Hindawi Publishing Corporation
Advances in Fuzzy Systems
Volume 2012, Article ID 204864, 10 pages
doi:10.1155/2012/204864

Research Article

Excluded-Mean-Variance Neural Decision Analyzer for Qualitative Group Decision Making

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Received 25 May 2012; Accepted 16 July 2012

Academic Editor: Ashu M. G. Solo

Many qualitative group decisions in professional fields such as law, engineering, economics, psychology, and medicine that appear to be crisp and certain are in reality shrouded in fuzziness as a result of uncertain environments and the nature of human cognition within which the group decisions are made. In this paper we introduce an innovative approach to group decision making in uncertain situations by using a mean-variance neural approach. The key idea of this proposed approach is to compute the excluded mean of individual evaluations and weight it by applying a variance influence function (VIF); this process of weighting the excluded mean by VIF provides an improved result in the group decision making. In this paper, a case study with the proposed excluded-mean-variance approach is also presented. The results of this case study indicate that this proposed approach can improve the effectiveness of qualitative decision making by providing the decision maker with a new cognitive tool to assist in the reasoning process.

1. Introduction

In any professional field, such as law, engineering, economics, psychology, and medicine, we are often faced with ambiguous choices in our decision making processes. A decision making process involves *perceptions* relating to neurological processes of acquiring and mentally interpreting qualitative (fuzzy) information. Our cognitive process relating to the process of acquiring knowledge by the use of reasoning, intuition, and perception for the evaluation of suitable courses of action (decisions) among several alternative choices plays a very large part in human decision making [1]. Such decisions are made where human judgment is called upon to choose among competing and viable options. These decisions are often made under the environments of vague (fuzzy), incomplete, and conflicting evidences. Thus, our decision making process is often masked by fuzzy and statistical uncertainties. A change in the formulation of these cognitive processes can alter the results of our decision significantly.

In order to have the best possible decisions in an uncertain environment, it is necessary to have an opportunity to continuously adapt the decision parameters by using new information. Therefore, the decision making processes would benefit by incorporating some feedback mechanism that allows various outcomes to be assessed and evaluated. During the past, a number of methods to improve the quality of decisions have been developed by applying some novel approaches, both statistical and fuzzy [1].

In real life, the process of group decision making is subject to the different perceptions and cognitive abilities of individual panel members (evaluators), and thus, cognitive uncertainty is introduced into the group decision making environment. These cognitive uncertainties (fuzziness) arise

due to the different background and experience of individual panel members in the group and are thus the function of individual thinking processes. Thus, the opinions of evaluators in a group are masked by both cognitive (fuzzy) and statistical uncertainties. Therefore, there is a continuous search for the development of a better approach to enhance the quality of the decision making processes.

In this paper, we present a novel approach for qualitative group decision making, namely *excluded-mean-variance neural decision analyzer* (E-MVNDA). Here, we introduce a new notion of excluded mean and excluded variance. This proposed excluded-mean-variance neural decision analyzer (E-MVNDA) consists of the following three stages:

(i) *quantifier* (an interface between human and computer): for computational purposes, the quantification of the perceived subjective opinion of individual evaluators on each piece of evidence is converted into some fuzzy number, say over the interval $[-10, 10]$;

(ii) *preprocessor*: first, the two statistical parameters, excluded mean ($\boldsymbol{\mu}$) and excluded variance ($\mathbf{v}$), of the perceived subjective opinions of evaluators expressed in the form of fuzzy numbers are computed. Then the neural input $\mathbf{u}$ for the neural decision processor is computed which is the product of $\boldsymbol{\mu}$ and a variance influence function (VIF);

(iii) *neural decision processor* (NDP): the NDP uses $\mathbf{u}$, the output of the preprocessor, under various classifications of evidences, and finally yields a decision $\mathbf{y}$.

In this paper, we evaluate the performance of the proposed excluded-mean-variance neural decision analyzer (E-MVNDA) on a generic case study.

2. Design of Excluded-Mean-Variance Neural Decision Analyzer (E-MVNDA)

The group decision making analyzer in a qualitative (fuzzy) environment presented in this paper is based upon the confluence of fuzzy statistical and neural approaches [2]. For the formulation of this novel group decision making approach in a qualitative environment, we introduce three new statistical parameters: *excluded mean* ($\boldsymbol{\mu}$), *excluded variance* ($\mathbf{v}$), *and variance influencing function (VIF)*. In this VIF approach, we modify the excluded-mean value $\boldsymbol{\mu}$ of the quantified evaluations for each piece of evidence (factor). When we examine the qualitative language of decision making in a group, it becomes apparent that the opaqueness of the process hides the impact of this dimension of variance. It should be emphasized that without the incorporation of VIF, the decision making will be plagued by the large variance resulting from the undue influence of the unscreened subjective ranking tacitly present in any qualitative decision making process.

