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Research Article

Fuzzy Shortest Path Problem Based on Level λ-Triangular LR Fuzzy Numbers

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In problems of graphs involving uncertainties, the fuzzy shortest path problem is one of the most studied topics, since it has a wide range of applications in different areas and therefore deserves special attention. In this paper, algorithms are proposed for the fuzzy shortest path problem, where the arc length of the network takes imprecise numbers, instead of real numbers, namely, level λ-triangular LR fuzzy numbers. Few indices defined in this paper help to identify the shortest path in fuzzy environment.

1. Introduction

Many researchers have focused on fuzzy shortest path problem in a network, since it is important to many applications such as communications, routing, and transportation. In traditional shortest path problems, the arc length of the network takes precise numbers, but in the real-world problem, the arc length may represent transportation time or cost which can be known only approximately due to vagueness of information, and hence it can be considered a fuzzy number. The fuzzy set theory, proposed by Zadeh [1], is utilized to deal with uncertainty problems.

The fuzzy shortest path problem was first analysed by Dubois and Prade [2]. According to their approach, the shortest path length can be obtained, but the corresponding path in the network may not exist. Klein [3] proposed a dynamic programming recursion-based fuzzy algorithm. Lin and Chern [4] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada and Soper [5] proposed a fuzzy algorithm, which was based on multiple-labelling methods to offer nondominated paths to a decision maker. Chuang and Kung [6] proposed a fuzzy shortest path length procedure that can find a fuzzy shortest path length among all possible paths in a network. Yao and Lin [7] presented two new types of fuzzy shortest path network problems. The main results obtained from their studies were that the shortest path in the fuzzy sense corresponds to the actual paths in the network, and the fuzzy shortest path problem is an extension of the crisp case. Nayeem and Pal [8] have proposed an algorithm based on the acceptability index introduced by Sengupta and Pal [9] which gives a single fuzzy shortest path or a guideline for choosing the best fuzzy shortest path according to the decision maker's viewpoint. Thus, numerous papers have been published on the fuzzy shortest path problem (FSPP).

This paper is organized as follows. In Section 2, some elementary concepts and operations of fuzzy set theory have been reviewed. Also new indices have been defined for level λ-triangular LR fuzzy numbers. In Section 3, new algorithms have been proposed for fuzzy shortest path problem based on level λ-triangular LR indices. Finally, the paper is concluded in Section 4.

2. Prerequisites

Definition 1 (Acyclic digraph). A digraph is a graph each of whose edges are directed. Hence, an acyclic digraph is a directed graph without cycle.

Definition 2 (Level λ-triangular LR fuzzy number). Level λ-triangular LR fuzzy number is shown in Figure 1 and it

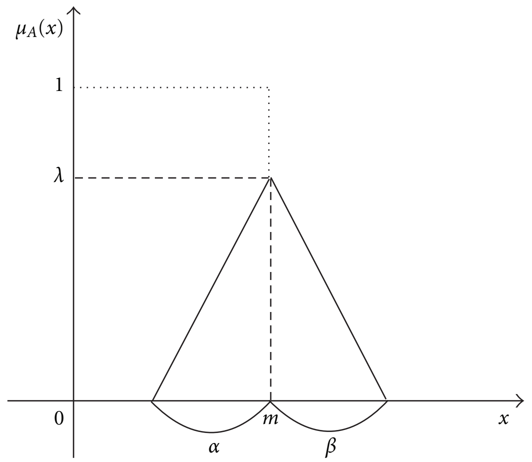

Figure 1: Level λ-triangular LR fuzzy number A.

is represented by $A = (m, \alpha, \beta; \lambda)_{\mathrm{LR}}$ with the membership function

$$\mu_A(x) = \begin{cases} 0, & x \le m - \alpha, \\ \dfrac{\lambda[x - (m - \alpha)]}{\alpha}, & m - \alpha \le x \le m, \\ \lambda, & x = m, \\ \dfrac{\lambda[(m + \beta) - x]}{\beta}, & m \le x \le m + \beta, \\ 0, & x \ge m + \beta. \end{cases} \tag{1}$$

Here, m is the point whose membership value is λ, where $0 < \lambda \le 1$ and α, β are the left hand and right hand spreads, respectively.

In addition, let $(m, \alpha, \beta; 1)_{\mathrm{LR}}$ be the triangular LR fuzzy number, and it is denoted by $(m, \alpha, \beta)_{\mathrm{LR}}$.

