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Research Article

Optimum Fuzzy Design of Ecological Pressurised Containers

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In this study, the basic engineering principles, goals, and constraints are all combined to fuzzy methodology and applied to design of optimally pressurised containers emphasising the ecological and durability merits of various materials. The present fuzzy heuristics approach is derivable from generalisation of conventional analytical optimisation method into fuzzy multitechnical tasks. In the present approach, first the goals and constraints of the end-user are identified. Then decision variables are expressed as functions of the design variables. Their desirable ranges and biases are defined using the same fuzzy satisfaction function form. The optimal result has highest total satisfaction. These are then checked and fine-tuned by finite element method FEM. The optimal solution is the ecoplastic vessel, and aluminium was close. The method reveals that optimum depends strongly on the preset goals and values of the producer, society, and end-user.

1. Introduction

Production and use of small nonreusable containers, vessels, and bins has increased greatly worldwide. Thus the goal of ensuring their ecological sustainability has become acute as attested by many global megatrends of swings, cycles of the global economy as discussed by Marchetti [1].

The function of a container is to protect the usability of the material contained within it for a desired storage time. The contents may be food, beverages, medicine, or others. The material and geometry of the container should be designed optimally to endure the intended lifetime loads and also be sustainably biodegradable or recyclable after use. In nature, there are many ingenious optimal designs of containers. All of them are fully sustainable and ecological.

The conventional analytical and analytical optimisation methods are cumbersome and not easy to use for engineering design work. Diaz [2] has shown that use of fuzzy logic-based goal aggregation in design optimisation is a suitable tool for engineering design. Hundal [3] has shown that systematic mechanical designing, considering ecology, is important to achieve optimal ecodesign engineered products. Halada [4] discusses the importance of using ecomaterials to obtain a sustainable society. Hassan [5] has discussed how application of value-focused thinking on the environmental selection of wall structures is an effective tool for ecodesign tasks. Okubo and Fujii [6] use of ecocomposites with natural fibre reinforcement can be used to obtain products which have good mechanical properties and desired sustainability. Martikka and Katajisto [7] have studied possibilities of obtaining natural fibre reinforced biodegradable composite materials.

These tasks require use of cellular solid mechanics theory as discussed by Gibson and Ashby [8]. Optimal material selection in mechanical design is discussed by Ashby [9]. The methods for analysis of performance of fibre composites are discussed by Agarwal and Broutman [10]. Composite structures have many failure modes whose activation should be taken into account in an optimally prescient way. Basic fuzzy formulation of multiobjective optimisation by technical laws and heuristics is discussed by Martikka and Pöllänen [11]. The solutions need to be verified by FEM models [12]. Conventional formulations as discussed by Haftka and

Gurdal [13] and Rao [14] focus on exact numerical solutions of well-defined tasks.

In this study, the same fuzzy formulation is used consistently to describe the goals and constraints of the end user. One of them is minimisation of material cost and others are functional geometric and strength constraints. This goal aggregate functions like genes for the product. Many gene combinations can be made resulting in product individuals with different behaviour under loading and at market competition. By using purposeful ecoengineering design, a fit for purpose optimum solution is quickly created, designed, and manufactured. But without a good goal, no optimum is found with any amount of random selections.

The aim is to formulate optimum design methodology which takes into account all known requirements in a consistent way. In engineering optimisation at concept stage, most tasks are highly nonlinear and design variables are few and discrete. These tasks can be solved easily and fast using exhaustive or learning-enhanced search methods. The next aim is to verify the optimality of the results using FEM models [12]. Future aim is to use biomimicry to copy the abundant optima designs already existing in nature to obtain useful design innovations.

2. Megatrends as Guidelines for Design

Before any long-term technical development is started, it is necessary to observe the global megatrends and figure out which and why some are rising and some are falling. Some examples are shown in Figure 1 as modified from Marchetti [1].

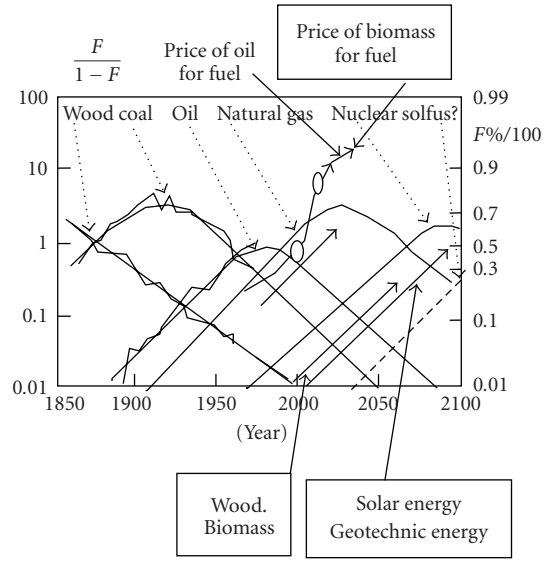

Figure 1: Globally most used energy sources. During 2001–2005 the oil price doubled and also in 2007-2008. Modified from [1].