In a group decision making process on a given case, each member of the group expresses his/her opinion (evaluation), this opinion is influenced by individual perception, which, in turn, is influenced by his/her experience. The evaluation employs fuzzy linguistic quantifiers which quantify linguistic expressions such as "*good*," "*very good*," "*fair*," or "*bad*" [3, 4]. In order to express the opinion (evaluation) of each evaluator, the qualitative (fuzzy) opinions are quantified into some fuzzy numbers, say over the interval of $[-10, 10]$. For example, if someone was assessing the quality of various tea brands, the assessor may regard the quality of the tea as being "very tasty" whereas another may score it as "good, but tart." During the process of quantification of their assessments, the first tester may give a score of his taste as "9" over the interval of, say, $[-10, 10]$, while the second may give a score as "6." These scores are subjective (fuzzy). The quantified group opinions are masked by both fuzzy and statistical (random fluctuations) uncertainties. In this collected opinion, fuzziness arises because of the subjectivity (perception and cognition) of individual evaluators, and the statistical uncertainty arises because of the collection of fuzzy evaluations from the group of decision makers. Figure 1 shows an example of the fuzzy uncertainty associated with a quantified number, $s = 5$, from a qualitative opinion of an evaluator.

The procedure in the excluded-mean-variance neural decision analyzer (E-MVNDA) for a group decision making, as shown in Figure 2, is as follows: first, each subjective (qualitative) opinion is quantified into a fuzzy number (evaluation), say s. The preprocessor computes excluded-mean ($\boldsymbol{\mu}$) and excluded-variance ($\mathbf{v}$) for each piece of evidence (factor) using the fuzzy data provided by n evaluators. Then, a decision is made by a neural decision processor (NDP). In the following subsections, we give the details of the preprocessor and neural decision processor (NDP).

2.1. Preprocessor: Excluded Mean, Excluded Variancen and Variance Influence Function. Let us consider a group decision-making case with m evidence/factors and n evaluators as shown in Table 1.

The individual pieces of evidence/factors are the features of a case to be evaluated, and the evaluators of the group are asked to quantify their opinions for each piece of evidence with a fuzzy number over an interval, say $[-10, 10]$. The preprocessor of E-MVNDA computes statistical values (excluded mean, $\boldsymbol{\mu}$, and excluded variance, $\mathbf{v}$) for each piece of evidence/factor using the evaluation of the group evaluators.

Let us consider a situation with a group of n evaluators and m pieces of evidence/factors. In the evaluation process, each evaluator (j, $j = 1, 2, \ldots, n$) is asked to quantify his/her opinion on each piece of evidence (factor, i, $i = 1, 2, \ldots, m$) using a fuzzy number s_{ij} over an interval, say $[-10, 10]$. In some cases, some p evaluators do not wish to mark a score and instead mark "X" for some evidence (factor, i) for any of the following reasons:

(i) the evaluator feels that this particular evidence is not relevant for this decision; and/or

(ii) the evaluator considers himself unqualified to judge this particular evidence appropriately.

Table 1: Evaluation matrix ($m \times n$) and preprocessing for E-MVNDA. (m = number of evidences; n = number of evaluators.)

	Evaluation							
	Evaluators					Preprocessing		
Evidences (factors)	E_1	$\cdots$	E_j	$\cdots$	E_n	Excluded mean (μ)	Excluded variance (v)	Variance influence function (VIF) ($f(\alpha v)$)
F_1	s_{11}	$\cdots$	s_{1j}	$\cdots$	s_{1n}	μ_1	v_1	$f(\alpha v_1)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
F_i	s_{i1}	$\cdots$	s_{ij}	$\cdots$	s_{in}	μ_i	v_i	$f(\alpha v_i)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
F_m	s_{m1}	$\cdots$	s_{mj}	$\cdots$	s_{mn}	μ_m	v_m	$f(\alpha v_m)$

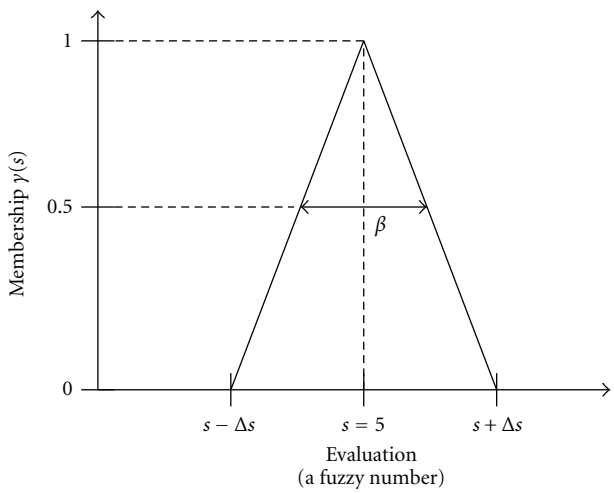

Figure 1: A triangular membership function ($\gamma \in [0,1]$) for a fuzzy number (evaluation) $s = 5$. At the membership $\gamma = 0.5$, the width $\beta = \Delta s$ of the triangular membership function represents the measure of uncertainty associated with the fuzzy number s.

In the computation of mean and variance, the number of p opinions marked "Xs" are excluded. Thus, the excluded-mean (μ_i) and the excluded-variance (v_i) are computed which are defined as follows.

