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Research Article

On Lower Separation and Regularity Axioms in Fuzzy Topological Spaces

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We use the concepts of the quasicoincident relation to introduce and investigate some lower separation axioms such as αT_0, αT_1, $\alpha T_{1/2}$, and αT_2 as well as the regularity axioms αR_0 and αR_1. Further we study some of their properties and the relations among them in the general framework of fuzzy topological spaces.

1. Introduction

The fundamental concept of a fuzzy set was introduced by Zadeh in 1965, [1]. Subsequently, in 1968, Chang [2] introduced fuzzy topological spaces (in short, fts). In Chang's fuzzy topological spaces, each fuzzy set is either open or not. Later on, Chang's idea was developed by Goguen [3], who replaced the closed interval $I = [0, 1]$ by a more general lattice L. In 1985, Kubiak [4], and Šostak [5], in separated works, made topology itself fuzzy besides their dependence on fuzzy sets. In 1991, from a logical point of view, Ying [6] studied Hohles topology and called it fuzzifying topology. This fuzzification opened a rich field for research. As it is well known, the neighborhood structure is not suitable to I-topology, and Pu and Liu [7] broke through the classical theory of neighborhood system and established the strong and powerful method of quasicoincident neighborhood system in I-topology. Zhang and Xu [8] established the neighborhood structure in fuzzifying topological spaces. Considering the completeness and usefulness of theory of I-fuzzy topologies, Fang [9] established I-fuzzy quasicoincident neighborhood system in I-fuzzy topological spaces and gave a useful tool to study I-fuzzy topologies.

In ordinary topology, α-open sets were introduced and studied by Njastad [10]. Bin Shahna [11], in the same spirit, defined fuzzy α-open and fuzzy α-closed. Separation is an essential part of fuzzy topology, on which a lot of work has been done. In the framework of fuzzifying topologies, Shen [12], Yue and Fang [13], Li and Shi [14], and Khedr et al. [15] introduced some separation axioms and their separation axioms are discussed on crisp points not on fuzzy points. In 2004, Mahmoud et al. [16] introduced fuzzy semicontinuity and fuzzy semiseparation axioms and examined the validity of some characterization of these concepts. Further, they also defined fuzzy generalized semiopen set and introduced fuzzy separation axioms by using thew semiopen sets concept. In the same paper, the authors also discussed fuzzy semiconnected and fuzzy semicompact spaces and some of their properties.

The present paper is organized as follows. It consists of four sections. After this introduction, Section 2 is devoted to some preliminaries. In Section 3, we introduce the notions of some lower separation axioms such as the αT_0, αT_1, $\alpha T_{1/2}$, and αT_2 axioms with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces. In Section 4, we introduce the notions of some lower regularity axioms such as the αR_0 and αR_1 with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces.

2. Preliminaries

Throughout this paper, X represents a nonempty fuzzy set and fuzzy subset A of X, denoted by $A \leq X$, then it is characterized by a membership function in the sense of Zadeh [1]. The basic fuzzy sets are the empty set, the whole set, and the class of all fuzzy sets of X which will be denoted by 0_X, 1_X, and I^X, respectively. A subfamily τ of I^X is called a fuzzy topology described by Chang [2]. Moreover, the pair $(X,\tau) := (I^X,\tau)$ will be meant as a fuzzy topological space, on which no separation axioms are assumed unless explicitly stated. The fuzzy closure, the fuzzy interior, and the fuzzy complement of any set A in (X,τ) are denoted by $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$, and $1 - A$, respectively. A fuzzy set which is a fuzzy point [17] with support $x \in X$ and value t $(0 < t \leq 1)$ is denoted by x_t, and $\mathrm{Pt}(X)$ will denote the family of all point fuzzy sets $x_t \in I^X$. For any two fuzzy sets A and B in (X,τ), $A \leq B$ if and only if $A(x) \leq B(x)$ for each $x \in X$.

Definition 1 (see [18]). In a fuzzy topological space (X,τ), a fuzzy set A is called a quasicoincident with a fuzzy set B, denoted by AqB, if $A(x) + B(x) > 1$ for some $x \in X$. A fuzzy point $x_t \leq A$ is called quasicoincident with the fuzzy set A, denoted by x_tqA, if $t + A(x) > 1$. Relation "does not quasicoincide with" or "not quasicoincident with" is denoted by $\neg q$. A fuzzy set A in (X,τ) is called quasi-neighborhood of x_t if there is a fuzzy open set U such that $x_tqU \leq A$.

Definition 2 (see [11]). A fuzzy subset A of a fuzzy topological space (X,τ) is said to be fuzzy α-open set in (X,τ) if $A \leq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ and the fuzzy complement of fuzzy α-open set is fuzzy α-closed set.

$F\alpha O(X,\tau)$ will denote the family of all fuzzy α-open sets in (X,τ), and $F\alpha C(X,\tau)$ will denote the family of all fuzzy α-closed sets in (X,τ).

Definition 3 (see [11]). Let A be a fuzzy set in fuzzy topological space (X,τ). $\mathrm{Int}_\alpha(A) = \bigvee\{B \in F\alpha O(X,\tau) : B \leq A\}$ is called the α-interior of A, and $\mathrm{Cl}_\alpha(A) = \bigwedge\{B \in F\alpha C(X,\tau) : A \leq B\}$ is called the α-closure of A.

Theorem 1 (see [11]). *Let (X,τ) be a fuzzy topological space, and let A, B be two fuzzy sets in X. Then the following holds.*

(1) $A \leq \mathrm{Cl}_\alpha(A)$.

(2) $A \in F\alpha O(X,\tau)$ *if and only if* $A = \mathrm{Cl}_\alpha(A)$.

