Research Article

Theoretical Analysis of the Shaft

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This paper represents the dynamic response of a steel shaft which is fixed at both ends by bearing. The shaft is subjected to both axial and bending loads. The behavior of the shaft in the presence of two transverse cracks subjected to the same angular position along longitudinal direction is observed by taking basic parameters such as nondimensional depth (b_i/D), nondimensional length (L_i/L), and three relative natural frequencies with their relative mode shapes. The compliance matrix is calculated from the stress intensity factor for two degrees of freedom. The dynamic nature of the cracked shaft at two cracked locations at a different depth is observed. The compliance matrix is a function of crack parameters such as depth and location of crack from any one of the bearings. The three relative natural frequencies and their mode shapes at a different location and depth obtained analytical and experimental method. Multiple adaptive neurofuzzy inference system (MANFIS) methodology (an inverse technique) is used for locating the cracks at any depth and location. The input of the MANFIS is provided with the first three natural frequencies and the first three mode shapes obtained from analytical method. The predicted result of the MANFIS (relative crack location and depth) has been validated using the results from the developed experimental setup.

1. Introduction

Generation and propagation of transverse crack in a shaft or rotor is a common phenomenon for every machine. Nowadays, it becomes a challenging task in front of a designer to identify the crack and take steps for precaution before damaging the whole structure. Zheng et al. [1] have formulated a learning algorithm based on radial basis function neural network for structural damage diagnosis. In their work, they have used fuzzy logic, genetic algorithm, and neural network for development of the proposed model. The model has been designed with the data (modal frequencies obtained from finite element analysis). Their model is capable of predicting the delimitation location and size of the composite laminated beam successfully. They have found that the developed model is capable of predicting the results within an error of 18 percent. Bachschmid et al. [2] have analyzed the vibration response of a shaft line used in a turbo generator unit. They have used the quasilinear approach and finite element method to obtain the vibration signature (modal frequencies and amplitude of vibration). They have found out the effect

of a crack along the shaft line on the dynamic response of the system at different speed of the shaft. They have expressed that the crack forces depend on the depth of crack and are proportional to static bending moments and inversely proportional to the third power of the shaft diameter. The maximum response of the shaft has been found during the resonant condition. Buezas et al. [3] have employed a genetic algorithm optimization method for crack detection in structural elements. The developed model has used the contact between the interfaces of the crack to address the damage diagnosis problem. They have simulated the crack as a notch or a wedge with a unilateral Signorini contact model. They have compared the dynamic response of the damage structure with the finite element model using the least square approach and claimed that the GA-based algorithm can detect the damage parameter in the structure system effectively. Zhang et al. [4] have used strain gauge and a servo hydraulic machine, a drop weight impact device to find out loading rate effect on the crack velocity in high strength concrete. They have conducted a three-point bend test on the high strength concrete specimen with strain gauges mounted

on them to measure the crack velocity. They found that under high loading rate the crack propagates with slightly increasing velocity but under low loading rate the crack advances with increasing velocity. They have claimed that the peak strain crack velocity and strain rate will help in validating the numerical model to evaluate the rate dependence of the fracture behavior of high strength concrete. Singh and Tiwari [5] have developed a methodology to identify the number of cracks present in the shaft and to estimate their position and size. The technique has been developed using a transverse forced response of the shaft system which has been computed using finite element method and the Timoshenko beam theory. They have taken measures to reduce the effect of noise on the vibration response. Genetic algorithm has been used for predicting the position of the cracks and depths for a simply supported having two cracks. A study has been performed by Chomette and Sinou [6] to develop an optimal control and identification methodology for truss structure with multiple cracks. They have stated that in a control system, the presence of transverse crack can destabilized the performance of the system. In their investigation, they have fabricated a method to detect the presence of multiple cracks using the rational fraction polynomial algorithm. The efficiency of the proposed method has been validated by considering the crack parameter (location and depth) along with a crack orientation. Tlaoui et al. [7] have investigated the bending and torsional vibration response numerically and experimentally for uncracked and cracked shafts to develop online monitoring system for damage identification. In their work, the dynamics response of the over hung shaft has been calculated taking into account the effect of ball bearings and test frames which support the shaft structure with the help of modal analysis software and finite element analysis. They have claimed that the effect of a different depth on bending and torsional frequency gives a better insight to the vibratory behavior. Rubio et al. [8] have used closed-form polynomial expression to derive the flexibility function of crack shaft to measure the modal response both in bending and tension. They have considered the tip of the crack as an elliptical front for more realistic configuration of the crack present in a shaft. The validation of the developed analytical method has been done by comparing the result obtained from finite element method and experimental technique. They have found a very good agreement between the results from experimental and the proposed methodology. Naik and Maiti [9] presented formulation of a crack in a Timoshenko and an Euler-Bernoulli shafts based on compliance technique to analyse the free vibration taking into consideration the angular orientation. The analytical method has been developed using strain energy release rate and stress intensity factors. The equation proposed has been used to derive free transverse and torsional vibration. The results from the analytical approach have been compared with the frequencies obtained from finite element for different configuration of crack, and they found that the results are well in agreement. Saeed et al. [10] has presented single artificial neural network (ANN) and multiple adaptive neuro fuzzy interface system (ANFIS) for crack diagnosis in the curvilinear beams using changes in vibration parameter. They have used finite element method