Excluded-Mean:

$$\mu_i = \left(\frac{1}{n-p}\right)\left[\sum_{j=1}^{n} s_{ij}\right], \quad \text{for } s_{ij} \neq X. \tag{1}$$

Excluded-Variance:

$$v_i = \left(\frac{1}{n-p}\right)\left[\sum_{j=1}^{n} \left(s_{ij} - \mu_j\right)^2\right], \quad \text{for } s_{ij} \neq X. \tag{2}$$

The excluded mean of opinions indicates the average opinions of the group, whereas the extended-variance in the scores indicates the degree of inconsistency (disagreement or fluctuations) of the opinions of the evaluators. The *low* variance indicates a *high* consistency in the opinions, whereas the *high* variance implies a *low* consistency. Thus, the mean opinion (μ_i) alone of each evaluator is *not* a true indicator of the group opinion. Therefore, the excluded mean is further weighted by a function of excluded-variance, a variance influence function (VIF). One can formulate many types of VIF. One of the VIFs that we use in this paper is defined as

$$f(\alpha v_i) = \exp(-\alpha v_i), \quad (i = 1, 2, \ldots, m), \tag{3}$$

where $\exp(-\alpha v_i)$ is an exponential function, and α is the gain that provides the importance to VIF as shown in Figure 3.

It should be noted that a large difference of opinions will yield a large excluded-variance v_i and, as shown in Figure 3, as excluded-variance v_i increases, the significance of the evidence ($f(\alpha v_i)$) decreases. Also, as shown in this diagram, as the gain α changes, the significance of the relationship changes. The preprocessor of E-MVNDA is illustrated in Figure 4.

As found in the literature, current decision-making processes consider only the conventional mean of the individual scores, which does not quite reflect the nature of the evaluations. In this proposed approach, in the preprocessor we compute excluded mean (μ_i), excluded variance (v_i), and VIF ($f(\alpha v_i)$) for each individual piece of evidence, and then finally compute $\mathbf{u}$, the input to neural decision processor (NDP), which is a product of μ_i and $f(\alpha v_i)$. In group decision making, this process of computing neural input $\mathbf{u}$ more accurately represents the nature of decision making, in the statistical sense. The excluded-variance v_i for each piece of evidence is determined by the spread of the evaluators' subjective evaluations for some particular evidence. In a group decision making process, it is commonly observed that some evaluations are very close (low variance), whereas some evaluations are very much diverse (high variance). It is also to be noted that the *diversity* (*fluctuation*) of evaluations for certain evidence arises due to the different experiences, subjectivity and cognitions of individual evaluators. In order to assess the agreement of evaluations for each piece of evidence, it is necessary that some weight be assigned to each piece of evidence in the evaluation process. This basic principle of variance influence function (VIF) is expressed as follows.

(i) If the evaluations for a piece of evidence are in general agreement, then the excluded-variance is relatively low, and, thereby, the influence of that evidence should be high.

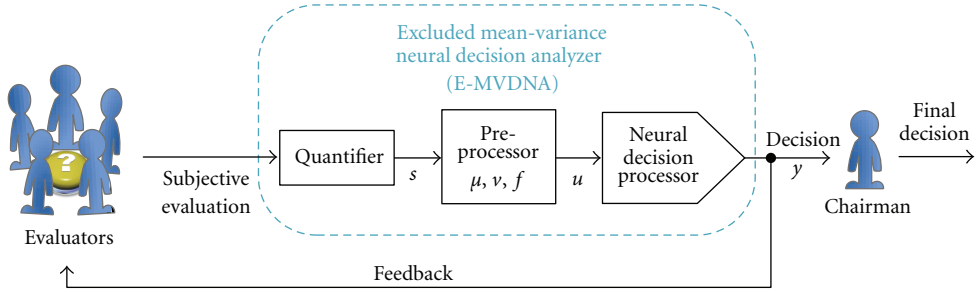

Figure 2: Flow chart of excluded-mean-variance neural decision analyzer (E-MVNDA).

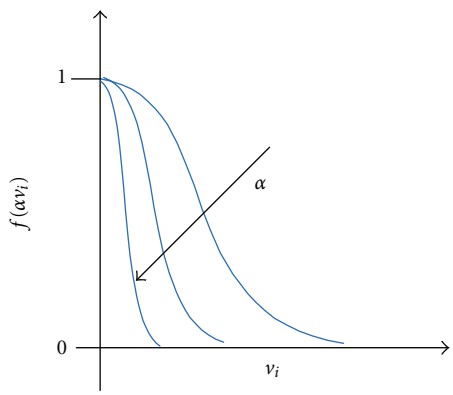

Figure 3: A variance influence function (VIF, $f(\alpha v_i)$) that decreases exponentially with increasing excluded-variance v_i. The gain α represents the importance of the variance, (3). As α increases, the slope of the VIF becomes steeper.

(ii) If the evaluations for a piece of evidence are not in general agreement, then the excluded-variance is relatively high, and, thereby, the influence of that evidence should be low.

2.2. Neural Decision Processor (NDP). In general a neural network (NN) is composed of many neural layers, and each neural layer has many neural units. NN is one of the most powerful tools for classification. It provides a superior procedure for identification and classification and has therefore been used widely in this type of research [5]. A neural unit was inspired by the study of biological neurons such as the synaptic operation and the somatic operation. In the synaptic operation, new input information is perceived through the memory cell (neural weight, w). In the somatic operation, the perceived information is processed by a linear or nonlinear mapping function ($\Phi[\cdot]$). A sigmoidal function is commonly applied for a mapping function due to its special characteristics exhibiting a progression from a small beginning to an accelerated end as natural processes [6].

Considering the special features of conventional NN, we propose a neural decision processor (NDP) for group decision making as shown in Figure 5. The NDP presented in this paper is composed of two neural layers. The first neural layer is the category layer, and the second one is the decision layer [7, 8].