(3) *If* $A \leq B$, *then* $\mathrm{Cl}_\alpha(A) \leq \mathrm{Cl}_\alpha(B)$.

(4) $\mathrm{Cl}_\alpha(A \vee B) = \mathrm{Cl}_\alpha(A) \vee \mathrm{Cl}_\alpha(B)$.

(5) $\mathrm{Cl}_\alpha(A \wedge B) \leq \mathrm{Cl}_\alpha(A) \wedge \mathrm{Cl}_\alpha(B)$.

3. α-Separation Axioms

In this section, we introduce the notions of some lower separation axioms such as the αT_0, αT_1, $\alpha T_{1/2}$, and αT_2 axioms. Furthermore, we instigate some of their properties and the relations between them in the general framework of fuzzy topological spaces.

Definition 4. A fuzzy topological space (X,τ) is called

(1) fuzzy αT_0-space if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exist $U \in F\alpha O(X,\tau)$ such that $x_tqU \leq 1 - y_r$ or $y_rqU \leq 1 - x_t$,

(2) fuzzy αT_1-space if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exist $U, V \in F\alpha O(X,\tau)$ such that $x_tqU \leq 1 - y_r$ and $y_rqV \leq 1 - x_t$,

(3) fuzzy αT_2-space if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exist $U, V \in F\alpha O(X,\tau)$ such that $x_tqU \leq 1 - y_r$, $y_rqV \leq 1 - x_t$ and $U\neg qV$.

Theorem 2. *Let (X,τ) be a fuzzy topological space. If (X,τ) is fuzzy αT_i-space, then it is fuzzy αT_{i-1}-space, where $i = 1, 2$.*

Proof. Obvious. □

Theorem 3. *A fuzzy topological space (X,τ) is fuzzy αT_0-space if and only if for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$.*

Proof. Suppose that (X,τ) is αT_0-space. Then for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exists $U \in F\alpha O(X,\tau)$ such that $x_tqU \leq 1 - y_r$ or $y_rqU \leq 1 - x_t$. If $x_tqU \leq 1 - y_r$, then $x_t \nleq 1 - U$ and $U \leq 1 - y_r$, that is, $x_t \nleq 1 - U$ and $y_r \leq 1 - U$. Since $1 - U$ is fuzzy α-closed and $\mathrm{Cl}_\alpha(y_r)$ is the smallest fuzzy α-closed containing y_r, then $\mathrm{Cl}_\alpha(y_r) \leq 1 - U$. Since $x_t \nleq 1 - U$ and $x_t \leq \mathrm{Cl}_\alpha(x_t)$, then $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$.

Conversely, suppose that x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$ and $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$. Let $z_\lambda \in Pt(X)$ such that $z_\lambda \leq \mathrm{Cl}_\alpha(x_t)$ and $z_\lambda \nleq \mathrm{Cl}_\alpha(y_r)$. We claim that $x_t \nleq \mathrm{Cl}_\alpha(y_r)$. For, if $x_t \leq \mathrm{Cl}_\alpha(y_r)$, then $\mathrm{Cl}_\alpha(x_t) \nleq \mathrm{Cl}_\alpha(y_r)$. This contradicts the fact that $z_\lambda \nleq \mathrm{Cl}_\alpha(y_r)$. Hence $x_t \nleq \mathrm{Cl}_\alpha(y_r)$, that is, $x_tq(1 - \mathrm{Cl}_\alpha(y_r))$. And since $U := 1 - \mathrm{Cl}_\alpha(y_r) \in F\alpha O(X,\tau)$ and $1 - y_r \leq 1 - \mathrm{Cl}_\alpha = U$, then $x_tqU \leq 1 - y_r$. That is, (X,τ) is fuzzy αT_0-space. □

Theorem 4. *A fuzzy topological space (X,τ) is fuzzy αT_1-space if and only if every singleton fuzzy points x_t in X is fuzzy α-closed in X.*

Proof. Suppose that (X,τ) is αT_1-space. Let $y_r \leq 1 - x_t$ $(x \neq y)$. Then there exist $U, V^y \in F\alpha O(X,\tau)$ such that $x_tqU \leq 1 - y_r$ and $y_rqV^y \leq 1 - x_t$. In part $y_rqV^y \leq 1 - x_t$, we have $V^y \leq 1 - x_t$. Let $A = \bigvee\{V^y : y_rq(1 - x_t)\}$. One may easily verify that $A = 1 - x_t$. Hence $1 - x_t$ is fuzzy α-open set, that is, x_t is fuzzy α-open set.

Conversely, let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Then x_t and y_r are fuzzy α-closed sets. Consequently, $1 - x_t$ and $1 - y_r$ are fuzzy α-open sets. Hence $y_rq(1 - x_t) \leq (1 - x_t)$ and $x_tq(1 - y_r) \leq (1 - y_r)$. Therefore, (X,τ) is fuzzy αT_1-space. □

Definition 5. A fuzzy subset A of fuzzy topological space (X,τ) is called fuzzy α-symmetric if for every pair of fuzzy points x_t, y_r in X, $x_t \leq \mathrm{Cl}_\alpha(y_r)$ implies $y_r \leq \mathrm{Cl}_\alpha(x_t)$ (i.e., $x_t \leq \mathrm{Cl}_\alpha(y_r)$ implies $\mathrm{Cl}_\alpha(y_r) = \mathrm{Cl}_\alpha(x_t)$).

Definition 6. A fuzzy subset A of fuzzy topological space (X,τ) is called fuzzy α-generalized closed set in X (briefly α-g-closed) if $\mathrm{Cl}_\alpha(A) \leq U$ whenever $A \leq U$ and $U \in F\alpha O(X,\tau)$.