FIGURE 1: Schematic diagram for multiple crack shafts.

FIGURE 2: Shaft with multiple cracks.

(FEM) to calculate the natural frequency and frequency response functions for intact and crack beam. They have found that the multiple ANN and the multiple ANFIS prediction errors are less than the single ANN. From their study, they have concluded that the multiple ANFIS can diagnose the crack presenting the beam with less error even in the presence of noise. Abraham [11] has discussed various neurofuzzy models for various applications. He has discussed three types of cooperative neurofuzzy models that have evolved during last decades.

2. Theoretical Analyses of Shaft

With the help of linear fracture mechanics theory, taking strain energy release rate and stress intensity factor, the natural frequencies and mode shapes of steel cracked shaft (fixed-fixed) can be calculated as follows.

2.1. Local Flexibility of a Cracked Shaft under Bending and Axial Loading. The presence of a transverse surface crack of depth b_1 and b_2 on shaft of diameter D and length L introduces a local flexibility, which can be defined in matrix form, the dimension of which depends on the degrees of freedom. Here, a 2×2 matrix is considered. A shaft is subjected to axial force (F_1) and bending moment (F_2), shown in Figure 1, which gives coupling with the longitudinal and transverse motion. Figure 2 represents the position of crack from one of the fixed ends.

The strain energy release rate at the fractured section can be written as (Tada et al., 1973):

$$J_e = \frac{1}{E'} (C_{11} + C_{12})^2, \quad (1)$$

FIGURE 3: Relative crack depth (β) versus dimensionless compliance ($(\ln(\bar{S}_{i=1,2j=1,2}))$).

where

$$\frac{1}{E'} = \frac{1 - \nu^2}{E'} \quad (\text{condition for plane strain}), \quad (2)$$

$$\frac{1}{E'} = \frac{1}{E} \quad (\text{condition for plane stress}),$$

C_{11} , C_{12} are the stress intensity factors of mode I (opening of the crack) for load F_1 and F_2 , respectively. The values of local stiffness matrix from earlier studies by (Tada et al., 1973) obtained by taking the inversion of compliance matrix.

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1}. \quad (3)$$

2.2. Analysis of Vibration Characteristics of the Cracked Shaft. A steel shaft of length “ L ” diameter “ D ,” with two cracks of depth “ b_1 ” and “ b_2 ” at a distance “ L_1 ” and “ L_2 ,” respectively, from the fixed end is considered. Taking $v_1(x, t)$, $v_2(x, t)$, and $v_3(x, t)$ as the responses of the longitudinal vibration for the sections and $y_1(x, t)$, $y_2(x, t)$, and $y_3(x, t)$ as the responses of the bending vibration for the same sections, the function for the system can be defined as

$$\begin{aligned} \bar{v}_1(\bar{x}) &= A_1 \cos(\bar{C}_v \bar{x}) + A_2 \sin(\bar{C}_v \bar{x}), \\ \bar{v}_2(\bar{x}) &= A_3 \cos(\bar{C}_v \bar{x}) + A_4 \sin(\bar{C}_v \bar{x}), \\ \bar{v}_3(\bar{x}) &= A_5 \cos(\bar{C}_v \bar{x}) + A_6 \sin(\bar{C}_v \bar{x}), \\ \bar{y}_1(\bar{x}) &= A_7 \cosh(\bar{C}_y \bar{x}) + A_8 \sinh(\bar{C}_y \bar{x}) \\ &\quad + A_9 \cos(\bar{C}_y \bar{x}) + A_{10} \sin(\bar{C}_y \bar{x}), \end{aligned}$$

