Research Article

A New Time-Invariant Fuzzy Time Series Forecasting Method Based on Genetic Algorithm

Erol Eğrioglu

Department of Statistics, Faculty of Arts and Science, University of Ondokuz Mayıs, 55139 Samsun, Turkey

Correspondence should be addressed to Erol Eğrioglu, erole@omu.edu.tr

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In recent years, many fuzzy time series methods have been proposed in the literature. Some of these methods use the classical fuzzy set theory, which needs complex matricial operations in fuzzy time series methods. Because of this problem, many studies in the literature use fuzzy group relationship tables. Since the fuzzy relationship tables use order of fuzzy sets, the membership functions of fuzzy sets have not been taken into consideration. In this study, a new method that employs membership functions of fuzzy sets is proposed. The new method determines elements of fuzzy relation matrix based on genetic algorithms. The proposed method uses first-order fuzzy time series forecasting model, and it is applied to the several data sets. As a result of implementation, it is obtained that the proposed method outperforms some methods in the literature.

1. Introduction

The fuzzy time series were firstly defined in Song and Chissom [1]. Many time series in the real life have uncertainty observations. This kind of the time series is called fuzzy time series. For example, some of these time series are stock index data, air pollution data, enrollment data, and temperature data. The observations of these time series are convertible to fuzzy sets. The fuzzy time series separate two classes which are time-variant and time-invariant. The time-invariant fuzzy time series have time-invariant relationship of lagged fuzzy time series variables. This relationship is proved from an “R” matrix, which is invariant in the time space. The first method for forecasting time-invariant time series is proposed in Song and Chissom [2], in which the membership of observations is determined, subjectively. Chen [3] proposed fuzzy time series method which does not need complex matricial operations, and it uses fuzzy group relationship tables. Since these use order of fuzzy sets, the membership functions of fuzzy sets are not taken into consideration. Sullivan and Woodall [4] proposed fuzzy time series method based on Markov Chains. All of these methods forecast fuzzy time series based on first-order fuzzy time series forecasting model.

In the literature, many fuzzy time series forecasting method are based on high-order fuzzy time series model, bivariate fuzzy time series model, and multivariate fuzzy time series model. Some of these methods are due to Chen [5], Aladag et al. [6], and Eğrioglu et al. [7]. Generally, fuzzy time series methods are based on three stages. These are fuzzification, determination of fuzzy relation, and defuzzification. In the literature, the fuzzy time series methods are improved by employing various artificial intelligence techniques in these three stages. The genetic algorithms, particle swarm optimization, and fuzzy c-means methods are used in the fuzzification stage. Feed forward neural networks are used determining fuzzy relation and defuzzification stage.

Many proposed methods in the literature neglected membership values of fuzzy sets. In this study, we proposed new fuzzy time series forecasting method for first order fuzzy time series forecasting model. The proposed method takes into account of membership values. Moreover, the proposed method does not need complex matricial operations and outperforms the well-known methods in the literature.

In Section 2 are given some important definitions. In Section 3, the fuzzy c-means method is summarized. In Section 4, genetic algorithm is briefly explained. In Section 5, we present details of our proposed methods. In Section 6,

the proposed method is applied to some data sets, and we present results of implementation.

2. The Some Definitions Related to Fuzzy Time Series

The fuzzy time series were firstly defined in Song and Chissom [1]. The time-variant and time-invariant fuzzy time series definitions are given below.

Definition 1. Let $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), a subset of real numbers, be the universe of discourse on which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \dots$, then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2. Suppose $F(t)$ is implied by $F(t-1)$ only, that is, $F(t-1) \rightarrow F(t)$. Then this relation can be expressed as $F(t) = F(t-1) \circ R(t, t-1)$, where $R(t, t-1)$ is the fuzzy relationship between $F(t-1)$ and $F(t)$, and $F(t) = F(t-1) \circ R(t, t-1)$ is called the first order model of $F(t)$.

Definition 3. Suppose $R(t, t-1)$ is a first-order model of $F(t)$. If for any t , $R(t, t-1)$ is independent of t , that is, for any t , $R(t, t-1) = R(t-1, t-2)$, then $F(t)$ is called a time-invariant fuzzy time series, otherwise it is called a time-variant fuzzy time series.