3. Useful Application Area of Classical Optimum Design Method

Classical method of formulating and solving optimisation tasks is well grounded. But compared to heuristic and fuzzy formulations and solutions, it has a restricted applicability due to need of much mathematical manipulations. This comparison is illustrated using the case study model shown in Figure 2. Since now high pressure is proposed, then a container with spherical ends is reasonable. The tangential stress in the cylinder is double its axial stress and also double the sphere stress.

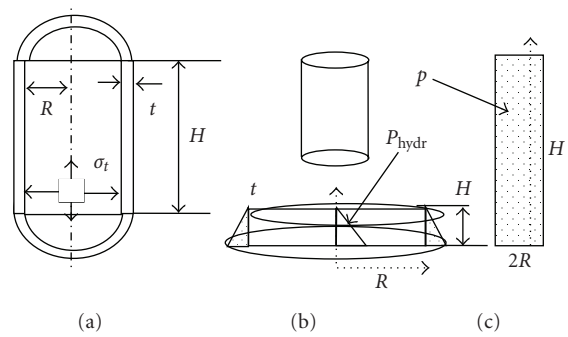

Figure 2: (a) The case study model. Resulting geometry of analytical solution. (b) Hydrostatic pressure gives a flat vessel $H/R = 1/2$. (c) Even pressure gives a long pipe, $H \gg R$.

3.1. Use of the Classical Method. The present task is to maximise the useful volume V. In order to conform to common practice, the task is converted to goal minimisation as follows:

$$\begin{aligned} \text{minimise} \quad & f(\mathbf{x}) = -V \quad \text{or } f = 1/V \\ \text{subject to constraints} \quad & g_1 = P(\text{Cost}) - P_{\text{cost}\cdot\min} > 0, \\ & g_2 = P(\text{Performance}) - P_{\text{Perf}\cdot\min} > 0. \end{aligned} \tag{1}$$

The classical method is to define an aggregate function Q covering the constraints also. The constraints are multiplied with unknown Lagrangian multipliers.

These are discussed by Haftka and Gurdal [13] and Rao [14].

$$\begin{aligned} Q &= f(x) - \sum_{j=1}^{n_g} \lambda_j g_j(\mathbf{x}), \quad i = 1, \ldots, n, \\ g_j(\mathbf{x}) &\geq 0, \quad j = 1, \ldots, n_g. \end{aligned} \tag{2}$$

Kuhn-Tucker conditions for obtaining optimum are

$$\frac{\partial}{\partial x_i} Q = \frac{\partial}{\partial x_i} f(x) - \sum_{j=1}^{n_g} \lambda_j \frac{\partial}{\partial x_i} g_j(\mathbf{x}) = 0, \quad i = 1, \ldots, n. \tag{3}$$

This method is applied to the vessel design task to illustrate the underlying principles. First, three design variables

are chosen, radius R, height H, and wall thickness t. Then using these, the aggregate function is

$$Q = -V + \lambda_1 g_1 + 2\pi\lambda_2 g_2 \Longrightarrow \text{minimum}$$
$$V = \pi R^2 H \tag{4}$$
$$Q = Q(\mathbf{x}) = Q(x_1, x_2, x_3) = Q(R, H, t).$$

The volume of material V_m is a function of basic design variables. It is constrained to be less than the allowed amount of volume V_{mat}. The stress σ is constrained to be lass than allowable stress σ_{all}. Two equality constraints are now for the available material and stress

$$g_1 = 1 - \frac{V_m}{V_{\text{mat}}} = 1 - \frac{2\pi t(R^2 + RH)}{V_{\text{mat}}} = 0, \tag{5}$$

$$g_2 = 1 - \frac{\sigma}{\sigma_{\text{all}}} = 1 - \frac{pR/t}{\sigma_{\text{all}}} = 0. \tag{6}$$

Thus, substitution of these in (3) gives three nonlinear equations.

$$\begin{aligned} \frac{\partial}{\partial R} f + \lambda_1 \frac{\partial}{\partial R} g_1 + 2\pi\lambda_2 \frac{\partial}{\partial R} g_2 &= 0, \\ \frac{\partial}{\partial H} f + \lambda_1 \frac{\partial}{\partial H} g_1 + 2\pi\lambda_2 \frac{\partial}{\partial H} g_2 &= 0, \\ \frac{\partial}{\partial t} f + \lambda_1 \frac{\partial}{\partial t} g_1 + 2\pi\lambda_2 \frac{\partial}{\partial t} g_2 &= 0. \end{aligned} \tag{7}$$

The number of design variables is five R, H, t, λ_1, and λ_2. Thus the five nonlinear equations can be solved in principle to give the optimal slenderness ratio x

$$x = \frac{H}{R}, \qquad \frac{1 - (1/2)x}{1 + x} = -\frac{1}{2} + \frac{p_{,H}}{p} H. \tag{8}$$

Two loading options can be applied to two principal load cases, constant, and hydrostatic pressure loading.