Definition 3 (Addition operation on level λ-triangular LR fuzzy numbers). Let $L_1 = (m_1, \alpha_1, \beta_1; \lambda)_{\mathrm{LR}}$ and $L_2 = (m_2, \alpha_2, \beta_2; \lambda)_{\mathrm{LR}}$ be the two level λ-triangular LR fuzzy numbers, then

$$\begin{aligned} L_1 + L_2 &= (m_1, \alpha_1, \beta_1; \lambda)_{\mathrm{LR}} + (m_2, \alpha_2, \beta_2; \lambda)_{\mathrm{LR}} \\ &= (m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2; \lambda)_{\mathrm{LR}}. \end{aligned} \tag{2}$$

Definition 4 (Minimum operation on level λ-triangular LR fuzzy numbers [10]). Let $L_1 = (m_1, \alpha_1, \beta_1; \lambda)_{\mathrm{LR}}$ and $L_2 = (m_2, \alpha_2, \beta_2; \lambda)_{\mathrm{LR}}$ be the two level λ-triangular LR fuzzy numbers, then

$$\begin{aligned} L_{\min} &= \min(L_1, L_2) \\ &= (\min(m_1, m_2), \max(\alpha_1, \alpha_2), \min(\beta_1, \beta_2); \lambda)_{\mathrm{LR}}. \end{aligned} \tag{3}$$

In this paper, we introduce the following definitions.

Definition 5 (Level λ-triangular LR intersection index). Let the level λ-triangular LR ith fuzzy path length be $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{\mathrm{LR}}$, and let the level λ-triangular LR fuzzy shortest length be $L_{\min} = (m, \alpha, \beta; \lambda)_{\mathrm{LR}}$, $0 < \lambda \le 1$ where $m - \alpha \le m_i - \alpha_i$, $m \le m_i$, $m + \beta \le m_i + \beta_i$, then the level λ-triangular LR intersection index between $L_{\min}$ and L_i is calculated as

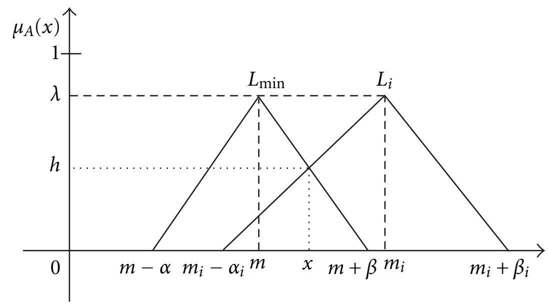

Figure 2: Level λ-triangular LR intersection index.

$$T_{\mathrm{LRI}}(L_{\min}, L_i) = \frac{\lambda[(m + \beta) - (m_i - \alpha_i)]}{\alpha_i + \beta} = h. \tag{4}$$

The above formula is obtained as follows.

Consider Figure 2:

Let y_d be the membership function.

For $m \le x \le m + \beta$,

$$\begin{aligned} y_d = \frac{\lambda[(m + \beta) - x]}{(m + \beta) - m} &\Longrightarrow \beta y_d = \lambda(m + \beta) - \lambda x, \\ \lambda x &= \lambda(m + \beta) - \beta y_d, \\ x &= \frac{\lambda(m + \beta) - \beta y_d}{\lambda}. \end{aligned} \tag{5}$$

For $m_i - \alpha_i \le x \le m_i$,

$$\begin{aligned} y_d = \frac{\lambda[x - (m_i - \alpha_i)]}{m_i - m_i + \alpha_i} &\Longrightarrow \alpha_i y_d = \lambda x - \lambda(m_i - \alpha_i), \\ \lambda x &= \alpha_i y_d + \lambda(m_i - \alpha_i), \\ x &= \frac{\alpha_i y_d + \lambda(m_i - \alpha_i)}{\lambda}. \end{aligned} \tag{6}$$

Equating (5) and (6),

$$\begin{aligned} \frac{\lambda(m + \beta) - \beta y_d}{\lambda} &= \frac{\alpha_i y_d + \lambda(m_i - \alpha_i)}{\lambda}, \\ \lambda[(m + \beta) - (m_i - \alpha_i)] &= (\alpha_i + \beta) y_d, \\ T_{\mathrm{LRI}}(L_{\min}, L_i) = y_d &= \frac{\lambda[(m + \beta) - (m_i - \alpha_i)]}{\alpha_i + \beta} = h. \end{aligned} \tag{7}$$

In the level λ-triangular LR intersection index, we have $L_1 < L_2$ if and only if $T_{\mathrm{LRI}}(L_{\min}, L_1) > T_{\mathrm{LRI}}(L_{\min}, L_2)$.