A general structure of a neural unit is illustrated in Figure 6. A neural unit is composed of two operations, the synaptic operation and the somatic operation. The synaptic operation is the sum of products of neural inputs ($\mathbf{u}$) and neural weights ($\mathbf{w}$) which represent the past experiences. The somatic operation is a nonlinear mapping process. In the following, we briefly define the synaptic and somatic operations.

Let us consider a set of neural input vector, $\mathbf{u}$, consisting of m neural inputs, defined as

$$\mathbf{u} = \{u_1, u_2, \ldots, u_m\} \in R^m \tag{4}$$

and, a set of corresponding neural synaptic weight vector, $\mathbf{w}$, defined as

$$\mathbf{w} = \{w_1, w_2, \ldots, w_m\} \in R^m. \tag{5}$$

In synaptic operation, we consider a threshold (bias) as $u_0 = 1$ with a weight w_0. Combining this threshold (u_0) with the neural inputs (u_i, $i = 1, 2, 3, \ldots, m$), an augmented neural input vector $\mathbf{u_a}$ and an augmented neural weight vector $\mathbf{w_a}$ are defined as

$$\begin{gathered} \mathbf{u_a} = \{u_0, u_1, u_2, \ldots, u_m\} \in R^{m+1}, \quad u_0 = 1, \\ \mathbf{w_a} = \{w_0, w_1, w_2, \ldots, w_m\} \in R^{m+1}. \end{gathered} \tag{6}$$

Now, the synaptic and the somatic operations are defined as follows.

Synaptic operation:

$$z = \sum_{i=0}^{m} w_i \cdot u_i = \mathbf{w_a^T} \cdot \mathbf{u_a}. \tag{7}$$

Somatic operation:

$$y = \Phi[g \cdot z], \tag{8}$$

where g is the somatic gain

The input to the NDP is the output vector from the preprocessor. As explained in the previous section, the statistical fluctuations in group opinions can be dealt with by the variance influence function (VIF) in the preprocessor.

In this neural decision processor (NDP), *the category layer* is determined by the process of group decision making. In most cases, the evidence identified as relevant will have

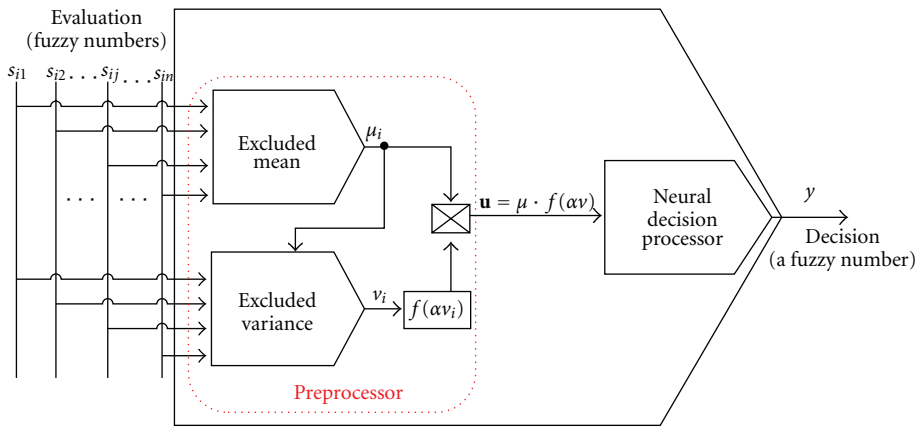

Figure 4: Schematic diagram of the preprocessor for E-MVNDA.

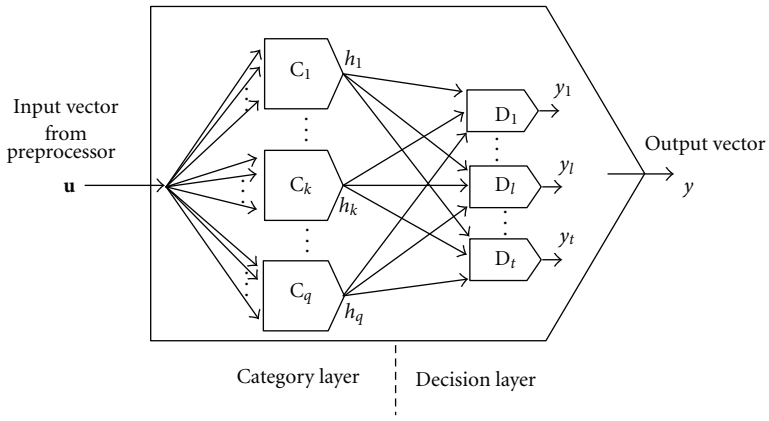

Figure 5: Neural decision processor (NDP) for a group decision making process. This NDP is composed of a category layer and a decision layer.

some common features. Therefore, the pieces of evidence that have common features are grouped into a class of category. In this study, we define a category layer which classifies evidence in q categories $(C_1, C_2, \ldots, C_q)$. Furthermore, each such evidence is weighted by different values (weight, w) in different categories. For example, the evidence F_1 can be classified into category C_1 as well as in C_3; however, the corresponding weights of F_1 in C_1 and C_3 may be different due to the degree of importance of F_1 in each category. The weights (w) of the neural units in the category layer are preassigned by unanimous group assessments. The number of neural units in *the category layer* is determined by the number of categories of evidence.