We easily observe that every fuzzy α-closed set is fuzzy α-g-closed set.

Theorem 5. *A fuzzy topological space (X,τ) is fuzzy α-symmetric if and only if for every fuzzy point x_t in X is fuzzy α-g-closed set.*

Proof. Suppose that (X,τ) is fuzzy α-symmetric and suppose that $x_t \leq U \in F\alpha O(X,\tau)$ and $\mathrm{Cl}_\alpha(x_t) \nleq U$. This implies that there is fuzzy point y_r in X such that $y_r \leq \mathrm{Cl}_\alpha(x_t) \wedge (1-U)$. Then $y_r \leq \mathrm{Cl}_\alpha(x_t)$ and $y_r \leq (1-U)$, that is, $\mathrm{Cl}_\alpha(y_r) \leq \mathrm{Cl}_\alpha(1-U) = 1-U$. Since (X,τ) is fuzzy α-symmetric and $y_r \leq \mathrm{Cl}_\alpha(x_t)$, then $x_t \leq \mathrm{Cl}_\alpha(y_r) \leq 1-U$. But this is a contradiction with $x_t \leq U$ and $t \in (0,1]$. Hence, $\mathrm{Cl}_\alpha(x_t) \leq U$.

Conversely, suppose that for every fuzzy point x_t in X is fuzzy α-g-closed set. Suppose that $x_t \leq \mathrm{Cl}_\alpha(y_r)$ and $y_r \nleq \mathrm{Cl}_\alpha(x_t)$. That is, $y_r \leq 1-\mathrm{Cl}_\alpha(x_t)$. Since $1-\mathrm{Cl}_\alpha(x_t) \in F\alpha O(X,\tau)$ and y_r is a fuzzy α-g-closed set, then $\mathrm{Cl}_\alpha(y_r) \leq 1-\mathrm{Cl}_\alpha(x_t)$. This implies that $x_t \leq 1-\mathrm{Cl}_\alpha(x_t) \leq 1-x_t$. This is a contradiction. Hence (X,τ) is fuzzy α-symmetric. □

Corollary 1. *If fuzzy topological space (X,τ) is fuzzy αT_1-space, then it is α-symmetric.*

Proof. By Theorem 4, in fuzzy αT_1-space (X,τ), every fuzzy point is fuzzy α-closed set. By facts, every fuzzy α-closed set is fuzzy α-g-closed set, and by Theorem 5, (X,τ) is fuzzy α-symmetric. □

Corollary 2. *A fuzzy topological space (X,τ) is fuzzy α-symmetric and fuzzy αT_0-space if and only if it is fuzzy αT_1-space.*

Proof. If (X,τ) is fuzzy αT_1-space, then by Theorem 2 and Corollary 1, it is fuzzy α-symmetric and fuzzy αT_0-space. Conversely, suppose that x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Then by fuzzy αT_0-space, we may assume that there exists $x_t q U \leq 1-y_r$ for some $U \in F\alpha O(X,\tau)$, hence $x_t \nleq \mathrm{Cl}_\alpha(y_r)$, which implies, by α-symmetric, $y_r \nleq \mathrm{Cl}_\alpha(x_t)$. That is, $y_r q(1-\mathrm{Cl}_\alpha(x_t)) \leq 1-\mathrm{Cl}_\alpha(x_t) \leq 1-x_t$. Hence, (X,τ) is fuzzy αT_1-space. □

Definition 7. A fuzzy topological space (X,τ) is called fuzzy $\alpha T_{1/2}$-space if every α-g-closed set is α-closed set.

Theorem 6. *For fuzzy α-symmetric topological space (X,τ), the following properties are equivalent:*

(1) *(X,τ) is fuzzy αT_0-space;*

(2) *(X,τ) is fuzzy $\alpha T_{1/2}$-space;*

(3) *(X,τ) is fuzzy αT_1-space.*

Proof. Obvious. □

Theorem 7. *For fuzzy α-symmetric topological space (X,τ), the following properties are equivalent:*

(1) *(X,τ) is fuzzy αT_2-space;*

(2) *for every pair of fuzzy points x_t, y_r in X $(x \neq y)$, there exists $U \in F\alpha O(X,\tau)$ such that $x_t q U \leq 1-y_r$ and $y_r \nleq \mathrm{Cl}_\alpha(U)$;*

(3) *for every fuzzy point x_t in X, $\bigwedge\{\mathrm{Cl}_\alpha(U) : U \in F\alpha O(X,\tau),\ x_t \leq U\} = x_t$.*

Proof. (1)→(2): Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Then there exist $U, V \in F\alpha O(X,\tau)$ such that $x_t q U \leq 1-y_r$, $y_r q V \leq 1-x_t$ and $U \neg q V$, hence, $x_t \leq U$. Since $U \leq 1-V$ and $1-V$ is fuzzy α-closed, then $\mathrm{Cl}_\alpha(U) \leq \mathrm{Cl}_\alpha(1-V) = 1-v$. And since $y_r \nleq 1-V$, then $y_r \nleq \mathrm{Cl}_\alpha(U)$.