Definition 6 (Indices based on $L_{\min}$ and L_i). Let the level λ-triangular LR ith fuzzy path length be $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{\mathrm{LR}}$, and let the level λ-triangular LR fuzzy shortest length be $L_{\min} = (m, \alpha, \beta; \lambda)_{\text{LR}}$, $0 < \lambda \le 1$ where $m - \alpha \le m_i - \alpha_i$, $m \le m_i$, $m + \beta \le m_i + \beta_i$, then

(a) the level λ-triangular LR weighted average index between $L_{\min}$ and L_i is calculated as

$$Z_i^*(L_{\min}, L_i) = \frac{\lambda(m) + \lambda(m_i)}{\lambda + \lambda} = \frac{\lambda(m) + \lambda(m_i)}{2\lambda}. \quad (8)$$

Here, we have $L_1 < L_2$ if and only if $Z_1^*(L_{\min}, L_1) < Z_2^*(L_{\min}, L_2)$,

(b) the level λ-triangular LR Minkowski distance index between $L_{\min}$ and L_i is calculated as

$$d(L_{\min}, L_i) = \sqrt[w]{\mathfrak{f} + |(m+\beta) - (m_i + \beta_i)|^w}, \quad (9)$$

where $\mathfrak{f}$ denotes $|(m-\alpha) - (m_i - \alpha_i)|^w + |m - m_i|^w$ and $w \in [1, \infty]$. Here, we have $L_1 < L_2$ if and only if $d(L_{\min}, L_1) < d(L_{\min}, L_2)$.

Generalizing Hamming distance and Euclidean distance results in Minkowski distance. It becomes the Hamming distance (HD) for $w = 1$, while the Euclidean distance (ED) for $w = 2$.

Definition 7 (Indices based on L_i). Let the level λ-triangular LR ith fuzzy path length be $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{\text{LR}}$, then

(a) the level λ-triangular LR mean index for L_i is calculated as

$$T_{\text{Mean}}(L_i) = \left[m_i + \left(\frac{\beta_i - \alpha_i}{2} \right) \right]. \quad (10)$$

It is obtained as follows:

$$T_{\text{Mean}}(L_i) = \frac{\int_{m_i-\alpha_i}^{m_i+\beta_i} \lambda x\, dx}{\int_{m_i-\alpha_i}^{m_i+\beta_i} \lambda\, dx} = \frac{2m_i + \beta_i - \alpha_i}{2}, \quad (11)$$

(b) the level λ-triangular LR centroid index for L_i is calculated as

$$T_{\text{Centroid}}(L_i) = \left[m_i + \left(\frac{\beta_i - \alpha_i}{3} \right) \right]. \quad (12)$$

It is obtained as follows:

$$T_{\text{Centroid}}(L_i) = \frac{\int_{m_i-\alpha_i}^{m_i} [\lambda[x - (m_i - \alpha_i)]/\alpha_i] x\, dx + \int_{m_i}^{m_i+\beta_i} [\lambda[(m_i + \beta_i) - x]/\beta_i] x\, dx}{\int_{m_i-\alpha_i}^{m_i} [\lambda[x - (m_i - \alpha_i)]/\alpha_i] dx + \int_{m_i}^{m_i+\beta_i} [\lambda[(m_i + \beta_i) - x]/\beta_i] dx} = \frac{3m_i + \beta_i - \alpha_i}{3}. \quad (13)$$

Crisp Graph with Fuzzy Weights (Type V) (Blue et al. [11]). A fifth type of graph fuzziness occurs when the graph has known vertices and edges, but unknown weights (or capacities) on the edges. Thus, only the weights are fuzzy.

Applications. To plan the quickest automobile route from one city to another. Unfortunately, the map gives distances, not travel times, so it is not known exactly how long it takes to travel any particular road segment.

Definition 8 (The signed distance [7]). $d(b, 0) = b$ when $b, 0 \in \mathbb{R}$, where $d(b, 0)$ means the signed distance of b measuring from 0.