The symbol “ $\circ$ ” stands for max-min composition of fuzzy sets. Song and Chissom [2] firstly introduced an algorithm based on the first order model for forecasting time invariant $F(t)$. In Song and Chissom [2], the fuzzy relationship matrix $R(t, t-1) = R$ is obtained by many matricial operations. The fuzzy forecasts are obtained based on max-min composition as follows:

$$F(t) = F(t-1) \circ R. \quad (1)$$

The dimension of R is depending on number of fuzzy sets which are partition number of universe and discourse. If we use more fuzzy sets, we need more matricial operations for obtaining R . In this situation, Song and Chissom's [2] method is getting more complex.

3. The Fuzzy C-Means Clustering Method: An Overview

Song and Chissom [1] method uses decomposition of the universe discourse in the stage of fuzzification. There are two problems: the number of intervals of arbitrary length and the arbitrary choice of membership degrees. In order to overcome these problems, Cheng et al. [8] and Li et al. [9] fuzzy c-means (FCM) clustering method was used. FCM clustering method was firstly introduced by Bezdek [10]. Let u_{ij} be the membership, v_i the center of cluster, n the number of variables, and c the number of clusters. Then the objective function, which is minimized in FCM, is

$$J_\beta(X, V, U) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^\beta d^2(x_j, v_i), \quad (2)$$

TABLE 1: Enrollment data.

Years	Actual	Years	Actual
1971	13055	1982	15433
1972	13563	1983	15497
1973	13867	1984	15145
1974	14696	1985	15163
1975	15460	1986	15984
1976	15311	1987	16859
1977	15603	1988	18150
1978	15861	1989	18970
1979	16807	1990	19328
1980	16919	1991	19337
1981	16388	1992	18876

where β is a constant greater than 1 and $d(x_j, v_i)$ is a distance between the observation and the center of the cluster. J_β is minimized with subject the constraints:

$$\begin{aligned} 0 &\leq u_{ij} \leq 1, \quad \forall i, j, \\ 0 &< \sum_{j=1}^n u_{ij} \leq n, \quad \forall i, \end{aligned} \quad (3)$$

$$\sum_{i=1}^c u_{ij} = 1, \quad \forall j.$$

In this method, the minimization is done by an iterative algorithm. In each repetition, the values of u_{ij} and v_i are updated by the formulas given in (4):

$$\begin{aligned} v_i &= \frac{\sum_{j=1}^n u_{ij}^\beta x_j}{\sum_{j=1}^n u_{ij}^\beta}, \\ u_{ij} &= \frac{1}{\sum_{k=1}^c \left(d(x_j, v_i) / d(x_j, v_k) \right)^{2/(\beta-1)}}. \end{aligned} \quad (4)$$

4. Genetic Algorithm: An Overview

Chen and Chung [11], Lee et al. [12] used genetic algorithm in fuzzification stage. The genetic algorithms were first proposed by Holland [13]. The genetic algorithms have population size, evaluation function, cross-over rate, mutation rate and maximum generation number. The genetic algorithm researches an optimal solution with many chromosomes. In a chromosome, there are many genes. Generally, the genetic algorithm starts with random population whose size is determined by user according to the problem under study. For example, Chen and Chung [11] used 50 as population size and the new generation is produced by various techniques in the iterations. Some of these techniques are crossover, mutation, and natural selection, summarized as follows.

Crossover. The system randomly selects two chromosomes from a population and randomly selects a crossover point

TABLE 2: centers of clusters are obtained from FCM algorithm in Step 1.

Centers of clusters						
v_1	v_2	v_3	v_4	v_5	v_6	v_7
13480,6299	15003,98	15455,4	15965,12	16837,86	18161,53	19144,4

TABLE 3: The memberships of the observations to clusters are obtained from FCM algorithm in Step 1.