3.2. Hydrostatic Pressure Loading

$$p = \rho g H \Longrightarrow p_{,H} = \rho g \Longrightarrow \tag{9}$$

the optimal slenderness ratio is

$$x = \frac{1}{2} \Longrightarrow R = 2H. \tag{10}$$

Use of equality constraint (5) gives height

$$g_1 = 0 \Longrightarrow H = \left[\frac{\sigma_{\text{all}} V_{\text{mat}}}{24\pi\rho g}\right]^{1/4}. \tag{11}$$

Use of the equality constraint (6) gives the wall thickness

$$g_2 = 0 \Longrightarrow t = \frac{\rho g 2H^2}{\sigma_{\text{all}}}. \tag{12}$$

3.3. Constant Pressure Loading. Another common technical loading option is constant pressure. The dimensioning (9) suggests a feasible solution to choose very large height to radius slenderness ratio. This result is a pressurised pipe.

$$p = \text{const} \Longrightarrow H \gg R, \qquad g_2 = 0 \Longrightarrow t = \frac{pR}{\sigma_{\text{all}}}. \tag{13}$$

3.4. Derivation of the Fuzzy Goal Formulation. Goal formulations and solutions are straightforward in mathematically simple crisp tasks. But their utility is restricted when goals are fuzzy. But when the goals are fuzzy the utility is restricted. The proposition is that fuzzy tasks can be solved using fuzzy formulation and tools. First step in this direction is to consider the equality constraints g_1 and g_2 as events which occur as a result of engineering work. The events are combined "event" = g_1 and g_2. Next step is to define fuzzy satisfaction on this event as $P(\text{event})$ and maximise it. In the case study, it is reasonable to use relative variables

$$\begin{gathered} Q = V \uparrow, \qquad \left(\frac{V_m \downarrow}{V_{\text{mat}}}\right), \qquad \left(\frac{\sigma_{\text{all}}}{\sigma \downarrow} = N \uparrow\right) \\ P(Q) \uparrow = P(V) \uparrow \cdot P(V_m) \uparrow \cdot P(N) \uparrow \Longrightarrow \max \Longrightarrow 1. \end{gathered} \tag{14}$$

Using this line of approach, any vague goals can be defined, easily and usefully. A desire no. k is also a decision variable s_k. The total event; decision variable s is intersection of other decision variables

$$s = s_1 \cap s_2 \cap s_3 \cap s_4 \cap s_5 \cap s_6. \tag{15}$$

Satisfaction on this event is

$$P(s) \Longrightarrow P(s) = P(s_1) \cdot P(s_2) \cdots P(s_{\text{n}}). \tag{16}$$

This may be expressed verbally as

$$\begin{aligned} P(s) = {} & P(s_1 = \text{factor of safety}) \\ & \cdot P(s_2 = \text{volume}) \cdot P(s_3 = \text{material mass}) \\ & \cdot P(s_4 = \text{Cost}) \cdot P(s_5 = \text{ecology}) \\ & \cdot P(s_6 = \text{corrosion resistance}). \end{aligned} \tag{17}$$

Maximal Total Satisfaction Is the Goal of the Customer. The end user makes his decisions by choosing the product which has the maximal $P(s)$.

Probable Net Profit Is Goal of the Producer. One simple model is

$$\text{Net} = \Pr\{\text{success of } R \,\&\, D\}(B - K). \tag{18}$$

Here $\Pr\{e\}$ is probability of occurrence of an event e, K is cost and B is brutto profit. The producer requires a positive result

$$B - K = \left(x_{\text{efficiency}} + x_{\text{subsidies}} - 1\right)K. \tag{19}$$

Here the $x_{\text{efficinecy}}$ means efficiency factor for the company, and $x_{\text{subsidies}}$ means societal subsidy factor to support sustainable production. Society can also express requirement of sustainability by legislation effectively biasing the satisfaction function.

Probabilities of success are related to fuzzy distribution, Figure 3. The producer is satisfied with reliable high and positive net profit

$$\text{Net} = qP\{s\}K \tag{20}$$

Here q is some factor. A simple example in Figure 3 makes this clearer. Height of a fuzzy function $P(s)$ is $p = 1$ and the initial rise from zero to unit in Δs is roughly linear.

Cumulative distribution cdf $= F(s) = \Pr\{s\}$ of pdf $p(s)$ is also roughly linear

$$P(s) = \frac{s - s_{\min}}{\Delta s} p \approx F(s) = \frac{s - s_{\min}}{\Delta s} h, \quad \Delta s = s_a - s_{\min}$$
$$P(\Delta s) = \frac{\Delta s}{\Delta s} p = 1 \Longrightarrow p = 1, \qquad F(\Delta s) = \frac{\Delta s}{\Delta s} h = 1. \tag{21}$$

4. Use of Sound Design Principles as Guidance to Goals

Sound principles are useful to get guidance at the start of designing. One is axiomatic design by Suh [15].