(2)→(3): It is clear that

$$x_t \leq \bigwedge\{\mathrm{Cl}_\alpha(U) : U \in F\alpha O(X,\tau),\ x_t \leq U\}. \tag{1}$$

Now if $x_t \neq y_r$, then there exists $U \in F\alpha O(X,\tau)$ such that $x_t q U \leq 1-y_r$ and $y_r \nleq \mathrm{Cl}_\alpha(U)$. This implies that

$$y_r \nleq \bigwedge\{\mathrm{Cl}_\alpha(U) : U \in F\alpha O(X,\tau),\ x_t \leq U\}. \tag{2}$$

Hence

$$\bigwedge\{\mathrm{Cl}_\alpha(U) : U \in F\alpha O(X,\tau),\ x_t \leq U\} = x_t. \tag{3}$$

(3)→(1): Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$. Since

$$\bigwedge\{\mathrm{Cl}_\alpha(U) : U \in F\alpha O(X,\tau),\ x_t \leq U\} = x_t, \tag{4}$$

then there is fuzzy $U \in F\alpha O(X,\tau)$ such that $x_t \leq U$ and $y_r \nleq \mathrm{Cl}_\alpha(U)$. Hence $x_t q U \leq \mathrm{Cl}_\alpha(U) \leq 1-y_r$. Put $V = 1-\mathrm{Cl}_\alpha(U)$, then $y_r q V \leq 1-V \leq 1-x_t$ and it is clear that $V \neg U$. Hence (X,τ) is fuzzy αT_2-space. □

4. α-Regularity Axioms

In this section, we introduce the notions of some lower regularity axioms such as the αR_0 and αR_1 with instigating some of their properties and the relations between them in the general framework of fuzzy topological spaces.

Definition 8. A fuzzy topological space (X,τ) is called fuzzy αR_0-space if for every $U \in F\alpha O(X,\tau)$ and for every fuzzy point $x_t \leq U$, $\mathrm{Cl}_\alpha(x_t) \leq U$.

Theorem 8. *A fuzzy topological space (X,τ) is fuzzy αR_0-space if and only if for every pair of fuzzy points x_t, y_r in X with $(x \neq y)$ and $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$, $\mathrm{Cl}_\alpha(x_t) \neg q\, \mathrm{Cl}_\alpha(y_r)$.*

Proof. Suppose that a fuzzy topological space (X,τ) is fuzzy αR_0-space. Let x_t, y_r be a pair of fuzzy points in X with $(x \neq y)$ and $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \leq \mathrm{Cl}_\alpha(x_t)$ and $z_\mu \nleq \mathrm{Cl}_\alpha(y_r)$. If $x_t \leq \mathrm{Cl}_\alpha(y_r)$, then $\mathrm{Cl}_\alpha(x_t) \leq \mathrm{Cl}_\alpha(y_r)$. Hence, $z_\mu \leq \mathrm{Cl}_\alpha(y_r)$, but this is a contradiction. Then $x_t \nleq \mathrm{Cl}_\alpha(y_r)$, that is, $x_t \leq 1-\mathrm{Cl}_\alpha(y_r)$. Since $1-\mathrm{Cl}_\alpha(y_r)$ is fuzzy α-open and (X,τ) is fuzzy αR_0-space, then $\mathrm{Cl}_\alpha(x_t) \leq 1-\mathrm{Cl}_\alpha(y_r)$. Hence $\mathrm{Cl}_\alpha(x_t) \neg q\, \mathrm{Cl}_\alpha(y_r)$.

Conversely, Let $V \in F\alpha O(X,\tau)$ and $x_t \le V$. We will prove that $\mathrm{Cl}_\alpha(x_t) \le V$. Let $y_r \not\le V$. Then $y_r \le 1 - V$ and $x \ne y$. This implies that $\mathrm{Cl}_\alpha(y_r) \le \mathrm{Cl}_\alpha(1-V) = 1 - V$. Since $x_t \le V$, then $x_t \not\le \mathrm{Cl}_\alpha$, that is, $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$. Then by assumption, $\mathrm{Cl}_\alpha(x_t) \neg q \, \mathrm{Cl}_\alpha(y_r)$. That is, $\mathrm{Cl}_\alpha(x_t) \le 1 - \mathrm{Cl}_\alpha(y_r) \le V$. Hence (X,τ) is fuzzy αR_0-space. □

Definition 9. Let A be a fuzzy subset of fuzzy topological space (X,τ). The fuzzy α-kernel of A, denoted by $F\,\mathrm{Ker}_\alpha(A)$, is defined to be the set

$$F\,\mathrm{Ker}_\alpha(A) = \bigwedge\{U \in F\alpha O(X,\tau) : A \le U\}. \tag{5}$$

In particular, the fuzzy α-kernel of fuzzy point $x_t \in Pt(X)$ is defined to be the set

$$F\,\mathrm{Ker}_\alpha(x_t) = \bigwedge\{U \in F\alpha O(X,\tau) : x_t \le U\}. \tag{6}$$

Lemma 1. *Let (X,τ) be a fuzzy topological space, and let A be a fuzzy subset of X. Then*

$$F\,\mathrm{Ker}_\alpha(A) = \bigvee\{x_t \in Pt(X) : Aq\,\mathrm{Cl}_\alpha(x_t)\}. \tag{7}$$

Proof. Suppose that $x_t \in Pt(X)$ and $A \neg q \, \mathrm{Cl}_\alpha(x_t)$. Then $A \le 1 - \mathrm{Cl}_\alpha(x_t)$. Since $x_t \not\le 1 - \mathrm{Cl}_\alpha(x_t)$ and $1 - \mathrm{Cl}_\alpha(x_t) \in F\alpha O(X,\tau)$ containing A, then $x_t \not\le F\,\mathrm{Ker}_\alpha(A)$. That is,

$$F\,\mathrm{Ker}_\alpha(A) \le \bigvee\{x_t \in Pt(X) : Aq\,\mathrm{Cl}_\alpha(x_t)\}. \tag{8}$$

Conversely, suppose that $x_t F\,\mathrm{Ker}_\alpha(A)$. That is, there is $U \in F\alpha O(X,\tau)$ such that $A \le U$ and $x_t \not\le U$. Hence $x_t \le 1 - U$ which implies that

$$\mathrm{Cl}_\alpha(x_t) \le \mathrm{Cl}_\alpha(1-U) = 1 - U \le 1 - A. \tag{9}$$