Finally *the decision layer* of NDP accumulates the information ($\mathbf{h}$) from each neural unit in the category layer. In this layer, the importance of the categories is ranked by assigning different weights. The neural weights ($\mathbf{w}$) in this decision layer are also preassigned by the members of a group.

The excluded-mean variance neural decision analyzer (E-MVNDA) proposed in this paper is an analytical tool to aid in a group decision making process. This is a generic mathematical decision analysis model that can be applied in a variety of real life decision making processes in a qualitative language (fuzzy) environment.

3. A Case Study: New Product Development

In industries, it is fundamental to determine if a new product can be competitive and/or successful in the market before deciding a new product line. Accurate decision making on the new product development becomes more important, and a committee of experts evaluates the idea of the new product under the confluence of various pieces of evidence/factors. Recently, many applications of fuzzy set theory employing evaluations and survey analysis have been used in industrial engineering research [9–11].

We apply E-MVNDA to a group decision for the development of a new product with 50 evaluators and 14 factors (characteristics of the product) on the new product. A survey (benchmark) was carried out using these 50 evaluators, and their evaluations were expressed by scores over the interval $[-10, 10]$ as shown in Table 2.

Table 2 shows the evaluation for each piece of evidence by individual evaluators as well as the calculated statistical parameters, excluded-mean ($\boldsymbol{\mu}$), excluded-variance ($\mathbf{v}$) and variance influence function (VIF), for each factor (evidence). "X" mark in the table implies "irrelevance", and it indicates "no evaluation" which may occur during the evaluation of factors (evidence) as discussed earlier.

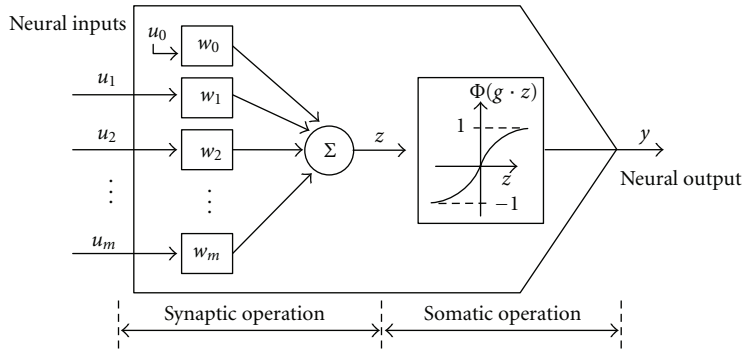

Figure 6: The structure of a neural unit consisting of synaptic operation and somatic operation.

Table 2: Evaluation of factors (evidences) and pre-processing (excluded mean, excluded variance, and variance influence function) of each factor (evidence). (number of factors (evidences), $m = 14$; number of evaluators, $n = 50$.)

	Evaluation							
	Evaluators					Preprocessing		
Evidences (factors)	E_1	E_2	$\cdots$	E_{49}	E_{50}	Excluded mean (μ)	Excluded variance (ν)	Variance influence function (VIF) ($f(\alpha\nu)$) ($\alpha = 1$)
F_1	5	X	$\cdots$	3	1	1.93	7.97	0.3
F_2	-1	X	$\cdots$	-2	-8	-2.26	6.82	1.1
F_3	-1	5	$\cdots$	X	1	2.61	9.98	0
F_4	X	1	$\cdots$	-1	X	0.05	2.27	102.8
F_5	-6	-3	$\cdots$	1	-4	-2.9	7.74	0.4
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
F_{13}	4	2	$\cdots$	2	X	3.54	11.54	0
F_{14}	3	7	$\cdots$	7	2	3.82	9.22	0

("X" represents "irrelevant". The complete table is *presented in Table 3*.)

In this case study, for each neural unit in the category layer, we assigned an equal weight, $w = 1$, for all the factors (evidence) in that category with the threshold $w_0 = 0$, and in the somatic operation, a linear mapping function with gain $g = 1$ was assigned.

For this particular case study having 14 pieces of evidence, we classified these pieces of evidence into four categories with the weights in the decision layer as follows:

(i) Category 1 (C_1): F_2, F_5, F_6, F_7, F_8, F_9 and F_{11}, $w_1 = 15$.

(ii) Category 2 (C_2): F_2, F_3, F_5, F_6, F_7, F_9, F_{11}, and F_{12}, $w_2 = 10$.

(iii) Category 3 (C_3): $F_2, F_5, F_6, F_7, F_9, F_{11}$, and F_{13}, $w_3 = 5$.

(iv) Category 4 (C_4): F_1, F_4, F_{10} and F_{14}, $w_4 = 1$.

It should be noted that some of the evidence is common to various categories, for example, F_2 is included in categories C_1, C_2 and C_3. In this case study, we assume that in the synaptic operation, the threshold $w_0 = 0$, and in the somatic operation, the mapping function $\Phi(gz) = 100 \tan h(gz)$ with gain $g = 0.5$.

The output Φ of E-MVNDA is a bipolar function over the interval $[-100, 100]$, where a negative value represents the "negative" decision with a confidence level between $[-100\%, 0\%]$, and a positive value implies the "positive" decision with a confidence level between $[0\%, 100\%]$, whereas $\Phi = 0$ implies a neutral decision. In our decision making process, to achieve some meaningful results, we define the confidence zone over the interval $\pm\rho\%$, for example, in this study we set $\rho = 30$.