Axiomatic design recommends use of axioms as guidelines towards rational alternatives in design.

Axiom 1 (independence axiom). Maintain the independence of the functional requirements (FR).

Axiom 2 (information axiom). Minimise the information content of design.

Applying these to the case study, it seems that there is a minimal number of decision variables or FRs. They are partly independent since increasing volume causes need for more shell material. Probability of success of design may be about $P = \exp(-\text{Info})$. Unnecessary information is detrimental to design success and causes harmful interference.

Too strict or irrelevant functional requirements may distort or exclude the solution from optimum.

TRIZ design ideas are another source of insightful rules.

The theory of inventive solving (TRIZ) has been developed by Altshuller [16].

Eight patterns are defined in this method. Next are some interpretations.

(1) *Stage of evolution of a technological system*. Megatrends show that major technological changes occur like mountain ranges.

(2) *Evolution towards increasing ideal solutions*. A strive exists to find out basic principles which are ideal. These can be expressed by fuzzy design goals.

(3) *Nonuniform development of system elements*. In this case, some material or manufacturing breakthrough may be paradigmatic.

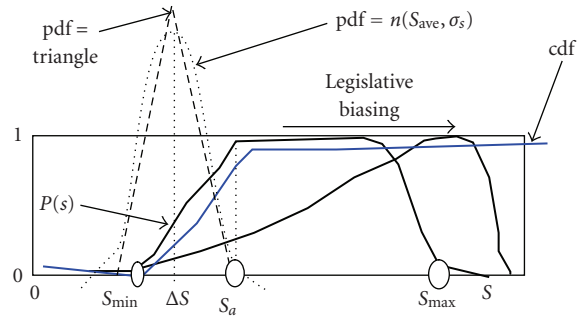

Figure 3: Relationship between simple fuzzy function and a pdf function.

(4) *Increase of dynamism and controllability*. It is not straightforward how to apply this to simple tasks.

(5) *Complexity increases first and then the essential features are found by simplification*. This is natural since the irrelevant factors are left out when they are found out.

(6) *Matching and mismatching of elements*. This may mean that functions may be joined or separated depending on the utility.

(7) *Increase advance to microelements and increased use of force fields*. Nanotechnology can change the strength of macromaterials and force field may supplant mechanical springs.

(8) *Less human involvement*. Machines and bacteria can be working autonomously.

The distinguishing feature of optimisation tasks is that there are some contradictory requirements. This results in a trade-off optimum. The Triz states the same saying that an invention may be obtained when several contradictions are overcome. Often these contradictions have been some hindrance, technological or psychological. When they are overcome, synergy effects may be found in the new areas of new possibilities.

5. The Total Design Control Loop as a Guide for Engineering Work

It is well known that all human activities start from a need. Then some control loop activity is started to fulfil the recognised need and stopped when there is no need. In Figure 4, the principle of a design loop activity is applied to the present innovation task.

5.1. Operation Steps within the Loop. The loop starts on the observation that there is a market need N and some attempt N' to satisfy it resulting in an unfulfilled need $\Delta N = N - N'$. This insufficiency is recognised by operation R to obtain the recognised need $X = R \cdot \Delta N$. Then X is compared with an attempt to satisfy recognised need $X' = H \cdot Y$ which is obtained by applying H activity to analyse and design a potential product concept Y. This concept $Y = G\Delta X$ has

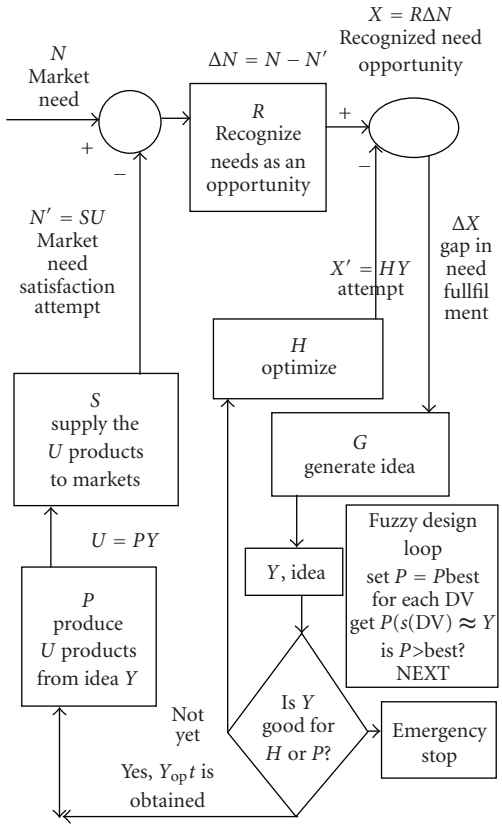

Figure 4: A typical design control loop for innovation tasks.