$$\begin{aligned} \bar{y}_2(\bar{x}) &= A_{11} \cosh(\bar{C}_y \bar{x}) + A_{12} \sinh(\bar{C}_y \bar{x}) \\ &\quad + A_{13} \cos(\bar{C}_y \bar{x}) + A_{14} \sin(\bar{C}_y \bar{x}), \\ \bar{y}_3(\bar{x}) &= A_{15} \cosh(\bar{C}_y \bar{x}) + A_{16} \sinh(\bar{C}_y \bar{x}) \\ &\quad + A_{17} \cos(\bar{C}_y \bar{x}) + A_{18} \sin(\bar{C}_y \bar{x}), \end{aligned} \quad (4)$$

where $\bar{x} = x/L$, $\bar{v} = v/L$, $\bar{y} = y/L$, $\alpha_1 = L_1/L$, $\alpha_2 = L_2/L$,

$$\bar{C}_v = \frac{\omega L}{C_v}, \quad \bar{S}_v = \left(\frac{E}{\rho} \right)^{1/2}, \quad \bar{C}_y = \left(\frac{\omega L^2}{S_y} \right)^{1/2}, \quad (5)$$

$$S_y = \left(\frac{EI}{\mu} \right)^{1/2}, \quad \mu = A\rho,$$

A_i ($i = 1, 18$). Constants are to be determined, from boundary conditions. The boundary conditions of the cantilever beam in consideration are

$$\begin{aligned} \bar{v}_1(0) &= 0, & \bar{y}_1(0) &= 0, & \bar{y}'_1(0) &= 0, \\ \bar{v}_1(1) &= 0, & \bar{y}_1(1) &= 0, & \bar{y}'_1(1) &= 0. \end{aligned} \quad (6)$$

At the cracked section:

$$\begin{aligned} \bar{v}'_1(\alpha_1) &= \bar{v}'_2(\alpha_1), & \bar{y}_1(\alpha_1) &= \bar{y}_2(\alpha_1), \\ \bar{y}''_1(\alpha_1) &= \bar{y}''_2(\alpha_1), & \bar{y}'''_1(\alpha_1) &= \bar{y}'''_2(\alpha_1), \\ \bar{v}'_2(\alpha_2) &= \bar{v}'_3(\alpha_2), & \bar{y}_2(\alpha_2) &= \bar{y}_3(\alpha_2), \\ \bar{y}''_2(\alpha_2) &= \bar{y}''_3(\alpha_2), & \bar{y}'''_2(\alpha_2) &= \bar{y}'''_3(\alpha_2). \end{aligned} \quad (7)$$

Also at the cracked section L_1 , we have

$$\begin{aligned} AE \frac{dv_1(L_1)}{dx} &= C_{11}(v_2(L_1) - v_1(L_1)) \\ &\quad + C_{12} \left(\frac{dy_2(L_1)}{dx} - \frac{dy_1(L_1)}{dx} \right). \end{aligned} \quad (8)$$

Due to the discontinuity of axial deformation to the left and right of the crack, the boundary conditions given in (8) arise.

Multiplying both sides of (15) by $AE/LC_{11}C_{12}$, we get

$$\begin{aligned} M_1 M_2 \bar{v}'_1(\alpha_1) &= M_2(\bar{v}_2(\alpha_1) - \bar{v}_1(\alpha_1)) \\ &\quad + M_1(\bar{y}'_2(\alpha_1) - \bar{y}'_1(\alpha_1)). \end{aligned} \quad (9)$$

Similarly, $EI(d^2 y_1(L_1)/dx^2) = C_{21}(v_2(L_1) - v_1(L_1)) + C_{22}((dy_2(L_1)/dx) - (dy_1(L_1)/dx))$.

Due to the discontinuity of slope to the left and right of the crack the boundary conditions given in (9) arises.

Multiplying both sides of (16) by $EI/L^2 C_{22} C_{21}$, we get

$$\begin{aligned} M_3 M_4 \bar{y}''_1(\alpha_1) &= M_3(\bar{v}_2(\alpha_1) - \bar{v}_1(\alpha_1)) \\ &\quad + M_4(\bar{y}'_2(\alpha_1) - \bar{y}'_1(\alpha_1)), \end{aligned} \quad (10)$$

where $M_1 = AE/LC_{11}$, $M_2 = AE/C_{12}$, $M_3 = AI/LC_{22}$, and $M_4 = EI/L^2C_{21}$.