3.1. Validation of E-MVNDA with Group Decision. In order to validate our proposed model, we compare the benchmark group decision and the result of E-MVNDA. In the group decision making, an individual member of the group provides his decision in the form of "*Yes*" or "*No*". If the number of "Yes" is Y, and the number of "No" is N, then we define the confidence level η as

$$\eta = \frac{Y - N}{Y + N} \times 100\ (\%). \tag{9}$$

A positive η implies a positive decision (Yes), and a negative η represents a negative decision (No).

In this benchmark group survey with 50 evaluators shown in the last column of Table 3 22 evaluators voted "*Yes*"

and 28 voted "*No*" for the proposed new production line. This implies that a "negative" decision was made with the confidence level, $((22-28)/50)\times 100 = -12\%$. This "negative" decision with low confidence (-12%) is not within the significant level. For the validation of E-MVNDA presented in this study, if the results from E-MVNDA are similar to that of the benchmark group decision, then, we can safely rely on our proposed E-MVNDA configuration.

For the E-MVNDA process, first, in the pre-processing, we computed excluded mean μ, excluded variance v, and variance influence function (VIF, f) with gain $\alpha = 1$. Following this, the outputs of the preprocessor were fed to the neural decision processor (NDP) with assigned weights (w) in the category and the decision layers.

As shown in Figure 7, after the decision making process of the E-MVNDA, the decision value (judgment) becomes -0.23 which implies that the E-MVNDA reflects a "negative" decision with the confidence level -11.6% which is very close to the decision of the benchmark group (-12%) as well.

As shown in Figure 7, the decisions from E-MVNDA as well as the benchmark group lie approximately in the same low confidence level, and the group decision may easily be changed from positive to negative or negative to positive when some of the evaluators change their opinions. In the lower confidence level, the decisions are not very well defined.

3.2. Fact-Finding Process of Individual Evaluator by E-MVNDA: A Feedback Process in Decision Making. After validating the E-MVNDA with the benchmark group decision, all of the evidence could be ranked by the values of neural weights in *the category and the decision layers*. The process of making such rankings in itself would aid the decision maker in his/her thinking. "Did the evaluator place too much emphasis on that evidence?" is a question that would then be open to meaningful review, either by others or individuals alone. The exploration of an evaluation is called the "fact-finding process" which is a feedback (review) process in decision making. Thus, we applied E-MVNDA to review the decision of an individual evaluator. In this review process, the decisions of individual evaluators were examined with feedback by varying their evaluations on certain pieces of evidence. The results of the pre-processing shown in Table 3 show that F_4 and F_{10} give the lowest values of excluded-variance v, thereby, the highest values of variance influence function (VIF, f), which indicates that these two pieces of evidence were the most dominating features in this decision making.

In this case study, we explored E_1's evaluation. Before changing the evaluations, we found that the decision confidence of E_1 was -14%. After changing the evaluations by varying the factors F_1 and F_{10}, the confidence level of E_1's decision varies. The changes that result in E_1 by varying the scores of F_1 and F_{10} are shown in Figure 8 and Figure 9, respectively.

The lower weighted evidence has little effect on the individual decision process. As shown in Figure 8, the decision changes of E_1 with different evaluations of F_1, the yellow (bigger) circles are accumulated closely at certain confidence levels, which indicates that the decisions of E_1 are mostly the same regardless of E_1's evaluations of F_1. The changes of F_2, F_3, F_5, F_6, F_7, F_8, F_{11}, F_{12}, F_{13}, and F_{14} result in similar outcomes. However, the other evidence (F_4, F_9, and F_{10}) played a significant role in E_1's decision making.

In Figure 9, the changes in decision of E_1 caused by varying F_{10} are described. The yellow (bigger) circles are widely distributed over the confidence levels of $[-98.26\%, 97.02\%]$, which indicates that the decisions of E_1 is actually being altered with different evaluations of F_{10}. The changes that arise by varying F_4 and F_9 result in similar outcomes. This observation implies that a change in the decisions by other evaluators in a group may result in a similar wide distribution over the confidence level.

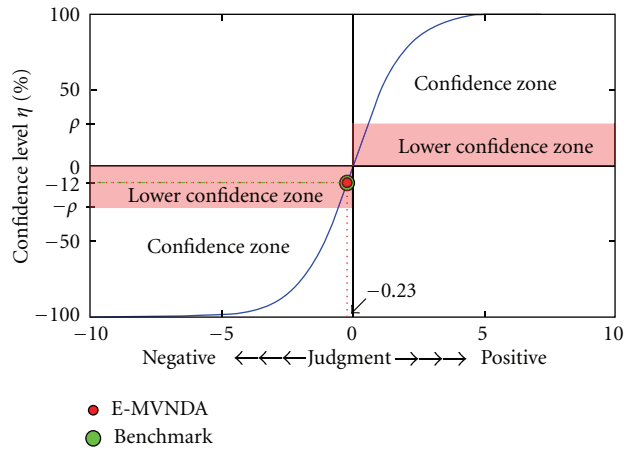

Figure 7: Group decisions by E-MVNDA and benchmark. In this study, both decisions of E-MVNDA and benchmark lie in the lower confidence zone.