been obtained by applying G or idea generation process to the unfulfilled recognised need $\Delta X = X - X'$. When Y is accepted then it is produced by process P to get total production $U = PY$. Then U products are supplied to market by supply process S as $N' = SU = SP \cdot Y$ which is a need satisfaction attempt. From this one obtains

$$Y_{\text{opt}} = G\left(R \cdot \left\{N - S \circ P \circ Y_{\text{opt}}\right\} - H \cdot Y_{\text{opt}}\right)$$
$$\longrightarrow [1 + GR \cdot SP + GH] \circ Y_{\text{opt}} = GR \circ N. \tag{22}$$

From this loop equation, the optimal design concept is solved.

$$Y_{\text{opt}} = \frac{GR}{1 + GH + GR \cdot SP} \circ N. \tag{23}$$

Market survey recognition is $R = 0.7$. First, one may desire one optimal and successful product concept, $Y_{\text{opt}} = 1$. This is obtained when $G = 1000$ concepts are generated by the fuzzy design loop and all of them are analysed for optimality by $H = 1/1000$ selection success. Now four optimal products of each material are obtained. One of them is the most satisfactory and it is chosen as the one to be produced. Supply efficiency $S = 1$ and number of products $U = PY = 1000$.

$$Y \Longrightarrow 1 \therefore N' = SPY = 1 \cdot P \cdot 1 \Longrightarrow N = N' = P$$
$$Y_{\text{opt}} = \frac{1000 \cdot 0.7}{1 + 1000 \cdot 0.001 + 1000 \cdot 0.7 \cdot 1 \cdot P} \circ N = 1 \tag{24}$$
$$\Longrightarrow N = (2 + 700P) = 700N \quad P \approx N.$$

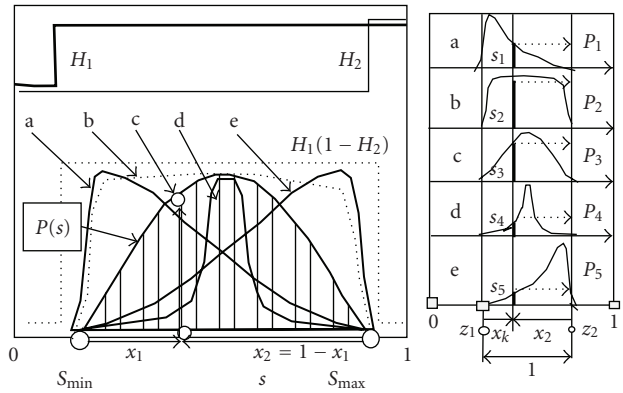

Figure 5: Principle of modelling of the general satisfaction functions. Its position and skewness can be varied.

Table 1: Skewness parameter values.

	a	b	c	d	e
p_1	0.1	0.1	1	5	5
p_2	5	0.1	1	5	0.1
$x_{1\max}$	0.02	0.5	0.5	0.5	0.98

6. Goals and Constraints as Fuzzy Functions

In engineering tasks the phase of defining the goals and constraints is important. Design variables are few and discrete but their relationship is highly nonlinear. In these tasks, a satisfactorily fast method is exhaustive or somewhat learning search. Designers can shift their emphasis on the work itself when all goals and constraints can be formulated consistently and easily by one flexible fuzzy function of a standard form. This is illustrated in Figure 5 and Table 1.

In the design algorithm, the satisfaction function is defined for each decision variable s by inputting the left and right limits and two bias parameters p. The left skewed option is useful to get low cost designs. Flat shape allows indifferent choice of s. The location of maximum can be shifted. The call is CALL pzz($s_{\min}, s_{\max}, p_1, p_2, s, P(s)$). The output is the satisfaction function $P(s)$ and it varies in the range $0, \ldots, 1$. The decision variables s are changed to an internal dimensionless variable x_1.

$$x_1 = \frac{s - s_{\min}}{s_{\max} - s_{\min}} \Longrightarrow x_2 = 1 - x_1. \tag{25}$$

The satisfaction function depends on one variable x_1

$$P(x_1) = (p_1 + p_2)^{p_1 + p_2} \left(\frac{x_1}{p_1}\right)^{p_1} \left(\frac{1 - x_1}{p_2}\right)^{p_2} H_{12}. \tag{26}$$

Here

$$H_{12} = H_1(s)(1 - H_2(s)). \tag{27}$$

Two step functions are used to define the desired range of the decision variable

$$H_1(s) = \frac{1}{2}[1 + \operatorname{sgn}(s - s_{\min})],$$
$$H_2(s) = \frac{1}{2}[1 + \operatorname{sgn}(s - s_{\max})]. \tag{28}$$

Outside of the desired range, a small nonzero seed value is added to the satisfaction function to promote search drive for improvement. Maximum value of $P = 1$ is at $x_{1\max}$

$$\frac{d}{dx_1}P(x_1) = K\frac{d}{dx_1}\left\{\left(\frac{x_1}{p_1}\right)^{p_1}\left(\frac{1 - x_1}{p_2}\right)^{p_2}\right\} = 0. \tag{29}$$