Similarly, at the crack section L_2 , we can have the following expression

$$\begin{aligned} M_1 M_2 \bar{v}_2'(\alpha_2) &= M_2 (\bar{v}_3(\alpha_2) - \bar{v}_2(\alpha_2)) \\ &+ M_1 (\bar{y}_3'(\alpha_2) - \bar{y}_2'(\alpha_2)), \\ M_3 M_4 \bar{y}_2''(\alpha_2) &= M_3 (\bar{v}_3(\alpha_2) - \bar{v}_2(\alpha_2)) \\ &+ M_4 (\bar{y}_3'(\alpha_2) - \bar{y}_2'(\alpha_2)). \end{aligned} \quad (11)$$

The normal functions, (8) to (11) along with the boundary conditions as mentioned previously, yield the characteristic equation of the system as

$$|Q| = 0. \quad (12)$$

This determinant is a function of natural circular frequency (ω), the relative locations of the crack (α_1, α_2), and the local stiffness matrix (C) which in turn is a function of the relative crack depth ($b_1/D, b_2/D$).

The results of the theoretical analysis for the first three mode shapes for uncracked and cracked beams are shown in Figure 4.

3. Analysis of MANFIS for Detection of Multiple Cracks

Introduction to MANFIS. Detection of fault in multistructural system is a challenge to the researcher before the system failure. It not only hampers to the production rate but also increase the maintenance cost. There are several methods developed to identify the transverse multiple crack in the shaft or beam before catastrophic failure. The development of new methods for health monitoring of structures are developed day by day. Neural network, fuzzy-logic, and genetic algorithm are some useful reverse techniques used for fault diagnosis of a damage structure for prediction of a crack in accurate position and depth which is a great achievement. The present paper represents an inverse technique using multiple adaptive neurofuzzy evolutionary systems (MANFIS) to identify multiple transverse cracks in the steel shaft fixed at both ends. The present shaft consists of two cracks at different locations and depths.

The present new technique known as MANFIS is one of the advanced methods based on fuzzy-logic and neural network which integrate the positive features of both techniques. It is a technique which can be applied to a system where multiple outputs are required. Therefore, it can be utilized for detecting the multicrack of a structure such as shaft.

3.1. Steps to MANFIS. MANFIS consists of five layers, the first input layer is known as adaptive layer which takes six inputs such as the first three relative natural frequencies with their relative mode shape difference. The input layer is designed in the basic knowledge of fuzzy inference system. The other four layers including output are based on neural

network. Fuzzy layer of the MANFIS is trend with various fuzzy linguistic terms with several fuzzy rules. The six parameters of the system such as the first three relative natural frequencies and the first three mode shape are input to the fuzzy layer. Both second and third layers are called fixed layers. Fourth and fifth layers of the MANFIS are also called adaptive layers. Relative crack location and relative crack depth are the outputs of the fifth layer of MANFIS. It is one of the best techniques employed for nonlinear and complex function. The extended work for ANFIS is known as MANFIS. The present advance technique is an appropriate method for predicting the position and depth of crack of a given shaft before failure. The result obtained from MANFIS is well in agreement with the experimental results. As it is a nondestructive technique (NDT) which can be applied to all types of vibrating systems.

3.2. Theoretical Analysis of Multiple Adaptive Neurofuzzy Inference System for Crack Detection. A number of ANFIS systems group together to form MANFIS. The controller of ANFIS is employed for designing the controller of MANFIS. In the present scenario, six parameters for the shaft are used as inputs to the MANFIS, and four parameters are used for outputs from the system. The input parameters of the shaft are $\inf(x_1)$, $\text{mnf}(x_2)$, $\text{fnf}(x_3)$, $\text{amd}(x_4)$, $\text{bmd}(x_5)$, and $\text{cmd}(x_6)$. The output parameters of shaft are as follows:

- (1) first relative crack location = " $\alpha_1(L_1/L)$ "; first relative crack depth = " $\beta_1(b_1/D)$ ";
- (2) second relative crack location = " $\alpha_2(L_2/L)$ "; second relative crack depth = " $\beta_2(b_2/D)$ ".

The following logistic systems are prepared for MANFIS controller. The output parameter obtained by MANFIS is based on the following logistic system.