Years	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5	Cluster 6	Cluster 7
1971	0,8889	0,0424	0,0279	0,0190	0,0113	0,0062	0,0043
1972	0,9925	0,0032	0,0019	0,0012	0,0006	0,0003	0,0002
1973	0,8072	0,0932	0,0478	0,0274	0,0137	0,0065	0,0043
1974	0,0486	0,7570	0,1245	0,0446	0,0157	0,0060	0,0036
1975	0,0000	0,0001	0,9998	0,0001	0,0000	0,0000	0,0000
1976	0,0048	0,1716	0,7757	0,0378	0,0069	0,0020	0,0011
1977	0,0039	0,0485	0,7993	0,1328	0,0114	0,0027	0,0014
1978	0,0017	0,0135	0,0601	0,9116	0,0104	0,0019	0,0009
1979	0,0001	0,0003	0,0005	0,0013	0,9971	0,0005	0,0002
1980	0,0005	0,0018	0,0030	0,0071	0,9821	0,0042	0,0013
1981	0,0093	0,0409	0,0900	0,4378	0,3869	0,0249	0,0103
1982	0,0001	0,0027	0,9950	0,0018	0,0003	0,0001	0,0000
1983	0,0004	0,0070	0,9835	0,0078	0,0009	0,0002	0,0001
1984	0,0057	0,7977	0,1647	0,0236	0,0055	0,0017	0,0010
1985	0,0066	0,7367	0,2179	0,0290	0,0066	0,0021	0,0012
1986	0,0001	0,0004	0,0013	0,9977	0,0005	0,0001	0,0000
1987	0,0000	0,0001	0,0002	0,0006	0,9987	0,0003	0,0001
1988	0,0000	0,0000	0,0000	0,0000	0,0001	0,9997	0,0001
1989	0,0010	0,0018	0,0023	0,0032	0,0063	0,0438	0,9416
1990	0,0009	0,0017	0,0022	0,0029	0,0052	0,0239	0,9632
1991	0,0010	0,0019	0,0024	0,0031	0,0057	0,0258	0,9601
1992	0,0021	0,0041	0,0052	0,0072	0,0147	0,1196	0,8472

from the two selected chromosomes to exchange genes after the crossover point. The crossover operation is depending on crossover rate. The random number generates uniform distribution, then the crossover operation is applied if random number is bigger than crossover rate.

Mutation. The user must determine a mutation rate. Then a chromosome is randomly selected. If the system randomly generates a real value between zero and one, which is smaller than or equal to the mutation rate, then the system performs the mutation operation with a randomly selected gene from the chromosomes.

Natural Selection. Each chromosome of any generation is evaluated according to evaluation function. All chromosomes are ordered according to evaluation function value. The best chromosomes are transferred to the next generation. Some worst chromosomes are discarded from generations, and the new chromosomes are admitted to the new generation.

5. The Proposed Method for Forecasting First Order Fuzzy Time Series Method

Here, we study forecasting time-invariant fuzzy time series. The FCM is used for fuzzification of time series, and genetic

algorithm is used for finding the fuzzy relation. The proposed method has the following important advantages:

- (i) the membership values of the fuzzy sets are taken into consideration;
- (ii) there is no necessity of complicated matricial operations;
- (iii) FCM no needs subjective decisions by using partition of the universe of discourse.

The algorithm is given below in step by step as follows.

Step 1 (Time series are fuzzified by FCM). Let c be the number of fuzzy sets such that $2 \leq c \leq n$. FCM initially assigns crisp values to the fuzzy sets. After this application, the center of each cluster L_r is determined and denoted by v_r , $r = 1, 2, \dots, c$.

Step 2 (Determination of the parameters of genetic algorithm). The evaluation function is the RMSE as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (x_t - \hat{x}_t)^2}{n}}, \quad (5)$$

TABLE 4: The forecasts of the proposed method.