That is, $Aq\,\mathrm{Cl}_\alpha(x_t)$. Hence $\bigvee\{x_t \in Pt(X) : Aq\,\mathrm{Cl}_\alpha(x_t)\} \le F\,\mathrm{Ker}_\alpha(A)$. □

Lemma 2. *Let (X,τ) be a fuzzy topological space and $x_t, y_r \in Pt(X)$. Then $y_r \le F\,\mathrm{Ker}_\alpha(x_t)$ if and only if $x_t \le \mathrm{Cl}_\alpha(y_r)$.*

Proof. Suppose that $x_t \le \mathrm{Cl}_\alpha(y_r)$ and $y_r \not\le F\,\mathrm{Ker}_\alpha(x_t)$. Then there is $U \in F\alpha O(X,\tau)$ such that $x_t \le U$ and $y_r \not\le U$. Hence $y_r \le 1 - U$, which implies that $\mathrm{Cl}_\alpha(y_r) \le \mathrm{Cl}_\alpha(1-U) = 1 - U$. But this is a contradiction with $x_t \le \mathrm{Cl}_\alpha(y_r)$ and $x_t \le U$. Hence $y_r \le F\,\mathrm{Ker}_\alpha(x_t)$. Conversely, suppose that $y_r \le F\,\mathrm{Ker}_\alpha(x_t)$ and $x_t \not\le \mathrm{Cl}_\alpha(y_r)$. Then $x_t \le 1 - \mathrm{Cl}_\alpha(y_r) \in F\alpha O(X,\tau)$. Since $y_r 1 - \mathrm{Cl}_\alpha(y_r)$, then $y_r \le F\,\mathrm{Ker}_\alpha(x_t)$. But this is a contradiction. Hence $x_t \le \mathrm{Cl}_\alpha(y_r)$. □

Lemma 3. *Let (X,τ) be a fuzzy topological space and $x_t, y_r \in Pt(X)$. Then $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$ if and only if $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$.*

Proof. Suppose that $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \le F\,\mathrm{Ker}_\alpha(x_t)$ and $z_\mu \not\le F\,\mathrm{Ker}_\alpha(y_r)$. In the part $z_\mu \le F\,\mathrm{Ker}_\alpha(x_t)$, by Lemma 2, $x_t q\,\mathrm{Cl}_\alpha(z_\mu)$. This implies that $x_t \le \mathrm{Cl}_\alpha(z_\mu)$, that is, $\mathrm{Cl}_\alpha(x_t) \le \mathrm{Cl}_\alpha(z_\mu)$. And similarly, in the part $z_\mu \not\le F\,\mathrm{Ker}_\alpha(y_r)$ we get $y_r \neg q \, \mathrm{Cl}_\alpha(z_\mu)$. This implies $y_r \not\le \mathrm{Cl}_\alpha(z_\mu)$. Since $\mathrm{Cl}_\alpha(x_t) \le \mathrm{Cl}_\alpha(z_\mu)$, then $y_r \not\le \mathrm{Cl}_\alpha(x_t)$. Hence $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$.

Conversely, suppose that $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$. Then there exists fuzzy point z_μ in X such that $z_\mu \le \mathrm{Cl}_\alpha(x_t)$ and $z_\mu \not\le \mathrm{Cl}_\alpha(y_r)$. If $x_t \le \mathrm{Cl}_\alpha(y_r)$, then $\mathrm{Cl}_\alpha(x_t) \le \mathrm{Cl}_\alpha(y_r)$. Hence $z_\mu \le \mathrm{Cl}_\alpha(y_r)$ but this is a contradiction. Then $x_t \not\le \mathrm{Cl}_\alpha(y_r)$, that is, $x_t \le 1 - \mathrm{Cl}_\alpha(y_r)$. Hence $1 - \mathrm{Cl}_\alpha(y_r) \in F\alpha O(X,\tau)$ containing x_t and not y_r. Then $y_r \not\le F\,\mathrm{Ker}_\alpha(x_t)$ and $y_r \le F\,\mathrm{Ker}_\alpha(y_r t)$. Hence $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$. □

Theorem 9. *A fuzzy topological space (X,τ) is fuzzy αR_0-space if and only if for every pair of fuzzy points x_t, y_r in X with $(x \ne y)$ and $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$,*

$$F\,\mathrm{Ker}_\alpha(x_t) \neg q F\,\mathrm{Ker}_\alpha\left(y_r\right). \tag{10}$$

Proof. Suppose that a fuzzy topological space (X,τ) is αR_0-space. Let x_t, y_r be a pair of fuzzy points in X with $(x \ne y)$ and $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$. By Lemma 3, $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$. Suppose that $F\,\mathrm{Ker}_\alpha(x_t) q F\,\mathrm{Ker}_\alpha(y_r)$ for some $z \in X$. Take

$$\mu = F\,\mathrm{Ker}_\alpha(x_t)(z) \vee F\,\mathrm{Ker}_\alpha(y_r)(z) \in (0,1]. \tag{11}$$

Then

$$z_\mu \le F\,\mathrm{Ker}_\alpha(x_t), \qquad z_\mu \le F\,\mathrm{Ker}_\alpha(y_r). \tag{12}$$

In the part $z_\mu \le F\,\mathrm{Ker}_\alpha(x_t)$, by Lemma 2 we get that $x_t \le \mathrm{Cl}_\alpha(z_\mu)$, which implies $\mathrm{Cl}_\alpha(x_t) \le \mathrm{Cl}_\alpha(z_\mu)$. Then by Theorem 8, $\mathrm{Cl}_\alpha(x_t) = \mathrm{Cl}_\alpha(z_\mu)$. Similarly, in the part $z_\mu \le F\,\mathrm{Ker}_\alpha(y_r)$, we get that $\mathrm{Cl}_\alpha(y_r) = \mathrm{Cl}_\alpha(z_\mu) = \mathrm{Cl}_\alpha(x_t)$. This is a contradiction. Therefore, $F\,\mathrm{Ker}_\alpha(x_t) \neg q F\,\mathrm{Ker}_\alpha(y_r)$.