Definition 9 (Signed distance of level λ-triangular fuzzy number A [7]). For each $\lambda \in (0, 1]$ and $A = (a, b, c; \lambda)_{\text{LR}}$, the signed distance from 0 to A is defined by

$$d(A, 0) = \frac{1}{4}(2b + a + c). \quad (14)$$

The ranking of level λ-triangular fuzzy numbers is defined by $A < B$ if and only if $d(A, 0) < d(B, 0)$.

For the sake of verification, Definition 9 is rewritten as the signed distance from 0 to $L_1 = (m_1, \alpha_1, \beta_1; \lambda)_{\text{LR}}$, and it is defined by

$$d(L_1, 0) = \frac{1}{4}[2m_1 + (m_1 - \alpha_1) + (m_1 + \beta_1)]. \quad (15)$$

Here, $L_1 < L_2$ if and only if $d(L_1, 0) < d(L_2, 0)$.

Definition 10 (Nayeem and Pal acceptability index [8]). Nayeem and Pal extended the acceptability index originally proposed by Sengupta and Pal [9] for interval numbers to triangular LR fuzzy numbers as follows.

If $a = (m_1, \alpha_1, \beta_1)_{\text{LR}}$ and $b = (m_2, \alpha_2, \beta_2)_{\text{LR}}$ are two triangular LR fuzzy numbers with $m_1 - \alpha_1 \le m_2 - \alpha_2$, $m_1 \le m_2$, $m_1 + \beta_1 \le m_2 + \beta_2$, then the acceptability index of the proposition "a" preferred to "b" is given by $A(a < b) = (m_2 - m_1)/(\beta_1 + \alpha_2)$. Here, $a < b$ if and only if $A(a < b) > A(b < a)$.

To find the fuzzy shortest path, the above acceptability index was slightly modified in [10] as follows:

$$A(L_{\min} < L_i) = \frac{m_i - m}{\beta + \alpha_i}, \quad (16)$$

where $L_i = (m_i, \alpha_i, \beta_i)_{\text{LR}}$ is the triangular LR ith fuzzy path length, and $L_{\min} = (m, \alpha, \beta)_{\text{LR}}$ is the triangular LR fuzzy shortest length. Here, $L_1 < L_2$ if and only if $A(L_{\min} < L_1) < A(L_{\min} < L_2)$.

The acceptability index defined in [10] was obtained as follows.

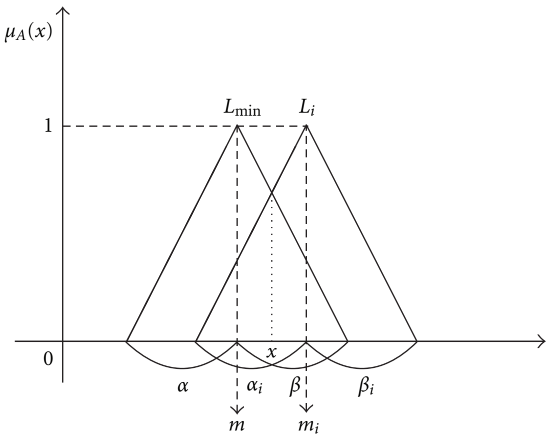

FIGURE 3: Triangular LR acceptability index.

Definition 11 (Triangular LR fuzzy number A [12]). A fuzzy number A is of LR type if there exists a reference function L (for left) and R (for right) and scalar $\alpha > 0$, $\beta > 0$ with

$$\mu_A(x) = \begin{cases} L\left[\left(\dfrac{m-x}{\alpha}\right)\right], & x < m, \\ 1, & x = m, \\ R\left[\left(\dfrac{x-m}{\beta}\right)\right], & x > m, \end{cases} \tag{17}$$

where "m" is a real number whose membership value is 1, and α, β are called the left and right spreads, respectively. Symbolically, A is represented by $A = (m, \alpha, \beta)_{\mathrm{LR}}$.

Consider Figure 3:
if $x < m_i$,

$$y_d = \frac{m_i - x}{\alpha_i} \Longrightarrow x = m_i - \alpha_i y_d\ . \tag{18}$$

If $x > m$,

$$y_d = \frac{x - m}{\beta} \Longrightarrow x = \beta y_d + m\ . \tag{19}$$

Equating (18) and (19),

$$\begin{aligned} m_i - \alpha_i y_d &= \beta y_d + m, \\ m_i - m &= (\beta + \alpha_i)\, y_d, \\ y_d &= \frac{m_i - m}{\beta + \alpha_i}. \end{aligned} \tag{20}$$

This acceptability index can also be defined for level λ-triangular LR fuzzy numbers as follows:

$$A(L_{\min} < L_i) = \frac{\lambda(m_i - m)}{\beta + \alpha_i}, \tag{21}$$

where $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{\mathrm{LR}}$ and $L_{\min} = (m, \alpha, \beta; \lambda)_{\mathrm{LR}}$, for the sake of verification. If $\lambda = 1$, we obtain the acceptability index defined in [10].

