\pi_{A_1}^2(x_i)+\pi_{A_2}^2(x_i)\\
&+\pi_{A_3}^2(x_i)+\pi_{A_4}^2(x_i)\}<1.
\end{aligned}\tag{21}$$

This is contradictory, and so A is a nonfuzzy set. □

4. WA and WG Operators for TzFIFNs

On the foundation of the definitions discussed by Wang [13], the weighted arithmetic operators for TzFIFNs are given as follows.

Definition 10. Let $\tilde{a}_j = \langle[\mu_{1j}, \mu_{2j}, \mu_{3j}, \mu_{4j}], [\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, \gamma_{4j}]\rangle$, $j = 1, 2, \ldots, n$, be a collection of TzFIFN values. The trapezoidal fuzzy intuitionistic fuzzy weighted averaging (TzFIFWA) operator, TzFIFWA : $Q^n \to Q$, is defined as

$$\begin{aligned}
&\mathrm{TzFIFWA}_\omega(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \\
&= \sum_{j=1}^{n} \omega_j \tilde{a}_j = \left(1 - \prod_{j=1}^{n}(1-\mu_j)^{\omega_j}, \prod_{j=1}^{n}(\gamma_j)^{\omega_j}\right) \\
&= \left(\left[1 - \prod_{j=1}^{n}(1-\mu_{1j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\mu_{2j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\mu_{3j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\mu_{4j})^{\omega_j}\right],\right. \\
&\qquad \left.\left[\prod_{j=1}^{n}(\gamma_{1j})^{\omega_j}, \prod_{j=1}^{n}(\gamma_{2j})^{\omega_j}, \prod_{j=1}^{n}(\gamma_{3j})^{\omega_j}, \prod_{j=1}^{n}(\gamma_{4j})^{\omega_j}\right]\right),
\end{aligned} \tag{22}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_t)^T$ is the weight vector of $\tilde{a}_j = \langle[\mu_{1j}, \mu_{2j}, \mu_{3j}, \mu_{4j}], [\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, \gamma_{4j}]\rangle$, such that $\omega_j > 0$ and $\sum_{j=1}^{n}\omega_j = 1$.

Definition 11. Let $\tilde{a}_j = \langle[\mu_{1j}, \mu_{2j}, \mu_{3j}, \mu_{4j}], [\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, \gamma_{4j}]\rangle$, $j = 1, 2, \ldots, n$, be a collection of TzFIFN values. The trapezoidal fuzzy intuitionistic fuzzy weighted geometric (TzFIFWG) operator, TzFIFWG : $Q^n \to Q$, is defined as

$$\begin{aligned}
&\mathrm{TzFIFWG}_\omega(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \\
&= \sum_{j=1}^{n} \omega_j \tilde{a}_j = \left(\prod_{j=1}^{n}(\mu_j)^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\gamma_j)^{\omega_j}\right) \\
&= \left(\left[\prod_{j=1}^{n}(\mu_{1j})^{\omega_j}, \prod_{j=1}^{n}(\mu_{2j})^{\omega_j}, \prod_{j=1}^{n}(\mu_{3j})^{\omega_j}, \prod_{j=1}^{n}(\mu_{4j})^{\omega_j}\right],\right. \\
&\qquad \left.\left[1 - \prod_{j=1}^{n}(1-\gamma_{1j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\gamma_{2j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\gamma_{3j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\gamma_{4j})^{\omega_j}\right]\right),
\end{aligned} \tag{23}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_t)^T$ is the weight vector of $\tilde{a}_j = \langle[\mu_{1j}, \mu_{2j}, \mu_{3j}, \mu_{4j}], [\gamma_{1j}, \gamma_{2j}, \gamma_{3j}, \gamma_{4j}]\rangle$, such that $\omega_j > 0$ and $\sum_{j=1}^{n}\omega_j = 1$.