The $x_{1\max}$ value giving maximum value of 1 for $P(x_{1\max})$ is

$$x_{1\max} = \frac{1}{1 + (p_2/p_1)}, \qquad P(x_{1\max}) = 1. \tag{30}$$

The desired range for each decision variable s_k is

$$R(s_k) = s_{k\min} < s_k < s_{k\max}. \tag{31}$$

The total event s is intersection of separate events

$$s = s_1 \cap s_2 \cap s_3 \cap s_4 \cap s_5 \cap s_6. \tag{32}$$

Satisfaction on this event

$$P(s) \Longrightarrow P(s) = P(s_1) \cdot P(s_2) \cdots P(s_n). \tag{33}$$

The design goal is to maximise this product function.

The satisfaction function selection can be made as the customer wishes. Two of three parameters can be freely chosen as desired p_1, p_2 or $x_{1\max}$. The following options are available:

(i) Choose (p_1, p_2) and calculate $x_{1\max}$ (p_2/p_1), or

(ii) Choose p_2 and $x_{1\max}$ and calculate p_1.

7. Design Variables of the Case Study

Design variables are of three main types.

7.1. Geometric Design Variables. Geometric design variables are selected from a list of discrete feasible options shown in Table 2.

7.2. Material Design Variables. Material design variables are selected from a list of discrete feasible options shown in Table 3.

In this case study, the volume was about 1 litre. Then standard choice is diameter $d = .08$ m, height $H = 0.02$ for the cylindrical part.

Table 2: Geometrical design variables.

index	Radius R(ir)	Wall t(itt)	Height H(ih)
1	0.006	0.00008	0.01:
2	0.008	0.0001	0.015
3	0.01	0.00012	0.02
4	0.013	0.00015	0.03
5	0.015	0.00018	0.035
6	0.018	0.0002	0.04
7	0.02	0.00022	0.05
8	0.025	0.0003	0.06
9	0.03	0.0004	0.07
10	0.04	0.0005	0.08

Table 3: Material design variables. sall(im) is allowed stress of material im.

Material code	M\$(1) = "Al"	M\$(2) = "St"
All. stress (Pa)	sall(1) = 1 E + 08	sall(2) = 1.5 E + 08
Material unit cost	cm(1) = 20	cm(2) = 5
Density (kg/m^3)	rho(1) = 4000	rho(2) = 8000
Ecological merit	eco(1) = .1	eco(2) = .7
Corr. resistance	corres(1) = .8	corres(2) = .15
Elastic modulus	E(1) = 6 E + 10	E(2) = 2.1 E + 11
Material code	M\$(3) = "PVC"	M\$(4) = "ecopl"
All. stress (Pa)	sall(3) = 2 E + 07	sall(4) = 4 E + 07
Material unitcost	cm(3) = 15	cm(4) = 20
Density (kg/m^3)	rho(3) = 1800	rho(4) = 1800
Ecological merit	eco(3) = .15	eco(4) = .8
Corr. resistance	corres(3) = .5	corres(4) = .3
Elastic modulus	E(3) = 5 E + 09	E(4) = 5 E + 09

8. Decision Variables of the Case Study

Decision variable $s_1 = N =$ factor of safety against internal pressure.

This is now defined simply as the ratio of allowed stress to hoop stress σ_t the cylinder.

$$s_1 = N = \frac{\sigma_{\text{all}}(\text{im})}{\sigma}, \quad \sigma = p\frac{R(\text{ir})}{t(\text{itt})}, \quad P_s(1) = P(s_1). \tag{34}$$

Desired range and bias are

$$s_{1,\min} = 0.3 \le s_1 \le s_{1,\max} = 4$$
$$\text{bias } p_1 = 0.5, \qquad p_2 = 0.1. \tag{35}$$

This choice favours N close to unity.

Decision variable $s_2 = V =$ useful volume inside

$$s_2 = V = \pi R(\text{ir})^2 H(\text{ih}), \qquad P_s(2) = P(s_2). \tag{36}$$

Now in this case study a small volume $V = V_0 = .001\,\text{m}^3$ is chosen.

Desired range and bias are

$$s_{2,\min} = 0.01\,V_0 \le s_2 \le s_{2,\max} = 2\,V_0,$$
$$\text{bias } p_1 = 0.2, \qquad p_2 = 0.2. \tag{37}$$

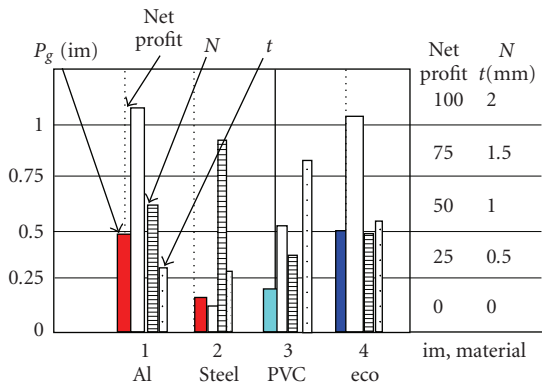

Figure 6: Results of optimisation.