Years	Fuzzy forecasts							Defuzzified forecasts
1972	0,8694	0,8279	0,7866	0,3820	0,2837	0,1199	0,0430	13480,63
1973	0,8694	0,8279	0,7866	0,3820	0,2837	0,1199	0,0430	13480,63
1974	0,8072	0,8072	0,7866	0,3820	0,2837	0,1199	0,0478	14242,00
1975	0,3643	0,5920	0,7570	0,7570	0,6596	0,4888	0,1245	15710,00
1976	0,5000	0,9998	0,3584	0,9998	0,8535	0,3184	0,2807	15484,55
1977	0,5000	0,7757	0,3584	0,7757	0,7757	0,3184	0,2807	15935,65
1978	0,5000	0,7993	0,3584	0,7993	0,7993	0,3184	0,2807	15935,65
1979	0,4219	0,7284	0,2815	0,7736	0,9116	0,0601	0,9043	16837,86
1980	0,0022	0,9009	0,0558	0,3246	0,9971	0,9971	0,5000	17499,69
1981	0,0071	0,9009	0,0558	0,3246	0,9821	0,9821	0,5000	17499,69
1982	0,4219	0,4378	0,2815	0,4378	0,4378	0,3869	0,4378	16737,00
1983	0,5000	0,9950	0,3584	0,9950	0,8535	0,3184	0,2807	15484,55
1984	0,5000	0,9835	0,3584	0,9835	0,8535	0,3184	0,2807	15484,55
1985	0,3643	0,5920	0,7977	0,7977	0,6596	0,4888	0,1647	15710,00
1986	0,3643	0,5920	0,7367	0,7367	0,6596	0,4888	0,2179	15710,00
1987	0,4219	0,7284	0,2815	0,7736	0,9977	0,0496	0,9043	16837,86
1988	0,0022	0,9009	0,0558	0,3246	0,9987	0,9987	0,5000	17499,69
1989	0,0001	0,0001	0,5000	0,5000	0,5000	0,5000	0,9997	19144,40
1990	0,0032	0,0063	0,0438	0,0438	0,0438	0,5000	0,5621	19144,40
1991	0,0029	0,0052	0,0239	0,0239	0,0239	0,5000	0,5621	19144,40
1992	0,0031	0,0057	0,0258	0,0258	0,0258	0,5000	0,5621	19144,40

TABLE 5: The RMSE values of the some first order methods and proposed method.

Methods	RMSE
Song and Chissom [2]	642,26
Sullivan and Woodall [4]	621,33
Chen [3]	638,36
Proposed method	484,61

being x_t the crisp time series, $\hat{x}_t$ the defuzzified forecast, and n is the number of forecasts. The other parameters are selected as follows:

- population size: 30;
- crossover rate: 1;
- mutation rate: 0.01;
- maximum generation: 300;
- number of discarded chromosomes in the natural selection: 10.

Step 3 (Initialization). The genes are the elements of the crisp relation (Figure 1):

$$R = [r_{ij}], \quad i = 1, 2, \dots, c, \quad j = 1, 2, \dots, c. \quad (6)$$

Step 4 (Crossover operation). The crossover operation is applied if the random number is bigger than the crossover rate. The system randomly selects two chromosomes from a population and randomly selects a crossover point from

r_{11}	r_{12}	$\dots$	r_{1c}	r_{21}	r_{22}	$\dots$	r_{2c}	$\dots$	r_{c1}	r_{c2}	$\dots$	r_{cc}
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FIGURE 1: The structure of one chromosome.

FIGURE 2: An example of crossover operation.

the two selected chromosomes to exchange genes after the crossover point. The crossover operation is shown in Figure 2.

Step 5. If the system randomly generates a real value between zero and one which is smaller than or equal to the mutation rate, then the system performs the mutation operation with a randomly selected gene from the chromosomes. The mutation operation is demonstrated in Figure 3. x_{new} is randomly generated from $[x_{11}, x_{13}]$ interval.

Step 6. The method for calculating RMSE for any chromosome is given in Substeps 6.1–6.4.

TABLE 6: The results of the methods for IMKB 100 data.

Method		Number of fuzzy sets								Min	Mean
		7	8	9	10	11	12	13	14		
Chen [3]	RMSE (training)	1420	1178	1340	1164	1281	1233	1152	1205	1090	1252
	RMSE (test)	1238	1132	1340	1191	1155	1131	1218	1125	1186	1210
Song and Chissom [1, 2]	RMSE (training)	1462	1178	1340	1164	1281	1233	1140	1209	1105	1257
	RMSE (test)	1571	1132	1340	1191	1155	1131	1249	1121	1132	1240
Proposed Method	RMSE (training)	1379	1271	1391	1360	1176	1176	1152	1089	1107	1234
	RMSE (test)	1565	1317	1291	1154	1118	1126	1039	1036	1022	1185

FIGURE 3: An example of mutation operation.