Conversely, we will use Theorem 8 to prove that (X,τ) is fuzzy αR_0-space. Let x_t, y_r be a pair of fuzzy points in X with $(x \ne y)$ and $\mathrm{Cl}_\alpha(x_t) \ne \mathrm{Cl}_\alpha(y_r)$. Then by Lemma 3, $F\,\mathrm{Ker}_\alpha(x_t) \ne F\,\mathrm{Ker}_\alpha(y_r)$. Hence by assumption, we get that $F\,\mathrm{Ker}_\alpha(x_t) \neg q F\,\mathrm{Ker}_\alpha(y_r)$. Suppose that $\mathrm{Cl}_\alpha(x_t) q\,\mathrm{Cl}_\alpha(y_r)$ for some $z \in X$. Take

$$\mu = \mathrm{Cl}_\alpha(x_t)(z) \vee \mathrm{Cl}_\alpha(y_r)(z) \in (0,1]. \tag{13}$$

Then $z_\mu \le \mathrm{Cl}_\alpha(x_t)$ and $z_\mu \le \mathrm{Cl}_\alpha(y_r)$. Hence by Lemma 2, $x_t \le F\,\mathrm{Ker}_\alpha(z_\mu)$ and $y_r \le F\,\mathrm{Ker}_\alpha(z_\mu)$. Then by Lemma 1,

$$F\,\mathrm{Ker}_\alpha(x_t) \le F\,\mathrm{Ker}_\alpha\left(z_\mu\right), \qquad F\,\mathrm{Ker}_\alpha\left(y_r\right) \le F\,\mathrm{Ker}_\alpha\left(z_\mu\right), \tag{14}$$

that is,

$$F\,\mathrm{Ker}_\alpha(x_t) q F\,\mathrm{Ker}_\alpha\left(z_\mu\right), \qquad F\,\mathrm{Ker}_\alpha\left(y_r\right) q F\,\mathrm{Ker}_\alpha\left(z_\mu\right). \tag{15}$$

Hence by assumption,

$$F\,\mathrm{Ker}_\alpha(x_t) = F\,\mathrm{Ker}_\alpha\left(z_\mu\right), \qquad F\,\mathrm{Ker}_\alpha\left(y_r\right) = F\,\mathrm{Ker}_\alpha\left(z_\mu\right). \tag{16}$$

Hence $F\,\mathrm{Ker}_\alpha(x_t) = F\,\mathrm{Ker}_\alpha(y_r)$, that is, $F\,\mathrm{Ker}_\alpha(x_t) q F\,\mathrm{Ker}_\alpha(y_r)$. But this is a contradiction. Hence $\mathrm{Cl}_\alpha(x_t) \neg q \, \mathrm{Cl}_\alpha(y_r)$. Therefore, by Theorem 8, (X,τ) is fuzzy αR_0-space. □

Theorem 10. *For fuzzy topological space (X,τ), the following properties are equivalent:*

(1) *(X,τ) is fuzzy αR_0-space;*

(2) *for every fuzzy set $A \neq 0_X$ and $U \in F\alpha O(X,\tau)$ such that AqU, there exists $V \in F\alpha C(X,\tau)$ such that AqV and $V \leq U$;*

(3) *for every $U \in F\alpha O(X,\tau)$, $U = \bigvee\{V \in F\alpha O(X,\tau) : V \leq U\}$;*

(4) *for every $U \in F\alpha O(X,\tau)$, $U = \bigwedge\{V \in F\alpha O(X,\tau) : U \leq V\}$;*

(5) *for every fuzzy point $x_t \in Pt(X)$, $\mathrm{Cl}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(x_t)$.*

Proof. (1)→(2): Let $A \neq 0_X$ be fuzzy set in X and $U \in F\alpha O(X,\tau)$ such that AqU for some $z \in X$. Take $\mu = A(z) \vee U(z)$. Then $z_\mu \leq A$ and $z_\mu \leq U$. Since $U \in F\alpha O(X,\tau)$ and (X,τ) is fuzzy αR_0-space, then $\mathrm{Cl}_\alpha(z_\mu) \leq U$. Take $V = \mathrm{Cl}_\alpha(z_\mu)$. Then $V \in F\alpha C(X,\tau)$ and $V \leq U$. Since $z_\mu \leq A$, then $\mathrm{Cl}_\alpha(z_\mu)qA$, that is, AqV.

(2)→(3): It is clear that $\bigvee\{V \in F\alpha O(X,\tau) : V \leq U\} \leq U$. Let $x_t \leq U$. Since $U \in F\alpha O(X,\tau)$ and $x_t \neq 0_X$, then there exists $V \in F\alpha C(X,\tau)$ such that $x_t \leq V$ and $V \leq U$. Then $x_t \leq \bigvee\{V \in F\alpha O(X,\tau) : V \leq U\}$, that is, $U \leq \bigvee\{V \in F\alpha O(X,\tau) : V \leq U\}$.

(3)→(4): Obvious.