Rules for MANFIS Controller. Suppose that $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$, and Δ_6 are the fuzzy memberships shades defined for the input variables $\inf(x_1)$, $\text{mnf}(x_2)$, $\text{fnf}(x_3)$, $\text{amd}(x_4)$, $\text{bmd}(x_5)$, and that $\text{cmd}(x_6)$ and p_1, p_2, p_3, p_4, p_5 , and p_6 are fuzzy membership functions for the inputs x_1, x_2, x_3, x_4, x_5 , and x_6 , respectively. The rules for the MANFIS are defined as follows:

$$\begin{aligned} \text{if } x_1 \text{ is } (\Delta_1)_f, x_2 \text{ is } (\Delta_2)_g, x_3 \text{ is } (\Delta_3)_h, x_4 \text{ is } (\Delta_4)_j, \\ x_5 \text{ is } (\Delta_5)_k, x_6 \text{ is } (\Delta_6)_m, \text{ then} \end{aligned} \quad (13)$$

$$\begin{aligned} F_{e,i} &= q_{e,i}x_1 + r_{e,i}x_2 + s_{e,i}x_3 + t_{e,i}x_4 \\ &+ u_{e,i}x_5 + v_{e,i}x_6 + z_{e,i}, \end{aligned}$$

where

$$\begin{aligned} F_{1,i} &= \alpha_{1,i} = q_{1,i}x_1 + r_{1,i}x_2 + s_{1,i}x_3 + t_{1,i}x_4 + u_{1,i}x_5 \\ &+ v_{1,i}x_6 + z_{1,i}, \text{ for relative crack location } \left(\frac{L_1}{L}\right), \\ F_{2,i} &= \beta_{1,i} = q_{2,i}x_1 + r_{2,i}x_2 + s_{2,i}x_3 + t_{2,i}x_4 + u_{2,i}x_5 \\ &+ v_{2,i}x_6 + z_{2,i}, \text{ for relative crack depth } \left(\frac{b_1}{D}\right), \end{aligned}$$

FIGURE 4: (a) Relative amplitude versus distance from the fixed end (1st mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/D = 0.1$, and $b_2/D = 0.1$. (b) Relative amplitude versus distance from the fixed end (2nd mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/H = 0.1$, and $b_2/H = 0.1$. (c) Relative amplitude versus distance from the fixed end (3rd mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/D = 0.1$, and $b_2/D = 0.1$.

$$\begin{aligned}
 F_{3,i} &= \alpha_{2,i} = q_{1,i}x_1 + r_{1,i}x_2 + s_{1,i}x_3 + t_{1,i}x_4 + u_{1,i}x_5 \\
 &\quad + v_{1,i}x_6 + z_{1,i}, \text{ for relative crack location } \left(\frac{L_2}{L}\right), \\
 F_{4,i} &= \beta_{2,i} = q_{2,i}x_1 + r_{2,i}x_2 + s_{2,i}x_3 + t_{2,i}x_4 + u_{2,i}x_5 \\
 &\quad + v_{2,i}x_6 + z_{2,i}, \text{ for relative crack depth } \left(\frac{b_1}{D}\right),
 \end{aligned}$$

$$\begin{aligned}
 e &= 1 \text{ to } 4, & f &= 1 \text{ to } p_1, & g &= 1 \text{ to } p_2, \\
 h &= 1 \text{ to } p_3, & j &= 1 \text{ to } p_4, & k &= 1 \text{ to } p_5, \\
 m &= 1 \text{ to } p_6, & i &= 1 \text{ to } p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6,
 \end{aligned} \tag{14}$$

and “ α_1 ”, “ α_2 ”, “ β_1 ”, and “ β_2 ” are the linear consequent functions defined in terms of the inputs ($x_1, x_2, x_3, x_4,$

FIGURE 5: Bell-shaped membership function.

FIGURE 6: Multiple ANFIS (MANFIS) for crack detection.

x_5 , and x_6). Consider that $q_{1,i}$, $r_{1,i}$, $s_{1,i}$, $t_{1,i}$, $u_{1,i}$, $v_{1,i}$, $z_{1,i}$, $q_{2,i}$, $r_{2,i}$, $s_{2,i}$, $t_{2,i}$, $u_{2,i}$, $v_{2,i}$, and $z_{2,i}$ are the consequent parameters of the ANFIS fuzzy model. In the ANFIS model, nodes of the same layer have similar functions. The nodes present in a given layer of ANFIS model represent the similar functions. The results obtained from the output of one layer of ANFIS model are provided to input results for subsequent layer and show on.