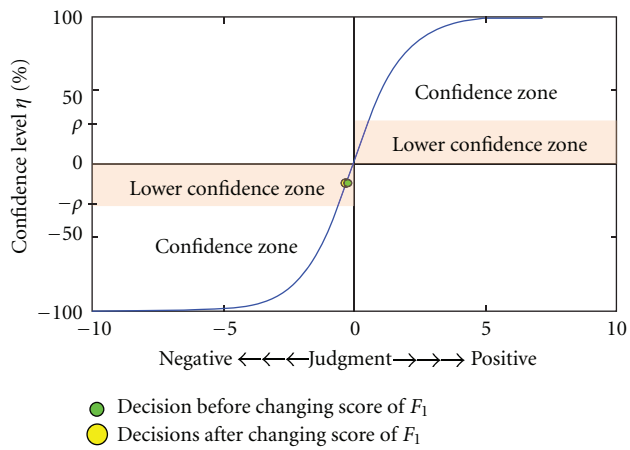

Figure 8: Fact finding of evaluator E_1 by varying the evaluation of factor F_1 over the interval $[-10, 10]$. Before changing the score of the factor F_1, the confidence of the decision of evaluator E_1 is -14%. After changing the score of the factor F_1, the confidence level of the decision of evaluator E_1 varies only over a small range of $[-13.96\%, -13.37\%]$.

Table 3: Evaluation of factors (evidences) and pre-processing (excluded mean, excluded variance, and variance influence function) of each factor (evidence), and individual decisions. (number of factor (evidence), $m = 14$; number of evaluators, $n = 50$.)

Evaluators	Evidence/factors														Individual decision
	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	
E_1	5	−1	−1	X	−6	−5	1	3	X	X	−2	1	4	3	*No*
E_2	X	X	5	1	−3	−2	−2	5	−5	−1	4	5	2	7	*No*
E_3	−3	−4	9	2	−2	−1	−7	9	X	1	−2	6	−1	4	*Yes*
E_4	5	−2	X	1	−6	−4	1	10	X	−1	−3	2	6	5	*Yes*
E_5	1	−1	4	X	X	−5	−2	−1	−2	X	X	5	X	−1	*No*
E_6	2	−1	5	−3	−1	−6	X	2	X	X	−1	6	1	5	*No*
E_7	−2	−2	3	1	−6	1	−2	2	−2	2	−4	3	6	4	*No*
E_8	3	−1	2	X	−4	2	−4	5	1	X	1	5	−1	X	*No*
E_9	4	−3	2	1	X	−1	X	8	−4	1	1	−7	8	2	*Yes*
E_{10}	4	−2	−2	−1	−7	1	−1	6	−1	X	3	5	4	7	*Yes*
E_{11}	2	1	5	1	−4	6	−4	5	−5	X	−6	4	−2	X	*Yes*
E_{12}	5	−5	3	1	−3	X	−4	3	−2	X	X	1	6	5	*Yes*
E_{13}	X	−3	−2	1	X	X	−3	4	−1	2	−3	2	6	3	*Yes*
E_{14}	−1	2	X	2	−5	1	2	6	−1	1	7	5	7	5	*Yes*
E_{15}	3	−5	−1	−1	−7	2	−2	10	−2	1	1	X	2	9	*Yes*
E_{16}	X	−2	8	X	−6	−1	X	7	1	−1	X	X	7	7	*Yes*
E_{17}	1	3	4	−1	4	−4	5	5	−2	1	−4	1	6	7	*Yes*
E_{18}	5	1	−2	1	−4	−2	1	−1	3	1	−1	3	2	9	*Yes*
E_{19}	−2	−5	5	1	−6	4	−1	3	−3	1	−1	8	8	4	*Yes*
E_{20}	3	−4	7	3	−5	1	−6	10	−2	X	X	−2	−2	3	*No*
E_{21}	6	−3	5	−1	X	3	−4	10	−1	1	2	−6	8	−1	*Yes*
E_{22}	−1	−2	−1	X	X	−2	−2	3	X	1	X	−4	9	3	*No*
E_{23}	3	X	1	1	−3	−1	4	5	X	X	1	3	10	4	*Yes*
E_{24}	X	X	6	1	X	3	−3	X	X	1	1	−4	5	2	*No*
E_{25}	7	−7	1	1	3	−2	−6	1	−5	X	X	−1	7	3	*No*
E_{26}	4	2	2	X	−6	−6	X	7	−1	2	4	3	5	1	*No*
E_{27}	5	−2	−2	2	−6	−2	−7	2	−4	X	X	−2	1	6	*No*
E_{28}	2	−7	2	−1	X	−1	−8	9	1	1	5	5	X	6	*No*
E_{29}	X	−6	1	−1	X	X	3	7	−3	2	−1	3	2	5	*No*
E_{30}	1	−3	1	−1	−4	2	3	4	4	X	−6	7	1	9	*No*
E_{31}	2	−3	2	X	X	−1	−10	10	−1	1	−1	2	1	2	*No*
E_{32}	1	−1	1	−1	−3	1	−3	7	1	−1	−3	X	8	5	*Yes*
E_{33}	3	4	3	−2	−3	−2	−4	8	−4	X	−4	2	6	−1	*No*
E_{34}	−1	−2	3	X	−1	−1	−1	8	−2	1	−2	−1	−1	2	*No*
E_{35}	−1	−5	3	X	1	2	−5	8	X	X	3	1	1	7	*Yes*
E_{36}	X	−5	7	X	−5	3	3	3	−2	X	4	−4	4	−3	*No*
E_{37}	−3	−4	X	1	−2	−1	−2	3	X	X	6	X	−1	7	*No*
E_{38}	X	1	6	1	−3	−4	−7	9	2	X	2	9	4	X	*Yes*
E_{39}	3	−1	2	−2	−3	X	3	−2	−1	X	−4	4	9	−1	*No*
E_{40}	X	X	1	X	−2	−1	−2	3	−3	−1	−1	4	2	X	*No*
E_{41}	−2	−1	−1	X	1	−4	−6	10	−2	X	1	5	3	X	*No*
E_{42}	5	−6	−3	2	−4	−3	−2	7	−2	1	−2	4	6	−3	*No*
E_{43}	X	−2	4	−1	−2	−3	−8	1	4	X	−1	−2	−1	1	*No*
E_{44}	4	−1	2	−3	2	X	−2	5	−1	X	1	8	3	X	*Yes*
E_{45}	−1	−1	4	−1	−3	X	−2	6	X	X	1	4	X	6	*Yes*
E_{46}	−4	−2	8	−2	2	X	1	4	1	X	1	3	1	3	*Yes*
E_{47}	6	−1	9	1	−2	X	−8	6	−1	2	−3	−3	1	4	*Yes*