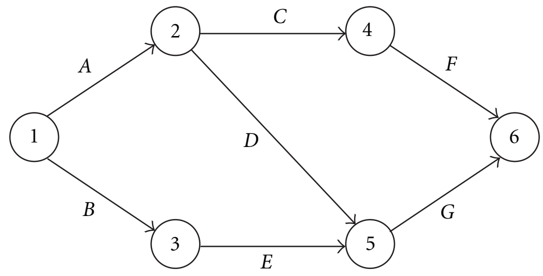

FIGURE 4: The classical network.

3. Proposed Algorithm for Fuzzy Shortest Path Problem Based on Level λ-Triangular LR Fuzzy Numbers

In many practical situations, we often need to employ a measurement tool to distinguish between two similar sets or groups. For this purpose, several similarity measures had been presented in [13–19].

In this paper, we introduce a new method called intersection index which acts as the measurement tool between $L_{\min}$ and L_i. The larger the height of the intersection area of two triangles, the higher the intersection index will be between them. If there is no intersection area between $L_{\min}$ and L_i, then the intersection index is treated as zero. As mentioned previously, the similarity measure, namely, intersection index defined in Definition 5 will help the decision makers to decide which path is the shortest one. Using this concept, we now propose Algorithm 1 for FSPP.

Example 12.

Step 1. Construct a network with 6 vertices and 7 edges as cited in Figure 4.

let $\lambda = 0.7$.

Assume the arc lengths as

A(1-2) $= (30, 2, 4;\ 0.7)_{\mathrm{LR}}$, B(1-3) $= (20, 5, 7;\ 0.7)_{\mathrm{LR}}$, C(2-4) $= (25, 2, 2;\ 0.7)_{\mathrm{LR}}$, D(2-5) $= (15, 4, 2;\ 0.7)_{\mathrm{LR}}$, E(3-5) $= (21, 4, 6;\ 0.7)_{\mathrm{LR}}$, F(4-6) $= (14, 1, 4;\ 0.7)_{\mathrm{LR}}$, G(5-6) $= (17, 4, 2;\ 0.7)_{\mathrm{LR}}$.

Step 2. The possible paths and the corresponding path lengths are as follows:

P_1: 1-2-4-6 with $A + C + F = (69, 5, 10;\ 0.7)_{\mathrm{LR}} = L_1$,
P_2: 1-2-5-6 with $A + D + G = (62, 10, 8;\ 0.7)_{\mathrm{LR}} = L_2$,
P_3: 1-3-5-6 with $B + E + G = (58, 13, 15;\ 0.7)_{\mathrm{LR}} = L_3$.

Step 3. $L_{\min} = (58, 13, 8;\ 0.7)_{\mathrm{LR}}$.

Step 4. See Table 1.

Step 5. Path P_3, that is 1-3-5-6, is the fuzzy shortest path since it has the highest level λ-triangular LR intersection index, and the corresponding shortest path length is $L_3 = (58, 13, 15;\ 0.7)_{\mathrm{LR}}$.

Input: $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{LR}$, $i = 1$ to n where L_i denotes the level λ-triangular LR fuzzy path length.
Output: $L_{\min} = (m, \alpha, \beta; \lambda)_{LR}$, where $L_{\min}$ denotes the level λ-triangular LR fuzzy shortest length.
Step 1: Construct a Network $G = (V, E)$ where V is the set of vertices and E is the set of edges. Here G is an acyclic digraph and the arc length takes the level λ-triangular LR fuzzy numbers.
Step 2: Calculate all the possible paths P_i and the corresponding path lengths L_i, using Definition 3. Set $L_i = (m_i, \alpha_i, \beta_i; \lambda)_{LR}$, $i = 1$ to n and $0 < \lambda \leq 1$.
Step 3: Calculate the fuzzy shortest length $L_{\min}$ using Definition 4 and set $L_{\min} = (m, \alpha, \beta; \lambda)_{LR}$.
Step 4: Calculate the level λ-triangular LR intersection index between $L_{\min}$ and L_i using Definition 5 for $i = 1$ to n.
Step 5: The path having the highest level λ-triangular LR intersection index is identified as the shortest path.