This favours the middle range.

Decision variable $s_3 = M =$ material mass

$$s_3 = M(\text{im}) = \rho(\text{im})V_m(\text{im}),$$
$$V_m(\text{im}) = 2\pi R(\text{ir})t(\text{itt})(R(\text{ir}) + H(\text{ih})) \tag{38}$$
$$P_s(3) = P(s_3).$$

Desired range and bias are

$$s_{3,\min} = 0.001 \le s_3 \le s_{3,\max} = 1,$$
$$\text{bias } p_1 = 0.1, \qquad p_2 = 3\,(\text{small mass}). \tag{39}$$

Decision variable $s_4 = K =$ cost of material

$$s_4 = c(\text{im})M(\text{im}), \quad P_s(4) = P(s_4). \tag{40}$$

Desired range and bias are

$$s_{4,\min} = 0.001\,\text{Cos t}_{\max}, \qquad s_{4,\max} = 10\,\text{Cos t}_{\max}$$
$$\text{bias } p_1 = 0.1, \qquad p_2 = 2 \tag{41}$$
$$\text{Cos t}_{\max} = 1.$$

Decision variable $s_5 =$ eco $=$ ecological merit

$$s_5 = \text{eco}(\text{im}), \qquad P_s(5) = P(s_5). \tag{42}$$

Desired range and bias are

$$s_{5,\min} = 0.001 \le s_5 \le s_{5,\max} = 1,$$
$$\text{bias } p_1 = 1, \qquad p_2 = 0.1\,(\text{high eco}). \tag{43}$$

Decision variable $s_6 =$ Corres $=$ material mass

$$s_6 = \text{corres}(\text{im}), \qquad P_s(6) = P(s_6). \tag{44}$$

Desired range and bias are

$$s_{6,\min} = 0.001 \le s_6 \le s_{6,\max} = 1,$$
$$\text{bias } p_1 = 2, \qquad p_2 = 0.1\,(\text{high corres}). \tag{45}$$