Substep 6.1. c -gens constitute in the rows of R .

Substep 6.2. Fuzzy forecasts are obtained using (1). After R is estimated, fuzzy forecasts can be obtained by using FCM algorithm. For example, for any t , the $(t - 1)$ th fuzzy observation is given as follows:

$$F(t - 1) = [1 \ 0.5 \ 0.3 \ 0 \ 0], \quad R = \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad (7)$$

The fuzzy forecast for t time is computed by (1) as follows:

$$\begin{aligned} \hat{F}(t) &= F(t - 1) \circ R = [1 \ 0.5 \ 0.3 \ 0 \ 0] \circ \begin{bmatrix} 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &= [1 \ 0.5 \ 0.5 \ 0.3 \ 0]. \end{aligned} \quad (8)$$

Substep 6.3. If the membership values of a fuzzy forecast have only one maximum, then select the center of this cluster as the defuzzified forecasted value. If the membership values of an fuzzy forecast has two or more maximum, then select the arithmetic mean of centers of the corresponding clusters as the defuzzified forecasted value. For example, if $\hat{F}(t) = [0.7 \ 0.5 \ 0.3 \ 0.2 \ 0.4]$, then the maximum is 0.7 and then the fuzzified forecast is (v_1) , center of L_1 . If $\hat{F}(t) = [0.7 \ 0.5 \ 0.7 \ 0.2 \ 0.4]$, then the maximum is 0.7 and then

FIGURE 4: The IMKB 100 data between 28/12/2010 and 29/02/2012.

fuzzified forecast is arithmetic mean of v_1 center of L_1 cluster and v_3 , center of L_3 .

Substep 6.4. RMSE is computed by (5).

Step 7 (Natural selection operation). All chromosomes are ordered according to the RMSE value. The best 20 chromosomes are transferred to the next generation. 10 chromosomes are discarded from the generation. The new 10 randomly generated chromosomes are replaced in the new generation.

Step 8. Steps 4 and 7 are repeated 300 times.

6. Implementation

The proposed method is firstly applied to Alabama University Enrollment data (1971–1992) which is well known in the literature. The Enrollment data are given Table 1.

The proposed method is programmed in Matlab 7.9 version. In the application, we use seven fuzzy sets ($c = 7$) in Step 1. The centers of clusters and the membership values from FCM algorithm are given Tables 2 and 3, respectively. The fuzzy and defuzzified forecasts are given in Table 4.

Consider

$R =$

$$\begin{bmatrix} 0,8694 & 0,8279 & 0,7866 & 0,3820 & 0,2837 & 0,1199 & 0,0430 \\ 0,3643 & 0,5920 & 1,0000 & 1,0000 & 0,6596 & 0,4888 & 0,0000 \\ 0,5000 & 1,0000 & 0,3584 & 1,0000 & 0,8535 & 0,3184 & 0,2807 \\ 0,4219 & 0,7284 & 0,2815 & 0,7736 & 1,0000 & 0,0496 & 0,9043 \\ 0,0022 & 0,9009 & 0,0558 & 0,3246 & 1,0000 & 1,0000 & 0,5000 \\ 0,0000 & 0,0000 & 0,5000 & 0,5000 & 0,5000 & 0,5000 & 1,0000 \\ 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,0000 & 0,5000 & 0,5621 \end{bmatrix}.$$

(9)

The RMSE of well-known methods [2–4] is given in Table 5.

The proposed method is successively applied to daily Istanbul stock market time series data between 28/12/2010 and 29/02/2012 dates (IMKB 100). The graph of the time series is given in Figure 4. The data between 28/12/2010 and 10/10/2011 dates are used as training data, the data between 11/10/2011 and 29/02/2012 dates are used as test data.

The number of fuzzy clusters is changed from 7 to 15. The obtained results from three methods are given in Table 6.

The RMSE values of the proposed method are smaller than Chen's [3] and Song and Chissom's [1, 2] methods.

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