Layer 1. Every node in the first layer is taken as a rectangular load (adaptive node) characterized by fuzzy membership function representing the degree to which inputs satisfy the quantifier. For six input parameters, the outputs obtained from corresponding nodes are as follows:

$$N_{1,g,e} = \mu(\Delta_1)_g(x) \text{ for}$$

$$g = 1, \dots, p_1 \text{ (for input } x_1),$$

$$N_{1,g,e} = \mu(\Delta_2)_g(x) \text{ for}$$

$$g = p_1 + 1, \dots, p_1 + p_2 \text{ (for input } x_2),$$

$$N_{1,g,e} = \mu(\Delta_3)_g(x) \text{ for}$$

$$g = p_1 + p_2 + 1, \dots, p_1 + p_2 + p_3 \text{ (for input } x_3),$$

$$N_{1,g,e} = \mu(\Delta_4)_g(x) \text{ for}$$

$$g = p_1 + p_2 + p_3 + 1, \dots, p_1$$

$$+ p_2 + p_3 + p_4 \text{ (for input } x_4),$$

$$N_{1,g,e} = \mu(\Delta_5)_g(x) \text{ for}$$

$$g = p_1 + p_2 + p_3 + p_4 + 1, \dots, p_1$$

$$+ p_2 + p_3 + p_4 + p_5 \text{ (for input } x_5),$$

$$N_{1,g,e} = \mu(\Delta_6)_g(x) \text{ for}$$

$$g = p_1 + p_2 + p_3 + p_4 + p_5 + 1, \dots, p_1$$

$$+ p_2 + p_3 + p_4 + p_5 + p_6 \text{ (for input } x_6).$$

(15)

Here, the membership functions are $\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5$, and Δ_6 considered to be bell-shaped function. Which is defined as follows (Figure 5):

$$\mu(\Delta_1)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = 1, \dots, p_1,$$

$$\mu(\Delta_2)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = p_1 + 1, \dots, p_1 + p_2,$$

$$\mu(\Delta_3)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = p_1 + p_2 + 1, \dots, p_1 + p_2 + p_3,$$

$$\mu(\Delta_4)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = p_1 + p_2 + p_3 + 1, \dots, p_1 + p_2 + p_3 + p_4,$$

$$\mu(\Delta_5)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = p_1 + p_2 + p_3 + p_4 + 1, \dots, p_1 + p_2 + p_3 + p_4 + p_5,$$

$$\mu(\Delta_6)_g(x) = \frac{1}{1 + \left\{ \left((x - c_g) / a_g \right)^2 \right\}^{bg}},$$

$$g = p_1 + p_2 + p_3 + p_4 + p_5 + 1, \dots, p_1$$

$$+ p_2 + p_3 + p_4 + p_5 + p_6,$$

(17)

FIGURE 7: Adaptive neurofuzzy inference system (ANFIS) for crack detection.

where a_g , b_g , and c_g are the parameters for the fuzzy membership function. The value of a_g , b_g , and c_g will change depending upon input parameters of the shaft.

Layer 2. It is a fixed layer of octagonal shape denoted as “ Π .” The output obtained from the second layer is denoted by $N_{2,i,e}$. The present output is a product of all incoming signal:

$$N_{2,i,e} = w_{i,e} = \mu(\Delta_1)_g(x) \mu(\Delta_2)_g(x) \mu(\Delta_3)_g(x) \mu(\Delta_4)_g(x) \times \mu(\Delta_5)_g(x) \mu(\Delta_6)_g(x), \quad (18)$$

for $i = 1, \dots, p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6$, $g = 1, \dots, p_1 + p_2 + p_3 + p_4 + p_5 + p_6$, and $w_{i,e}$ is called the firing strength. It can be obtained by T-nom operator algebraic product as $\{\text{Tap}(a, b) = ab\}$. The output node of the second layer represents the firing strength.

Layer 3. The node represented in the third layer is fixed, and octagonal shaped is represented by the symbol “ N .” The

output of the third layer can be calculated by the ratio of firing strength of i th. Rule ($w_{i,e}$) which is the sum of all firing strength:

$$N_{3,i,e} = \bar{w}_{i,e} = \frac{w_{i,e}}{\sum_{r=1}^{r=p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6} w_{r,e}}, \quad (19)$$

where $N_{3,i,e}$ given as a normalized firing strength.