Algorithm 1

Step 1 and **Step 2** are same as in the Algorithm 1.
Step 3: Calculate the level λ-triangular LR mean and centroid index for each possible path lengths L_i, using Definition 7.
Step 4: The path having the minimum level λ-triangular LR mean and centroid index is identified as the shortest path and the corresponding path length is the shortest path length.

Algorithm 2

Table 1: Results of the network based on level λ-triangular LR intersection index.

Paths	$T_{\text{LRI}}(L_{\min}, L_i)$	Ranking
P_1: 1-2-4-6	0.11	3
P_2: 1-2-5-6	0.54	2
P_3: 1-3-5-6	0.7	1

Table 2: Results of the network based on level λ-triangular LR weighted average index.

Paths	$Z_i^*(L_{\min}, L_i)$	Ranking
P_1: 1-2-4-6	63.5	3
P_2: 1-2-5-6	60	2
P_3: 1-3-5-6	58	1

Path P_3, that is 1-3-5-6, is the fuzzy shortest path since it has the minimum level λ-triangular LR weighted average index.

Table 3: Results of the network based on level λ-triangular LR distance index.

Paths	$HD\ (L_{\min}, L_i)$ $w = 1$	$ED\ (L_{\min}, L_i)$ $w = 2$	$d\ (L_{\min}, L_i)$ $w = 3$	Ranking
P_1: 1-2-4-6	43	25.5	21.82	3
P_2: 1-2-5-6	15	9	7.78	2
P_3: 1-3-5-6	7	7	7	1

Path P_3, that is 1-3-5-6, is the fuzzy shortest path since it has the minimum level λ-triangular LR distance index.

3.1. Comparing the Results Based on Level λ-Triangular LR Intersection Index with Few Indices Defined in This Paper. See Tables 2 and 3, and Algorithm 2.

Example 13.

Steps 1 and 2. They are the same as in Example 12.

Table 4: Results of the network based on level λ-triangular LR mean and centroid index.

Paths	$T_{\text{Mean}}\ (L_i)$	$T_{\text{Centroid}}\ (L_i)$	Ranking
P_1: 1-2-4-6	71.5	70.67	3
P_2: 1-2-5-6	61	61.33	2
P_3: 1-3-5-6	59	58.67	1

Table 5: Results of the network based on signed distance ranking index.

Paths	$d(L_i, 0)$	Ranking
P_1: 1-2-4-6	70.25	3
P_2: 1-2-5-6	61.5	2
P_3: 1-3-5-6	58.5	1

Path P_3, that is 1-3-5-6, is the fuzzy shortest path since it has the minimum signed distance ranking index and the corresponding shortest path length is L_3.

Table 6: Results of the network based on acceptability index.

Paths	$A(L_{\min} < L_i)$ $\lambda = 0.7$	$A(L_{\min} < L_i)$ $\lambda = 1$	Ranking
P_1: 1-2-4-6	0.59	0.85	3
P_2: 1-2-5-6	0.16	0.22	2
P_3: 1-3-5-6	0	0	1

Path P_3, that is 1-3-5-6, is the fuzzy shortest path since it has the minimum acceptability index, and the corresponding shortest path length is L_3.

Step 3. See Table 4.

Step 4. Path P_3, that is 1-3-5-6, is identified as the shortest path, since it has the minimum level λ-triangular LR mean and centroid index, and the corresponding shortest path length is $L_3 = (58, 13, 15; 0.7)_{LR}$.

3.2. Results and Discussions. One way to verify the solution obtained is to make an exhaustive comparison, now comparing the result obtained in this paper with the existing results to generalize our proposed approach. See Tables 5 and 6.

Hence, we find that the solution obtained for FSPP in this paper coincides with the solution of the existing methods.

4. Conclusion

Fuzzy shortest path length and the shortest path are the useful information for the decision makers in a fuzzy shortest path problem. Due to its practical application, many researchers have focused on the fuzzy shortest path problem, and some algorithms were developed for the same. Hence, in this paper, we have defined few indices and developed the new algorithms based on them and verified with the existing methods. The ranking given to the paths is helpful for the decision makers as they make decision in choosing the best of all the possible path alternatives. Hence, we conclude that the algorithms developed in the current research are the simplest and the alternative method for getting the shortest path in fuzzy environment.

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