Table 3: Continued.

Evaluators	Evidence/factors														Individual decision
	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	
E_{48}	1	−2	−2	−1	−2	−5	−3	5	1	X	−3	−4	−2	4	*No*
E_{49}	3	−2	X	−1	1	1	1	−1	2	−1	−3	−1	2	7	*No*
E_{50}	1	−8	1	X	−4	1	−4	3	−2	−2	−4	3	X	2	*No*
μ	1.93	−2.26	2.61	0.05	−2.90	−0.86	−2.37	5.14	−1.15	0.62	−0.37	1.98	3.54	3.82	
ν	7.97	6.82	9.98	***2.27***	7.74	8.13	12.77	10.58	5.26	***1.37***	9.91	14.69	11.54	9.22	
f ($\times 10^{-3}$)	0.3	1.1	0	***102.8***	0.4	0.3	0	0	5.2	***255.1***	0	0	0	0	

"X" represents "irrelevant".
(Note that the two highest values of the VIF corresponding to the two lowest variances are indicated in bold and italic).

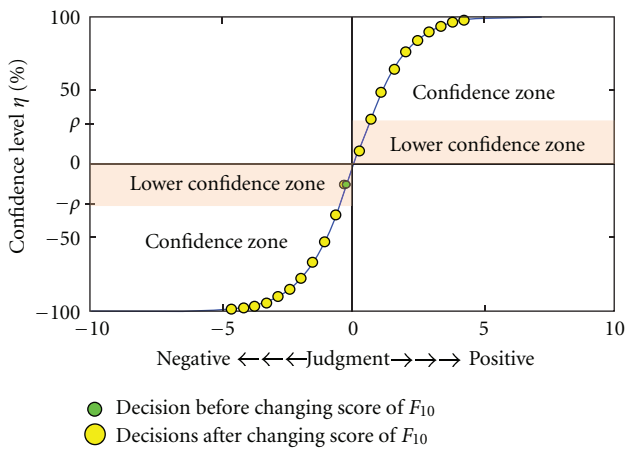

Figure 9: Fact finding of evaluator E_1 by varying the evaluation of factor F_{10} over the interval $[-10, 10]$. Before changing the score of the factor F_1, the confidence of the decision of evaluator E_1 is −14%. After changing the score of the factor F_{10}, the confidence level of the decision of evaluator E_1 varies over a large range of [−98.26 %, 97.02%].

4. Conclusions

In this paper, we have introduced a novel approach for a group decision making process, namely the excluded-mean-variance neural analyzer (E-MVNDA). This is accomplished by employing a new neural approach for quantitative group decision making. The key idea of this proposed approach is to update the output of the preprocessor by applying a variance influence function (VIF) which emphasizes the importance of each piece of evidence, thereby reducing the statistical uncertainty that occurs in the process of transforming qualitative expressions to quantitative scoring. This improves the evaluations in group decision making processes.

A case study of a group decision making of a "*Yes*" or "*No*" variety on a new product development was carried out to demonstrate the application of E-MVNDA. The proposed decision analysis algorithm outlined in this paper provided more information and a more secure outcome. The results from this study show that E-MVNDA can assist such decision making processes. The result of E-MVNDA reveals and identifies which variances of which evidence can impact the process in group decision making. Further, E-MVNDA was applied to an individual fact-finding process as a review process using the outcomes from the group decision, which represents a feedback process during a decision making process. E-MVNDA shows how the individual decision could be changed by altering the evaluation of certain evidence. After changing individual decisions, the group decision may also be changed. The advantage of applying E-MVNDA in a process of group decision making is that it allows for more precisiation [12, 13], without losing the fuzzy richness of the reality under consideration.

In summary, E-MVNDA can assist decision makers by this process of weighting and ranking the evidence they rely upon. E-MVNDA is a useful tool to be applied for decision making in qualitative language environments such as business, law, or public policy.

Acknowledgment

The authors wish to acknowledge the financial support from the Natural Sciences and Engineering Research Council of Canada through Discovery, Strategic, and Collaborative Research & Development grants (NSERC Grants RGPIN-170464, STPGP-365290, STPGP-350861, and NNAPJ-376336).

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