Layer 4. The nodes represented in the fourth layer are rectangular shaped (adaptive node) and are represented with a node function:

$$N_{4,i,e} = \bar{w}_{i,e} F_{e,i} = \bar{w}_{i,e} (q_{e,i}x_1 + r_{e,i}x_2 + s_{e,i}x_3 + t_{e,i}x_4 + u_{e,i}x_5 + v_{e,i}x_6 + z_{e,i}), \quad (20)$$

FIGURE 8: Experimental setup.

where $\bar{w}_{i,e}$ is the normalized firing strength form (output) of the third layer and $q_{e,i}$, $r_{e,i}$, $s_{e,i}$, $t_{e,i}$, $u_{e,i}$, $v_{e,i}$, and $z_{e,i}$ are the set of parameters for crack location and depth.

Layer 5. The single node in this layer is a fixed node of octagonal nature represented by “ Σ ”; the results obtained from fifth layer are taken as the summation of all incoming signals:

$$N_{5,1,e} = \sum_{i=1}^{i=p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6} \bar{w}_{i,e} F_{e,i} = \frac{\sum_{i=1}^{i=p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6} w_{i,e} F_{e,i}}{\sum_{i=1}^{i=p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot p_5 \cdot p_6} w_{i,e}} \quad (21)$$

In the present ANFIS structure, six-dimensional space are provided to six shaft input parameter specified by “ $p_1 \times p_2 \times p_3 \times p_4 \times p_5 \times p_6$ ” region. Every region is governed by fuzzy rule. The first layer of ANFIS is meant for fuzzy subspace. The parameters present in the fourth layer are used for optimization purpose. The parameter obtained in the fourth layer can be estimated by the least square method during forward pass of nodes of hybrid algorithm. In the backward, pass error signals propagate backwards and premise the parameter and are updated by gradient descent method. The MANFIS architectures are presented in Figures 6 and 7.

3.3. Results and Discussions of MANFIS. The following discussions can be made from the analysis of the results of the multiple adaptive neurofuzzy inference system to predict the relative crack locations and relative crack depths.

The simulation results in current analysis indicate that the impact of crack locations and depths on the vibration characteristics of the shaft is quite evident. This is an important outcome of the numerical and experimental analysis which is used as a baseline for formulation of a multiple crack diagnostic tool using MANFIS technique. The bell-shaped membership function used for designing the ANFIS

system has been shown in Figure 5. The proposed MANFIS for multiple crack diagnosis and the detailed showing the different layers of the ANFIS system for crack detection have been presented in Figures 6 and 7, respectively. The suitability of the MANFIS technique has been checked by comparing the results with that of experimental analysis, and the comparison has been presented in Table 1. Ten sets of inputs (relative first three natural frequencies and relative first three mode shape differences) out of the several hundred inputs have been considered for the previously mentioned techniques, and the corresponding outputs in terms of relative first crack location (α_1), relative second crack location (α_2), relative first crack depth (β_1), and relative second crack depth (β_2) are presented in the Table 1. The first six columns of Table 1 present the inputs for the previously mentioned methodologies, that is, relative 1st natural frequency (inf), relative 2nd natural frequency (mnf), relative 3rd natural frequency (fnf), relative 1st mode shape difference (amd), relative 2nd mode shape difference (bmd), and relative 3rd mode shape difference (cmd), respectively. The rest of the columns from the table represent the outputs such as relative crack locations and relative crack depths from the respective techniques. From the analysis of the results presented in Table 1, it is found that the percentage deviation of the results of MANFIS is 2.45%.

4. Experimental Setup

Experiments are performed to determine the natural frequencies and mode shapes for different crack depths on steel shaft specimen of length 1000 mm and diameter 150 mm. The experimental setup is shown in Figure 8. The amplitude of transverse vibration at different locations along the length of the steel shaft is recorded by positioning the vibration pickup and tuning the vibration generator at the corresponding resonant frequencies. The results for the first three models are plotted in Figure 9.

5. Discussions

From the analysis of the results obtained from the different techniques for crack diagnosis in the current paper, it is observed that the presence of cracks on the structure in Figure 1 has a substantial effect on its vibration response. The variations of dimensionless compliance with the relative crack depth have been presented in Figure 2. The effects of cracks on the first three mode shapes of the beam structure have been presented in Figure 3. Experimental analysis for the cracked beam carried out with the given setup is shown in Figure 8. The vibration signatures obtained from the theoretical and experimental analysis are used to design the MANFIS for hybrid technique. Methodology has been shown in Figures 4 and 5, respectively. A comparison of results from MANFIS and numerical and experimental analysis are presented in Table 1.