(4)→(5): Let $x_t \in Pt(X)$ and $y_r \nleq F\,\mathrm{Ker}_\alpha(x_t)$. Then there exits $V \in F\alpha O(X,\tau)$ such that $x_t \leq V$ and $y_r \nleq V$. Hence $y_r \leq 1-V$, which implies that $\mathrm{Cl}_\alpha(y_r) \leq \mathrm{Cl}_\alpha(1-V) = 1-V$. That is, $\mathrm{Cl}_\alpha(y_r) \leq 1 - \bigwedge\{U \in F\alpha O(X,\tau) : V \leq U\}$. Hence there exists $U \in F\alpha O(X,\tau)$ such that $x_t \nleq U$ and $\mathrm{Cl}_\alpha(y_r) \leq U$. Hence $\mathrm{Cl}_\alpha(x_t) \leq 1 - U$. Therefore, $y_r \nleq \mathrm{Cl}_\alpha(x_t)$. That is, $\mathrm{Cl}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(x_t)$.

(5)→(1): Let $U \in F\alpha O(X,\tau)$ and $x_t \leq U$. Then $\mathrm{Cl}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(x_t) \leq U$. Hence (X,τ) is fuzzy αR_0-space. □

Corollary 3. *A fuzzy topological space (X,τ) is fuzzy αR_0-space if and only if $\mathrm{Cl}_\alpha(x_t) = F\,\mathrm{Ker}_\alpha(x_t)$ for all $x_t \in Pt(X)$.*

Proof. Suppose that (X,τ) is fuzzy αR_0-space. Then by Theorem 10, $\mathrm{Cl}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(x_t)$ for all $x_t \in Pt(X)$. Let $y_r \leq F\,\mathrm{Ker}_\alpha(x_t)$. Then by Lemma 2, $x_t \in \mathrm{Cl}_\alpha(y) \leq F\,\mathrm{Ker}_\alpha(y_r)$. Hence $x_t \leq F\,\mathrm{Ker}_\alpha(y_r)$, which implies, by the same lemma, that $y_r \in \mathrm{Cl}_\alpha(x_t)$. Therefore, $\mathrm{Cl}_\alpha(x_t) = F\,\mathrm{Ker}_\alpha(x_t)$ for all $x_t \in Pt(X)$. Conversely, it is obvious by Theorem 10. □

Theorem 11. *For fuzzy topological space (X,τ), the following properties are equivalent:*

(1) *(X,τ) is fuzzy αR_0-space;*

(2) *$x_t \leq \mathrm{Cl}_\alpha(y_r)$ if and only if $y_r \leq \mathrm{Cl}_\alpha(x_t)$ for all $x_t, y_r \in Pt(X)$.*

Proof. (1)→(2): Let $x_t \leq \mathrm{Cl}_\alpha(y_r)$. Since (X,τ) is fuzzy αR_0-space, then, by Corollary 3, $\mathrm{Cl}_\alpha(y_r) = F\,\mathrm{Ker}_\alpha(y_r)$. Hence by Lemma 2, $y_r \leq \mathrm{Cl}_\alpha(x_t)$. Similarly, we examine the converse.

(2)→(1): Let $U \in F\alpha O(X,\tau)$ and $x_t \leq U$. If $y_r \nleq U$, then $x_t \nleq \mathrm{Cl}_\alpha(y_r)$. Then by (2), $y_r \nleq \mathrm{Cl}_\alpha(x_t)$. Hence $\mathrm{Cl}_\alpha(x_t) \leq U$. That is, (X,τ) is fuzzy αR_0-space. □

Theorem 12. *For fuzzy topological space (X,τ), the following properties are equivalent:*

(1) *(X,τ) is fuzzy αR_0-space.*

(2) *$G = F\,\mathrm{Ker}_\alpha(G)$ whenever $G \in F\alpha C(X,\tau)$.*

(3) *if $G \in F\alpha C(X,\tau)$ and $x_t \leq G$, then $F\,\mathrm{Ker}_\alpha(x_t) \leq G$.*

(4) *$F\,\mathrm{Ker}_\alpha(x_t) \leq \mathrm{Cl}_\alpha(x_t)$ for all $x_t \in Pt(X)$.*

Proof. (1)→(2): Let $G \in F\alpha C(X,\tau)$. It is clear that $G \leq F\,\mathrm{Ker}_\alpha(G)$. Let $x_t \nleq G$. Then $x_t \leq 1 - G \in F\alpha O(X,\tau)$. Since (X,τ) is fuzzy αR_0-space, then $\mathrm{Cl}_\alpha(x_t) \leq 1 - G$. Then $\mathrm{Cl}_\alpha(x_t)\neg qG$, and by Lemma 1 we get that $x_t \nleq F\,\mathrm{Ker}_\alpha(G)$. Therefore $G = F\,\mathrm{Ker}_\alpha(G)$.

(2)→(3): In general, $A \leq B$ implies that $F\,\mathrm{Ker}_\alpha(A) \leq F\,\mathrm{Ker}_\alpha(B)$. Therefore, it follows from (2) that $F\,\mathrm{Ker}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(G) = G$.

(3)→(4): Since $x_t \leq \mathrm{Cl}_\alpha(x_t)$ and $\mathrm{Cl}_\alpha(x_t) \in F\alpha C(X,\tau)$, then $F\,\mathrm{Ker}_\alpha(x_t) \leq \mathrm{Cl}_\alpha(x_t)$.