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FOR im ′ material class is selected by the user
FOR ir = ir1 TO ir2 ′ Radius R(ir)
FOR itt = itt1 TO itt2, t(it), wall thickness class
FOR iH = iH1 to iH2 · H(iH) height
Each k = 1, 2, . . ., decision variable s_k is calculated, the range
and bias pair p_1 and p_2 are given. Then the satisfaction
function P(s) is obtained by a call
CALL pzz(s_min, s_max, p_1, p_2, s, P(s)).
The total satisfaction is product of partial satisfactions.
P_s = 1, the initialisation first, before the loop
FOR i = 1 TO 3
   Ps = Ps ∗ Ps(i)
NEXT I: then P_g = P_s
IF P_g > P_gbest THEN
  ′new optimum is found better than previous
ELSE search is continued. END IF
NEXT
```

Algorithm 1: Total satisfaction is first initialised to a low value $P_{\text{gbest}} = .0000001$.

Table 4: Total satisfaction for the customer. Pg(im) in products of various materials. Inner radius $R = 0.04$ m, inner height $H = 0.2$ m, t: wall thickness (mm), N: factor of safety, Net: net profit = Pg·K for the producer.

Material	Pg · 1000	Cost K	Net	t wall	N
im = 1, Al	46	2.42	111	0.5	1.25
im = 2, Steel	12	1.2	15	0.5	1.88
im = 3, PVC	18	2.4	43	1.7	0.75
im = 4, eco	48	2,17	104	1.2	1

9. Algorithm for Optimisation

In engineering optimisation at concept stage, most tasks are highly nonlinear and also the design variables are few and discrete. For these tasks, the exhaustive or learning enhanced search method is satisfactory. User can select the face material from the list of available selections. See Algorithm 1.

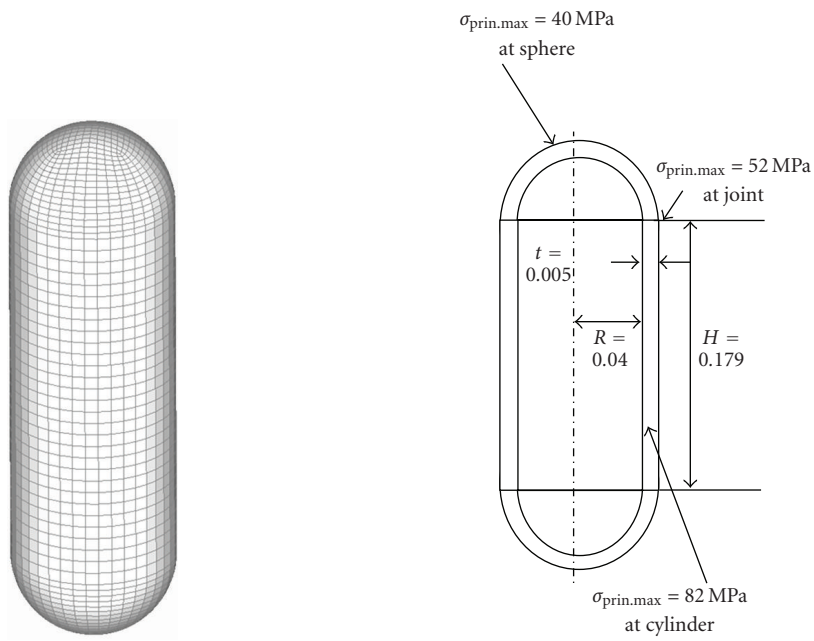
10. Results of Optimisation

Some results are shown in Figure 6 and Table 4.

One can see that the best choice for the customer is eco followed by Al choice. The producer may favour the Al product which gives 10% more probable profit than eco bin. Small shift in societal opinions and subsidy policy may shift the preferences. The shifts are not free but stiffly predestined by the global megatrends which prevail over the markets and allow only small variations.

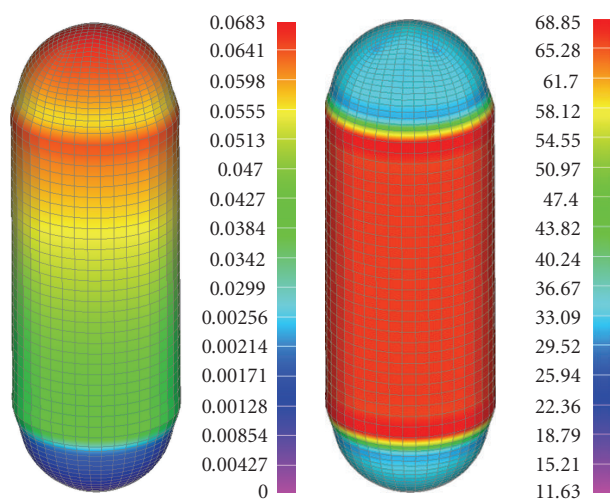
Some results of optimisation are shown in Figure 6 showing total satisfaction, net profit, factor of safety N, and wall thickness t for each material selection.

Steel gives the highest N but the lowest net profit. The optimality of the ecoproduct can be achieved by developing high enough strength, cost-effective manufacturing and societal will expressed with subsidies.



(a) (b)

Figure 7: (a) FEM mesh model of the vessel using shell elements. (b) Main dimensions and principal maximum stresses at the outer surface are shown.



(a) (b)

Figure 8: (a) Deformations and displacements. (b) Principal maximum stresses at the outer surface. The stresses are obtained with wall 0.6 mm.

11. Results of FEM Checking

Results are shown in Figures 7 and 8 with wall 0.6 mm. In order to compare with analytical optimum $t = 0.5$ mm the stresses are multiplied with 1.2 and shown in Figure 7(b).

The results agree reasonably well. The results are elastic and apply to the model with wall 0.5 mm. The simple tangential stress formula gave 80 MPa and the principal maximum stress was 82 by FEM and 83 at joint. This includes the effect of the axial stress also. FEM model revealed clearly that the circular plate ends are feasible only with small pressures.

12. Conclusion

We have combined in this approach basic mechanics with new goal formulations and heuristics into an ecodesign engineering to design optimally small volume large quantity containers. In nature, these ingeniously designed and bio manufactured structures are already widely and successfully

used by animals and plants. In the latest decades, the idea of bio mimicry has become important.

The present analytical and analytical optimisation methods are cumbersome and not well suited for engineering design work since not all goals and constraints can be considered well and fast enough.

This hindrance is overcome by the fuzzy-based method since all goals and constraints can be formulated by one flexible fuzzy standard function. In engineering optimisation at concept stage, most tasks are highly nonlinear and also the design variables are few and discrete. For this reason, the exhaustive or learning enhanced search methods are deemed to be satisfactory.

One evident advantage of the present method over the conventional ones is that more useful and reasonable optima can be found since all relevant desires are considered simultaneously during the design. Drawback is that one has to formulate all critical locations analytically whereas EM takes all into account.

The recommended optima need to be explored and verified using FE method to obtain a reliable product model.

The optimal solution was the ecoplastic vessel, and aluminium was close. The method reveals that the optimum depends on the company policy preferences, societal needs and values, and subsidies. But the global megatrends stiffly dictate them to fit into the trend.

The future aim of this work is to generalise this approach so that it can be flexibly applied to many kinds of tasks where optimal selections are needed.

One evident global megatrend is that the number of possible options for materials, geometry, and the several functional options and also the societal preferences and legislation are changing fast, while time for correct decisions is becoming shorter.

Using these design methodology designers of machines can better mimic the ingenious solutions of nature and thus also the important future trends of technology and vague information in their design and development work.

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