5. MADM Algorithm for Trapezoidal Fuzzy IFS

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, and let $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes; $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weighting vector of the attribute G_j, $j = 1, 2, \ldots, n$, where $\omega_j \in [0, 1]$, such that $\sum_{j=1}^{n}\omega_j = 1$. Suppose that

$$\begin{aligned}
\tilde{R}_k &= \left(\tilde{r}_{hij}^{(k)}\right)_{m\times n} = \left(\mu_{hij}^{(k)}, \gamma_{hij}^{(k)}\right)_{m\times n} \\
&= \left(\left[\mu_{1ij}^{(k)}, \mu_{2ij}^{(k)}, \mu_{3ij}^{(k)}, \mu_{4ij}^{(k)}\right],\right. \\
&\qquad \left.\left[\gamma_{1ij}^{(k)}, \gamma_{2ij}^{(k)}, \gamma_{3ij}^{(k)}, \gamma_{4ij}^{(k)}\right]\right)_{m\times n}
\end{aligned} \tag{24}$$

be the trapezoidal fuzzy intuitionistic fuzzy number decision matrix, where $[\mu_{1ij}^{(k)}, \mu_{2ij}^{(k)}, \mu_{3ij}^{(k)}, \mu_{4ij}^{(k)}]$ is the degree of the membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $[\gamma_{1ij}^{(k)}, \gamma_{2ij}^{(k)}, \gamma_{3ij}^{(k)}, \gamma_{4ij}^{(k)}]$ is the degree of nonmembership value for the alternative A_i, where $\mu_{hij}^{(k)}, \gamma_{hij}^{(k)} \subset [0, 1]$ and $\mu_{4ij}^{(k)}, \gamma_{4ij}^{(k)} \leq 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $k = 1, 2, \ldots, t$.

The developed model of MADM is given as follows.

Step 1. Utilize the decision information given in the matrix $\tilde{R}$ and the TzFIFWA or TzFIFWG operator which has the associated weighting vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ and aggregate the given decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m\times n}$ into group integrated attribute values $\tilde{r}_i$:

$$\begin{aligned}
\tilde{r}_i &= \mathrm{TzFIFWA}_\omega(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \\
&= \sum_{j=1}^{n} \omega_j \tilde{a}_j = \left(1 - \prod_{j=1}^{n}(1-\mu_j)^{\omega_j}, \prod_{j=1}^{n}(\gamma_j)^{\omega_j}\right) \\
&= \left(\left[1 - \prod_{j=1}^{n}(1-\mu_{1j})^{\omega_j}, 1 - \prod_{j=1}^{n}(1-\mu_{2j})^{\omega_j},\right.\right.
\end{aligned}$$

$$1-\prod_{j=1}^{n}\left(1-\mu_{3j}\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\mu_{4j}\right)^{\omega_j}\Bigg],$$

$$\left[\prod_{j=1}^{n}\left(\gamma_{1j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\gamma_{2j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\gamma_{3j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\gamma_{4j}\right)^{\omega_j}\right]\Bigg) \quad (25)$$

or

$$\begin{aligned}\tilde{r}_i &= \mathrm{TzFIFWG}_{\omega}\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right)\\ &= \sum_{j=1}^{n}\omega_j\tilde{a}_j = \left(\prod_{j=1}^{n}\left(\mu_j\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\gamma_j\right)^{\omega_j}\right)\\ &= \Bigg(\Bigg[\prod_{j=1}^{n}\left(\mu_{1j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\mu_{2j}\right)^{\omega_j},\\ &\qquad \prod_{j=1}^{n}\left(\mu_{3j}\right)^{\omega_j},\prod_{j=1}^{n}\left(\mu_{4j}\right)^{\omega_j}\Bigg],\\ &\qquad \Bigg[1-\prod_{j=1}^{n}\left(1-\gamma_{1j}\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\gamma_{2j}\right)^{\omega_j},\\ &\qquad 1-\prod_{j=1}^{n}\left(1-\gamma_{3j}\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-\gamma_{4j}\right)^{\omega_j}\Bigg]\Bigg).\end{aligned} \quad (26)$$

Step 2. Utilize the correlation coefficient (8) to (11) of TzFIFNs to derive the closeness between the overall group integrated attribute values $\tilde{r}_i$ and the TzFIFN positive ideal value $\tilde{r}^+$, where $\tilde{r}^+ = \langle[1,1,1,1],[0,0,0,0]\rangle$.

Step 3. Rank alternatives A_i, $i = 1,2,\ldots,m$, and select the best in accordance with the highest closeness obtained from Step 2.

6. Numerical Illustration

A company intends to select one person to take the position of Assistant Manager from four candidates. Five indicators (attributes) must be evaluated. They are shown as follows:

(i) technical skill (G_1),

(ii) professional ability (G_2),

(iii) creative ability (G_3),

(iv) analytical skill (G_4),

(v) leadership ability (G_5).