(4)→(1): We show the implication by using the par (5) of Theorem 10. Let $x_t \in Pt(X)$ and $y_r \leq \mathrm{Cl}_\alpha(x_t)$. Then by Lemma 2, $x_t \leq F\,\mathrm{Ker}_\alpha(y_r)$ and by (4), $F\,\mathrm{Ker}_\alpha(y_r) \leq \mathrm{Cl}_\alpha(y_r)$. Hence $x_t \leq \mathrm{Cl}_\alpha(y_r)$ which implies, by Lemma 2, $y_r \leq F\,\mathrm{Ker}_\alpha(x_t)$. Then $\mathrm{Cl}_\alpha(x_t) \leq F\,\mathrm{Ker}_\alpha(x_t)$. Therefore, by Theorem 10, (X,τ) is fuzzy αR_0-space. □

Definition 10. A fuzzy topological space (X,τ) is called fuzzy αR_1-space if for every pair of fuzzy points x_t, y_r in X with $\mathrm{Cl}_\alpha(x_t) \neq \mathrm{Cl}_\alpha(y_r)$, there exists $U, V \in F\alpha O(X,\tau)$ such that $\mathrm{Cl}_\alpha(x_t) \leq U$, $\mathrm{Cl}_\alpha(y_r) \leq V$ and $U\neg qV$.

Theorem 13. *A fuzzy topological space (X,τ) is fuzzy αR_1-space if and only if for every pair of fuzzy points x_t, y_r in X with $F\,\mathrm{Ker}_\alpha(x_t) \neq F\,\mathrm{Ker}_\alpha(y_r)$, there exists $U, V \in F\alpha O(X,\tau)$ such that $\mathrm{Cl}_\alpha(x_t) \leq U$, $\mathrm{Cl}_\alpha(y_r) \leq V$ and $U\neg qV$.*

Proof. Obvious, by Lemma 3. □

Open Problem. Recently, the several authors studied the notion of pairwise openness and closedness as well as pairwise covers in order to study pairwise Lindelöf spaces [19], pairwise continuity and mappings [20], pairwise nearly Lindelöf spaces [21], pairwise weakly regular-Lindelf spaces [22], and the pairwise almost Lindelöf spaces in bitopological setting, see for example, [23–25] which were the extensions of some results due to Balasubramanian [26], Cammaroto and Santoro [27], and Fawakhreh and Kılıçman [28, 29]. It is an open problem to extend these new concepts to the bitopological spaces.

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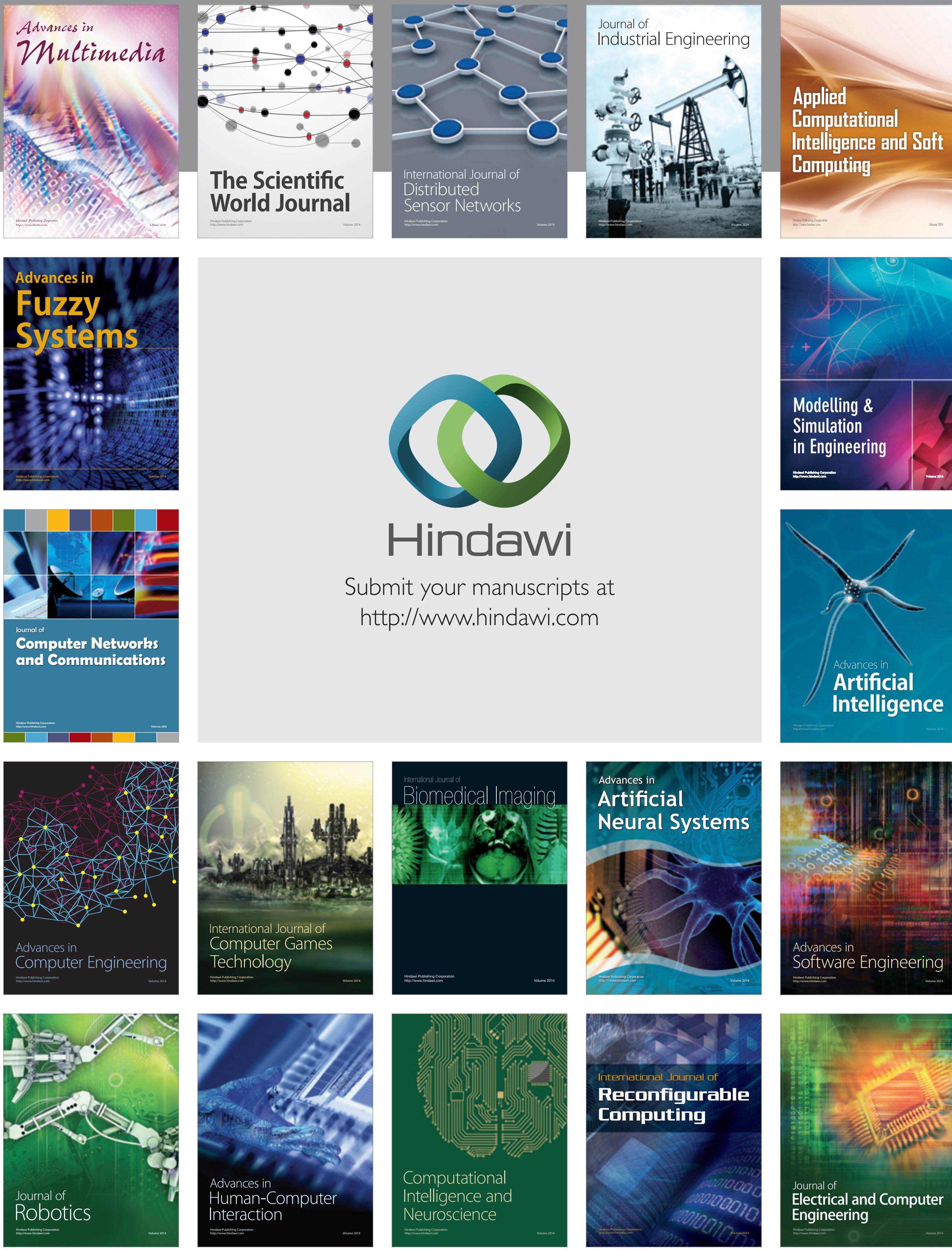
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