TABLE 1: Comparison of results between MANFIS, experimental analysis, and numerical analysis.

Relative first natural frequency "inf"	Relative second natural frequency "mnf"	Relative third natural frequency "fnf"	Average relative first mode shape difference "amd"	Average relative second mode shape difference "bmd"	Average relative third mode shape difference "cmd"	MANFIS relative				Experimental analysis relative				Numerical analysis relative			
						β_5	α_5	β_6	α_6	β_3	α_3	β_4	α_4	β_1	α_1	β_2	α_2
0.9964	0.9994	0.9988	0.030495	0.031595	0.085795	0.23859	0.23958	0.47628	0.2401	0.10098	0.10197	0.198156	0.100058	0.099	0.099	0.196	0.098
0.9854	0.998	0.991	0.029175	0.0314	0.085365	0.241	0.46948	0.47628	0.4851	0.102	0.19982	0.198156	0.202158	0.1	0.194	0.196	0.198
0.9812	0.9924	0.9948	0.028585	0.03074	0.085919	0.241	0.72358	0.47142	0.735	0.102	0.30797	0.196134	0.3063	0.1	0.299	0.194	0.3
0.965	0.9828	0.9936	0.0257165	0.02954	0.0853925	0.22895	0.93896	0.47871	0.9506	0.0969	0.39964	0.199167	0.396148	0.095	0.388	0.197	0.388
0.9918	0.9996	0.9912	0.03092	0.025875	0.0666	0.46272	0.23232	0.95499	0.4851	0.19584	0.09888	0.397323	0.202158	0.192	0.096	0.393	0.198
0.9946	0.9994	0.9962	0.070885	0.0317	0.109835	0.70372	0.46222	0.98	0.7007	0.29784	0.19673	0.504489	0.292006	0.292	0.191	0.499	0.286
0.97	0.9898	0.9816	0.0278	0.030695	0.085695	0.22895	0.69454	0.60021	0.94815	0.0969	0.29561	0.249717	0.395127	0.095	0.287	0.247	0.387
0.9868	0.9984	0.9898	0.036535	0.08095	0.13391	0.57358	0.23232	0.95	0.94325	0.24276	0.09888	0.450906	0.393085	0.238	0.096	0.446	0.385
0.9888	0.9992	0.9884	0.005305	0.10795	0.10395	0.69408	0.46706	0.971	0.9408	0.29376	0.19879	0.533808	0.392064	0.288	0.193	0.528	0.384
0.9832	0.9938	0.995	0.04377	0.030645	0.08668	0.47718	0.96074	0.991	0.24255	0.20196	0.40891	0.450906	0.101079	0.198	0.397	0.446	0.099

FIGURE 9: (a) Relative amplitude versus distance from the fixed end (1st mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/D = 0.1$, and $b_2/D = 0.1$. (b) Relative amplitude versus distance from the fixed end (2nd mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/D = 0.1$, and $b_2/H = 0.1$. (c) Relative amplitude versus distance from the fixed end (3rd mode of vibration) $L_1 = 0.015$ m, $L_2 = 0.85$ m, $b_1/D = 0.1$, and $b_2/D = 0.1$.

6. Conclusions

Based on the results from the MANFIS technique, the following conclusions are drawn for multiple crack diagnosis in the steel shaft.

In the current investigation, a methodology based on measurement of natural frequencies and mode shapes of the system has been presented for identification of crack locations and their severities in a shaft using MANFIS having one input (fuzzy) layer, four hidden layers, and one output layer. Analyzing the results obtained from experimental and numerical methods, it is clear that the natural frequencies and

mode shapes show a noticeable change due to the presence of cracks on the shaft. The first three relative natural frequencies and mode shape differences from the numerical and experimental analysis are used as inputs to the fuzzy segment (input layer) of the MANFIS. Relative crack locations and relative crack depths are the output from the developed controller. The predicted results of the MANFIS have been validated using the results from the developed experimental setup, and the results are found to be in close agreement. From the analysis of the results obtained from the newly designed controller, it is observed that the MANFIS predicts the position and severities of cracks with more accuracy than

the other AI techniques discussed in this paper and can be suitably utilized for online multiple crack diagnosis in the dynamically vibrating structures.

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