The weights of the indicators are = $(\omega_1,\omega_2,\ldots,\omega_n)^T$ = $(0.18,0.26,0.15,0.19,0.22)^T$. The individual attributes of each candidate are to be evaluated in order to come to a good decision. The decision matrix $\tilde{R} = (\tilde{r}_{ij})_{m\times n}$ is given as follows:

$$\tilde{R} = \begin{pmatrix} G_1 & G_2 & G_3 & G_4 & G_5 \\ \langle[0.15,0.21,0.32,0.42],[0.19,0.23,0.42,0.51]\rangle & \langle[0.11,0.19,0.23,0.31],[0.13,0.24,0.29,0.35]\rangle & \langle[0.17,0.27,0.37,0.43],[0.11,0.19,0.23,0.30]\rangle & \langle[0.25,0.35,0.45,0.55],[0.15,0.20,0.32,0.41]\rangle & \langle[0.15,0.23,0.34,0.47],[0.25,0.28,0.34,0.45]\rangle \\ \langle[0.21,0.33,0.45,0.49],[0.12,0.19,0.23,0.35]\rangle & \langle[0.13,0.15,0.25,0.34],[0.19,0.21,0.26,0.32]\rangle & \langle[0.21,0.33,0.42,0.51],[0.25,0.32,0.41,0.45]\rangle & \langle[0.21,0.32,0.43,0.56],[0.12,0.23,0.33,0.39]\rangle & \langle[0.18,0.25,0.36,0.43],[0.26,0.37,0.47,0.51]\rangle \\ \langle[0.13,0.19,0.31,0.39],[0.11,0.23,0.27,0.31]\rangle & \langle[0.21,0.23,0.29,0.42],[0.28,0.34,0.41,0.46]\rangle & \langle[0.15,0.20,0.35,0.49],[0.22,0.26,0.35,0.42]\rangle & \langle[0.13,0.15,0.25,0.29],[0.11,0.25,0.34,0.41]\rangle & \langle[0.19,0.23,0.35,0.43],[0.27,0.31,0.41,0.52]\rangle \\ \langle[0.18,0.25,0.32,0.43],[0.17,0.23,0.34,0.41]\rangle & \langle[0.22,0.25,0.39,0.43],[0.33,0.35,0.41,0.45]\rangle & \langle[0.16,0.23,0.36,0.39],[0.27,0.35,0.41,0.46]\rangle & \langle[0.15,0.23,0.41,0.44],[0.21,0.32,0.39,0.46]\rangle & \langle[0.24,0.34,0.45,0.61],[0.15,0.25,0.28,0.30]\rangle \end{pmatrix}. \quad (27)$$

Step 1. Utilize the decision information given in the matrix $\tilde{R}$, the TzFIFWA operator, and the weighting vector of the attributes $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T = (0.18,0.26,0.15,0.19,0.22)^T$ and aggregate the decision matrix $\tilde{R}$. The collective values are as follows:

$$\begin{aligned}\tilde{r}_1 &= \langle[0.1629,0.2470,0.3375,0.4346],\\ &\quad [0.1610,0.2297,0.3159,0.3986]\rangle,\\ \tilde{r}_2 &= \langle[0.1832,0.2673,0.3744,0.4598],\\ &\quad [0.1789,0.2532,0.3245,0.3937]\rangle,\\ \tilde{r}_3 &= \langle[0.1677,0.2036,0.3092,0.4057],\\ &\quad [0.1896,0.2813,0.3584,0.4248]\rangle,\\ \tilde{r}_4 &= \langle[0.1956,0.2642,0.3914,0.4720],\\ &\quad [0.2192,0.2963,0.3729,0.4078]\rangle.\end{aligned} \quad (28)$$

Step 2. Utilize the correlation coefficient of TzFIFNs to derive the closeness between the overall group integrated attribute values $\tilde{r}_i$ and the TzFIFN positive ideal value $\tilde{r}^+$, where

$\tilde{r}^+ = \langle[1,1,1,1],[0,0,0,0]\rangle$. Using (8) to (11), the correlation coefficients are calculated using the following formula:

$$K_{\text{TzFIFS}}(A,B) = \frac{C_{\text{TzFIFS}}(A,B)}{\sqrt{E_{\text{TzFIFS}}(A)\cdot E_{\text{TzFIFS}}(B)}},\qquad 0 \le K_{\text{TzFIFS}}(A,B) \le 1. \tag{29}$$

Hence the calculated values are given as follows:

$$\begin{aligned}
C_{\text{TzFIFS}}(\tilde{r}_1,\tilde{r}^+) &= 0.2955, & K_{\text{TzFIFS}}(\tilde{r}_1,\tilde{r}^+) &= 0.4663,\\
C_{\text{TzFIFS}}(\tilde{r}_2,\tilde{r}^+) &= 0.3212, & K_{\text{TzFIFS}}(\tilde{r}_2,\tilde{r}^+) &= 0.5140,\\
C_{\text{TzFIFS}}(\tilde{r}_3,\tilde{r}^+) &= 0.2715, & K_{\text{TzFIFS}}(\tilde{r}_3,\tilde{r}^+) &= 0.4332,\\
C_{\text{TzFIFS}}(\tilde{r}_4,\tilde{r}^+) &= 0.3308, & K_{\text{TzFIFS}}(\tilde{r}_4,\tilde{r}^+) &= 0.5347.
\end{aligned} \tag{30}$$

Step 3. Rank alternatives A_i, $i = 1,2,\ldots,m$, from the highest closeness value (correlation coefficient) obtained from Step 2; the result is obtained as follows:

$$A_4 > A_2 > A_1 > A_3. \tag{31}$$

Hence the best alternative is A_4.

Following the MADM model using the TzFIFWA operator, we have the following numerical results for the MADM model using TzFIFWG operator.

Step 1. Utilize the decision information given in the matrix $\tilde{R}_k$, the TzFIFWG operator, and the weighting vector of the attributes $\omega = (\omega_1,\omega_2,\ldots,\omega_n)^T = (0.18, 0.26, 0.15, 0.19, 0.22)^T$ and aggregate the same decision matrix $\tilde{R}$. The collective values are as follows:

$$\begin{aligned}
\tilde{r}_1 &= \langle[0.1553, 0.2388, 0.3245, 0.4202],\\
&\qquad [0.1695, 0.2325, 0.3236, 0.4088]\rangle\\
\tilde{r}_2 &= \langle[0.1792, 0.2514, 0.3608, 0.4467],\\
&\qquad [0.1906, 0.2654, 0.3431, 0.4045]\rangle\\
\tilde{r}_3 &= \langle[0.1635, 0.2006, 0.3059, 0.3973],\\
&\qquad [0.2094, 0.2857, 0.3385, 0.4347]\rangle\\
\tilde{r}_4 &= \langle[0.1917, 0.2600, 0.3874, 0.4596],\\
&\qquad [0.2331, 0.3024, 0.3670, 0.4163]\rangle.
\end{aligned} \tag{32}$$

Step 2. Utilize the correlation coefficient of TzFIFNs to derive the closeness between the overall group integrated attribute values $\tilde{r}_i$ and the TzFIFN positive ideal value $\tilde{r}^+$, where $\tilde{r}^+ = \langle[1,1,1,1],[0,0,0,0]\rangle$. Using (8) to (11), the correlation coefficients are calculated as follows:

$$\begin{aligned}
C_{\text{TzFIFS}}(\tilde{r}_1,\tilde{r}^+) &= 0.2847, & K_{\text{TzFIFS}}(\tilde{r}_1,\tilde{r}^+) &= 0.4493,\\
C_{\text{TzFIFS}}(\tilde{r}_2,\tilde{r}^+) &= 0.3095, & K_{\text{TzFIFS}}(\tilde{r}_2,\tilde{r}^+) &= 0.4966,\\
C_{\text{TzFIFS}}(\tilde{r}_3,\tilde{r}^+) &= 0.2673, & K_{\text{TzFIFS}}(\tilde{r}_3,\tilde{r}^+) &= 0.4281,\\
C_{\text{TzFIFS}}(\tilde{r}_4,\tilde{r}^+) &= 0.3246, & K_{\text{TzFIFS}}(\tilde{r}_4,\tilde{r}^+) &= 0.5270.
\end{aligned} \tag{33}$$

Step 3. Rank alternatives A_i, $i = 1,2,\ldots,m$, from the highest closeness (correlation coefficient) obtained from Step 2; the result obtained is

$$A_4 > A_2 > A_1 > A_3. \tag{34}$$

Hence the best alternative is A_4.

7. Conclusion

In this paper an MADM model was proposed based on the correlation coefficient of TzFIFS for ranking the alternatives together with WA and WG operators. The trapezoidal fuzzy intuitionistic fuzzy weighted averaging (TzFIFWA) operator and the trapezoidal fuzzy intuitionistic fuzzy weighted geometric (TzFIFWG) operator were used to aggregate the trapezoidal fuzzy intuitionistic fuzzy information given in a decision matrix. A numerical illustration was given to show the effectiveness of the proposed approach in using correlation coefficient of TzFIFSs because it preserved the linear relationship between the variables.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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